Lab 4: Regression

In this lab, we will work on a regression task to predict a continuous value as the target. We will use the Boston housing dataset. The dataset contains information about the housing values in suburbs of Boston.

Prepare and Explore Data

In [18]:
```
# Import pandas library
import pandas as pd

# Read csv data file
bos = pd.read_csv('boston.csv')
```

Let us first check out the number of rows and columns.

In [19]:
```
# Find out the number of instances and number of attributes
bos.shape
```

Out[19]:
```
(506, 14)
```

In [20]:
```
# View the first 5 rows
bos.head()
```

Out[20]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.90	5.33	36.2

Knowing the feature or attribute names alone is not very helpful we need to check out more detailed description of the data.

The Boston housing dataset contains 506 instances and 14 attributes.

Boston house prices dataset

Data Set Characteristics:

```
:Number of Instances: 506

:Number of Attributes: 13 numeric/categorical predictive. Median Value (attribute 14) is usually
the target.

:Attribute Information (in order):
    - CRIM     per capita crime rate by town
    - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
    - INDUS    proportion of non-retail business acres per town
    - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
    - NOX      nitric oxides concentration (parts per 10 million)
    - RM       average number of rooms per dwelling
    - AGE      proportion of owner-occupied units built prior to 1940
    - DIS      weighted distances to five Boston employment centres
    - RAD      index of accessibility to radial highways
    - TAX      full-value property-tax rate per $10,000
    - PTRATIO  pupil-teacher ratio by town
    - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
    - LSTAT    % lower status of the population
    - MEDV     Median value of owner-occupied homes in $1000's
```

```
:Missing Attribute Values: None

:Creator: Harrison, D. and Rubinfeld, D.L.
```

This is a copy of UCI ML housing dataset. https://archive.ics.uci.edu/ml/machine-learning-databases/housing/

This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic prices and the demand for clean air', J. Environ. Economics & Management, vol.5, 81-102, 1978. Used in Belsley, Kuh & Welsch, 'Regression diagnostics ...', Wiley, 1980. N.B. Various transformations are used in the table on pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address regression problems.

.. topic:: References

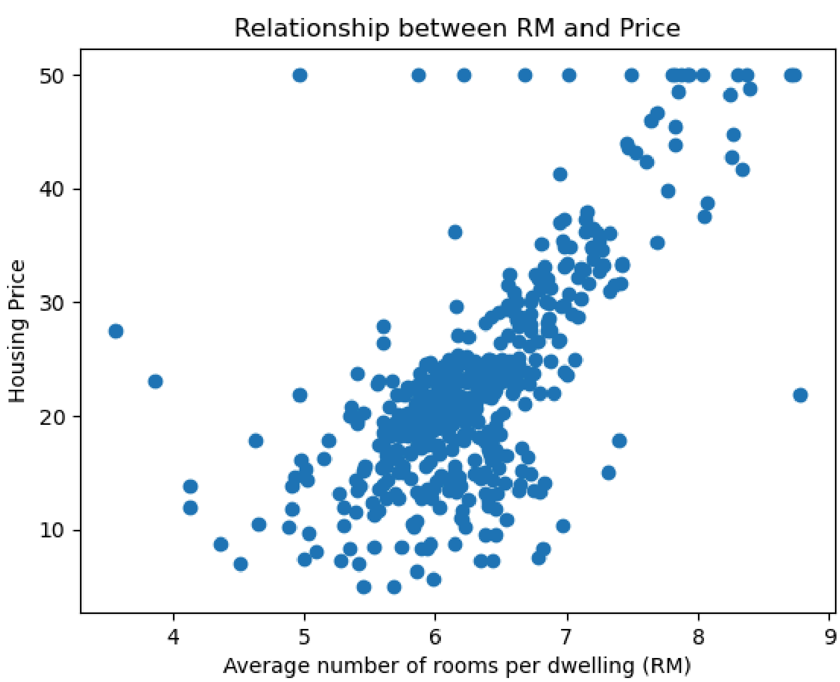
- Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of Collinearity', Wiley, 1980. 244-261.
- Quinlan,R. (1993). Combining Instance-Based and Model-Based Learning. In Proceedings on the Tenth International Conference of Machine Learning, 236-243, Universsier for us to view the data in a tabular format.

The target we want to predict is MEDV, which is the median value of owner-occupied homes in $1000's (i.e., housing price in Boston region).

It may be a bit difficult to visualize all 13 features with the target but we can study the correlation between each feature to the target by plotting a scatter plot.

In [21]:
```python
# Import matplotlib library
import matplotlib.pyplot as plt

# Plot RM against PRICE
plt.scatter(bos.RM, bos.MEDV)
plt.xlabel('Average number of rooms per dwelling (RM)')
plt.ylabel('Housing Price')
plt.title('Relationship between RM and Price')
plt.show()
```



From the scatter plot, we can see a positive correlation between RM and housing prices.

We will split the dataset into training and test sets. We will use 80% of the data to fit the model and allocate 20% in our test set. We will use all 13 features to fit a linear regression model.

In [22]:
```
# Indicate the target column
target = bos['MEDV']

# Indicate the columns that will serve as features
features = bos.drop('MEDV', axis = 1)

# Split data into train and test sets

# Import train_test_split function
from sklearn.model_selection import train_test_split

# Split the dataset into training and test sets
x_train, x_test, y_train, y_test = train_test_split(features, target, \
                                                    test_size = 0.2, random_state = 0)
```

Linear Regression

We will fit a linear regression model to predict the Boston housing prices. LinearRegression, in its simplest form, fits a linear model to the data set by adjusting a set of parameters in order to make the sum of the squared residuals of the model as small as possible.

y = βx + c

x : predictor variable(s)

y : target variable

β : coefficients

c : intercept

For more information about the parameters we can tune for linear regression, check out: https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html. If we want to tune the hyperparameters of the model, we can run k-fold cross validation or split x_train further into a training set and validation set. For k-fold cross validation, make sure you set the **scoring** parameter to a regression performance metric: https://scikit-learn.org/stable/modules/model_evaluation.html#scoring-parameter.

In this lab, we will just fit the model using the default linear regression parameters and run the model on the test set.

In [23]:
```
# Import linear regression
from sklearn.linear_model import LinearRegression

# Create a linear regression model
lm = LinearRegression()

# Fits a linear model
lm.fit(x_train, y_train)

# To print the coefficients of the model
lm.coef_
```

Out[23]:
```
array([-1.19443447e-01,  4.47799511e-02,  5.48526168e-03,  2.34080361e+00,
       -1.61236043e+01,  3.70870901e+00, -3.12108178e-03, -1.38639737e+00,
        2.44178327e-01, -1.09896366e-02, -1.04592119e+00,  8.11010693e-03,
       -4.92792725e-01])
```

Since there are 13 features (multivariate regression), each variable has one estimated coefficient so there is a total of 13 coefficients. We can also print the estimated intercept.

In [24]:
```
lm.intercept_
```

Out[24]:
```
38.09169492630233
```

If we want to perform some analysis on the coefficients, we can construct a data frame that contains features and estimated coefficients. The zip() function joins two tuples together. In our case, we are joining the feature names with the estimated coefficients.

In [25]:
```
pd.DataFrame(zip(bos.columns, lm.coef_), columns = ['features', 'estimated_coefficients'])
```

Out[25]:

	features	estimated_coefficients
0	CRIM	-0.119443
1	ZN	0.044780
2	INDUS	0.005485
3	CHAS	2.340804
4	NOX	-16.123604
5	RM	3.708709
6	AGE	-0.003121
7	DIS	-1.386397
8	RAD	0.244178
9	TAX	-0.010990
10	PTRATIO	-1.045921
11	B	0.008110
12	LSTAT	-0.492793

In [26]:
```
# Predict target using the linear model with estimated coefficients
test_predict = lm.predict(x_test)

# Print the first 5 predicted housing prices
test_predict[0:5]
```

Out[26]:
```
array([24.88963777, 23.72141085, 29.36499868, 12.12238621, 21.44382254])
```

We can map the predicted prices and actual prices from the test set in a scatter plot for comparison.

In [27]:
```
plt.scatter(y_test, test_predict)
plt.xlabel('Actual Housing Price')
plt.ylabel('Predicted Housing Price')
plt.title('Actual Housing Price versus Predicted Housing Price')
plt.show()
```

From the scatter plot, we can observe that the model tends to underpredict when the actual housing prices are on the lower and higher ends.

We can also generate regression performance metrics to make comparison between models easier.

In [28]:
```
# Returns R squared as a performance measure
lm.score(x_test, y_test)
```

Out[28]:
```
0.5892223849182534
```

The score() function returns the coefficient of determination (R squared) of the prediction. R squared measures if the model is a good fit for the data. R squared is always going to be between $-\infty$ and 1. The ideal value for R squared is 1. The closer the value of R squared to 1, the better is the model fitted.

In [29]:
```
# Import scikit-learn metrics module for RMSE and MAE calculation
from sklearn import metrics
# Import the math module
import math

# Compute the MSE
print("MSE (Test): ", metrics.mean_squared_error(y_test, test_predict))

# Compute the RMSE
print("RMSE (Test): ", math.sqrt(metrics.mean_squared_error(y_test, test_predict)))
```

```
MSE (Test):  33.44897999767632
RMSE (Test):  5.783509315085117
```

In [30]:
```
# If you are using scikit-learn 0.22.2, you can run this command to compute RMSE
# Compute the RMSE
print("RMSE (Test): ", metrics.mean_squared_error(y_test, test_predict, squared = False))
```

```
RMSE (Test):  5.783509315085117
```

In [31]:
```
# Compute the MAE
print("MAE (Test): ", metrics.mean_absolute_error(y_test, test_predict))
```

```
MAE (Test):  3.8429092204444997
```

From the performance metrics computed, we can conclude the linear model is a moderate fit for the data with R squared of 0.59. From the MAE = 3.84, we can conclude that the prediction made by the model is off by 3843 dollars (target is median value of owner-occupied homes in 1000's dollars so we get 3843 dollars by multiplying the MAE by 1000) on average. MAE is less sensitive to outliers but from the earlier plot of actual housing prices versus predicted housing prices, we can see that there are extreme cases of errors at the lower and higher ends. RMSE is more sensitive to large errors, so RMSE = 5.78 produces a higher error value (the housing prices prediction is off by 5784 dollars on average). To select the best regression model, we will relatively compare R squared, RMSE and MAE between different models.