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Assignment 2a:

y_i: actual value

$\hat{y}_i$: predicted value

	x_1	x_2	y
1)	1	2	0
2)	2	3	0
3)	3	4	1
4)	3	2	1
5)	2	3	1
6)	1	4	0

Given Parametes: $w_1 = 0.5$
$w_2 = 1.0$
$b = 0.0$

Question 1: Calculate loss using Mean Squared Error (MSE):

$$MSE = \frac{1}{n} \sum_{i=1}^{n=6} (y_i - \hat{y}_i)^2$$

$$\hat{y}_i = w_1 x_{1,i} + w_2 x_{2,i} + b$$

w_1 $x_{1,1}$ w_2 $x_{2,1}$ → 1st row

$$\hat{y}_1 = 0.5 \times 1 + 1.0 \times 2 + 0.0 = 0.5 + 2 = \boxed{2.5}$$

$$\hat{y}_2 = w_1 \times x_{12} + w_2 \times x_{22} + b$$

$\hat{y}_2 = 0.5 \times 2 + 1 \times 3 + 0.0 = \boxed{4.0}$

$\hat{y}_3 = 0.5 \times 3 + 1 \times 4 + 0.0 = \boxed{5.5}$

$\hat{y}_4 = 0.5 \times 3 + 1 \times 2 + 0.0 = \boxed{3.5}$

$\hat{y}_5 = 0.5 \times 2 + 1 \times 3 + 0.0 = \boxed{4.0}$

$\hat{y}_6 = 0.5 \times 1 + 1.0 \times 4 + 0.0 = \boxed{4.5}$

Compute MSE (loss):

$$MSE = \frac{1}{n} \sum_{i=1}^{n=6} (\hat{y}_i - y_i)^2$$

$$= \frac{1}{6} \sum_{i=1}^{n=6} (\hat{y}_i - y_i)^2$$

Calculating each term separately:

$(\hat{y}_i - y_i)^2 = (2.5 - 0)^2 = 6.25$

$= (4.0 - 0)^2 = 16.0$

$= (5.5 - 1.0)^2 = 20.25$

$= (3.5 - 1)^2 = 6.25$

3

$\frac{2}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)$ and

$-\frac{2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)$ are same

$(4.0 - 1)^2 = 9.0$

$(4.5 - 0)^2 = 20.25$

$= 6.25 + 16.0 + 20.25 + 6.25 + 9.0 + 20.25$

$= \boxed{78.0}$

$MSE = \frac{78.0}{6} = \boxed{13.0}$

Question 2: Compute the gradients of loss with respect to w_1, w_2 and b.

Gradient with respect to w_1:

$$\frac{\partial MSE}{\partial w_1} = \frac{2}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)x_{1,i}$$

total no of values

$$= \frac{2}{6}\sum_{i=1}^{n}(\hat{y}_i - y_i)x_{1,i}$$

Calculate $\sum_{i=1}^{n}(\hat{y}_i - y_i)x_{1,i}$ for each value separately:

$=(2.5-0)\times 1 = \boxed{2.5}$

$=(4.0-0)\times 2 = \boxed{8.0}$

$=(5.5-1)\times 3 = \boxed{13.5}$

$=(3.5-1)\times 3 = \boxed{7.5}$

$=(4.0-1)\times 2 = \boxed{6.0}$

$=(4.5-0)\times 1 = \boxed{4.5}$

$= 2.5+8.0+13.5+7.5+6.0+4.5$

$= \boxed{42.0}$

$\therefore \frac{2}{6}\sum_{i=1}^{n}(\hat{y}_i - y_i)x_{1i} = \frac{2}{6}\times 42.0$

$\frac{\partial MSE}{\partial w_1} = \boxed{14.0}$

Gradient with respect to w_2:

$\frac{\partial MSE}{\partial w_2} = \frac{2}{N}\sum_{i=1}^{n}(\hat{y}_i - y_i)\times x_{2i}$

Calculate $\sum_{i=1}^{n}(\hat{y}_i - y_i)x_{2i}$:

$=(2.5-0)2 = \boxed{5}$

$=(4.0-0)3 = \boxed{12.0}$

$= (5.5 - 1) \times 4 = \boxed{18.0}$

$= (3.5 - 1) \times 2 = \boxed{5.0}$

$= (4.0 - 1) \times 3 = \boxed{9.0}$

$= (4.5 - 0) \times 4 = \boxed{18.0}$

$\therefore \sum_{i=1}^{n} (\hat{y}_i - y_i) x_2 i = 5 + 12.0 + 18.0 + 5.0 + 9.0 + 18.0$

$= \boxed{67.0}$

$\therefore \frac{\partial MSE}{\partial w_2} = \frac{2}{6} \times 67.0 = \boxed{22.33}$

Gradient with respect to b: x_i isn't multiplied here

$$\frac{\partial MSE}{\partial b} = \frac{2}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)$$

Calculate $\sum_{i=1}^{n} (\hat{y}_i - y_i)$:

$\hat{y}_i - y_i = (2.5 - 0) = 2.5$

$= (4.0 - 0) = 4.0$

$= (5.5 - 1) = 4.5$

$= (3.5 - 1) = 2.5$

$= (4.0 - 1) = 3.0$

$= (4.5 - 0) = 4.5$

$= 2.5 + 4.0 + 4.5 + 2.5 + 3.0 + 4.5$

$= \boxed{21.0}$

$$\frac{\partial MSE}{\partial b} = \frac{2}{6} \times 21.0 = \boxed{7.0}$$

Part (b): Update Parameters (w_1, w_2, b) using Gradient Descent:

$\alpha = 0.01$

$$w_1 = w_1 - \alpha \frac{\partial MSE}{\partial w_1}$$

$$\Rightarrow w_1 = 0.5 - 0.01 \times 14.0 = \boxed{0.36}$$

$$\Rightarrow w_2 = w_2 \times \alpha \frac{\partial MSE}{\partial w_2}$$

$$\Rightarrow w_2 = 1.0 - 0.01 \times 22.23 = \boxed{0.7767}$$

$$\Rightarrow b_3 = b \times \alpha \frac{\partial MSE}{\partial w_3}$$

$$\Rightarrow b = 0.0 - 0.01 \times 7.0 = \boxed{-0.07}$$

New Parameter Values:

$w_1 = 0.36$, $w_2 = 0.7767$, $b = -0.07$