Relationship b/w spacetime-diagrams of two inertial observers:

→ (i) $ct - x$ as coordinate axes of S, light ray cone

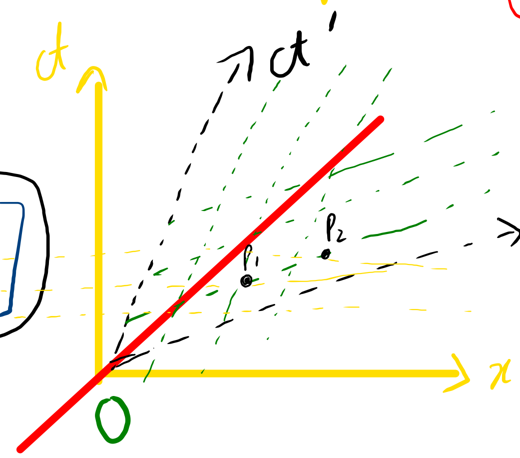
→ (ii) For ct'-x' axes of S'

→ $x' = \gamma(x - vt)$, $ct' = \gamma\left[ct - \frac{vx}{c}\right]$

Now for x'-axis → $ct' = 0$

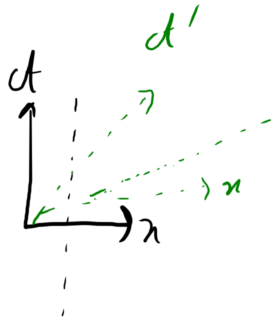
∴ $0 = \gamma\left[ct - \frac{v}{c}x\right]$

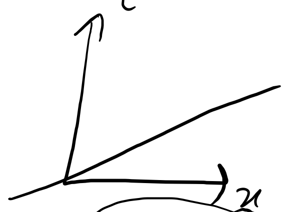
$$\boxed{ct = \frac{v}{c}x}$$



P_1 & P_2 are simultaneous in S'-frame

UWAGA ! they are not simult- in S.





$x = vt$

Now for t'-axis → $x' = 0$ ∴

$0 = \gamma(x - vt)$

$c \times x = vt \times c$

$cx = vt \cdot c$

$$\boxed{ct = \frac{c}{v}x}$$

I) line parallel to (Oct') are the worldlines of fixed points in S'.

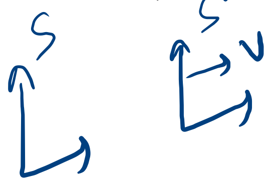
II) The lines parallel to Ox' are the lines corresponding to points at a fixed time acc. to S'

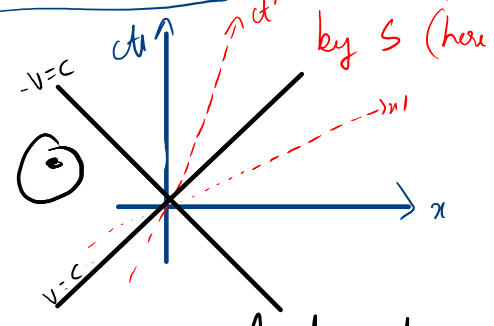
lines of simultaneity for S'

→ Spacetime diag for S

→ the axes of S' as measured by S (here S' vel $V = 0.6c$)

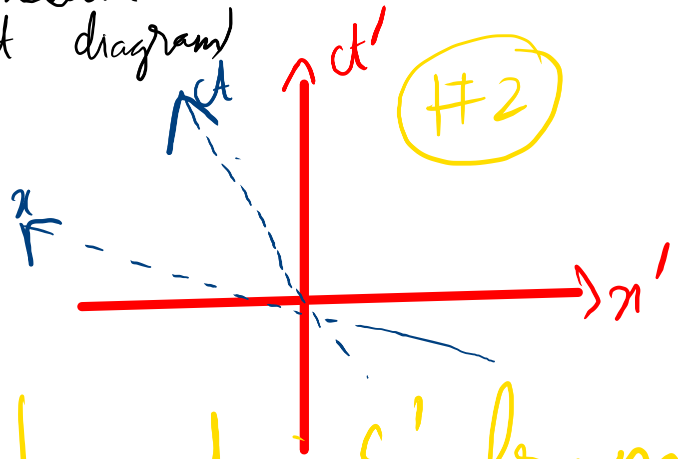
$x' = \gamma(x - vt)$ #1





One can ask → what does the spacetime diag. for S' looks like (what do axes of observer S looks like on that diagram)

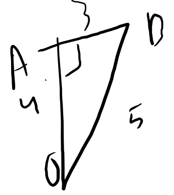
Use inverse L.T $x = \gamma(x' + vt')$



∵ since

∴ Consider time passed in S' frame, we know ∵ $\boxed{S'}$ is moving, her time should move less rapidly (t . . i . . c . . k) than time measured in S

Via #1 ∵ graph of ct'-x' axis is slanted, what appears to be the "distance" along that slanted line would be longer



$OA < OB \rightarrow$ via

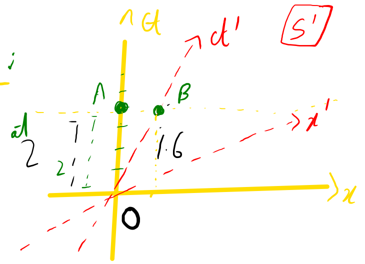
$\boxed{OA < OB}$

① S' is moving at $\boxed{v = 0.6c}$

(i) In S-frame;

an event happens at 2 hours (A)

$t = 2\text{hours}$



line of simultaneity for S, : $(x, ct) = (0, 2)$

① This cuts the $\boxed{S'}$ at $x = \frac{v}{c} \cdot t$ (B) $= \frac{0.6\cancel{c}}{\cancel{c}} \cdot \cancel{1}^{2} \Rightarrow 1.2$

∴ on diagram, this means for $B \rightarrow (1.2)$

(*) At this event, on the world line for S', our observer S measures $(x, t)(1.2, 2)$

(x) Now we would like to know, what is $\boxed{S'}$ measuring

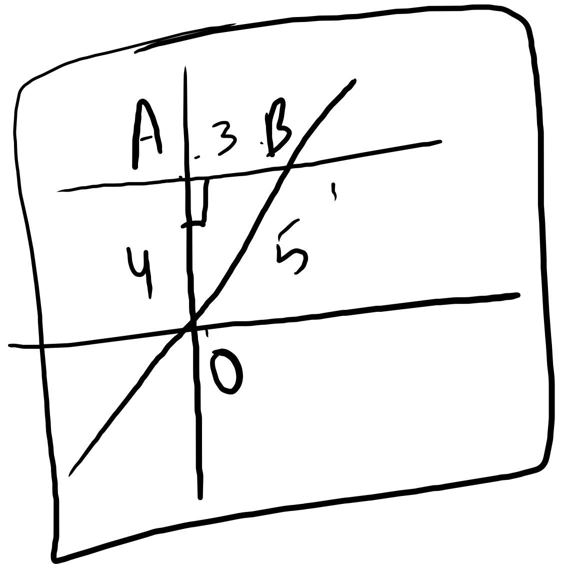
$v = 0.6$

$\gamma = 1.25$

$$x' = \gamma[x - vt]$$
$$= 1.25[1.2 - 0.6.2] = 0$$
$$t' = \gamma\left[t - \frac{vx}{c^2}\right] = 1.6$$

$$\boxed{t' = 1.6\text{hours}}$$

✓ ∴ time-dialation



$OB^2 = AB^2 + -$

$\boxed{OA > OB}$

we calculated $OA = 2\text{hours}$

$OB = 1.2\text{hours}$

Distance of spacetime diagram with (ct-x) $\neq$ distance on x-y axis on a normal sheet of paper.

→ Appropriate notion of "distance" in spacetime has for $(x)^2-(ct)^2$ / $(ct)^2-(x)^2$ ⇒ Invariant interval

$c^2dt^2 - dx^2 \ldots$

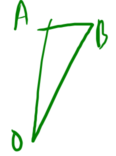
↳ $dx^2 + dy^2 + dz^2 - c^2dt^2$

$$I = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2$$

$$= (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (c\Delta t')^2$$

$\cancel{x^2 + y^2 = d^2}$ → $x^2 - c^2t^2$

Use this interval to create a new method to visualize distances!

Pythagor 

OB>OA

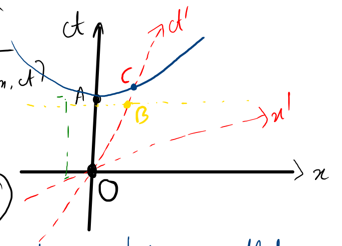
Sense :) in Euclidean space

going to S.R.T → Minkowskian

$\Delta s^2 : cdt^2 - dx^2 \ldots$

$\Delta s^2 = x^2 - ct^2 = x'^2 - ct'^2$

Spacetime diag for S



Point A = (0,2) → (x, ct)

∴ the value of Δs^2

$\Delta s^2 = 0^2 - 2^2 = -4$

OA = OC

OB < OC

∴ OB < OA

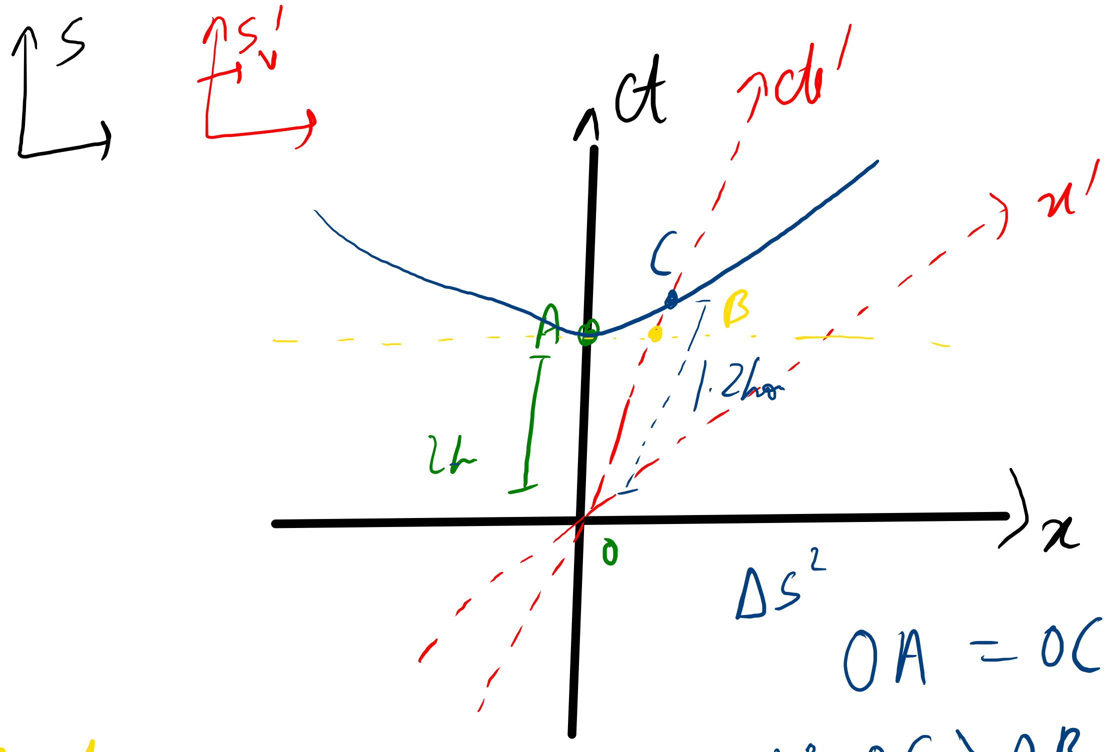
→ We can draw a hyperbola that goes through the point $x^2 - (ct)^2 = -4 = x'^2 - ct'^2$

Δs^2

→ All the observers will agree on this value (-) with this we can calibrate distances along worldlines of S' → Lable C

→ It lies further than B. ∵ C lies on S', then it has $x' = 0$, ∴ it lies of the hyperbola for $\Delta s = -4$, the it must be $ct' = 2$

→ Hence event C is that particular event when the other observer in S' has aged 2-hour.



Δs^2

OA = OC

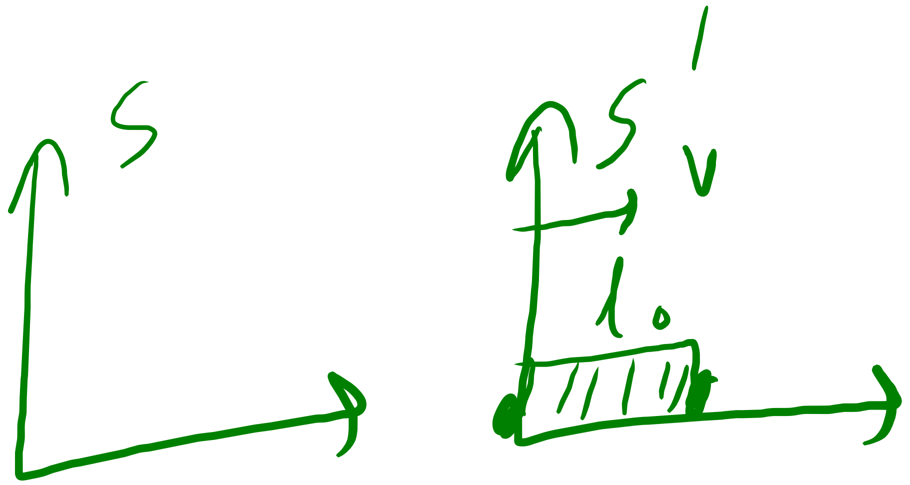
∴ OC > OB

∴ $\boxed{OA > OB}$

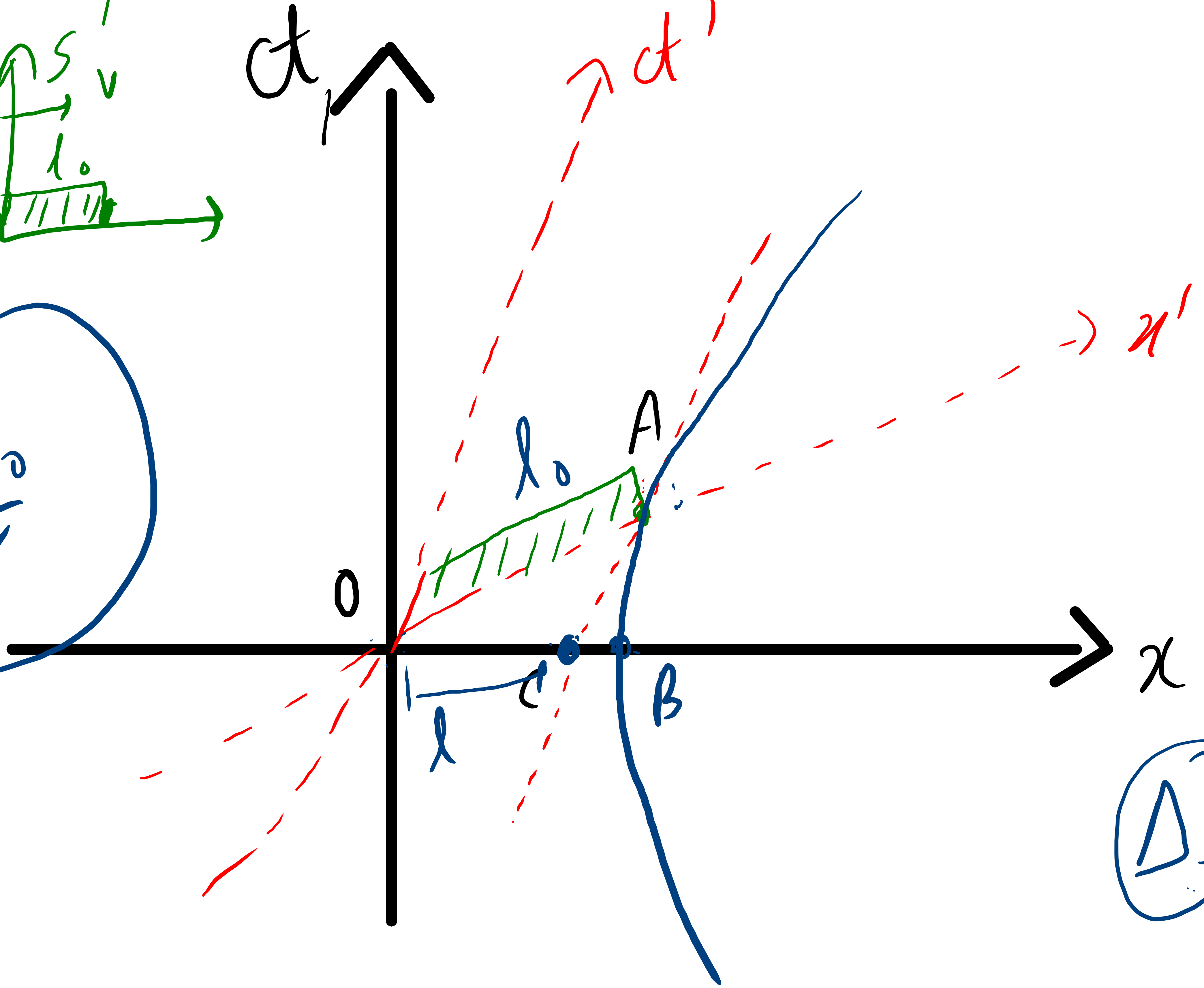
$\boxed{!}$ → (i) At every point where hyperbola intersects with observer's world line ⇒ measure same time

(ii) ∴ event A → $\boxed{S}$ time = 2 hours

event C → $\boxed{S'}$ time = 2 hours



$$l = \frac{l_0}{\gamma}$$



$$\Delta S \rightarrow \underline{OA = OB}$$

$$OC < OB$$

$$\therefore \; OC < OA$$