Introduction to Information & Communication Technologies

CL-1000

Lab 03

Arithmetic Conversions in Number System | Paper-Based Conversions Tasks & Techniques

National University of Computer & Emerging Sciences – NUCES – Karachi

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Course Code: CL-1000	Introduction to Information & Communication Technologies

Contents

1. What is Number System?

- A number system is a way of writing to define numbers.
- It uniquely represents each value through a specific "arithmetic and algebraic structure of the figures."
- The value of any digit in a number is determined by:
    i. The digit itself
    ii. Its position in the number
    iii. The base of the number system
- A number is a mathematical value used to measure and make sense of a quantity.

1.1 Examples of Variations of Numbers

- **Natural:** Natural numbers are the set of positive integers starting from 1.
$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$
- **Prime**: Numbers only divisible by themselves and 1.
$$\mathbb{P} = \{2, 3, 5, 7, 11, 13, 17, \dots\}$$
- **Odd**: Numbers that are not divisible by 2.
$$\mathbb{O} = \{\dots, -1, -3, -5, -7, -9, -11, -13, +1, +3, +5, +7, +9, +11, +13, \dots\}$$
- **Even:** Numbers that are Divisible by 2.
$$\mathbb{E} = \{\dots, -12, -10, -8, -6, -4, -2, 0, +2, +4, +6, +8, +10, +12, \dots\}$$

2. Types of Number Systems

There are various types of number systems in mathematics. The four most common number system types are:

- Decimal number system **(Base- 10)**
- Binary number system **(Base- 2)**
- Octal number system **(Base-8)**
- Hexadecimal number system **(Base- 16)**

The base defines how many unique symbols a number has.

3. Digits in different Number Systems

- Binary: 0 and 1
- Decimal: 0,1, 2, 3, 4, 5, 6, 7, 8, 9
- Octal: 0, 1, 2, 3, 4, 5, 6, 7
- Hexadecimal: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

4. Decimal Number System (Base 10 Number System)

The decimal number system has a base of 10 because it uses ten digits from 0 to 9. In the decimal number system, the positions successive to the left of the decimal point represent units, tens, hundreds, thousands and so on. This system is expressed in decimal numbers. Every position shows a particular power of the base (10).

4.1 Example of Decimal Number System

The decimal number **1457** consists of the digit 7 in the unit's position, 5 in the tens place, 4 in the hundreds position, and 1 in the thousands place whose value can be written as:

$$= (1 \times 10^3) + (4 \times 10^2) + (5 \times 10^1) + (7 \times 10^0)$$

$$= (1 \times 1000) + (4 \times 100) + (5 \times 10) + (7 \times 1)$$

$$= 1000 + 400 + 50 + 7$$

$$= 1457$$

5. Binary Number System (Base 2 Number System)

The base 2 number system is also known as the Binary number system wherein, only two binary digits exist, i.e., 0 and 1. Specifically, the usual base-2 is a radix of 2. The figures described under this system are known as binary numbers, which are the combination of 0 and 1. For example, 110101 is a binary number.

We can convert any system into binary and vice versa.

5.1 Example of Binary Number System

Write $(14)_{10}$ as a binary number:

2	**14**	
2	7	**0**
2	3	**1**
	1	**1**

Therefore,

$$(14)_{10} = (1110)_2$$

6. Octal Number System (Base 8 Number System)

In the octal number system, the base is 8 and it uses numbers from 0 to 7 to represent numbers. Octal numbers are commonly used in computer applications. Converting an octal number to decimal is the same as decimal conversion and is explained below using an example.

6.1 Example of Octal Number System

Convert $(215)_8$ into decimal:

$$= (2 \times 8^2) + (1 \times 8^1) + (5 \times 8^0)$$

$$= (2 \times 64) + (1 \times 8) + (5 \times 1)$$

$$= 128 + 8 + 5$$

$$= 141$$

Therefore,

$$(215)_8 = (141)_{10}$$

7. Hexadecimal Number System (Base 16 Number System)

In the hexadecimal system, numbers are written or represented with base 16. In the hexadecimal system, the numbers are first represented just like in the decimal system, i.e. from 0 to 9. Then, the numbers are represented using the alphabet from A to F.

The below-given table shows the representation of numbers in the hexadecimal number system.

Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

8. Number System Solved Examples

8.1 Example 01

Convert $(1056)_{16}$ to an octal number.

Solution:

Given, $(1056)_{16}$ is a hex number.

First, we need to convert the given hexadecimal number into decimal number

$$= (1 \times 16^3) + (0 \times 16^2) + (5 \times 16^1) + (6 \times 16^0)$$

$$= (1 \times 4096) + (0 \times 256) + (5 \times 16) + (6 \times 1)$$

$$= 4096 + 80 + 6$$

$$= 4182$$

$$= 141$$

Therefore,

$$(1056)_{16} = (4182)_{10}$$

Now we will convert this decimal number to the required octal number by repetitively dividing by 8.

Therefore, taking the value of the remainder from bottom to top, we get:

8	**4182**	
8	522	**6**
8	65	**2**
8	8	**1**
8	1	**0**
	0	**1**

Therefore,

$$(4182)_{10} = (10126)_8$$

8.2 Example 02

Convert $(1001001100)_2$ to a decimal number.

Solution:

$$= (1 \times 2^9) + (0 \times 2^8) + (0 \times 2^7) + (1 \times 2^6) + (0 \times 2^5)$$
$$+(0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$$

$$= (1 \times 512) + (0 \times 256) + (0 \times 128) + (1 \times 64) + (0 \times 32)$$
$$+(0 \times 16) + (1 \times 8) + (1 \times 4) + (0 \times 2) + (0 \times 1)$$

$$= 512 + 64 + 8 + 4$$
$$= (588)_{10}$$

Therefore,

$$(1001001100)_{16} = (588)_{10}$$

8.3 Example 03

Convert $(10101)_2$ into an octal number.

Solution:

As we know that the octal number system is represented by 7 digits, from 0-7, therefore, the following sequence add up to 7:

$$4, 2, 1 \rightarrow (4 + 2 + 1 = 7)$$

Hence, we can break the binary number starting from LSB into a pair of 3 digits for faster conversions.

So, the binary number $(10101)_2$, can be written in octal number as:

$\leftarrow$

		set			set		set
add extra $0_{(s)}$ *to complete the pair* $\rightarrow$	**0**	1	0		1	0	1
	4	**2**	**1**		**4**	**2**	**1**

Now add each pair (i.e., 4, 3, 1) of the sequence numbers associated with the set bits (i.e., 1 = set) individually and append the resultant.

$$= (2) \mid (4 + 1)$$

$$= 25$$

Therefore,

$$(10101)_2 = (25)_8$$

8.4 Example 04

Convert hexadecimal 2C to decimal number.

Solution:

$$= (2 \times 16^1) + (\mathrm{C} \times 16^0)$$

$$= (2 \times 16^1) + (12 \times 16^0)$$

$$= 32 + 12$$

$$= 44$$

Therefore,

$$(10101)_2 = (7)_8$$