Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

ANS : -

Given mean = 45 , standared deviation = 8

As per given condition the work will start after 10 min so mean will be 45+10 = 55

We have asked customer the car will be ready after 1 hour(x) = 60

Hence,

Z = (60 – 55) / 8 = 0.625

From z table

Z value will be 0.73237

the probability that the service manager cannot meet his commitment = 1 – 0.73237

= 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS :-

A. First find out the probability for employees older than 44 :

X = 44 , mean = 38, std = 6

1 - Stats.norm.cdf (44 , 38 , 6) = 1- 0.8413

= 0.1587

Now find the probability of employees between 38 and 44 :-

Stats.norm.cdf (44 , 38 , 6) - stats.norm.cdf (38,38, 6)

= 0.8413 - 0.5

= 0.3413

As we can clearly see that the probability of employees age between 38-44 is more than employees age more than 44.
So the given statement is False.

B. Lets calculate the probability of employees age under 30 :

X = 30 , mean = 38, std = 6

Stats.norm.cdf (30 , 38 , 6) = 0.0912

So the total number of emplyees age under 30 is 0.0912*400 = 36.48

So we can say that the statement is True.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS :-

Here, x1 and x2 are random variables which have same distribution and independent of each other

We have to find the sum of the mean and the variance

Sum of mean = 2μ
Sum of the variance = $2\sigma^2$

There is no any difference between 2x1 and x1+x2 as both of them have same distribution

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

ANS :-

Mean = 100 and std = 20

probability of the random variable taking a value between them is 0.99

hence,

Z value at 99 % =

Stats.norm.ppf (0.995) = 2.5758

1st value will be 2.5758*20 + 100 = 151.51
2nd value will be (-2.5758)*20 + 100 = 48.484

So option D is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5th percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS :-

Total profit = profit 1 + profit 2

Mean = profit 1 (mean) + profit 2 (mean)

= 5 + 7 = 12

Std = sqrt (9+16)

= sqrt (25)

= 5

Mean in rs = 12*45 = 540

Std in rs = 5* 45 = 225

A) Range for 95 % :-

Stats.norm.interval (0.95 , 540 , 225)

Range is rs (99.008 , 980.991) in millions

B) the 5th percentile :-

From z score we need to find the value of 0.5000-0.050 = 0.4500

We are getting the value of -1.645

the 5th percentile of profit = mean + (-1.645)*std

= 540 – (1.645 * 225)

= 540 – 370.125

= 169.87 = 170 in million

C)

Probability of 1st division making loss = stats.norm.cdf (0, 5 , 3)

= 0.0479

Probability of 2nd division making loss = stats.norm.cdf (0, 7, 4)

= 0.04005

We can see that 1st division can make more loss compared to 1st division.