

# Symbolic Model Checking of Timed Automata using LTSmin

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June 15, 2016

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# 1 Introduction

Timed automata [1] is a widely used modelling formalism. A recent usage of this formalism is the modelling of biological signalling pathways [2]. ANIMO is a tool that generates these timed automata from biological signalling pathways models. This leads however to large state spaces, and sometimes to models that are too large to handle by conventional methods. Therefore better model checking techniques for timed automata, that can handle larger state spaces are needed. We look into symbolic algorithms for timed automata.

BDDs (Binary Decision Diagrams) [3, 4] and variations like LDDs (List Decision Diagrams) [5] and MDDs (Multi-valued Decision Diagrams) [6] have proven their value in model checking algorithms. Due to advances in this field, models with much larger state spaces can be explored on the same machine. This progress has not been translated directly to more efficient methods for timed automata. Several methods have been proposed, like CDDs (Clock Difference Diagrams) [7], CMDs (Constraint Matrix Diagrams) [8], CRDs (Clock Restriction Diagrams) [9] and DDDs (Difference Decision Diagrams) [10, 11]. All of these methods show some extra difficulties or limitations over BDDs. Also after their introduction they have not been developed further.

LTSmin [12, 13] is a language independent on the fly model checker with several algorithmic back-ends. Its symbolic back-end uses BDDs to both represent the state space and the transition relations of models. These BDDs are generated on the fly by the search algorithms. LTSmin has a language module for the UPPAAL [14] through the Opaal [15] lattice model checker. Through this module UPPAAL models can be loaded into LTSmin. For this language currently, only the multi-core back-end can be used [16]. This multi-core approach showed efficient enough to compete with the latest version of the UPPAAL model checker. It showed significant speedups on multi-core machines, at the cost of some memory increase however. To tackle the memory increase a combination of the Opaal front-end and the symbolic back-end could be a solution.

The symbolic back-end of LTSmin provides both a memory reduction by using BDDs and a speedup by using multi-threaded search algorithms and the multi-threaded BDD package Sylvan [17]. Using this together with the UPPAAL language front-end will hopefully result in a model checker that can compete both on time and memory consumption with the UPPAAL model checker.

We will propose a symbolic reachability for timed automata that is capable of handling the models that are generated by the ANIMO tool.

## 2 Preliminaries

We will first define timed automata and zones, a method used to represent time in timed automata. Also a subsumption check over zones will be defined.

### 2.1 Timed Automata

Timed automata is a formalism that extends labelled transition systems with one or more, but finite, clocks. Guards over these clocks, denoted as  $G(C)$  can be used for transitions. Also reset actions for clock can be defined for transitions. All clocks in the system will increase at the same rate. As our work continues on [16] we use the same definition of timed automata.

**Definition 1** (Timed Automata). *An extended timed automaton is a 6-tuple  $A = \langle L, C, Act, s_0, \rightarrow, I_C \rangle$  where*

- $L$  is a finite set of locations, typically denoted by  $l$
- $C$  is a finite set of clocks, typically denoted by  $c$
- $Act$  is a finite set of actions
- $l_0 \in L$  is the initial location
- $\rightarrow \subseteq L \times G(C) \times Act \times 2^C \times L$  is the (non-deterministic) transition relation. We normally write  $l \xrightarrow{g,a,r} l'$  for a transition., where  $l$  is the source location,  $g$  is the guard over the clocks,  $a$  is the action, and  $r$  is the set of clocks reset.
- $I_C : L \rightarrow G(C)$  is a function mapping locations to downwards closed clock invariants.

With this definition we can combine timed automata to a network of timed automata, which is a parallel composition, to define larger systems.

**Definition 2** (Network of timed automata [16]). *Let  $Act = \{ch!, ch? | ch \in Chan\} \cup \{\tau\}$  be a finite set of actions, and let  $C$  be a finite set of clocks. Then the parallel composition of extended timed automata  $A_i = \langle L_i, C, Act, S_0^i, \rightarrow_i, I_C^i \rangle$  for all  $1 \leq i \leq n$ , where  $n \in \mathbb{N}$ , is a network of timed automata, denoted  $A = A_1 || A_2 || \dots || A_n$ .*

### 2.2 Zones

For basic transition systems the state space can grow exponentially for the size of the system. Timed automata are by definition infinite, as clocks have real values. If a state is defined between two points in time, an infinite amount of moments in time can happen during that state. Even when some

$$\begin{pmatrix} (0, \leq) & (0, \leq) & (0, \leq) \\ (5, \leq) & (0, \leq) & \infty \\ (4, \leq) & \infty & (0, \leq) \end{pmatrix}$$

Figure 1: DBM

granularity is used, that defines that clocks will only increase with certain step size the automata can still have infinite state space. To tackle this problem most model checkers use a notion of zones for the representation of time. A zone can be seen as a set of constraints over the clocks  $C$  of the form  $c_i \sim x$  and  $c_i - c_j \sim x$  where  $\sim \in \{<, \leq, =, \geq, >\}$  and  $x \in \mathbb{N}$ . To represent these zones several data structures have been developed. One of the most common used structures are Difference Bound Matrices (DBMs) [18, 19].

These matrices use both a column and a row for each clock, and on each position  $(i, j)$  an upper bound on the difference between the clocks  $c_i$  and  $c_j$  is given in the form  $c_i - c_j \preceq x$  where  $\preceq \in \{<, \leq\}$  and  $x \in \mathbb{Z}$ . For the constraints over the single clocks an extra clock  $\mathbf{O}$  with a constant value 0 is added. This way the upper and lower bound of a clock  $c_i$  can be given by  $c_i - \mathbf{O} \preceq x$  and  $\mathbf{O} - c_i \preceq y$ . The addition of this  $\mathbf{O}$  clock will give the matrix of a timed automaton always size  $(|C| + 1)^2$ . This way convex zones of clock variables can be represented. Each matrix can however only contain a single convex zone. Concave zones and multiple convex zones need multiple matrices to be represented. As a solution often a list of DBMs is used. In figure 1 we give an example of a DBM with two clocks:  $c_1$  and  $c_2$ , representing the zone  $0 \leq c_1 \leq 5 \wedge 0 \leq c_2 \leq 4$ .

### 2.3 Zone subsumption

In model checking an important function is to check if a certain state has been visited already earlier. For normal automata this can be done by comparing the newly found state to all states that have already been visited, and check if one of those states is equal to that new state. This is often done by more efficient methods, like hash functions, but the equality check remains. For states with zones this equality check does not satisfy. As two zones do not need to be equal, but the new zone needs to be a part of the already visited zones. In LTSmin this is done by a subsumption check [16] that is performed over the DBMs. This function checks if a new zone is a subset of the zone represented by a DBM, so that all of its' upper bounds are lower than the upper bounds of the already discovered zone.

### 3 Related Work

In this related work section we will discuss a number of methods used for model checking timed automata. We will choose a method to extend our work on, and go more into detail on that method.

#### 3.1 Methods

Already several model checkers for timed automata exist such as UPPAAL [14], KRONOS [20], RABBIT [21] and RED [9]. We focus mainly on the UPPAAL tool as we use the same input format. Opaal [15], the language module for LTSmin, uses the XML format that is created by the UPPAAL tools. This way we can use the UPPAAL user interface to create and adapt models. We also use the UPPAAL DBM library to represent zones. Several methods exist to represent the clock variables in a timed model. The most used methods are digitization and zones.

Digitization approximates the continuous values of clocks by using discrete values [22]. This approach is however very sensitive to the granularity of the values used and the upper bound of the clock values. When fine granularity or large upper bounds are used, the memory usage will increase too much. An advantage of this approach is that basic model checking approaches can be used and no extra complexity due to zone calculations is added. The method however only works for closed timed automata, meaning that no strict comparisons on clocks can be made in the model. In [23] a similar approach is proposed by using clock tick actions and removing clock variables altogether.

The most established method to represent clock zones are DBMs. We gave an introduction to this structure in the preliminaries section. Several methods based on BDDs have been developed to represent zones. All of these are similar to DBMs in the sense that they use clock constraints to represent the zones. They use a BDD-like structure to represent the zones more efficiently. CDDs [7] use single nodes for each variable and have disjoint intervals for that variable on the edges. This results in a node with a larger fanout and the upper and lower bound in a single node. DDDs [10,11] use a constraint on each node that can either be true or false, when a constraint is false a next node will have another constraint on the same variable. This requires a fixed ordering based on the variables, values and operators. CRDs [9] differ mainly from CDDs by using not disjoint intervals but possibly overlapping upper bounds for a variable pair on their edges. This diagram will have a larger fanout per node, like CDDs. They also use several normal forms for the diagrams which results in different performances. It is also shown that CRDs can be combined with BDDs into a single structure to fully symbolic represent state space. CMDs [8] combine CDDs, CRDs and DBMs into a single structure. This diagram type differs from the others by

having multiple constraints per edge, resulting in a diagram with few nodes. CMDs do not have a normal form so only reduced forms are proposed. In [24, 25] a method is proposed purely based on BDDs by translating the constraints directly into BDD nodes. This results in a unified structure for both the discrete variables and the clock constraints. The method is only a proof of concept and has not been implemented in a model checker and no performance results are known. In Figure 2 we have an example of all four BDD like structures representing the zone  $2 < c_1 - c_2 < 4 \vee 7 \leq c_1 - c_2 \leq 8$ .

A known difficulty in BDDs is the variable ordering. A bad ordering can lead to a BDD exponential in size, where a good ordering can sometimes lead to a significant smaller diagram. Of the zone diagrams named above, only for CRDs experiments with different orderings have been conducted, the other researches assume a given ordering on the variables and the ordering of the values is fixed. The CRD case shows that full interleaving and having related variables close to each other in the ordering is preferable and gives the best results, both on speed and memory. This is the same result as expected with BDDs, this suggests that similar orderings should be used with these techniques. In Table 1 we compare the different types of diagrams we discussed above.

### 3.2 LTSmin

LTSmin [12, 13] is a language independent model checker. It is built in a modular way such that new languages can be added by a PINS (Partitioned Next-State Interface) interface without too much effort, and new algorithms can be added easily. LTSmin offers four different algorithmic back-ends for model analysis: symbolic, multi-core, sequential and distributed. All of these back-ends support different types of reduction and model checking. Several language modules have already been built for LTSmin such as mCRL2, Promela, DVE and UPPAAL. The modular structure of LTSmin

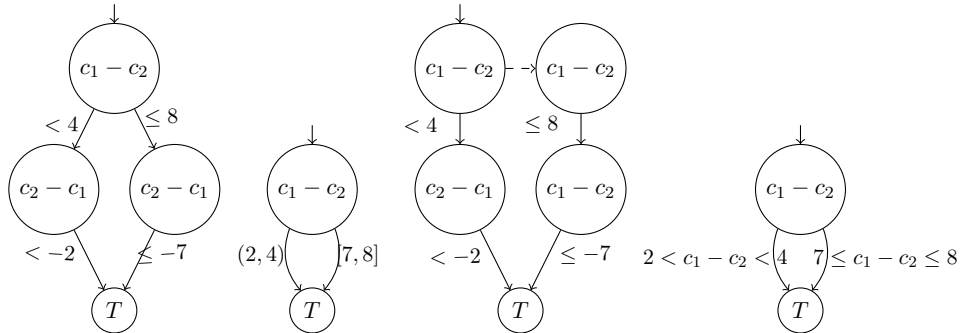


Figure 2: A CRD, CDD, DDD and CMD representation

Table 1: Comparing Diagrams

Type	Pro	Con
DBM	Canonical form for convex zones Existing library Inclusion check	Concave zones need multiple DBMs Not memory efficient
DDD	Structure like LDD Re-ordering of variables possible Apply same efficiency as BDDs Boolean variables also in DDD	Canonicity hard to obtain No on the fly canonicity Expensive normal form computation Only time performance tested Only reduction algorithms
CDD	Structure like MDD Inclusion check (intersection of complement)	No algorithm to get normal form Only high level algorithms given Methods don't maintain disjointness Expensive normal form computation No implementation results available Disjointness memory inefficient
CRD	Combination with BDD possible Variable reordering shows advantage Library available Some benchmarks exp better than CDD Extensive benchmarks Good performance backwards reach	3 possible canonical forms No algorithms in paper Some benchmarks linear worse than CDD
CMD	Benchmarks against RED and UPPAAL	Results differ per case Needs translation from vector to edges Two reduced forms
BDD discrete	Using existing BDD packages Good performance for small clock values	Performance decreases fast for large values Not possible with current Ppaal PINS Introducing additional 'tick' actions Only for closed timed automata
BDD zones	Using existing BDD packages All variable reorderings possible Only need direct translation DBM to state vector Easy to implement	Losing zone containment No implementation results



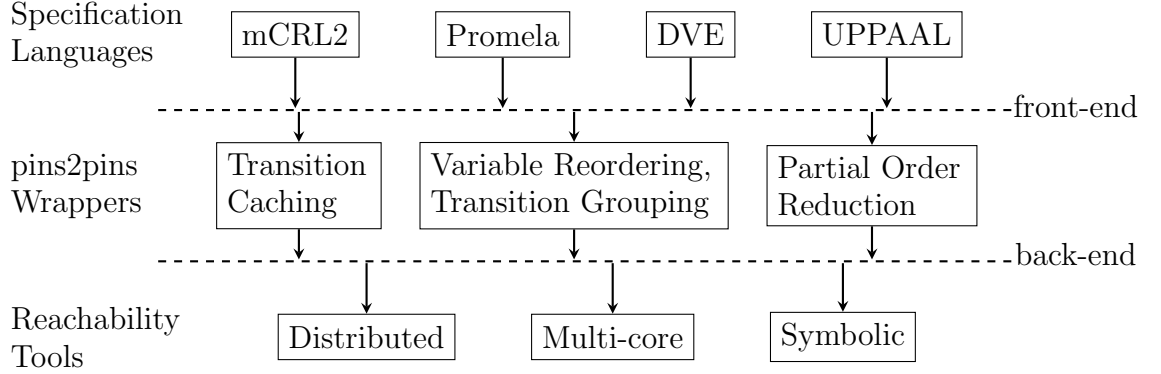


Figure 3: Modular structure of LTSmin

is shown in Figure 3. The PINS is the core of LTSmin. This interface abstracts as much as possible from the model without losing the structure. It represents states as fixed length integer arrays. The main function of the interface is a (partitioned) next-state function which returns the successor states. With these functions a state space can be generated on the fly. With the use of dependency matrices event locality can be determined statically [26]. Using these matrices, more efficient symbolic algorithms can be used, the number of next-state calls can be reduced, efficient variable reorderings can be used, and transition caching can be used. In the current UPPAAL PINS the next-state function is not partitioned and therefore no meaningful dependency matrix is created, and none of these algorithms can be used.

### 3.3 Difference Decision Diagrams

We have discussed several symbolic approaches for representing zones. All of these approaches have benefits and downsides over each other. We chose to develop one of these approaches in LTSmin. We wanted a diagram that can store both discrete states and zones, this can either be done in the diagram, or in a combination of the diagram and BDD or LDD nodes. Also a subsumption check on the diagram should be possible. We chose from the four zone representing diagrams discussed earlier. The CDD approach was not chosen due to the memory inefficient disjoint intervals and their algorithms not maintaining these intervals, the CMD approach is too similar to DBMs, on which we already have an approach. The choice between CRD and DDD was between two quite similar diagrams. We have decided to continue on the DDD. It is a diagram form that is closely related to LDDs, for which we already have a library, and it is also quite compatible to the current PINS structure and its next-state function. The method still has

some loose ends that need research, mostly on the algorithms and efficiently creating a canonical form. No results on the memory usage are available, which is normally the greatest benefit of a symbolic approach, so also on the results side we can extend the current research.

So DDDs are a diagram type that seems to fit well in the current structure we have, but there is still room for some more research. First we give the definition of a DDD.

**Definition 3** (Difference Decision Diagram [10]). *A difference decision diagram (DDD) is a directed acyclic graph  $(V, E)$ . The vertex set  $V$  contains two terminals 0 and 1 with out-degree zero, and a set of non-terminal vertices with out-degree two and the following attributes.*

Attribute	Type	Description
$pos(v), neg(v)$	<b>Var</b>	Positive variable $x_i$ , and negative variable $x_j$ .
$op(v)$	$\{<, \leq\}$	Operator $<$ or $\leq$ .
$const(v)$	$\mathbb{D}$	Constant $c$ .
$high(v), low(v)$	$V$	High-branch $h$ , and low-branch $l$ .

The set  $E$  contains the edges  $(v, low(v))$  and  $(v, high(v))$ , where  $v \in V$  is a non-terminal vertex.

In [10] a canonical form for DDDs is discussed, also called a fully reduced DDD. Only definitions are given here, no algorithms to reach this form. It is stated that it is difficult to reach this fully reduced form. It is not clear if they managed to make their apply function in such a way that it maintains canonicity, as the function for BDDs does. To reach canonicity, local reductions and ordering are a first step, but it is not enough due to dependencies among the constraints. For BDDs the local reductions and ordering are sufficient to reach a canonical form. First we give some notational shorthands and then we define an ordering and local reductions on DDDs.

$$\begin{aligned}
var(v) &= (pos(v), neg(v)) \\
bound(v) &= (const(v), op(v)) \\
cstr(v) &= (var(v), bound(v))
\end{aligned}$$

**Definition 4** (Ordered DDD [10]). *An ordered DDD (ODDD) is a DDD where each non-terminal vertex  $v$  satisfies:*

1.  $neg(v) \prec pos(v)$ ,
2.  $var(v) \prec var(high(v))$ ,
3.  $var(v) \prec var(low(v))$  or  
 $var(v) = var(low(v))$  and  $bound(v) \prec bound(low(v))$ .

After ordering a DDD some local reductions can be defined to reduce the size of a DDD.

**Definition 5** (Locally Reduced DDD [10]). *A locally reduced DDD ( $R_LDDD$ ) is an ODDD satisfying, for all non-terminals  $u$  and  $v$ :*

1.  $\mathbb{D} = \mathbb{Z}$  implies  $op(v) = '\leq'$ ,
2.  $(cstr(u), high(u), low(u)) = (cstr(v), high(v), low(v))$  implies  $u = v$ ,
3.  $low(v) \neq high(v)$ ,
4.  $var(v) = var(low(v))$  implies  $high(v) \neq high(low(v))$ .

These reductions are not enough to reach a canonical form. Here we define the other reductions and methods needed to reach a canonical form.

**Definition 6** (Path-reduced DDD [10]). *A path-reduced DDD ( $R_PDDD$ ) is a locally reduced DDD where all paths are feasible.*

**Definition 7** (Tightness [10]). *A dominating constraint  $\alpha = x_i - x_j \lesssim c$  is tight in a feasible path  $[p] = [p_1] \wedge \alpha \wedge [p_2]$  if for all tighter constraints  $(c', \lesssim') < (c, \lesssim)$ , the systems  $[p_1] \wedge (x_i - x_j \lesssim' c') \wedge [p_2]$  and  $[p]$  have different solutions. A path  $p$  is tight if it is feasible and all dominating constraints on it are tight. An  $R_LDDD u$  is tight if all paths from  $u$  are tight.*

**Definition 8** (Saturation [10]). *A tight path  $p$  from an  $R_PDDD$  is saturated if for all constraints  $\alpha$  not on  $p$ , if  $\alpha$  is added to  $p$  either (1)  $\alpha$  is not dominating and tight, or (2) the constraint system  $[p_1] \wedge \neg\alpha$  is infeasible when  $[p]$  is written  $[p] = [p_1] \wedge [p_2]$  with all constraints on  $p_1$  smaller than  $\alpha$  with respect to  $\prec$  and all constraints on  $p_2$  larger than  $\alpha$ . An  $R_PDDD u$  is saturated if all paths from  $u$  are saturated.*

**Definition 9** (Disjunctive vertex [10]). *Let  $p$  be a path leading to the vertex  $u$  in a DDD, and assume  $\alpha = cstr(u)$ ,  $h = high(u)$ , and  $l = low(u)$ . Then  $u$  is disjunctive in  $p$  if  $[p] \wedge (\alpha \rightarrow h, l)$  and  $[p] \wedge (h \vee l)$  have the same set of solutions.*

All of these definitions together lead to the following definition of a fully reduced DDD.

**Definition 10** (Fully reduced DDD [10]). *An  $R_PDDD u$  is a fully-reduced DDD ( $R_FDDD$ ) if it is tight, saturated and has no disjunctive vertices.*

DDDs are also used to represent the discrete variables in automata. This is done by translating the variable into a difference constraint. For example  $x_1 = 3$  will be translated into  $x_1 - 0 \leq 3 \wedge 0 - x_1 \leq -3$ , thus resulting into a DDD with two nodes.

So far we only found the results of two benchmark tests of DDDs, Milner's scheduler and Fischer's protocol [27]. Here the DDD approach has been compared with KRONOS and UPPAAL which were both slower than the DDD implementation. The results of these benchmarks show no memory usage.

## 4 Plan

We will implement first a method that will use the best of both worlds, the efficient algorithms from de DBMs and the memory efficiency of a symbolic approach. We will use the DBMs in the state exploration such that we can find a canonical representation of the clock zone of a newly explored state quite easily. For the symbolic representation of the state space, including the clock zones, and the transition relations, we will use normal BDDs. The DBMs will be flattened and put directly into the state vector and can then be handled by the symbolic BDD back-end. Therefore both the efficient algorithms and the memory efficient representation can be used. A downside to this approach is that a zone subsumption check is not possible anymore, as only equalities and no inequalities can be checked on BDDs, resulting in revisiting of some states. Further we will focus on efficient orderings of the BDDs, as both clock zones and states are contained in a single structure. We will also use this new method with the existing multi-core tool, such that we can still use the subsumption check that is implemented in LTSmin. After that we will continue towards a DDD model checker. First we will use the DDDs as the state space representation and still use the language module using the DBMs. We have not been able to find any literature on the combination of these techniques. There might be a significant memory improvement possible here. Eventually we aim at a complete symbolic solution with more operations on the DDD, such as the progress of time, then we can have a language module which does not use the DBMs any more. We will compare the different approaches we implement extensively to each other. All of these approaches will be implemented in the LTSmin toolset. This way we can really compare the methods and not just the tools.

Alongside this we will also have to make the Opaal PINS work with the UPPAAL models generated by ANIMO. The current versions do not work together because of global variables are used in the system declaration in the generated model, and this is a feature that Opaal does not support. We can make this work by either changing the models generated by ANIMO or by extending the Opaal PINS. At this time we do not know the best solution for this problem.

### 4.1 Questions

For the research we will state a couple of research questions:

- Is the combination of BDDs and flattened DBMs an efficient method for symbolic reachability analysis of timed automata? Both on memory usage and speed.
- Can improvements be achieved by using different orderings? Both by changing the order of only the clock variables and by mixing the clock

and state variables.

- Is the new language module needed for the symbolic approach also usable for the multi-core approach with subsumption?
- Can the BDD approach be generalized towards a method using DDDs?
- Is a fully symbolic reachability analysis using DDDs more efficient than the combination of DDDs and DBMs, both on memory and speed?

## 4.2 Algorithms

To create a DDD library we will implement a number of functions over DDDs. We will limit the functions to the ones needed for this purpose. Therefore it will not become a complete DDD package. One of the core operations on DDDs is the apply operation. This operation takes two DDDs and a binary operator and combines the two DDDs according to the operator. The apply function for DDDs is a generalisation of the function for BDDs. In [10] a general definition of the algorithm is given. We turned this more mathematical definition into an algorithm, we give pseudo-code in Algorithm 1. In Algorithm 2 we give the pseudo-code for the apply function with the or operator, or the union function, this way we can increase performance by not going down the entire diagram if we already found a terminal. All functions rely on a Mk function which checks if the node needed already exists, and otherwise creates a new node. The subsumption check, which we lost in the BDD approach, will be possible again with DDDs. This will be the same check as a state membership in an LDD. The only difference is that no equality, but upper bounds will be checked. Pseudo-code for this algorithm is given in Algorithm 3. If we combine DDDs with LDDs, only the correct check has to be adapted, checking for equalities, not inequalities, the algorithm will remain the same.

## 4.3 Planning

In this section we describe all things that need to be implemented to make model checking with a certain diagram possible.

To make symbolic model checking work we need to change the Opaal PINS. The PINS currently uses a pointer to a DBM. For the new approach we will put the values of the DBM directly into the state vector. This will increase the size of the state vector. All other references to the types and values of the state vector entries will need to be changed also. (1)

To make symbolic variable reordering possible we will need to partition the next-state function. In the code the next-state function is already split up per transition, but in a single transition group. Splitting this into multiple transition groups should not be too hard. (2)

---

**Algorithm 1** Apply

---

```
1: procedure APPLY( $v1, v2, op$ )
2:   if  $v1 \in \{0, 1\} \wedge v2 \in \{0, 1\}$  then
3:      $result \leftarrow (v1 \text{ } op \text{ } v2)$ 
4:   else if  $var(v1) \prec var(v2)$  then
5:      $high \leftarrow \text{APPLY}(high(v1), v2, op)$ 
6:      $low \leftarrow \text{APPLY}(low(v1), v2, op)$ 
7:      $result \leftarrow \text{MK}(cstr(v1), high, low)$ 
8:   else if  $var(v2) \prec var(v1)$  then
9:      $high \leftarrow \text{APPLY}(high(v2), v1, op)$ 
10:     $low \leftarrow \text{APPLY}(low(v2), v1, op)$ 
11:     $result \leftarrow \text{MK}(cstr(v2), high, low)$ 
12:   else if  $v1 \prec v2$  then
13:     $high \leftarrow \text{APPLY}(high(v1), high(v2), op)$ 
14:     $low \leftarrow \text{APPLY}(low(v1), v2, op)$ 
15:     $result \leftarrow \text{MK}(cstr(v1), high, low)$ 
16:   else if  $v2 \prec v1$  then
17:     $high \leftarrow \text{APPLY}(high(v1), high(v2), op)$ 
18:     $low \leftarrow \text{APPLY}(v1, low(v2), op)$ 
19:     $result \leftarrow \text{MK}(cstr(v2), high, low)$ 
20:   else if  $v1 = v2$  then
21:     $high(v1) \leftarrow \text{APPLY}(high(v1), high(v2), op)$ 
22:     $low(v1) \leftarrow \text{APPLY}(low(v1), low(v2), op)$ 
23:     $result \leftarrow \text{MK}(cstr(v1), high, low)$ 
24:   return  $result$ 
```

---

---

**Algorithm 2** Union

---

```
1: procedure UNION( $v1, v2$ )
2:   if  $v1 = v2$  then return  $v1$ 
3:   else if  $v1 = \text{false}$  then return  $v2$ 
4:   else if  $v2 = \text{false}$  then return  $v1$ 
5:   else if  $\text{var}(v1) \prec \text{var}(v2)$  then
6:      $high \leftarrow \text{UNION}(high(v1), v2)$ 
7:      $low \leftarrow \text{UNION}(low(v1), v2)$ 
8:      $result \leftarrow \text{MK}(cstr(v1), high, low)$ 
9:   else if  $\text{var}(v2) \prec \text{var}(v1)$  then
10:     $high \leftarrow \text{UNION}(high(v2), v1)$ 
11:     $low \leftarrow \text{UNION}(low(v2), v1)$ 
12:     $result \leftarrow \text{MK}(cstr(v2), high, low)$ 
13:   else if  $v1 \prec v2$  then
14:     $high \leftarrow \text{UNION}(high(v1), high(v2))$ 
15:     $low \leftarrow \text{UNION}(low(v1), v2)$ 
16:     $result \leftarrow \text{MK}(cstr(v1), high, low)$ 
17:   else if  $v2 \prec v1$  then
18:     $high \leftarrow \text{UNION}(high(v1), high(v2))$ 
19:     $low \leftarrow \text{UNION}(v1, low(v2))$ 
20:     $result \leftarrow \text{MK}(cstr(v2), high, low)$ 
21:   else if  $v1 = v2$  then
22:     $high(v1) \leftarrow \text{UNION}(high(v1), high(v2))$ 
23:     $low(v1) \leftarrow \text{UNION}(low(v1), low(v2))$ 
24:     $result \leftarrow \text{MK}(cstr(v1), high, low)$ 
25:   return  $result$ 
```

---

---

**Algorithm 3** Zone containment for DDDs

---

```
1: procedure CONTAINS( $v, z$ )
2:   if  $v \in 0, 1$  then return  $v$ 
3:   else if  $z[\text{var}(v)]$  correct in  $v$  then
4:     return CONTAINS( $high(v), z$ )
5:   else return CONTAINS( $low(v), z$ )
```

---



Once the next-state function is partitioned, also a sparse dependency matrix is needed. This will need to be created according to the transition groups. After this step the variable reordering steps in LTSmin need to work. (3)

To combine the new PINS with the multi-core LTSmin back-end the subsumption check will need to be changed. This check now relies on a pointer to a DBM, but it will now get the complete DBM, or state vector. Here the search algorithm or the subsumption check will need to know which variables are zone variables. It will also occur that different states will have the same discrete variables, but different zone variables, these all need to be checked for the subsumption check. (4)

For the combination with the multi-core back-end also the data structure will need to be adapted. The current structure stores a discrete state together with a set of pointers to DBMs. In the new situation each pair of discrete state and DBM will be stored explicitly. (5)

To use a DDD approach, the first step will be to create a minimal DDD library that has the functionality to save a state space. This library will probably miss some features for more advanced model checking techniques. (6)

Once we have a DDD library, we need to combine this with nodes to represent discrete state variables. The main candidate for this are LDD nodes, as they share the same structure with DDDs. (7)

For the combination of DDD and LDD nodes, the diagram will need to be able to identify the zone variables from the discrete state variables in the state vector. This will need an extra function in the PINS interface recognizing the different types of variables. (8)

For the DDD representation the values from the DBM will need to be translated to useful variables as ordering of the DDD is based on the values. Also to check for inequalities and set containment the meaningful values are needed. In the current DBM library both the value and the operator are saved in a single 32 bit integer. The DDD will need to know the value and the operator separately. In this section we describe all things that need to be implemented to make model checking with a certain diagram possible. (9)

Once we have a working DDD representation the first set of benchmark tests can be conducted. This will be the big set of tests for the BDD, DDD and multi-core approach, we can compare these to the old multi-core approach and to the newest version of UPPAAL. (10)

To continue towards a fully symbolic approach, we will need to extend the DDD library with some functions. We need for example a function to let time progress, and to set invariants over the states. (11)

For the fully symbolic approach again the language module will need to be adapted. We will no longer need the DBM in the language module. The module will only need to change the discrete state variables, the zone

variables will be adapted in the diagram. (12)

If we have this fully symbolic approach this will also need to be tested. We will conduct the same tests as we did for the other approaches, such that we can compare this diagram to all earlier approaches. (13)

In the table below we have put all actions that need to be done into tasks. In the second column we put which tool should work correctly for the Opaal language module after the task. This will give us intermediate points on which we can test the work that has been done to that point.

Task	Needs to function	Date
Start		01-02-2016
Flatten the DBMs (1)	Symbolic tool	15-02-2016
Partition the next-state function (2)	Symbolic tool	01-03-2016
Create dependency matrices (3)	Symbolic tool variable reordering	15-03-2016
Subsumption check for multi-core back end (4)		01-04-2016
Other adaptations multi-core approach (5)	Multi-core tool	01-04-2016
Create minimal DDD library (6)		15-04-2016
Combine DDD with LDD (7)		01-04-2016
Language module for DDD approach (8, 9)	Symbolic tool	08-04-2016
Benchmark tests for multiple approaches (10)		15-04-2016
Test with ANIMO models		15-04-2016
DDD library for fully symbolic approach (11)		15-05-2016
Language module for fully symbolic DDDs (12)	Symbolic tool	01-06-2016
Benchmark Testing fully symbolic approach (13)		15-06-2016
Writing Report		01-07-2016

## 5 Notes

### 5.1 Flattening DBM

We flattened the DBM, to make the opaal language module work with any algorithmic backend in LTSmin. We only did this on the edges of the successor function. So this function reads a flattened DBM as input and returns it as successor states, internally the original DBM representation is still used. This way the code had to be adapted the least. In this flattening we removed the diagonal elements of each DBM. By the way DBMs are constructed this will always represent the difference between a clock and itself. This difference is by definition always 0, so it can be removed, and hard coded be set to 0 internally. This reduces the number of state variables in the state vector by one for each clock.

### 5.2 Dependency Matrices

To get the best possible result of the regrouping algorithms, the dependency matrices had to be made as sparse as possible. This has been done for both the read matrix and may-write matrix. We had to use the may-write, instead of the must write matrix, as it is not always clear if a variable will really be written, or that it may stay untouched. First of all, all C-like code is parsed. Here it is stored per function which variables are read and written, and which other functions are called. Next all transitions are parsed, here some variables are read and written directly. Transitions can also call functions, then from the earlier parsed functions the read and written variables are used. In the final step we need to look at the time extrapolation. This extrapolation is based on the value of the location variable, so it results in a read dependency. In some cases, there is no difference between all possible location values, so a location does not need to be read.

Split the next state function in groups. Removed top level switch of state variables and changed into ifs. Created switch statement on groups. Added check for urgent transitions per group.

Building the dependency matrices. Both read and may-write. Parsing all functions for read/write variables and all functions called. Also parsing for all groups the guards and updates. Getting from these also the functions that are called, get the read/write vars from those and from the functions called in that function etc. Also all location variables get read dependency due to clock extrapolations based on the locations. Also the locations that are used in the urgency check are always read.

For the animo models I got the error that receiving channels should not have clock guards. This is not a UPPAAL message, but a opaal message, UPPAAL does support this. Changed this in the generated code. Making sure that first all guard on clocks are done. After those it is checked that the DBM is not empty. If it is not empty, clock updates are done and the

callback function is called. Also if both guards are on the same clock, the stronger guard will still be the dominant guard.

---

**Algorithm 4** Reduce

---

```

1: procedure REDUCE( $dbm, dim$ )
2:   for  $i \in dim$  do
3:     for  $j \in dim$  do
4:       for  $k \in dim$  do
5:         if  $!(dbm[i, k] \vee dbm[k, j] \vee dbm[i, j] \text{ on diagonal})$  then
6:           if  $dbm[i, k] + dbm[k, j] \leq dbm[i, j]$  then
7:              $dbm[i, j] := \infty$ 

```

---

We use a path reduced version of the DDD. This reduced version is not needed for all operations. Maybe we can improve performance by use the reduce function more or less often. For the count of the number of states we will use the amount of paths in the reduced DDD. States are not well defined for timed automata, as it is a set of discrete locations together with a zone of clocks. As the zones can have different representations, this gives some strange results. We will use the DDD function Biimplication for the equal operation and we will use implication for the subsumption check.

---

**Algorithm 5** Reduce

---

```
1: procedure REDUCEZERO( $dbm, dim$ )
2:    $placed[dim]$  all 0
3:    $red[dim, dim]$  all 0
4:    $eq[dim, dim]$  all 0
5:    $cl := 0$ 
6:    $newDBM[dim, dim]$  diagonal  $\infty$  rest 0
7:   for  $i \in dim$  do
8:     if  $placed[i] = 0$  then
9:       for  $j \in dim$  do
10:        if  $dbm[i, j] + dbm[j, i] = 0$  then
11:           $placed[j] := 1$ 
12:           $eq[cl, j] := 1$ 
13:         $cl ++$ 
14:    $repr[cl]$ 
15:   for  $i \in cl$  do
16:     for  $j \in dim$  do
17:       if  $eq[i, j] = 1$  then
18:          $repr[i] := j$ 
19:       break
20:    $clg[cl, cl]$ 
21:   for  $i \in cl$  do
22:     for  $j \in cl$  do
23:        $clg[i, j] := dbm[repr[i], repr[j]]$ 
24:   REDUCE( $clg, cl$ )
25:   for  $i \in cl$  do
26:     for  $j \in dim$  do
27:       if  $eq[i, j] = 1$  then
28:         for  $k \in dim$  do
29:           if  $eq[i, k]$  then
30:              $newDBM[j, k] = dbm[j, k]$ 
31:       for  $j \in cl$  do
32:          $newDBM[repr[i], repr[j]] := clg[i, j]$ 
33:   return newDBM
```

---

---

**Algorithm 6** MK

---

```
1: procedure MK( $var, bound, h, l$ )
2:   if  $bound = "< \infty"$  then
3:     return  $h$ 
4:   if member hashtable then
5:     return lookup
6:   else if  $l = h$  then
7:     return  $l$ 
8:   else if  $var = l.var \wedge h = l.high$  then
9:     return  $l$ 
10:  else
11:    return insert(new Node)
```

---

## 6 Time Extrapolation

In the successor generator step a time extrapolation is used. This extrapolation step reduces the number of DBMs created and makes sure that this number is finite. The most coarse abstraction as described in [28] is used. This extrapolation reduces the number of zones that are explored significantly. It also makes that less improvements can be made on the representation of the zones, for some models all states are extrapolated to the same zone, so nothing interesting happens at the timed side of the model any more. In opaal this algorithm is implemented in such a way that all UPPAAL locations are always read. The maximum extrapolation is based on the values of these locations. Only if there is no difference between all values for a certain location, it is not needed to read this. This results into an densely populated dependency matrix.

## 7 Benchmarks

Below we describe the different models we used to run the benchmarks. We tried to find models that scale up for a number of nodes or processes, so that we can also check the behaviour of our approaches for different sizes of the same model. In this section we use the terminology 'locations' and other 'discrete variables'. The definition of timed automata does not have this difference, but we use it to describe models, because the time extrapolation is dependent on locations, and not on the other discrete variables. This dependency fills a large part of the dependency matrices. In this section, a location is a state in the UPPAAL transition system editor, the other discrete variables are declared in the C-like syntax that UPPAAL uses.

### 7.1 Viking

The set of Viking tests models a torch that can be taken from one side to the other by either one or two vikings. By some timing guards, the vikings have to hold one to the torch for a minimal amount of time, given by guards. The model has a low number of discrete variables, one per viking, one for the torch and an indicator for the side on which the torch is. It has a global clock and a clock per viking.

The model results in a densely filled dependency matrix. The torch and all viking variables are always read for the time extrapolation. Only the side indicator is not read always. The write matrix is sparser.

The difference between the LDD and DDD representation is quite small for this model. In the extrapolation step all clock zones are set to  $[0..\infty]$  for all states, so in both diagrams the zones are represented by a single path. So the interesting things are only happening in the discrete parts.

### 7.2 Fischer

Fischer's mutual exclusion protocol [29] is modelled for a number of processes. There is no synchronization between processes, only blocking of actions can occur. This model has a slightly higher number of discrete variables compared to the Viking tests. Each process has a location and an id. The model also has 2 global discrete variables. Each process has a local clock, no global clock is used.

The dependency matrix of this model has some sparse rows, as each model has an id, which is a constant and can only be read. Again all the location variables are always read due to the time extrapolation.

### 7.3 CSMA-CD

The Carrier Sense Multiple Access/Collision Detection [20] is modelled for a number of senders. The model has a few discrete variables. Each sender



only has a location and only one global counter is used. Each sender and the bus have a local clock. The model uses a lot of synchronizations between the senders and the bus.

#### **7.4 Animo**

We could not use the ANIMO models, only the smallest model with no synchronizations was possible. As we started the project to work on ANIMO models, we still included that single model in the benchmark set. It is a model with only one node, so only one location variable. The model has two clocks, a global clock and a clock for the node. Further it does have quite a large number of discrete variables. Both the global declaration and the node have a portion of c-like code with a number of global variables.

This results in a model with a quite sparse dependency matrix, as only the single location is used for the time extrapolation. We expected this model to have good performance for the LDD method with variable re-ordering.

#### **7.5 Lynch-Shavit**

The Lynch-Shavit mutual exclusion protocol [30] is modelled for different number of processes. The structure of the model is quite like the Fischer model. It only uses one global variable more than Fischer.

#### **7.6 Milner**

Milners scheduler [31] is modelled for a number of nodes. The structure is like that of the CSMA-CD model, except that it does not use a bus. The model has a lot of synchronizations between the nodes, and between the node and a global process. Each node has two clocks, so the zone representation blows up quickly.

#### **7.7 Other models**

We also used some models that we could not scale up enough due to memory/time limitations, or that could not scale up due to the nature of the model. We will not describe those models into detail. These models were the critRegion, Critical, bocdp(-fixed) [32], bando and timelock model.

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