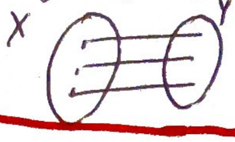
SETS → RELATIONS → FUNCTIONS

$A = (1, 2, 3)$
$B = (4, 5, 6)$

$(A \times B)$

$f : X \to Y$



Differential Calculus

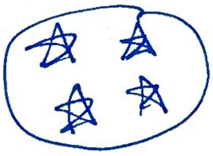
* Continuity and Differentiability
* Differentiation $\left(\frac{d}{dx}\right)$

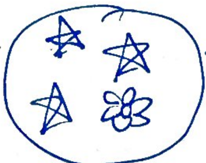
Integral Calculus

* indefinite integrals
* definite integrals
* area bounded $\left(\int_0^x f(x)\, dx\right)$

SET

* In mathematical language, ~~everthy~~ everything in this universe, is an object

*  → collection of stars ⇒ well-defined object

 → ! not ~~tw~~ well-defined.

* well-defined ~~obje~~ collection of object = SET.

A well-defined collection of object = SET.

↳ If a is an element of A : $a \in A$, if not an element $a \notin A$.

Eg) set A = { vowels in English alphabet }
A = { 2, 4, 6, 8 } : even numbers

Set-builder notation : { x : x, has properties P }

Eg) $A = \{1, 2, 3, 4, 5, 6, 7\} \to A = \{x : x \in N \text{ and } x < 8\}$

* $F = \{x : x = \frac{n}{n+1}$ where $n \in N$ and $1 \le n \le 8\}$

$F = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{8}{9}\}$

* Empty Set : { }.

* **SUBSET** : A set A is said to be a subset of set B, if every element of A is also an element of B, and we write $A \subseteq B$.

Eg) Let $A = \{2, 3, 5\}$ and $B = \{2, 3, 5, 7, 9\}$

So Then, every element of A is ~~so~~ an element of B $\therefore A \subseteq B$ $[A \neq B]$

Eg) $N = \{1, 2, 3 \ldots\}$ set of all natural numbers

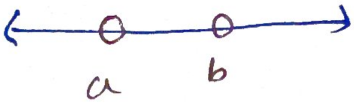
$Z = \{\ldots -4, -3, -2, \ldots 0, 1, 2\}$: set of all integers.

\# **INTERVAL** : Let $a, b \in R$ and $a < b$. Then, we define

(i) Closed Interval $[a, b] = \{x \in R : a \leq x \leq b\}$

$[a, b]$

(ii) Open Interval (a, b) or $]a, b[= \{x \in \mathbb{R} : a < x < b\}$



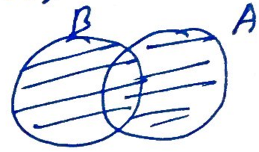
(a, b)

OPERATION ON SETS.

* <u>UNION</u> : $A \cup B$: set of all those elements which are either in A or in B or in both A & B

Eg) If $A = \{3, 4, 5, 6\}$ and $B = \{4, 6, 8, 10\}$

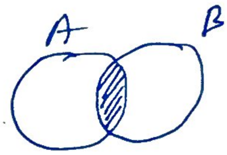
$A \cup B = \{3, 4, 5, 6, 8, 10\}$



* <u>INTERSECTION</u> : $A \cap B$: set of all elements which are common to A and B

Eg} $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 5, 7, 11, 13\}$

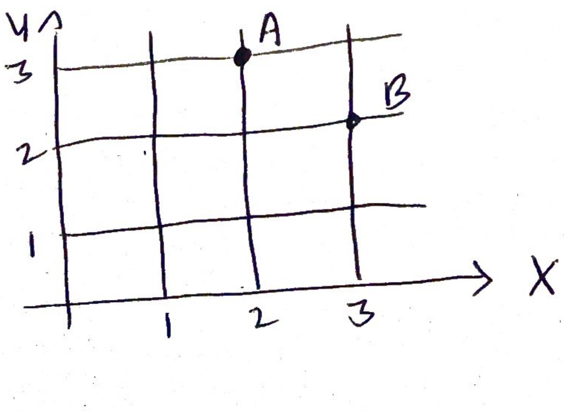
$A \cap B = \{3, 5, 7\}$.



RELATIONS

* <u>Ordered pair</u> : Two numbers a and b listed in a specific order and enclosed in parantheses form an ordered pair (a, b) components,

* by interchanging the position of the ordered pair is changed



$A(2, 3)$, $B(3, 2)$

$(a, b) \neq (b, a)$

Cartesian Product of Two Sets (A X B)

Let A and B be two non-empty ~~set~~ sets. Then $(A \times B)$: is the set consisting of all ordered pairs (a, b) such that $a \in A$ & $b \in B$.

$\therefore \quad A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

Eg) $A = \{1, 3, 5\}$ and $B = \{2, 3\}$

(i) $A \times B = \{1, 3, 5\} \times \{2, 3\} = \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$.

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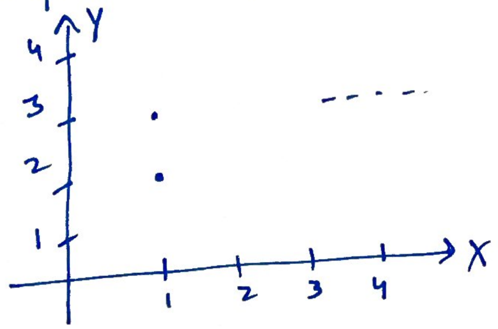
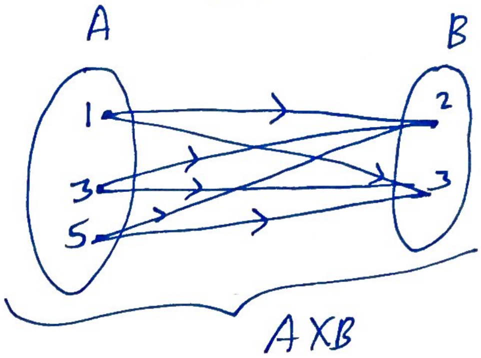
Graphical Representation



Arrow Representat'



RELATION: Let A and B be two nonempty sets. Then a relation R from A to B is a subset of A X B.

$\boxed{R \text{ is a relation from A to B} \Rightarrow R \subseteq A \times B}$

*) Let R be a relation from A to B.

(i) set of all first coordinates of R is called $\boxed{\text{domain of R}}$ dom(R).

(ii) set of all second coordinate " " " $\boxed{\text{range (R)}}$

(iii) set B is called $\boxed{\text{co-domain}}$ of R

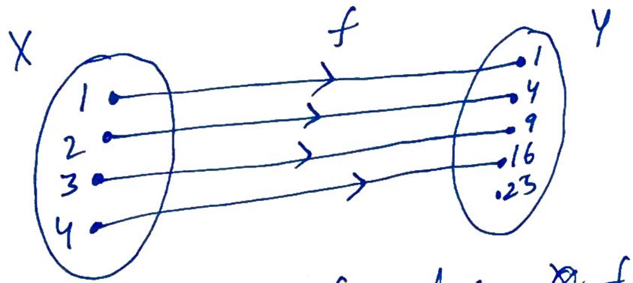
FUNCTIONS Let X and Y be two non-empty sets. Then a relation X to Y is called a function, if every element in X has a unique image in Y, we write $\boxed{f : X \to Y}$

! no two distinct ordered pairs in f has ~~son~~ same first coordinate.

$\boxed{*}$ If $(x, y) \in f$, we write $f(x) = y$ and there y is called image of x under f, x is called pre-image of y

Eg) Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 4, 9, 16, 25\}$

Let $f = \{(x, y) : x \in X, y \in Y$ and $y = x^2\}$



Real functions : A function $f : X \to Y$ is called a real function if $x \subseteq R$, $y \subseteq R$

Eg) If $f(x) = 3x^3 - 5x^2 + 10$ find $f(x-1)$

Replace x by $x-1 \Rightarrow f(x-1) = 3(x-1)^3 - 5(x-1)^2 + 10$

$$f(x-1) = 3(x-1)(x-1)^2 - 5(x-1)^2 + 10$$
$$= 3(x-1)(x^2-2x+1) - 5(x^2-2x+1) + 10$$
$$= 3(x-1)[x^2-2x+1]$$

Ask to D.
*

$$= 3[x^3 - 2x^2 + x - x^2 + 2x - 1] + 10 - 5[x^2-2x+1]$$
$$-5[x^2-2x+1] + 10$$
$$3[x^3 - 3x^2 + 3x - 1] - 5[x^2-2x+1] + 10$$
$$= 3x^3 - 9x^2 + 9x - 3 - 5x^2 + 10x - 5 + 10$$
$$= 3x^3 - 9x^2$$
$$\boxed{f(x-1) = 3x^3 - 14x^2 + 19x + 2}$$

Eg 2) If $f(x) = x + \frac{1}{x}$, show that $\left(f(x)\right)^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$ 5

L.H.S = R.H.S

L.H.S

$$\left(x+\frac{1}{x}\right)^3 = \underbrace{x^3 + \frac{1}{x^3}} + 3\left[x + \frac{1}{x}\right]$$

R.H.S

$$= x^3 + \frac{1}{x^3} + 3\left[x + \frac{1}{x}\right]$$

=

Problems based on Domains and Ranges of Real functions.

Eg 1) Find the domain and range of $f(x) = \frac{1}{x+3}$

Eg) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 4, 5\}$

Let R be a relation "is less than"

from A to B

(i) List the elements of R

$R = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

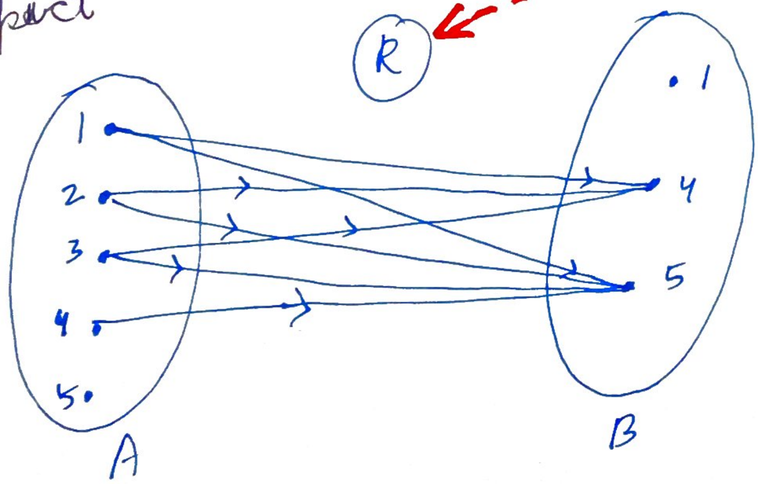
(ii) Find domain, co-domain and range of R

Dom (R) = $\{1, 2, 3, 4\}$, Range R = $\{4, 5\}$

and co-domain (R) = $\{1, 4, 5\}$.

(iii) Depict above via arrow diagram

R



Relation as a Particular Case of a Function

Eg) Let $X = \{2, 3, 4, 5\}$ and $Y = \{7, 9, 11, 13, 15, 17\}$

Define a relation f from X to Y by

$$f = \{(x, y) : x \in X, y \in Y \text{ and } y = 2x+3\}$$

→ $x = 2 \Rightarrow y = (2 \times 2 + 3) = 7$; → $x = 4 \Rightarrow y = (2 \times 4 + 3) = 11$

→ $x = 3 \Rightarrow y = (2 \times 3 + 3) = 9$ → $x = 5 \Rightarrow y = (2 \times 5 + 3) = 13$

(i) $f = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

(ii) $\text{dom}(f) = (2, 3, 4, 5)$; range $f = \{7, 9, 11, 13\}$

(iii) It is clear that no two distinct ordered pairs in f have the same first coordinate

$\therefore$ f is a function from X to Y

INVERSE of a FUNCTION

: Let $f : X \to Y$ and let $y \in Y$. Then we define ~~$f(y)$~~ $f^{-1}(y) = \{x \in X : f(x) = y\}$ = set of pre-images of y. ~~∅~~ f^{-1} is called inverse of f.

Eg) Let $f : R \to R : f(x) = x^2 + 1$. Find (i) $f^{-1}\{-4\}$

(ii) $f^{-1}\{5, 17\}$

Ans) It is given that $f(x) = x^2 + 1$

(i) Let $f^{-1}(-4) = x$ Then $f(x) = -4$

$x^2 + 1 = -4 \to x^2 = -5$

But there is no real value of x whose square is -5

$\therefore f^{-1}[-4] = \phi$ (empty)

(ii) Let $f^{-1}(5) = x$. Then, $f(x) = 5 \Rightarrow x^2 + 1 = 5 \Rightarrow x^2 = 4$

$x = \pm 2$

$\therefore f^{-1}\{5\} = \{-2, 2\}$.

Let $f^{-1}(17) = x$. Then, $f(x) = 17 \Rightarrow x^2 + 1 = 17 \Rightarrow x^2 = 16$

$x \Rightarrow \pm 4$

$\therefore f^{-1}\{17\} = \{-4, 4\}$