Iteration

There will be no worked-out solutions posted on Canvas. Working solutions should emerge from the discussion with your group and with the aid of the instructor and the TA. Groups will be assigned/recorded in the tutorial; these will act as your group for the term. **Once solutions are completed, one member of each group, must post the group's solutions on the appropriate Slack thread**. Submission of your group's solutions, as well as your individual engagement in discussions both in the tutorials and on Slack, counts toward your 10% participation mark. Working code from the tutorials will be essential to have available for quizzes and the final exam.

In the second lecture we saw the iterative method

$$x^{(k+1)} = \phi(x^{(k)}) = \frac{1}{2}\left(x^{(k)} + \frac{a}{x^{(k)}}\right)$$

We also saw that, starting from $x^{(0)} = 3$ with $a = 5$, after five iterations $x^{(k+1)} = x^{(k)}$ up to at least 15 digits.

Exercise A
In this exercise you will write a Python code that computes iterates of the function ϕ.

(**a**) Write a Python function for ϕ. Inputs should be x and a.

(**b**) Write a Python function that iterates ϕ. Inputs should be an initial point $x^{(0)}$, the parameter a and the maximal number of iterations $k_{\max}$. Your function should print the successive iterates $x^{(k)}$ to the screen.

(**c**) Now modify your function so that it terminates if the maximal number of iterations is reached **or** if the difference between successive iterates is below some threshold, i.e. if

$$|x^{(k+1)} - x^{(k)}| < \epsilon$$

where ϵ is an additional input.

(**d**) Use your function to see from which initial points the iterates converge to $\sqrt{a}$, depending on a. Also, try to formulate *how fast* the iterates converge. How many iterates do you need to compute to achieve a certain error ϵ? Hint: a graph may be useful for visualizing this.