6351 project1

zijing huang

January 2022

tracking error

$$e = \Phi_d - \Phi$$

$$\dot{e} = \dot{\Phi_d} - \dot{\Phi}$$

$$\ddot{e} = \ddot{\Phi_d} - \ddot{\Phi}$$

filtered tracking error

$$r = \dot{e} + \alpha e$$

$$\dot{r} = \ddot{e} + \alpha \dot{e}$$

multiply both sides by $M(\Phi)$ then

$$M(\Phi)\dot{r} = M(\Phi)\ddot{e} + M(\Phi)\alpha\dot{e}$$

since

$$\ddot{e} = \ddot{\Phi}_d - \ddot{\Phi}$$

then

$$M(\Phi)\dot{r} = M(\Phi)\ddot{\Phi}_d - M(\Phi)\ddot{\Phi} + M(\Phi)\alpha\dot{e}$$

since

$$M(\Phi)\ddot{\Phi} = -C(\Phi, \dot{\Phi}) - G(\Phi) + \tau$$

then

$$M(\Phi)\dot{r} = M(\Phi)(\ddot{\Phi}_d + \alpha\dot{e}) + C(\Phi, \dot{\Phi}) + G(\Phi) - \tau$$

define $\theta \in \mathbb{R}^p$ as the unknown constant parameters, $\tilde{\theta}(t) \in \mathbb{R}^p$ as the parametric error and $\hat{\theta}(t) \in \mathbb{R}^p$ as the estimated parameters(P is number of unknown parameters) then

$$\tilde{\theta} = \theta - \hat{\theta}$$

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

we know that $d\dot{\hat{\theta}}(t) \in \mathbb{R}^p$ is the parametric update law let

$$\zeta = \begin{bmatrix} e \\ r \\ \tilde{\theta} \end{bmatrix}$$

and consider

$$V(\zeta,t) = \frac{1}{2}e^Te + \frac{1}{2}r^TM(\Phi)r + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$$

then

$$\dot{V}(\zeta,t) = e^T\dot{e} + \frac{1}{2}r^TM(\dot{\Phi})r + r^TM(\Phi)\dot{r} + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}$$

since

$$r = \dot{e} + \alpha e$$

then

$$\dot{e} = r - \alpha e$$

now have

$$\dot{V}(\zeta,t) = e^T(r-\alpha e)+\frac{1}{2}r^TM(\dot{\Phi})r+r^T(M(\Phi)(\ddot{\Phi}_d+\alpha\dot{e})+C(\Phi,\dot{\Phi})+G(\Phi)-\tau)+\tilde{\theta}^T\Gamma^{-1}(-\dot{\hat{\theta}})$$

which is

$$\dot{V}(\zeta,t) = e^T(r-\alpha e)+r^T(M(\Phi)(\ddot{\Phi}_d+\alpha\dot{e})+C(\Phi,\dot{\Phi})+G(\Phi)+\frac{1}{2}M(\dot{\Phi})r-\tau)-\tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}})$$

with all the works from the $Two-LinkGradientAdaptiveController$ we can have

$$\dot{V}(\zeta,t) = -e^T\alpha e - r^T\beta r + r^T(Y\theta - \tau) - \tilde{\theta}\Gamma^{-1}\dot{\hat{\theta}}$$

where

$$Y\theta = Y_M(\phi,\psi) + Y_C(\phi,\dot{\phi}) + Y_G(\phi) + \frac{1}{2}Y_{\dot{M}}(\phi,r)$$

design the input as

$$\tau = Y\hat{\theta} + e + \beta r$$

substituting back in we get

$$\dot{V}(\zeta,t) = -e^T\alpha e - r^T\beta r + \tilde{\theta}^TY^Tr - \tilde{\theta}\Gamma^{-1}\dot{\hat{\theta}}$$

now we can design the update law as

$$\dot{\hat{\theta}} = \Gamma Y^T r$$

and substituting back in we get

$$\dot{V}(\zeta,t) = -e^T\alpha e - r^T\beta r$$

and now we could use Barbalat's Lemma to show $e \to 0$ and $r \to 0$ and we get GAT. Then we can do signal chasing to show τ is bounded.