

# STACKS

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NATIONAL UNIVERSITY OF TECHNOLOGY (NUTECH)

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LECTURE # 12



# STACKS

“A **Stack** is a special kind of list in which all insertions and deletions take place at one end, called the **Top**”

*A Stack is a recursive data structure.*

## Other Names

- Pushdown List
- Last In First Out (LIFO)

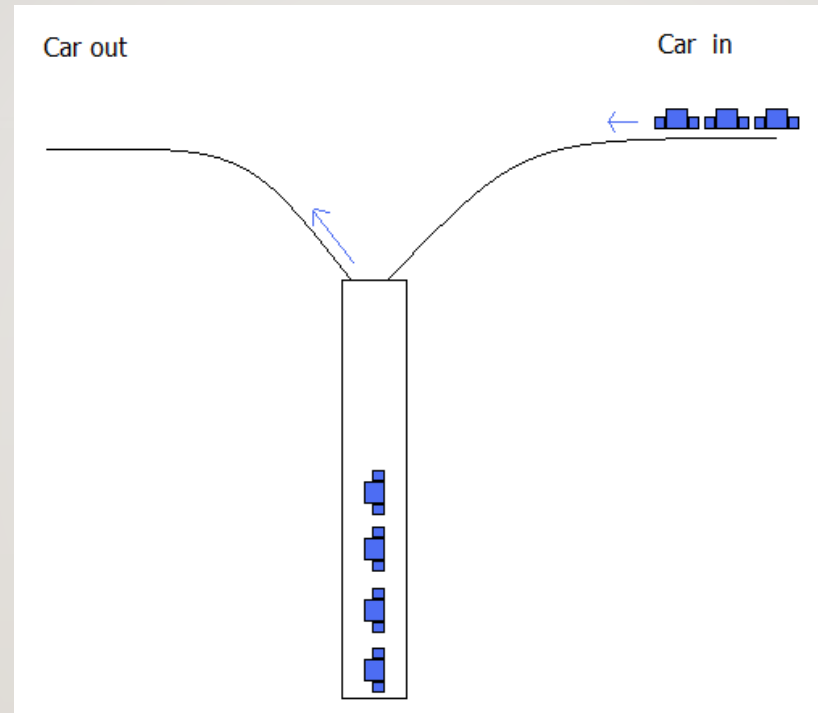


# STACKS (EXAMPLES)

- Books on a floor
- Dishes on a shelf



# STACKS (EXAMPLES)



Is there an appropriate data type to model this parking lot???

# APPLICATIONS

- Undo and Redo in MS Word
  - Last action would be undone/redone first
- Banking transactions
  - Last transaction would be viewed first
- Mathematical operations
  - Operators precedence



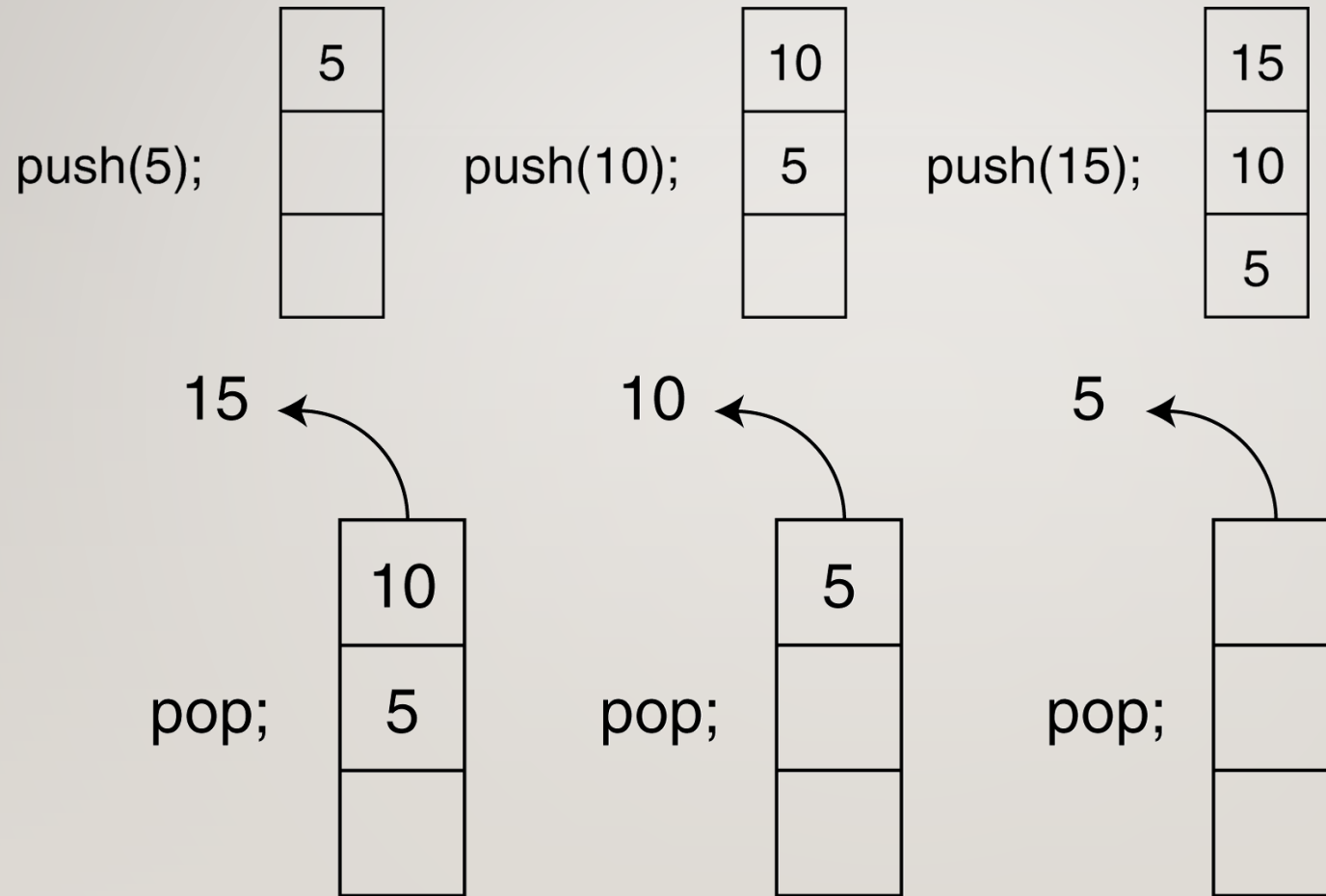
# COMMON OPERATIONS ON STACKS

1. **MAKENULL(S)**: Make Stack  $S$  be an empty stack.
2. **TOP(S)**: Return the element at the top of stack  $S$ .
3. **POP(S)**: Remove the top element of the stack.
4. **PUSH(S,x)**: Insert the element  $x$  at the top of the stack.
5. **EMPTY(S)**: Return true if  $S$  is an empty stack; return false otherwise.

# STATIC AND DYNAMIC STACKS

- There are two kinds of stack data structure -
  - a) **static**, i.e. they have a **fixed size**, and are *implemented as arrays*.
  - b) **dynamic**, i.e. they **grow in size** as needed, and *implemented as linked lists*

# PUSH AND POP OPERATIONS OF STACK

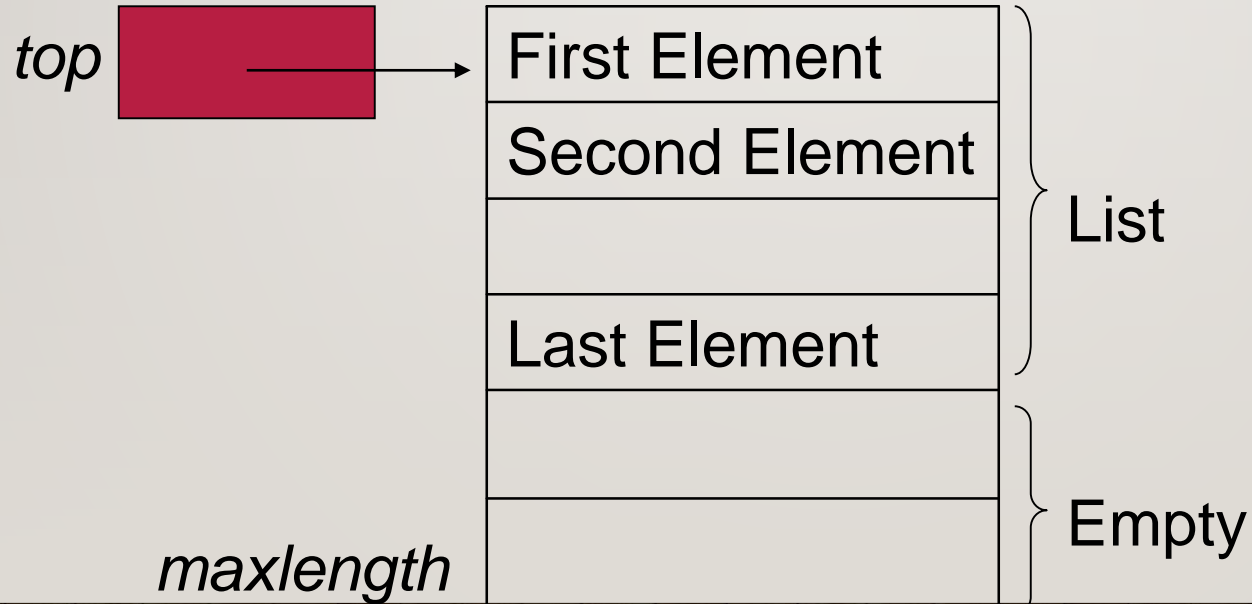




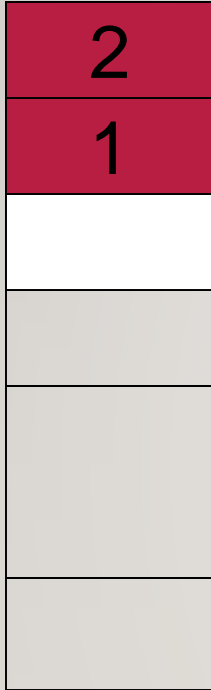
# AN ARRAY IMPLEMENTATION OF STACKS

## First Implementation

- Elements are stored in contiguous cells of an array.
- New elements can be inserted to the top of the list.



# AN ARRAY IMPLEMENTATION OF STACKS



## Problem with this implementation

- Every PUSH and POP requires moving the entire array up and down.

# OPERATIONS

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- **Push**
  - Stores a value to the **top** of the stack
- **Pop**
  - Retrieves a value from the **top** of the stack
- **IsFull**
  - Returns **true** if the stack is **full**
- **IsEmpty**
  - Returns **true** if the stack is **empty**

# IMPLEMENTATION

---

- Stack can be implemented using
  - **Array**
    - Static size
      - **Pro:** Best performance
      - **Con:** Fixed size
  - **Linked List**
    - Dynamic size
      - **Pro:** Variable size
      - **Con:** Bigger size can hurt the performance

# IMPLEMENTATION USING ARRAY

---

- **IntStack class**

```
class IntStack
{
    private:
        int *stackArray;
        int stackSize;
        int top;

    public:
        IntStack(int);    // Constructor
        void Push(int);
        void Pop(void);
        bool isFull(void);
        bool isEmpty(void);
        void Display(void);
};
```



# IMPLEMENTATION USING ARRAY

---

- **Constructor**

```
IntStack::IntStack(int size)
{
    stackArray = new int[size];
    stackSize  = size;
    top = -1;
}
```

# IMPLEMENTATION USING ARRAY

---

- **Push()**

```
void IntStack::Push(int num)
{
    if (isFull())
    {
        cout << "The stack is full.\n";
    }

    else
    {
        top++;
        stackArray[top] = num;
    }
}
```

# IMPLEMENTATION USING ARRAY

---

- **Pop()**

```
void IntStack::Pop(void)
{
    if (isEmpty())
    {
        cout << "The stack is empty.\n";
    }

    else
    {
        top--;
    }
}
```

# IMPLEMENTATION USING ARRAY

---

- **isFull()**

```
bool IntStack::isFull(void)
{
    bool status;

    if (top == stackSize - 1)

        status = true;
    else

        status = false;

    return status;
}
```

# IMPLEMENTATION USING ARRAY

---

- **isEmpty()**

```
bool IntStack::isEmpty(void)
{
    bool status;

    if (top == -1)

        status = true;
    else

        status = false;

    return status;
}
```



# IMPLEMENTATION USING ARRAY

---

- **Display()**

```
void IntStack::Display(void)
{
    if (isEmpty()) {
        cout << "The stack is empty.\n";
    }
    else {
        int t = top;
        while (t >= 0)
        {
            cout << "stackArray[" << t << "] = "
                << stackArray[t] << endl;
            t--;
        }
    }
}
```

# DEMONSTRATION

Push

Pop

Top

Stack  
Size

--	--	--	--	--	--	--

0

1

2

3

4

5

6

# DEMONSTRATION

---

Push

5

Top

-1

Pop

Stack  
Size

7



0

1

2

3

4

5

6

# DEMONSTRATION

Push

5

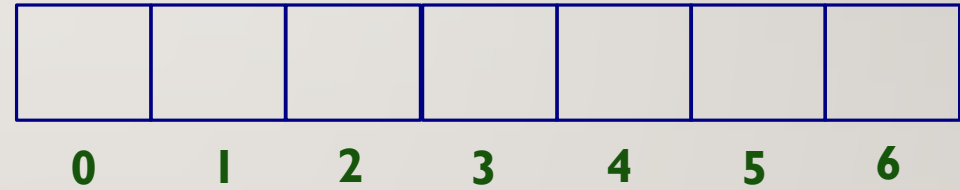
Top

-1

Pop

Stack  
Size

7



# DEMONSTRATION

Push

5

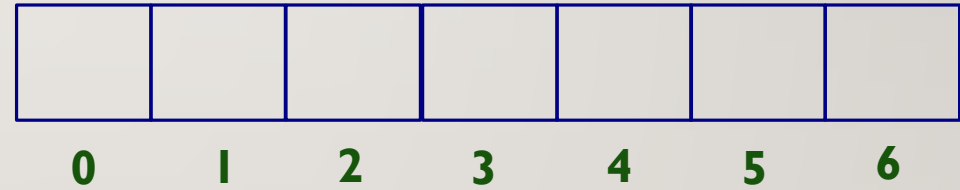
Top

0

Pop

Stack  
Size

7





# DEMONSTRATION

Push

Pop

Top

0

Stack  
Size

7

5

0

1

2

3

4

5

6

# DEMONSTRATION

Push

7

Top

0

Pop

Stack  
Size

7

5

0

1

2

3

4

5

6

# DEMONSTRATION

Push

7

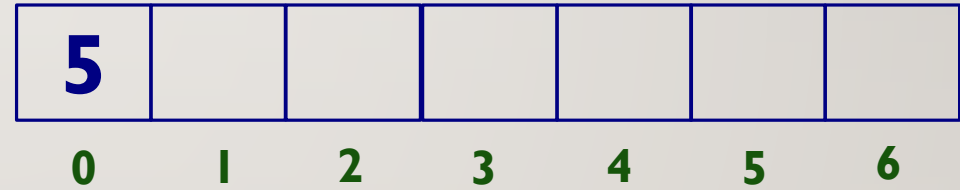
Top

0

Pop

Stack  
Size

7



# DEMONSTRATION

Push

7

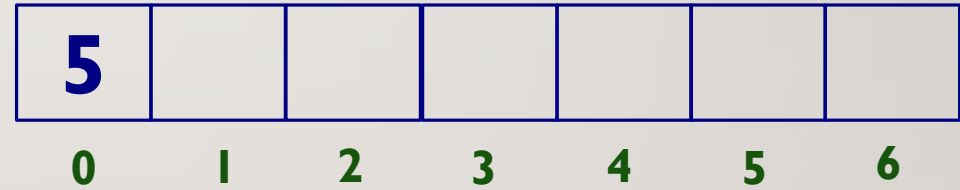
Pop

Top

1

Stack  
Size

7



# DEMONSTRATION

Push

Pop

Top

Stack  
Size

5	7					
0	1	2	3	4	5	6



# DEMONSTRATION

Push

3

Top

1

Pop

Stack  
Size

7

5

7

0

1

2

3

4

5

6

# DEMONSTRATION

Push

3

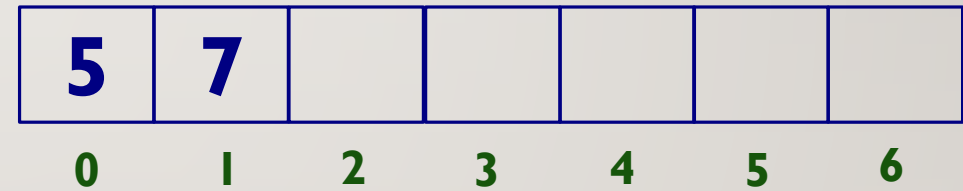
Top

1

Pop

Stack  
Size

7



# DEMONSTRATION

Push

3

Top

2

Pop

Stack  
Size

7

5

7

0

1

2

3

4

5

6

# DEMONSTRATION

Push

Pop

Top

2

Stack  
Size

7

5	7	3				
0	1	2	3	4	5	6

# DEMONSTRATION

Push

Pop

Top

2

Stack  
Size

7

5	7	3				
0	1	2	3	4	5	6



# DEMONSTRATION

Push

Pop

Top

2

Stack  
Size

7

5	7	3				
0	1	2	3	4	5	6

# DEMONSTRATION

Push

Pop

Top

Stack  
Size

5	7	3				
0	1	2	3	4	5	6

# DEMONSTRATION

Push

Pop

Top

Stack  
Size

5	7					
0	1	2	3	4	5	6

# DEMONSTRATION

Push

9

Top

1

Pop

Stack  
Size

7

5

7

0

1

2

3

4

5

6

# DEMONSTRATION

Push

9

Top

2

Pop

Stack  
Size

7

5

7

0

1

2

3

4

5

6



# DEMONSTRATION

Push

Pop

Top

2

Stack  
Size

7

5	7	9				
0	1	2	3	4	5	6

# DEMONSTRATION

Push

Top

2

Pop

Stack  
Size

7

5	7	9				
0	1	2	3	4	5	6

# DEMONSTRATION

Push

Pop

Top

2

Stack  
Size

7

5	7	9				
0	1	2	3	4	5	6

# DEMONSTRATION

Push

Pop

Top

Stack  
Size

5	7	9				
0	1	2	3	4	5	6

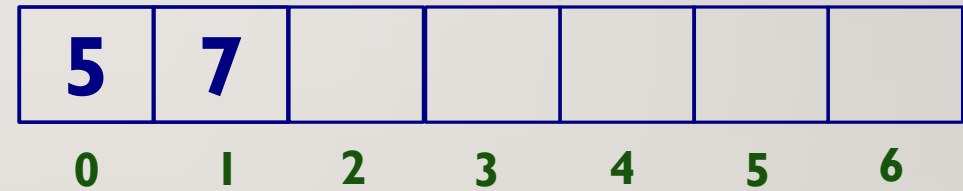
# DEMONSTRATION

Push

Pop

Top

Stack  
Size





# DEMONSTRATION

Push

Pop

Top

Stack  
Size

5	7					
0	1	2	3	4	5	6

# DEMONSTRATION

Push

Pop

Top

Stack  
Size

5	7					
0	1	2	3	4	5	6

# DEMONSTRATION

Push

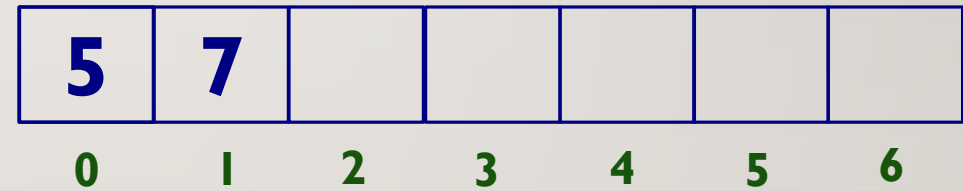
Top

0

Pop

Stack  
Size

7



# DEMONSTRATION

Push

Pop

Top

Stack  
Size

5						
0	1	2	3	4	5	6

# DEMONSTRATION

Push

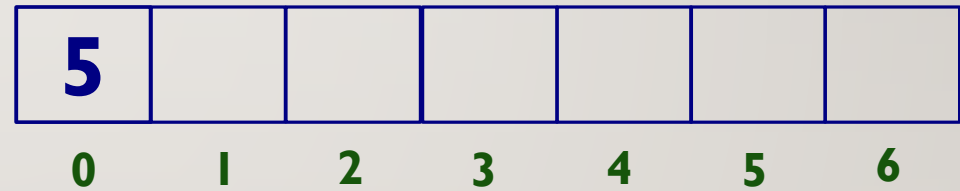
Pop

Top

0

Stack  
Size

7





# DEMONSTRATION

Push

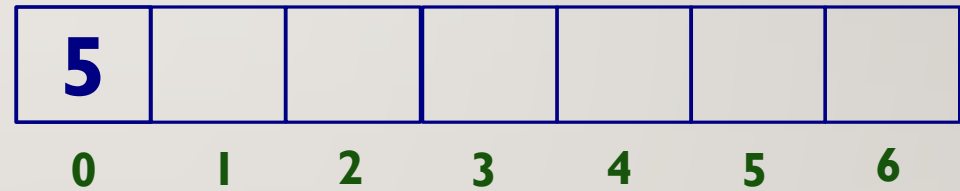
Top

0

Pop

Stack  
Size

7



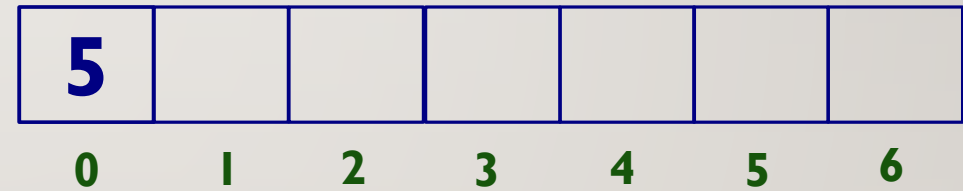
# DEMONSTRATION

Push

Pop

Top

Stack  
Size



# DEMONSTRATION

Push

Pop

Top

Stack  
Size

--	--	--	--	--	--	--

0

1

2

3

4

5

6

# DEMONSTRATION

Push

Pop

Top

Stack  
Size

--	--	--	--	--	--	--

0

1

2

3

4

5

6

# DEMONSTRATION

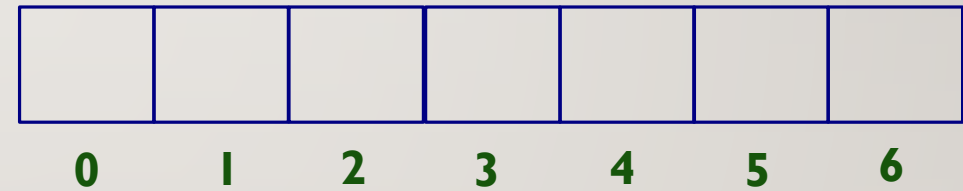
Push

Top

Pop

Stack  
Size

Empty  
Queue!



# DEMONSTRATION

Push

Pop

Top

6

Stack  
Size

7

5	7	3	9	6	2	1
0	1	2	3	4	5	6



# DEMONSTRATION

Push

8

Top

6

Pop

Stack  
Size

7

5	7	3	9	6	2	1
0	1	2	3	4	5	6

# DEMONSTRATION

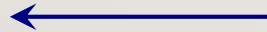
Push 8

Pop

Top 6

Stack Size 7

Full Queue!



5	7	3	9	6	2	1
0	1	2	3	4	5	6

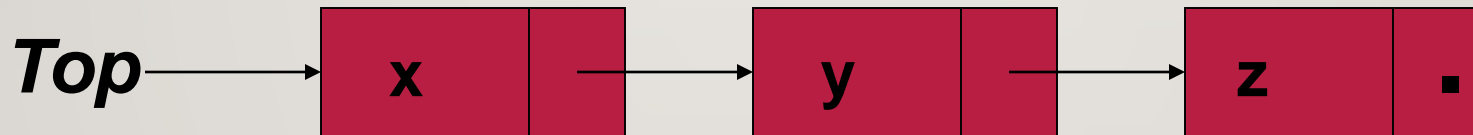
# STACK TEMPLATES

The stack class so far work with integers only. A stack template can be used to work with any data type.



# A LINKED-LIST IMPLEMENTATION OF STACKS

- Stack can *expand* or *shrink* with each PUSH or POP operation.
- PUSH and POP operate only on the header cell and the first cell on the list.



# LINKED LIST IMPLEMENTATION OF STACK

```
class Stack
{
    struct node
    {
        int data;
        node *next;
    }*top;
public:
    void Push(int newelement);
    void Pop(int &);
    bool IsEmpty();
};
```



```
void Stack::Push(int newelement)
{
    node *newptr;
    newptr=new node;
    newptr->data=newelement;
    newptr->next=top;
    top=newptr;
}
```

```
void Stack::Pop(int& returnvalue)
{
    if (IsEmpty()) { cout<<"underflow error"; return;}
    tempPtr=top;
    returnvalue=top->data;
    top=top->next;
    delete tempPtr;
}
```



```
bool Stack::IsEmpty()
{
    if (top==NULL)
        return true;
    else
        return false;
}
```

## Program 3

```
// This program demonstrates the dynamic stack  
// class DynIntClass.
```

```
#include <iostream.h>  
#include "dynintstack.h"
```

```
void main(void)  
{  
    DynIntStack stack;  
    int catchVar;  
  
    cout << "Pushing 5\n";  
    stack.push(5);  
    cout << "Pushing 10\n";  
    stack.push(10);  
    cout << "Pushing 15\n";  
    stack.push(15);
```

```
    cout << "Popping...\n";  
    stack.pop(catchVar);  
    cout << catchVar << endl;  
    stack.pop(catchVar);  
    cout << catchVar << endl;  
    stack.pop(catchVar);  
    cout << catchVar << endl;  
  
    cout << "\nAttempting to pop again... ";  
    stack.pop(catchVar);  
}
```

### Program Output

```
Pushing 5  
Pushing 10  
Pushing 15  
Popping...  
15  
10  
5
```

Attempting to pop again... The stack is empty.

# APPLICATIONS OF STACKS

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# ALGEBRAIC EXPRESSION

- An algebraic expression is a legal combination of operands and the operators.
  - Operand is the quantity on which a mathematical operation is performed.
  - Operator is a symbol which signifies a mathematical or logical operation.





# INFIX, POSTFIX AND PREFIX EXPRESSIONS

- **INFIX:** expressions in which operands surround the operator.
- **POSTFIX:** operator comes after the operands, also Known as Reverse Polish Notation (RPN).
- **PREFIX:** operator comes before the operands, also Known as Polish notation.
- Example
  - Infix:  $A+B-C$  Postfix:  $AB+C-$  Prefix:  $-+ABC$



# EXAMPLES OF INFIX TO PREFIX AND POSTFIX

Infix	PostFix	Prefix
$A+B$	$AB+$	$+AB$
$(A+B) * (C + D)$	$AB+CD+*$	$*+AB+CD$
$A-B/(C*D^E)$	?	?

# A+B\*C IN POSTFIX

- Applying the rules of precedence, we obtained

A+B\*C

A+(B\*C) Parentheses for emphasis

A+(BC\*) Convert the multiplication,

ABC\*+ **Postfix Form**

$( (A+B)*C-(D-E) ) \$ (F+G)$

Conversion to Postfix Expression

$( (AB+)*C-(DE-) ) \$ (FG+)$

$( (AB+C*)-(DE-) ) \$ (FG+)$

$(AB+C*DE--) \$ (FG+)$

$AB+C*DE- -FG+ \$$

Exercise: Convert the following to Postfix

$(A + B) * (C - D)$

$A \$ B * C - D + E / F / (G + H)$

# WHY DO WE NEED PREFIX/POSTFIX?

- Appearance may be misleading, **INFIX** notations are not as simple as they seem
- To evaluate an infix expression we need to consider
  - Operators' Priority
  - Associative property
  - Delimiters

# WHY DO WE NEED PREFIX/POSTFIX?

- Infix Expression Is Hard To Parse and difficult to evaluate.
- Postfix and prefix do not rely on operator priority and are easier to parse.



# WHY DO WE NEED PREFIX/POSTFIX?

- An expression in infix form is thus converted into prefix or postfix form and then evaluated without considering the operators priority and delimiters.





# CONVERSION OF INFIX EXPRESSION TO POSTFIX

$A \rightarrow B * C \rightarrow ABC * +$

There must be a precedence function. `prcd(op1, op2)`, where `op1` and `op2` are chars representing operators.

This function returns **TRUE** if `op1` has precedence over `op2` when `op1` appears to the left of `op2` in an infix expression without parenthesis. `prcd(op1,op2)` returns **FALSE** otherwise.

`prcd('*','+')` and `prcd('+','+')` are **TRUE** whereas `prcd('+','*')` is **FALSE**.

# ALGORITHM TO CONVERT INFIX TO POSTFIX

```
opstk = the empty stack;
while (not end of input) {
    symb = next input character;
    if (symb is an operand)
        add symb to the postfix string
    else {
        while (!empty(opstk) &&
            prcd(stacktop(opstk),symb) ) {
            topsymb = pop(opstk);
            add topsymb to the postfix string;
        } /* end while */
        push(opstk, symb);
    } /* end else */
} /* end while */
/* output any remaining operators */
while (!empty(opstk) ) {
    topsymb = pop(opstk);
    add topsymb to the postfix string;
} /* end while */
```

## Example-1: A+B\*C

sym b	Postfix string	opstk
<b>A</b>	<b>A</b>	
<b>+</b>	<b>A</b>	<b>+</b>
<b>B</b>	<b>AB</b>	<b>+</b>
<b>*</b>	<b>AB</b>	<b>+ *</b>
<b>C</b>	<b>ABC</b>	<b>+ *</b>
	<b>ABC*</b>	<b>+</b>
	<b>ABC*+</b>	

# ALGORITHM TO CONVERT INFIX TO POSTFIX

```
opstk = the empty stack;
while (not end of input) {
    symb = next input character;
    if (symb is an operand)
        add symb to the postfix string
    else {
        while (!empty(opstk) &&
            prcd(stacktop(opstk),symb) ) {
            topsymb = pop(opstk);
            add topsymb to the postfix string;
        } /* end while */
        push(opstk, symb);
    } /* end else */
} /* end while */
/* output any remaining operators */
while (!empty(opstk) ) {
    topsymb = pop(opstk);
    add topsymb to the postfix string;
} /* end while */
```

## Example-1: A\*B+C

sym b	Postfix string	opstk
<b>A</b>	<b>A</b>	
<b>*</b>	<b>A</b>	<b>*</b>
<b>B</b>	<b>AB</b>	<b>*</b>
<b>+</b>	<b>AB*</b>	<b>+</b>
<b>C</b>	<b>AB*C</b>	<b>+</b>
	<b>AB*C+</b>	