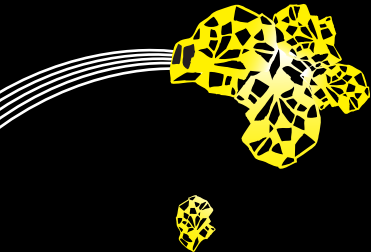


Symbolic Model Checking of Timed Automata using LTSmin

Sybe van Hijum





Overview

Timed Automata

Symbolic Model Checking

LTSmin

Working with $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$

Second Section

Third Section




Transition System

Definition (Labeled Transition System)

A labeled transition system is a 3-tuple $A = \langle S, Act, s_0 \rangle$ where

- ▶ *S is a finite set of states*
- ▶ *Act is a finite set of labelled actions*
- ▶ *$s_0 \in S$ is a finite set of actions*



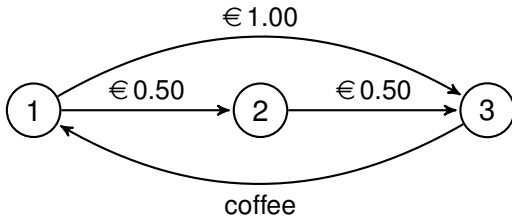
Transition System

1

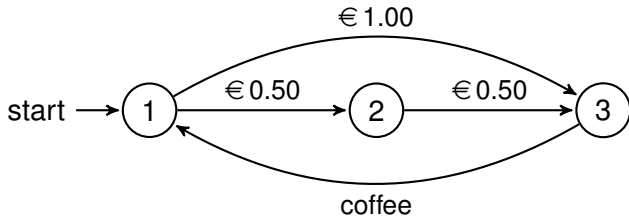
2

3

Transition System



Transition System





Timed Automata

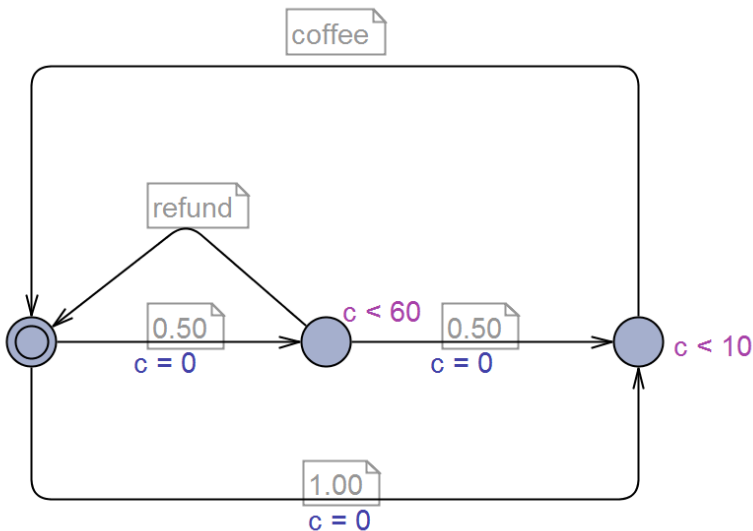
Definition (Timed Automata)

An extended timed automaton is a 6-tuple $A =$

$\langle L, C, Act, l_0, \rightarrow, I_C \rangle$ where

- ▶ *L is a finite set of locations, typically denoted by l*
- ▶ *C is a finite set of clocks, typically denoted by c*
- ▶ *Act is a finite set of actions*
- ▶ *$l_0 \in L$ is the initial location*
- ▶ *$\rightarrow \subseteq L \times G(C) \times Act \times 2^C \times L$ is the (non-deterministic) transition relation.*
- ▶ *$I_C : L \rightarrow G(C)$ is a function mapping locations to downwards closed clock invariants.*

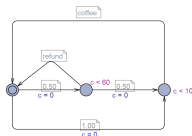
Timed Automata



Time Zones

Time not represented as a variable, but as a zone. Most used structure to represent zones: Different Bound Matrix (DBM)

- Only convex zones
- Memory inefficient



$$0 \leq c < 60$$

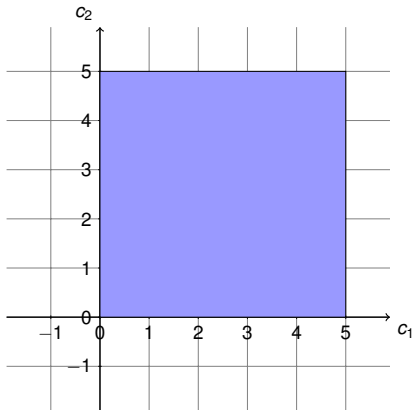
$$\Downarrow$$

$$\begin{aligned} c - 0 &< 60 \\ 0 - c &\leq 0 \end{aligned}$$

$$\begin{matrix} & 0 & c \\ \begin{matrix} 0 \\ c \end{matrix} & \begin{pmatrix} (0, \leq) & (0, \leq) \\ (60, <) & (0, \leq) \end{pmatrix} \end{matrix}$$



	O	c_1	c_2
O	$(0, \leq)$	$(0, \leq)$	$(0, \leq)$
c_1	$(5, <)$	$(0, \leq)$	$(5, <)$
c_2	$(5, <)$	$(5, <)$	$(0, \leq)$





Overview

Timed Automata

Symbolic Model Checking

LTSmin

Working with \LaTeX

Second Section

Third Section



Boolean Decision Diagram

- ▶ Expresses boolean expressions
- ▶ States can be seen as boolean expressions
- ▶ Memory efficient

Boolean Decision Diagram

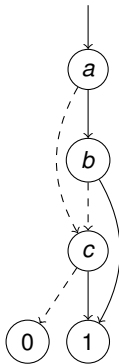
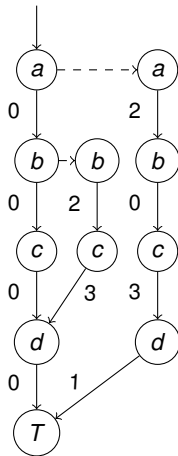


Figure: A BDD representing $(a \wedge b) \vee c$

List Decision Diagram





Overview

Timed Automata

Symbolic Model Checking

LTSmin

Working with \LaTeX

Second Section

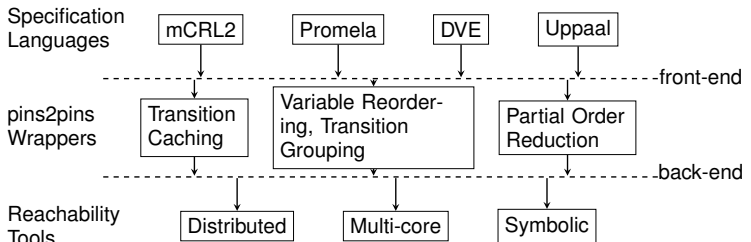
Third Section



LTSmin

- ▶ Language independent model checker
- ▶ Multiple algorithmic back ends
- ▶ Internal optimization wrappers

LTSmin






LTSmin

- ▶ States as integer vectors
- ▶ Partitioned next-state function
- ▶ Optimizations based on matrices
 - ▶ Read(r)
 - ▶ Must-write(w)
 - ▶ May-write(W)
 - ▶ Copy(-)

Matrices

- 1: $x = 1 \vee a[1] = 0 \rightarrow a[1] := 1, x := 0, y := 5$
2: $a[0] = 1 \vee y = 5 \rightarrow a[x] := 0, x := 1$

	x	y	$a[0]$	$a[1]$
1	+	w	-	+
2	+	r	+	W




Problem: Model checkers are designed for discrete variables (integers), clocks have real values.

- ▶ Can we use the LTSmin symbolic model checker for timed automata?
- ▶ Can we optimize the symbolic back end for clocks?



Current LTSmin Uppaal setup

States as a vector of discrete locations and a pointer to a DBM.
Implemented in explicit-state multi-core tool.
First approach: values from DBM directly into an LDD



	O	c₁	c₂
O	$(0, \leq)$	$(0, \leq)$	$(0, \leq)$
c₁	$(5, <)$	$(0, \leq)$	$(5, <)$
c₂	$(5, <)$	$(5, <)$	$(0, \leq)$

Old situation: $\{l_0, \dots, l_n, ptr\}$

New situation:

$\{l_0, \dots, l_n, (0, \leq), (0, \leq), (5, <), (5, <), (5, <), (5, <)\}$



LDD solution

- ▶ Correct, working solution
- ▶ Variable reordering possible
- ▶ All variables seen as discrete values
- ▶ No optimizations based on time

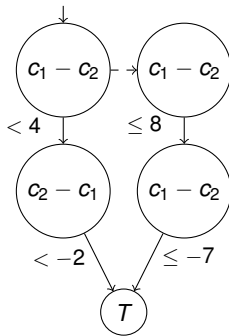
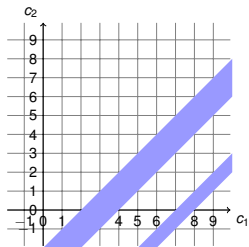
Difference Decision Diagram

Definition (Difference Decision Diagram)

A difference decision diagram (DDD) is a directed acyclic graph (V, E) . The vertex set V contains two terminals 0 and 1 with out-degree zero, and a set of non-terminal vertices with out-degree two and the following attributes.

Attribute	Type	Description
$pos(v), neg(v)$	Var	Positive variable x_i , and negative variable x_j .
$op(v)$	$\{<, \leq\}$	Operator $<$ or \leq .
$const(v)$	\mathbb{D}	Constant c .
$high(v), low(v)$	V	High-branch h , and low-branch l .

The set E contains the edges $(v, low(v))$ and $(v, high(v))$, where $v \in V$ is a non-terminal vertex.





Definition (Ordered DDD)

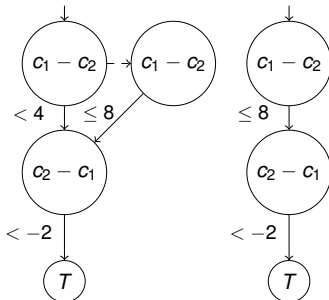
An ordered DDD (ODDD) is a DDD where each non-terminal vertex v satisfies:

1. $neg(v) \prec pos(v)$,
2. $var(v) \prec var(high(v))$,
3. $var(v) \prec var(low(v))$ or
 $var(v) = var(low(v))$ and $bound(v) \prec bound(low(v))$.

Definition (Locally Reduced DDD)

A locally reduced DDD (R_L DDD) is an ODDD satisfying, for all non-terminals u and v :

1. $\mathbb{D} = \mathbb{Z}$ implies $\forall v. op(v) = '\leq'$,
2. $(cstr(u), high(u), low(u)) = (cstr(v), high(v), low(v))$ implies $u = v$,
3. $low(v) \neq high(v)$,
4. $var(v) = var(low(v))$ implies $high(v) \neq high(low(v))$.





Overview

Timed Automata

Symbolic Model Checking

LTSmin

Working with $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$

Second Section


Third Section



Font Sizes

Table: The different font sizes within \LaTeX

tiny	sample text
scriptsize	sample text
footnotesize	sample text
small	sample text
normalsize	sample text
large	sample text
Large	sample text
LARGE	sample text
huge	sample text
Huge	sample text




Creation of a new frame

The text within the frame




Creation of a new frame - source

```
\begin{frame}{Creation of a new frame}  
    The text within the frame  
\end{frame}
```




Frame with pause itemes

- First item




Frame with pause itemes

- ▶ First item
- ▶ Second item




Frame with pause itemes

- ▶ First item
- ▶ Second item
- ▶ You get the point.



Frame with pause itemes - source


```
\begin{frame}{Frame with \texttt{pause} itemes}  
\begin{itemize}  
\item First item \pause  
\item Second item \pause  
\item You get the point.  
\end{itemize}  
\end{frame}
```



Frame with pause tables

Table: Caption

Class	A	B	C	D
X	1	2	3	4



Frame with pause tables


Table: Caption

Class	A	B	C	D
X	1	2	3	4
Y	3	4	5	6

Frame with pause tables

Table: Caption

Class	A	B	C	D
X	1	2	3	4
Y	3	4	5	6
Z	5	6	7	8



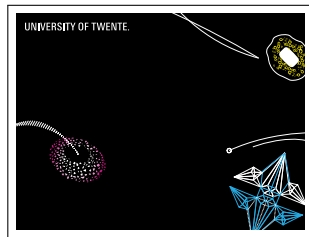
Frame with pause tables - source

```
\begin{frame}{Frame with \texttt{pause} tables}

\rowcolors[] {1} {blue!20} {blue!10}
\begin{table}
\caption{Caption}
\begin{tabular}{l!{\vrule}cccc}
Class & A & B & C & D \\\hline
X & 1 & 2 & 3 & 4 \pause \\
Y & 3 & 4 & 5 & 6 \pause \\
Z & 5 & 6 & 7 & 8
\end{tabular}
\end{table}
\end{frame}
```

Two Column Output

Text here.
Text here.
Text here.





Two Column Output - source

```
\begin{frame}{Two Column Output}
  \begin{columns}[c]
    \column{1.5in}
      Text here.\\
      Text here.\\
      Text here.
    \column{1.5in}
      \framebox{\includegraphics[width=1.5in]{img/back2}}
  \end{columns}
\end{frame}
```



Overview

Timed Automata

Symbolic Model Checking

LTSmin

Working with \LaTeX

Second Section

Third Section



First frame of the Second Section

Each new section starts with an Table Of Contents.



Overview

Timed Automata

Symbolic Model Checking

LTSmin

Working with \LaTeX

Second Section

Third Section



First frame of the Second Section

The Table Of Contents is clickable