Linear Algebra Midterm Note Sheet (Front)

Course coverage: 1.1, 1.2, 2.1, 2.2, 2.3, 3.1 + Canvas Additional Theorems.

CORE TRIGGERS (Write at top)

Row ops preserve solutions *(Add'l Thm 1C). Use whenever you swap to echelon/RREF.*
Free var ⇒ infinitely many solutions *(Add'l Thm 1A). Count pivots vs vars.*
Linearity *T(c■u■+…+c■u■)=c■T(u■)+…+c■T(u■) (Add'l Thm 3A).*
Dependence preserved*: {u■} dep ⇒ {T(u■)} dep (Add'l Thm 3B).*
1–1 preserves independence *(Add'l Thm 3C).*
Onto*: range(T)=■■ (Add'l Thm 3D). Matrix cues: 1–1 ⇔ pivot in each column; Onto ⇔ pivot in each row; rank + nullity = #cols.*

1) How many solutions? Give examples

YES/NO box: Unique? ***Yes*** *iff pivot in every variable column. Infinite?* ***Yes*** *iff ≥1 free var and consistent. None?* ***Yes*** *if a row [0 … 0 | b≠0].*
Justify: *Add'l Thm 1C (row ops) + Add'l Thm 1A (free var ⇒ ∞).*
Example write-up: *RREF → z free ⇒ (x,y,z)=(1–2s,s,0). Two sols: s=0,s=1.*

2) Build RREF from a parametric solution

YES/NO: Is z free? ***Yes*** *→ put pivot in x,y; leave z free.*
Justify: *Parametric form ↔ RREF reading (Sec 2.2) + Thm 1C.*
Example: *x=–1+s, y=2+s, z=s ⇒ rows: [1 –1 0 | –3], [0 1 –1 | 2].*

3) Linear independence fast tests

YES/NO: Are 5 vectors in ■■ independent? ***No*** *(too many).*
Justify: *Max rank in ■■ is 4 ⇒ dependence (Sec 2.1–2.3).*
YES/NO: Are 3 vectors in ■^{3} independent? ***Yes*** *iff pivot in every column in RREF of [v■ v■ v■].*

4) Do these vectors span ■■?

YES/NO: Fewer than n vectors span ■■? ***No*** *(rank ≤ #vecs).*
Yes *iff rank = n (pivots in all rows). (Sec 2.3).*

5) Make span grow strictly with each added vector (■^{2} example)

Pick u■=(1,0), u■=(2,0) (collinear) ⇒ span={ (t,0) }. Add u■=(0,1) ⇒ span=■^{2}.
Justify: *Rank increases when adding non-collinear vector (Sec 2.x).*

Linear Algebra Midterm Note Sheet (Back)

6) Extremal sets in ■■

Largest LI not spanning: size 3. Why? Any 4 LI vectors in ■■ span.
Smallest LD that spans: size 5. Why? To be dependent and still span 4-D, need ≥5.
Justify: *dimension & rank arguments (Sec 2.3).*

7) Two distinct linear transformations with given values

YES/NO: Can two maps differ if images on a basis are fixed?
No*: basis images determine T uniquely (3A).*
Strategy: *Solve for basis images via linearity; any unconstrained basis vector gives degrees of freedom ⇒ infinitely many choices.*
Justify: *Add'l Thm 3A (linearity).*

8) One-to-one vs Onto (matrix cues)

1–1: pivot in each column ⇔ nullspace={0}. Onto ■■: rank = m (pivot in each row).
Impossible cases: 1–1 ■^{3}→■^{2} (domain>codomain). Onto ■■ from ■^{3} (domain ***Justify:*** *Rank-nullity + Add'l Thm 3C/3D.*

9) Row-equivalent?

YES/NO: Same RREF? ***Yes*** *⇒ row-equivalent. Pivot pattern differs?* ***No****.*
Justify: *RREF uniqueness; Thm 1C for solution-set preservation.*

10) Can the same A give Ax=b with ∞ solutions but Ay=c with a unique solution? (4×2)

No. *If rank(A)<2 ⇒ any consistent RHS yields infinitely many; if rank(A)=2 ⇒ at most one solution for every RHS. One matrix can't do both behaviors.* ***Justify:*** *rank/ free-var logic (Sec 2.3).*

11) Kernel constraints for T(x)=Σ x■u■

If required T([1,–1,1,–1])=0 with u■,u■,u■ independent ⇒ set u■=u■+u■–u■ (works).
If T([1,0,1,0])=0 and u■,u■,u■ independent ⇒ ***impossible*** *since u■=–u■ contradicts independence.* ***Justify:*** *kernel = linear relations among u■.*

12) Smallest n with 1–1 but not onto for T:■^{3}→■■

n=4. *1–1 ⇒ rank=3; not onto requires codomain dim >3. Example T(x)=[I■;0]x.* ***Justify:*** *Rank/onto definition (3D).*