3 Tutorial Day 3

3.1 Preliminaries/Czterowymiarowy obraz świata

If you compare Lorentz transformation and Galilean transformation(see table), one notices that though both gives a connection between inertial observers , the concept of time is different. As in Galilean transformation, absolute time is invariant , in Lorentz transformation time and space gets mixed up.

Galilean Transformation	Lorentz transformation
$x' = x - vt$	$x' = \gamma(x - vt)$
$t' = t$	$t' = \gamma(t - \frac{vx}{c^2})$
$y' = y$	$y' = y$
$z' = z$	$z' = z$

In words of Minkowski:

> *... sama przestrzeń i sam czas skazane są na zniknięcie w cieniu...tylko rodzaj połączenia tych dwóch zachowa niezależną rzeczywistość*

In the new special relativity picture, time and space merge together into a four-dimensional continuum called **space-time**. In this picture square of **interval** between two events $(t_1, x_1, y_1, z1)$ and $(t_2, x_2, y_2, z2)$ is defined by :

$$s^2 = (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2 \tag{4}$$

and this quantity remains unchanged/invariant under a Lorentz Transformation. If you consider two events separated infinitesimally (t, x, y, z) and $(t + dt, x + dx, y + dy, z + dz)$ then we get :

$$ds^2 = (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \tag{5}$$

And a four-dimensional space-time continuum is which this form is invariant is called **Minkowski spacetime** and this is the background geometry for special relativity.

So far we have seen special Lorentz transformation connecting two inertial frames where one was moving in a specific direction. Generally there is a **full Lorentz transformation** which connects two frames in general position. One can show that a full Lorentz transformation can be decomposed into ordinary spatial rotation, followed by a boost, followed by another ordinary rotation. Physically, the first rotation lines up the $x - axis$ of S with the velocity v of S'. Then a boost in this direction with speed v transforms S to a frame which is at rest relative to S'. A final rotation lines up the coordinate frame with that of S'. The spatial rotations introduce no new physics. The only new physics is due to the boost and so we can , without loss of generality, restrict our attention to a special Lorentz transformation.

3.2 Problems

Basic Plotting in Mathematica:

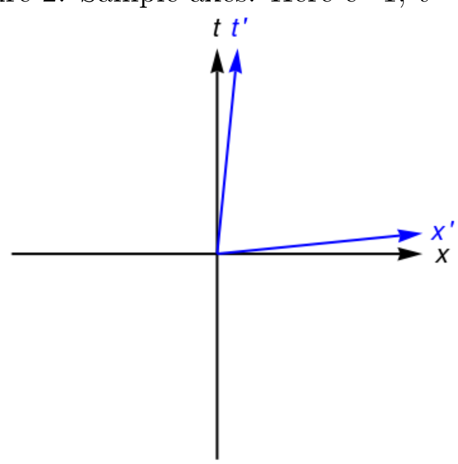
- Plot the *Lorentz* factor and relativistic Kinetic energy expression.
- In Tutorial 2, Problem 2.2. First try to plot you motion relative to Earth for $t = 0$ to $t = 100$. Then try to plot your trajectory in terms of proper time $\tau = 0$ to $\tau = 100$
- Try to plot the above two trajectories for various values of acceleration this time.

SpaceTime Axes : We try to generate a plot which depicts the space-diagram and use the *Manipulate* command in $Mathematica$ to see that if one increases the v parameter or in other words play around $\gamma - factor$ then one should be able to visualize the change happening in the coordinates in S' frame.

The idea is to first plot simple axes in Mathematica. Like how you have a $x - y$ axis in Euclidean space here you need $ct - x$ axis. You can consider that for S(frame at rest) the axes are $ct - x$ and for S'(moving frame) axes are $ct' - x$'.

1. First plot the $S - frame$ axes which would be simple perpendicular axes. Now we know to get $S' - frame$ axes we can use the inverse Lorentz transformation. Play around with this command in Mathematica :
   Graphics[{Thick, Black, Arrowheads[Large], Arrow[0, -1, 0, 1]}]
   UWAGA : You need to adjust the 'Arrowheads'. Essentially that is a coordinate point. So use the inverse Lorentz transformation for that point. And how to check if you are correct : at $v = 0$ both the axes, i.e , S and S' should coincide and at $v \to 1$ meaning ($v \to c$) you should get a 45 degree line.(See Fig 2)

Figure 2: Sample axes. Here c=1, $v = 0.1$

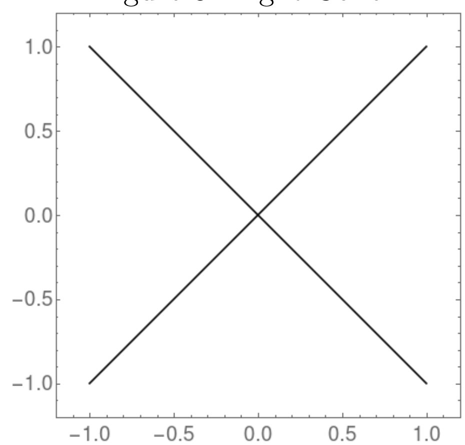


2. Plotting a light cone. Here you just need to play around with the plot command of Mathematica and adjust the *PlotRange* option.

3. Grid Lines. Since spacetime becomes a dynamic quantity in SRT, therefore if I plot a grid and when I observe a moving frame I should see the whole grid changing. So try to get Figure 4 by exploring the *ParametricPlot* function of *Mathematica* for eg:
   ParametricPlot[Table[x, s, x, -1, 1, 0.2], {s, -1, 1}]

   UWAGA : The values of the 'Table' for S' will be doing using the inverse Lorentz Transformation because you you want to see what is happening the the moving frame.

Figure 3: Light Cone



4. So by now , you have your spacetime axes for S and S' frame, your light cone and your grid lines for $ct - x$ and $ct' - x'$ ready. All you need is to use *Manipulate* command in Mathematica and get a diagram such that when you keep on increasing v you can see the axes in the moving frame go to the light-cone(and also you grid is changing-remember the whole spacetime is changing). Try to play around with *Manipulate* command in Fig 5. Essentially what you need is a function of v which you can manipulate and hence the space-time axes and the grid lines will change. **Also plot this diagram from the perspective of S', you only need to change $-v$ and some relabelling.**
   Manipulate[myfunc[v], ..., {v, 0, .99, 0.01}]
   UWAGA : It would be easier to use the *Module** command of *Mathematica* wherein you can define all you plots and then use that as the function which you can *Manipulate* for eg:
   Trial1[v_] := Module[{xL,tL,γ*, plot1, plot2...},* $\gamma = ...; xL = ...; plot1 = ..., plot2 = ...]$

5. The most difficult problem, try to finish this game: `https://testtubegames.com/velocityraptor.html`. It is a game involving a dinosaur trying to cross a level which has length contraction and time dilation effects :P ,

Figure 4: Gird Lines : On Left is the rest frame S and on right you have the moving frame S'

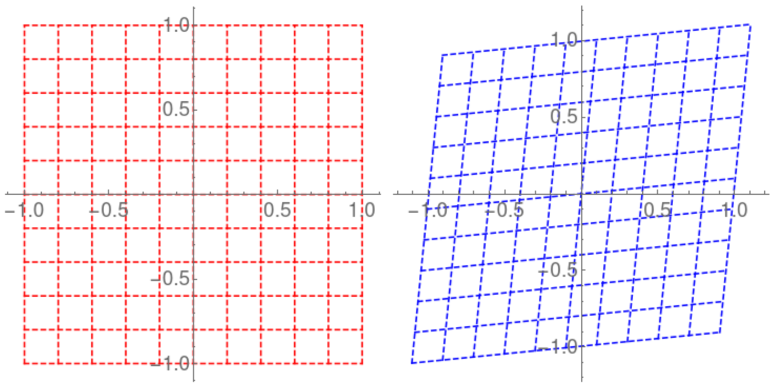
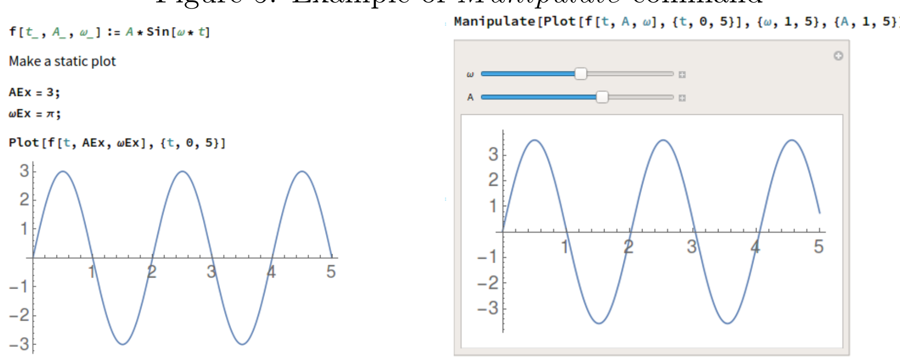


Figure 5: Example of $Manipulate$ command



*A module makes the variables in the curly brackets local to the module and does not change variables outside the module that happen to have the same name.