Computational Methods for Structural Analysis - Homework 10

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Introduction/Description:

The purpose of this homework is to use 3-noded triangular elements to construct a 2-D finite element code to determine nodal displacements, element stresses, and element strains for an arbitrary solid structure. The solid structure is a 2-D square plate with a circular hole in the center. The displacements will be scaled and viewed by creating and comparing deformed and undeformed plots. Stress and strain will be calculated and normalized. Axial stresses will be graphed in the x and y directions. There are 4 different sets of input files provided with the given nodes, elements, displacements, and forces. The input files represent the top right fourth of the 2-D plate with a whole in the center.

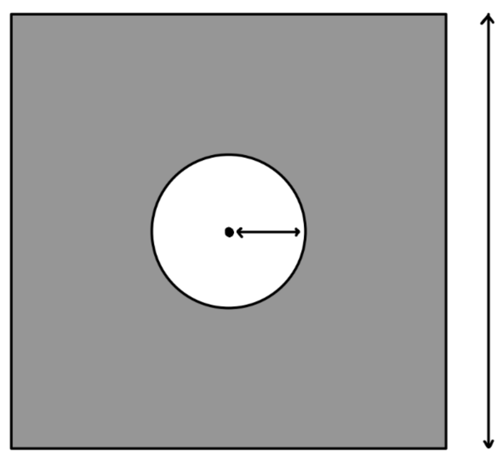


Figure 1: 2-D square plate with a circular hole in the middle.

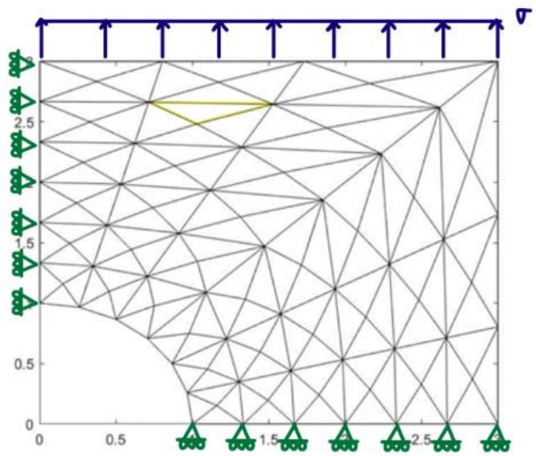


Figure 2: Top right corner ($1/4^{th}$) of the plate.

Figure 2 shows the 3-noded triangles, forces, and displacements. The forces applied are in blue, the displacements are due to the supports in green, and the highlighted yellow shows an example of one of the 3-noded triangles. Note that this is just an estimate of where the supports and forces might occur and does not necessarily correspond to the locations in the actual input files.

Provided Input Files:

Input Set 6	*Input Set 12*	*Input Set 24*	*Input Set R*
nodes6.txt	nodes12.txt	nodes24.txt	nodesR.txt
elements6.txt	elements12.txt	elements 24.txt	elements R.txt
displacements6.txt	displacements12.txt	displacements24.txt	displacementsR.txt
forces6.txt	forces12.txt	forces24.txt	forcesR.txt

Input Descriptions:

Each of the above sets of inputs provides a different number of 3-noded triangular elements at different positions to complete the finite element code. Since the structure is 2-D, the degrees of freedom will be either 1 or 2 (x or y direction) and the number of dimensions will be 2. Additionally, since we are using 3-noded triangular elements, the number of nodes per element will be 3, for each vertex of the triangle. The Input Set 6 has the least number of elements and the resulting mesh will be less detailed, while Input Set 24 has the most number of elements, and the resulting mesh will be more detailed. The other input sets lie in the middle.

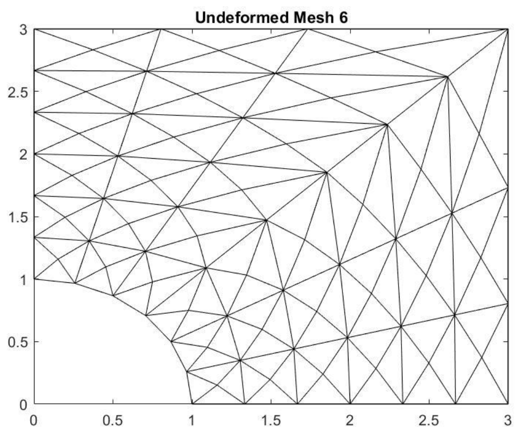
The nodes text file (e.g. nodes6.txt) contains the number of nodes, and the location of each node in the mesh. The first line of the file is the number of nodes, then from line 2 onward, the node number is in column 1, the x-coordinate is in column 2, and the y-coordinate is in column 3.

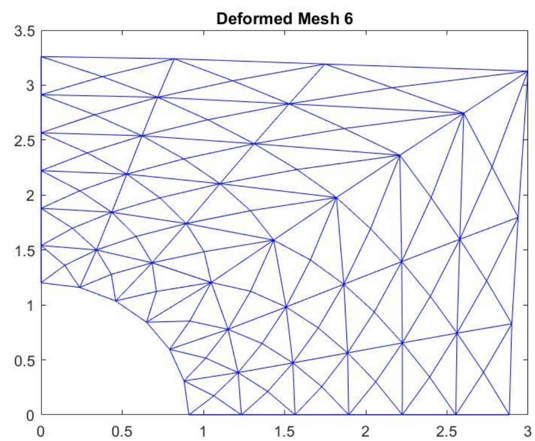
The elements text file (e.g. elements6.txt) contains information about the 3-noded elements. The first line in the file contains the number of elements, the value for Young's modulus, and the value for Poisson's ratio. Then from line 2 onward, the element is numbered in column 1 and the nodes that the element is defined by are in columns 2, 3, and 4.

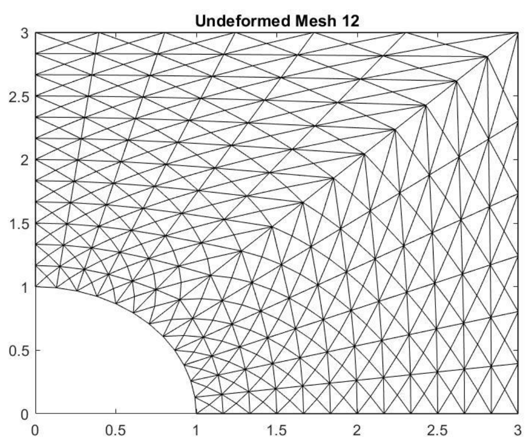
The displacements text file (e.g. displacements6.txt) contains the given displacements in the structure. The first line in the file is the number of displacements. Then from line 2 onward, the node that the displacement occurs is in column 1, the degree of freedom (the x or y direction of the displacement) is in column 2, and the value of the displacement is in column 3. Often the given displacements have a value of zero.

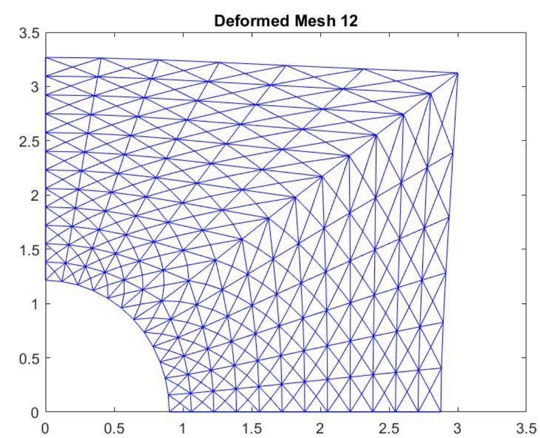
The forces text file (e.g. forces6.txt) contains the external forces acting on the structure. The first line in the file contains the number of forces. Then from line 2 onward, the node that the force occurs on is in column 1, the degree of freedom (the x or y direction that the force is applied in) is in column 2, and the value of the force is in column 3.

Meshes:









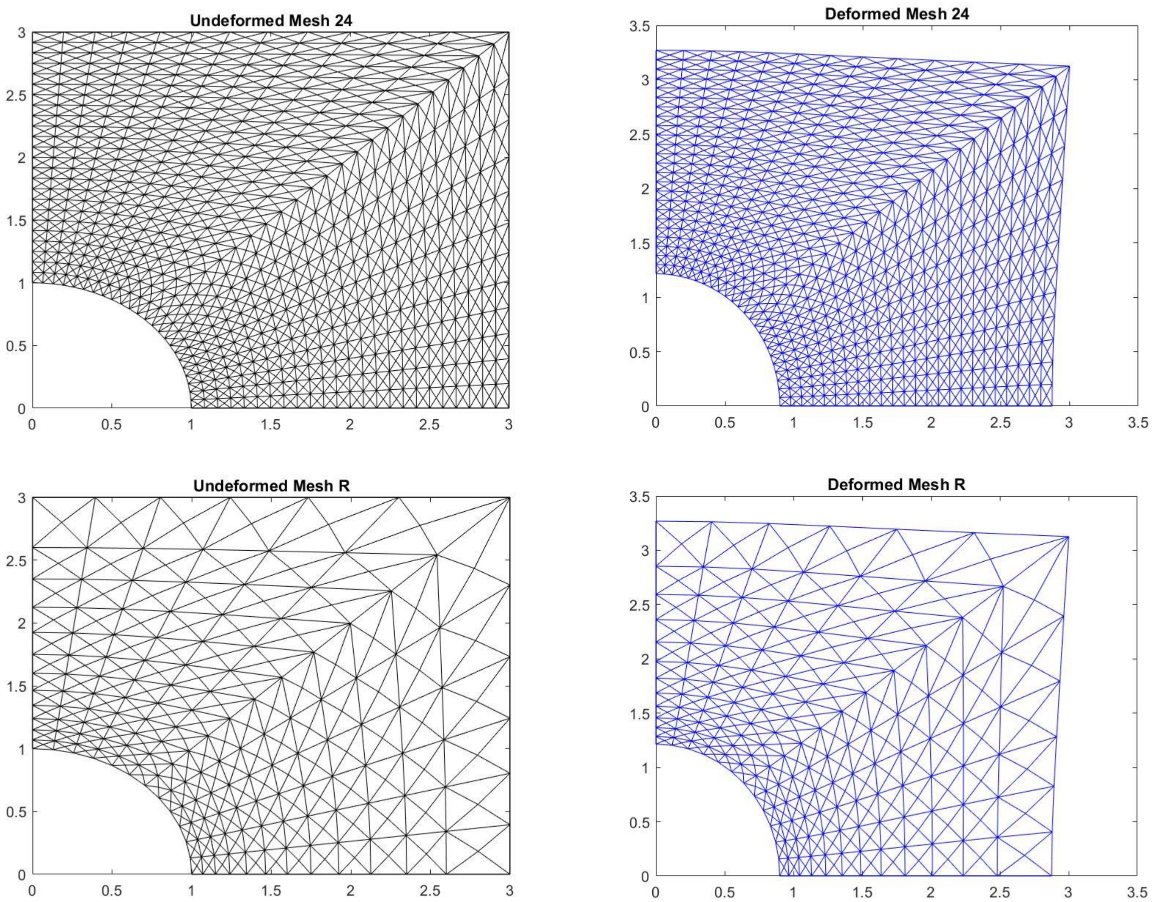


Figure 3: Deformed and Undeformed meshes for each input set.

Figure 3 contains eight graphs, 4 deformed, and 4 undeformed graphs for each input set. The undeformed meshes show the 3-noded triangular elements that make up the mesh. The deformed meshes show the structure with the same 3-noded triangular elements, and their displacements because of the forces on the structure. The deformed mesh is generated by scaling the displacements and adding the corresponding displacements to the existing node locations. For these meshes, I used a scaling factor of 0.05 on the nodal displacements. In looking at the resulting deformations, it seems that the top right corner moves inward and downward. The bottom of the right edge is more inward, and the left of the top edge is more outward.

Stress Graphs:

Stress is a measure of the internal forces in the structure. In the following figures, the stress in the x-direction and y-direction in the structure is plotted with respect to the x-coordinates and y-coordinates. Stress is defined as σ and is defined as the internal force divided by the cross-sectional area, or Young's multiplied by strain.

$$\sigma = \frac{F}{A}$$

where F = internal force, A = cross-sectional area

$$\sigma = E\varepsilon$$

where E = Young's modulus, ε = strain

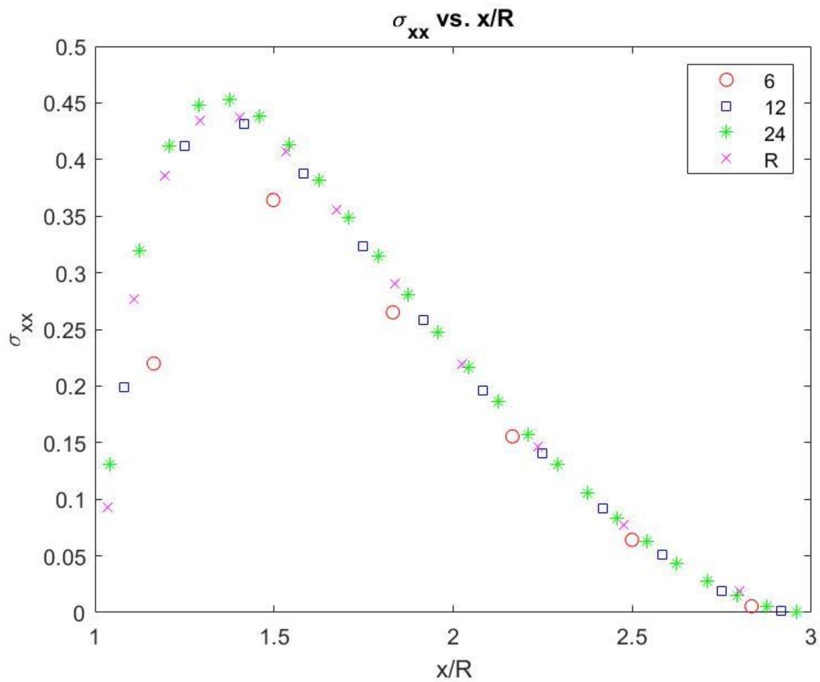


Figure 4: Axial stress in the x-direction vs. x/R

Figure 4 contains the plots of the axial stresses in the x-direction vs. x/R for each input set. The red circles are Input Set 6, the blue squares are Input Set 12, the green asterisks are Input Set 24, and the magenta crosses are Input Set R. The value of x is the interpolated x-value of the center of the triangle for that element, R is the radius (in this case 1), and σ_{xx} is axial stress in the x-direction. These 3-noded elements have a bottom, horizontal edge on the x-axis. The graphs for the different input sets show the same trend for stress in the x-direction. Each graph increases rapidly, then decreases more gradually. Input Set 24 has more elements, resulting in a smoother graph due to more data points.

$$\text{interpolated x-value} = \frac{x_1+x_2+x_3}{3}$$

where x_1= x-coordinate of node 1, x_2= x-coordinate of node 2, x_3= x-coordinate of node 3

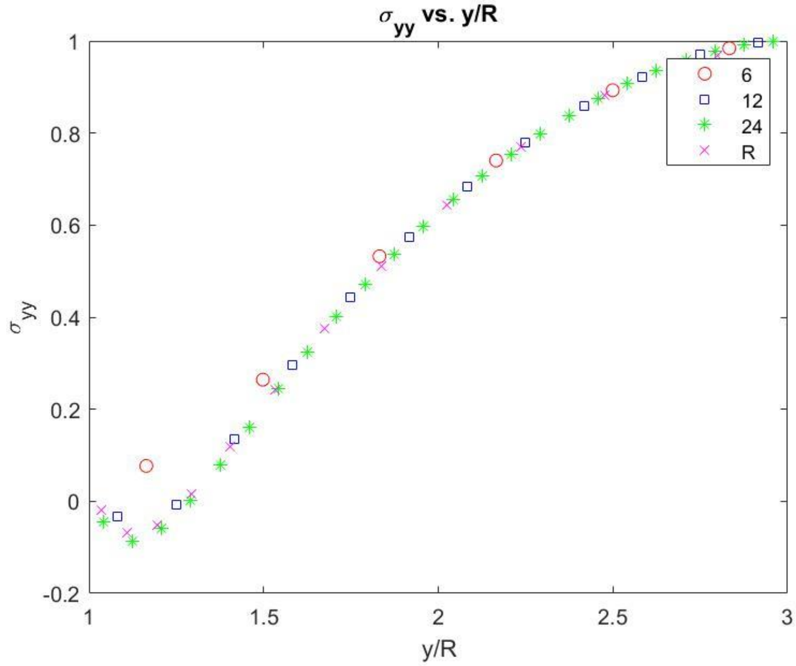


Figure 5: Axial stress in the y-direction vs y/R

Figure 5 contains the plots of the axial stresses in the y-direction vs. y/R for each input set. The red circles are Input Set 6, the blue squares are Input Set 12, the green asterisks are Input Set 24, and the magenta crosses are Input Set R. The value of y is the interpolated y value of the center of the triangle for that element, R is the radius (in our case 1), and σ_{yy} is axial stress in the y-direction. These 3-noded elements have a left, vertical edge on the y-axis. The graphs for the different input sets show the same trend for stress in the y-direction, they all increase steadily. Input Set 24 has more elements, resulting in a smoother graph due to more data points.

$$\text{interpolated y-value} = \frac{y_1+y_2+y_3}{3}$$

where y_1= y-coordinate of node 1, y_2= y-coordinate of node 2, y_3= y-coordinate of node 3

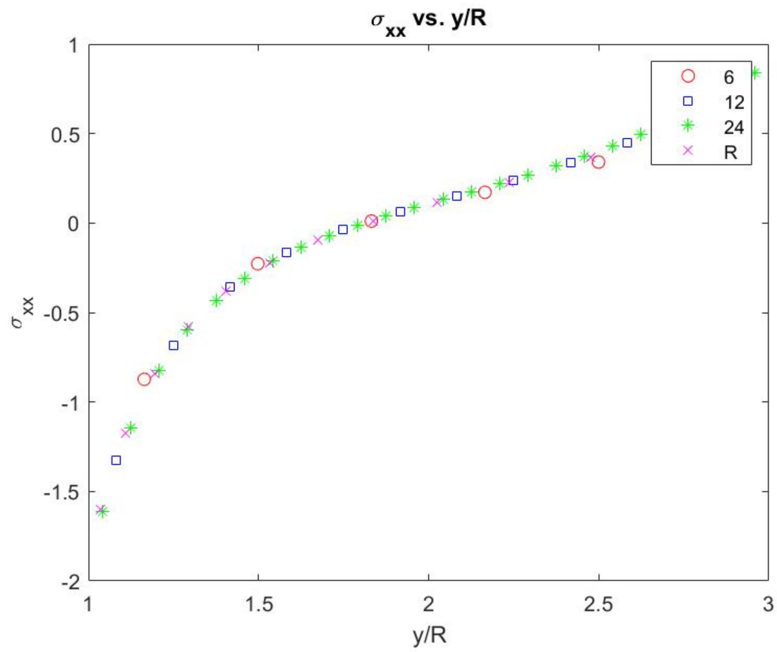


Figure 6: Axial stress in the x-direction vs. y/R

Figure 6 contains four graphs of the axial stresses in the x-direction vs. y/R for each input set. The red circles are Input Set 6, the blue squares are Input Set 12, the green asterisks are Input Set 24, and the magenta crosses are Input Set R. The value of y is the interpolated y-value of the center of the triangle for that element, R is the radius (in our case 1), and σ_{xx} is stress in the x-direction. These 3-noded elements have a left, vertical edge on the y-axis. The graphs for different input sets show the same trend that increases rapidly in the beginning, increases slowly in the middle, and increases steadily at the end. Input Set 24 has more elements, resulting in a smoother graph due to more data points.

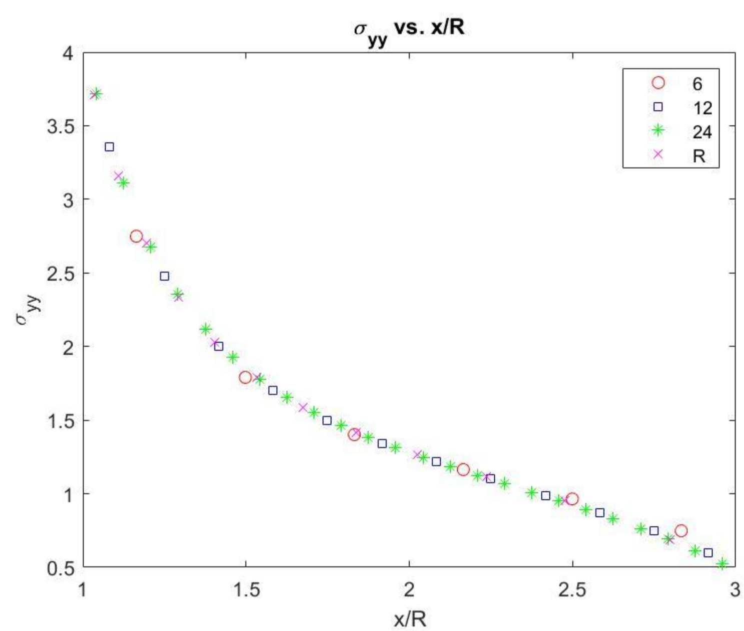


Figure 7: Axial stress in the y-direction vs. x/R for each input set.

Figure 7 contains plots of the axial stresses in the y-direction vs. x/R for each input set. The red circles are Input Set 6, the blue squares are Input Set 12, the green asterisks are Input Set 24, and the magenta crosses are Input Set R. The value of x is found using the interpolated x-value of the center of the triangle for that element, R is the radius (in our case 1), and σ_{yy} is stress in the y-direction. These stresses are from 3-noded elements have a bottom, horizontal edge on the x-axis. The graphs for different input sets show the same trend that decreases rapidly in the beginning and more steadily at the end. Input Set 24 has more elements, resulting in a smoother graph due to more data points.

Mesh with Stresses:

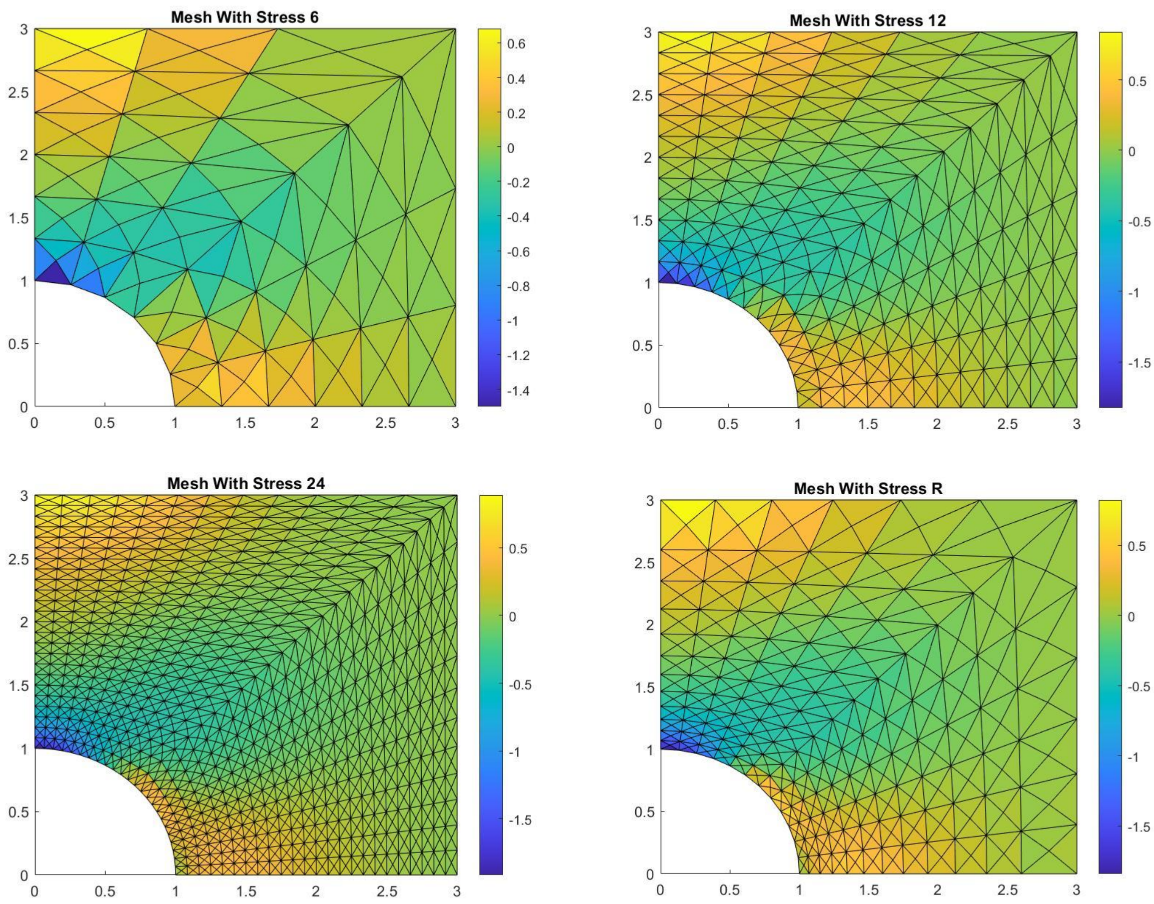


Figure 8: Stresses (σ_{xx}) on the structure for each input set.

Figure 8 contains four plots with color mapping for stresses in the x-direction on the structure for each input set. The color of each 3-noded triangular element represents the stress in that element. The color mapping corresponds with the color bar on the right side of the graph, where the dark blue color has a strain of around -2.0 and increases to around 0.6 in yellow. When strain increases, the color progresses from dark blue to light blue to green-blue to green to yellow-green to yellow. It can be observed that the color mapping corresponds to the graphs for the axial stress in the x directions on the x and y axes from the above figures. In Figure 4 the stress increases rapidly, then decreases more gradually. This is reflected on the x-axis as the color moves from green to very yellow, then back to green. In Figure 6 the stress increases steadily, and this is also reflected as the color progresses gradually from blue to yellow.

Extrapolated Stress:

The stress at the edges of the circle can be estimated using linear extrapolation. Based on two stresses next to the two points at the edge of the circle, (1,0) and (0,1), we can estimate a value for the actual stress.

Input Set	σ_{xx} vs. x/R	σ_{yy} vs. y/R	σ_{xx} vs. y/R	σ_{yy} vs. x/R
6	0.1490	-0.0153	-1.1906	3.2205
12	0.0930	-0.0473	-1.6496	3.7905
24	0.0371	-0.0238	-1.8469	4.0216
R	0.0082	0.0035	-1.7961	3.9641

The estimated value will likely be closest to the actual value with a input set with more elements. In this case, Input Set 24 might be the one to examine.

Summarize Results:

I created a 2-D finite element code with 3-noded elements to calculate nodal displacements, element stresses, and element strains in a 2-D plate with a hole in the middle. I used MATLAB to create the code and the file, *hw10_struct_a.m,* is attached in the submission. The code outputs displacements, stresses, and strains. The displacements are outputted as a two-column matrix where the first column is the x-direction and the second column is the y-direction. The stresses and strains are outputted as three column matrices. The outputted stresses and strains in the code are already going to be normalized due to the radius and Young's modulus being 1.

The nodal displacements can be viewed in Figure 3 in the deformed mesh. It can be observed that the top right corner moves inward and downward, the bottom of the right edge is more inward, and the left of the top edge is more outward. The location of the nodes in the deformed mesh are calculated by adding the corresponding nodal displacements to the original nodes. The displacements were multiplied by a reasonable scaling factor before they were added. In my code, I used a scaling factor of 0.05.

Stresses in the x-direction are plotted with respect to x/R and y/R, and in the y-direction with respect to x/R and y/R where x and y are interpolated points and R is the radius. From Figure 4, in the x-direction with respect to x/R, the stress increases rapidly, then decreases. From Figure 5, in the y-direction with respect to y/R, the stress increases gradually. From Figure 6, in the x-direction with respect to y/R, the stress increases gradually. From Figure 7, in the y-direction with respect to x/R, the stress decreases.