Midterm Theorem Quick Finder (Back)

Sec 1.1: Echelon Systems / Consistency

Thm: Consistent echelon system has infinitely many solutions iff there is a free variable (Canvas Add'l Thm 1A).

Use for: After RREF, to decide 0/1/inf solutions quickly. Count free vars.

Trigger words: how many solutions, consistent system, free variable.

Sec 1.2: Elementary Row Operations & RREF

Thm: Elementary row ops preserve the solution set (Canvas Add'l Thm 1C).

Thm: If A to B by deleting column j, the reduced A0 to B0 similarly (Canvas Add'l Thm 1D).

Facts: Each matrix has a unique RREF. Two matrices are row-equivalent iff they have the same RREF.

Use for: Replace A by RREF to analyze consistency, pivots, rank, nullspace; decide row-equivalence.

Recipe: Count Solutions
RREF of [A|b]: - Row [0 ... 0 | nonzero] -> No solution. - Pivot in all variable columns -> Unique solution. - Otherwise -> Free var(s) -> Infinitely many.

Sec 2.1: Linear Combinations & Span

Def: Span({v1,...,vk}) is all linear combinations of the vectors.

Facts: In R^m you need at least m vectors to possibly span R^m; zero vector never helps rank.

Use for: Span questions; construct examples/non-examples; growing span dimension.

Sec 2.2: Linear Independence / Dependence

Def: {v1,...,vk} independent iff the only solution to c1*v1+...+ck*vk=0 is all ci=0.

Quick tests: If k>m in R^m then dependent. If RREF of [v1 ... vk] has a pivot in every column then independent.

Use for: Max size of LI set, min size of spanning LD set, construct counterexamples.

Sec 2.3: Rank, Nullity, and Systems

Def: rank(A) = number of pivot columns; nullity(A) = #cols - rank(A).

Matrix cues: Columns span R^m iff rank(A)=m (pivot in each row).

Use for: Span of columns, existence/uniqueness, designing A to be 1-1 or onto when viewed as T(x)=Ax.

Decision Box: 1-1 vs Onto for T(x)=Ax
Domain R^n, Codomain R^m - 1-1 iff pivot in each column (nullspace = {0}). - Onto R^m iff rank = m (pivot in each row). - Impossible combos: * 1-1 when n>m. * Onto when n<m.

Sec 3.1: Linear Transformations

Add'l Thm 3A (Linearity): T(c1*u1+...+cr*ur)=c1*T(u1)+...+cr*T(ur).

Use for: Compute T(x) from values on basis/generators; show two T must agree if they agree on a basis.

Add'l Thm 3B: If {u1,...,ur} are linearly dependent, then {T(u1),...,T(ur)} are linearly dependent.

Use for: If images are independent, conclude preimages were independent (contrapositive).

Add'l Thm 3C: If T is one-to-one and {u1,...,ur} are independent, then {T(u1),...,T(ur)} are independent.

Use for: Rule out 1-1 when images are dependent; preserve independence.

Add'l Thm 3D (Onto): T is onto R^n iff range(T)=R^n (equals the codomain). For matrices: rank = #rows.

Use for: Decide onto by rank; construct 1-1 but not onto maps by choosing m>n and rank=n.

Unifying Row-Equivalence Theorem (Practical Form)

A row-equivalent to B iff RREF(A) = RREF(B). Left-multiplication by an invertible matrix preserves column relations (dependence/independence).

Use for: Decide if two matrices are equivalent; reason about column independence via the pivot structure in RREF.

When to Cite What (Problem Triggers)

Counting solutions or free variables -> Add'l Thm 1A + RREF (Sec 1.2).

Replacing a matrix by RREF -> Add'l Thm 1C.

Span R^m or rank tests -> Sec 2.3 rank=m.

Too many vectors to be independent -> Sec 2.2 size test.

Images under a linear map; building T from basis -> Add'l Thm 3A.

Show independence/dependence is preserved or impossible under T -> Add'l Thm 3B/3C.

Onto/Not onto design -> Add'l Thm 3D + rank.