Newton's Method for Systems of Nonlinear Equations

Consider the following system of equations:

$$\begin{aligned} x_2 - x_1 &= 0 \\ 2x_1 - x_2 - x_1 x_3 &= 0 \\ x_1 x_2 - 3x_3 &= 0 \end{aligned} \tag{1}$$

This is the Lorenz system (for a particular set of parameter values), which models the onset of convective motion in a fluid heated from below. There exists a solution $\mathbf{x} = (0, 0, 0)^t$, which corresponds to the fluid at rest, and there are two other (nontrivial) solutions that correspond to convective rolls. Your task is to find the solution corresponding to convective rolls that is nearest to $\mathbf{x} = (2, 2, 2)^t$.

1. In a file called `LorenzEqnsT8.py`, write a Python function called `LorenzEqns` that defines the system of equations (1), and in a file called `LorenzJacobianT8.py`, write another function called `LorenzJacobian` that defines its Jacobian. Push your code to the GitHub classroom repository.

2. Write a Python script called `LorenzSolve.py`, that performs Newton iteration for systems, to find the solution of the system of equations (1) that is nearest to $\mathbf{x} = (2, 2, 2)^t$. You'll find some useful code in the Course Code repository. In your script, include code that plots the approximate error versus the number of iterations on a semilogarithmic scale (log on the y axis). Push your code to the GitHub classroom repository.

3. Judging by the sequence of residuals, would you say that Newton iteration for systems has the same rate of convergence as Newton-Raphson iteration for a single equation? Explain.

   One member of your group should post your group's explanation with figure on the Slack #tutorial-8 channel