Network Science: Assignment 1

Syed Arham Naqvi

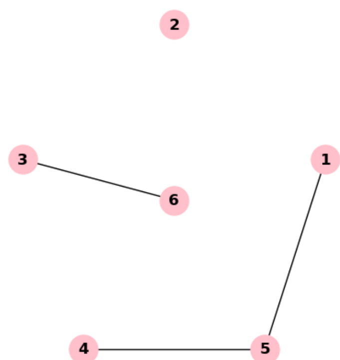
100590852

September 19, 2023

1.

a)

Graph of G^c:



Adjacency Matrix of G^c:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

b)

If p is the density of G then the density of G^c must be $1 - p$. This is because the amount of nodes remains constant in both graphs but $E(G^c)$ contains all those edges not in $E(G)$.

More formally, let $n = |V(G)|$, we know it also must then be true that $n = |V(G^c)|$ since the number of nodes is the same for both graphs. We can now use $|E(k_n)|$ where k_n is the complete graph on n nodes as the denominator for the densities of both G and its complement:

$$p = \frac{|E(G)|}{|E(k_n)|}, \quad p^c = \frac{|E(G^c)|}{|E(k_n)|}$$

Thus, if $E(G^c)$ are all those edges not in $E(G)$, then $E(k_n) = E(G) \cup E(G^c)$ and $E(G) \cap E(G^c) = \emptyset$.
So,

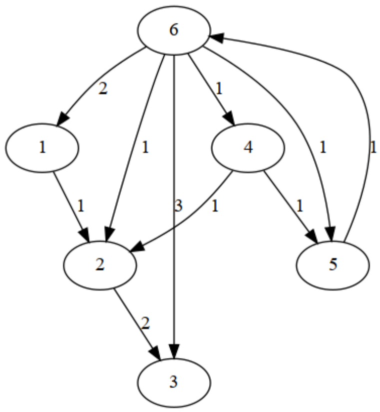
$$\begin{aligned} |E(k_n)| &= |E(G)| + |E(G^c)| \\ \frac{|E(k_n)|}{|E(k_n)|} &= \frac{|E(G)|}{|E(k_n)|} + \frac{|E(G^c)|}{|E(k_n)|} \\ 1 &= p + p^c \\ p^c &= 1 - p \end{aligned}$$

$\therefore$ the density of $G^c = 1 - p \quad \square$

2.

a)

Graph of G:



Adjacency Matrix of G:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 1 & 3 & 1 & 1 & 0 \end{bmatrix}$$

b)

The only node in G that qualifies as a sink node is node 3.

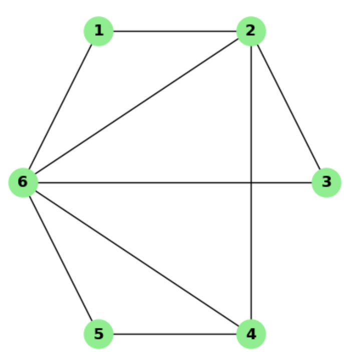
c)

$s_1^{in} = 2, \quad s_2^{in} = 3, \quad s_3^{in} = 5, \quad s_4^{in} = 1, \quad s_5^{in} = 2, \quad s_6^{in} = 1$

$s_1^{out} = 1, \quad s_2^{out} = 2, \quad s_3^{out} = 0, \quad s_4^{out} = 2, \quad s_5^{out} = 1, \quad s_6^{out} = 8$

d)

Undirected, Unweighted Network H:



e)

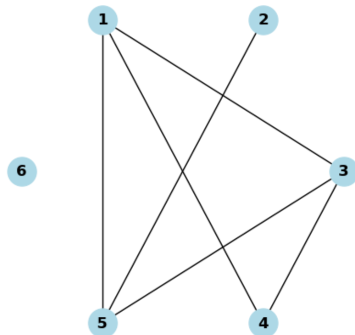
Average degree of H: $c = \frac{2m}{n} = \frac{2(9)}{6} = \frac{18}{6} = \mathbf{3}$

f)

Density of H: $p = \frac{2m}{n(n-1)} = \frac{2(9)}{6(5)} = \frac{18}{30} = \mathbf{0.6}$

g)

Complement of H, H^c:



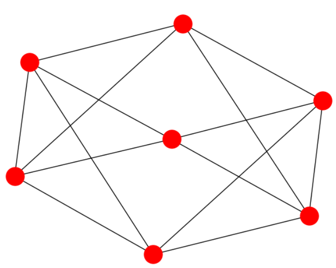
3.

a)

True. If a graph has 2 dominantating vertices, then every other vertex in the graph must have at the very least 2 adjacent vertices, namely, the two dominantating vertices. This means that it is impossible for a pendant vertex to exist in such a graph since a pendant vertex can can only have a single adjacent vertex.

b)

False. We can demonstrate that it is possible to draw a 4-regular graph on 7 vertices with the following counter example:



c)

Only statement 3 is **True**. This is beacuse every directed arch in the network simultaneously increases the total in-degree and total out-degree of the network by the same amount. Thus, counts for both attributes must always remain equal.