$$\tilde{R}^{\rho}{}_{\sigma\mu\nu} = \frac{1}{2}\eta^{\rho\alpha}\left(\partial_\mu \partial_\sigma \tilde{h}_{\alpha\nu} - \partial_\mu \partial_\alpha \tilde{h}_{\nu\sigma} - \partial_\nu \partial_\sigma \tilde{h}_{\alpha\mu} + \partial_\nu \partial_\alpha \tilde{h}_{\mu\sigma}\right.$$

$$\frac{\partial}{\partial \tilde{x}^\mu} \qquad \frac{\partial}{\partial x^\alpha}$$

$$\tilde{R}^{\rho}{}_{\sigma\mu\nu} = \frac{1}{2}\eta^{\rho\alpha}\begin{bmatrix} \partial_\mu \partial_\sigma \tilde{h}_{\alpha\nu} \\ - \quad \vdots \\ - \quad \vdots \\ + \quad \vdots \end{bmatrix} = \frac{1}{2}\eta^{\rho\sigma\alpha}\begin{bmatrix} \partial_\mu \partial_\sigma (h_{\alpha\nu} - \xi_{\alpha,\nu} - \xi_{\nu,\alpha}) \\ - \quad \vdots \\ \vdots \end{bmatrix}$$

Extra?

$$= \frac{1}{2}\eta^{\rho\alpha}\begin{bmatrix} \partial_\mu \partial_\sigma h_{\alpha\nu} - \xi_{\alpha,\nu\mu\sigma} - \xi_{\nu,\alpha\mu\sigma} \\ \vdots \\ \vdots \end{bmatrix}$$

$$\left\{ \text{Careful} \quad \frac{\partial \xi^\alpha}{\partial \tilde{x}^\mu} = \frac{\partial \xi^\alpha}{\partial x^\nu}\frac{\partial x^\nu}{\partial \tilde{x}^\mu} \xrightarrow{\frac{\partial x^\nu}{\partial \tilde{x}^\mu} = \delta^\nu_\mu - \frac{\partial \xi^\nu}{\partial x^\mu}} = \frac{\partial \xi^\alpha}{\partial x^\mu} \right\}$$

$$= \frac{1}{2}\eta^{\rho\alpha}\begin{bmatrix} \partial_\mu \partial_\sigma h_{\alpha\nu} - \cancel{\xi_{\alpha,\nu\mu\sigma}} - \cancel{\xi_{\nu,\alpha\mu\sigma}} \\ -\partial\partial h + \cancel{\xi} + \cancel{\xi} \\ -\partial\partial h + \cancel{} \quad \cancel{} \\ +\partial\partial h \quad \cancel{} \quad \cancel{} \end{bmatrix}$$

$$\Downarrow$$

$$= R^{\rho}{}_{\sigma\mu\nu}$$

Notation check — Metric Perturbat^n $\Rightarrow$ $h_{\alpha\beta}$

Trace-reversed Perturbation — $\bar{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} h$

$$\Downarrow$$

$\{ \bar{h}^a{}_a = -h \rightarrow$ trace-reversed $\}$

~~One can always find soln to $\Box \xi_\alpha = \partial^\beta \bar{h}_{\alpha\beta}$ ∴ achieving L·G ⟹ amount of gauge freedom has been reduced from~~

~~4 freely specifiable fun of 4 variatn $\longrightarrow \Box \xi^\alpha = 0 \rightarrow$ ✗ ?~~

In L·G → Einstein Tensor ⟹ reduces to wave-operator actn of trace-reversed metric perturbatn

$$\Box \bar{h}_{\alpha\beta} = -16\pi T_{\alpha\beta} \xrightarrow{\text{vacuum}} \boxed{\Box \bar{h}_{\alpha\beta} = 0}$$

⇓

admits wave like soln

I.M.P → Globally vacuum spacetime ⟹ Transverse Traceless gauge

↓

$T^{\alpha\beta} = 0$ everywhere

↓ $h\,? = 0$

Here along with L·G ⊕ $h\,? = 0$

⇓

(i) $h^{TT}_{\alpha\beta} = \bar{h}^{TT}_{\alpha\beta}$

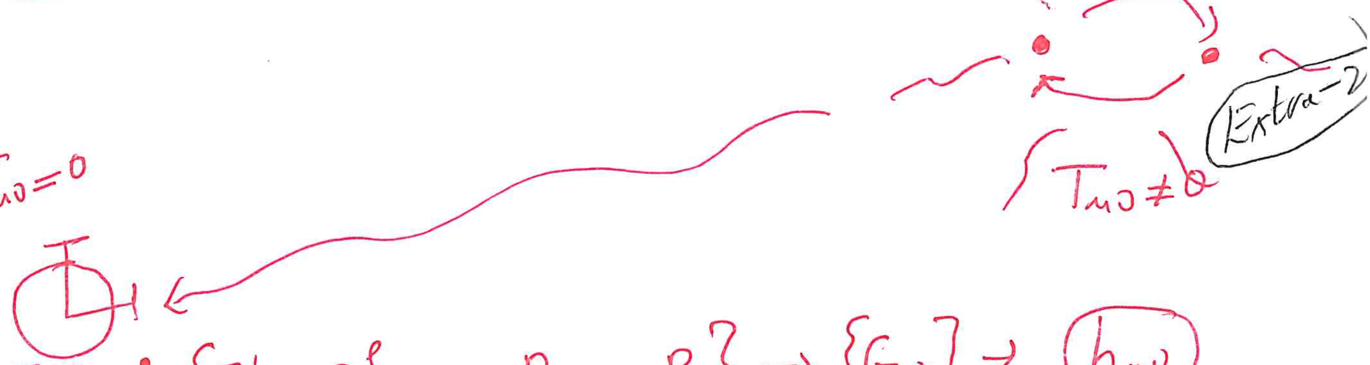
(ii) get the 2 polarisatn $h_+, h_\times$

Global spacetime with matter h_{ab} → gauge dof
→ physical, radiative dof
→ ~~cov~~ physical, non-radiative dof.

(i) only TT part of metric obey wave eqn radiative dof

(ii) Not TT part do not exhibit

RECAP $\Rightarrow$



$\{\Gamma's, R^{\rho}{}_{\sigma\mu\nu}, R_{\mu\nu}, R\} \Rightarrow \{G_{\mu\nu}\} \rightarrow h_{\mu\nu}$

$G_{\mu\nu}$: 6 terms $\downarrow$ $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h$ $\rightarrow$ 4 terms

$$G_{\mu\nu} = \frac{1}{2}\left(\partial\partial h + \partial\partial h \; \cdots \; - \Box \bar{h}\right)$$

$\downarrow$ Lorenz Gauge $\partial h = 0$

$$\boxed{\Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}}$$

$\downarrow$ vacuum $T_{\mu\nu}=0$

$$\boxed{\Box \bar{h}_{\mu\nu} = 0}$$

CKM: Gauge Transf^n $\longrightarrow$ GWs $(A_+, A_\times) \rightarrow$ Effect of GWs on matter

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* PLANE WAVE SOLN in Linearized Theory

$\Box \bar{h}_{\mu\nu} = \bar{h}_{\mu\nu,\alpha}{}^{\alpha}$ ~~$\bar{h}_{\mu\nu,\alpha}$~~ $= 0 \rightarrow \bar{h}_{\mu\nu} = \mathrm{R}\left[A_{\mu\nu}\, e^{ik_\alpha x^\alpha}\right]$

$\Downarrow$

Amplitude & Wave vector satisfy

$\rightarrow k_\alpha k^\alpha = 0$ (k is null)

$\rightarrow A_{\mu\alpha} k^\alpha = 0$ ($A \perp k$)

First sight $\rightarrow A_{\mu\nu} \longrightarrow 6$ dof $\longrightarrow 10 - 4$ [orthogonality]

(!) But Arbitariness due to gauge $\xi^\mu = ?$

$\downarrow$

4 arbitrary const $\rightarrow 6-4 = (2)$

Pick a 4-vel U : $\rightarrow$ Impose the condition

$$\boxed{A_{\mu\nu}U^{\nu}=0 \quad + \quad A^{\mu}{}_{\mu}=0}$$

One now has 8 constrains in all $A_{\mu\alpha}U^{\alpha}=A_{\mu\alpha}k$

$=A^{\alpha}{}_{\alpha}=0$

$\Downarrow$

on the 10 components of amplitude

$\Downarrow$

$\therefore$ remaining 2 dof $\Rightarrow$ 2 Pol of GW

Restate the 8 constraints in a Lorentz frame

$U^{0}=1,\ U^{i}=0$ (k^{α} independent form)

$$\boxed{h_{\mu 0}=0} \qquad \boxed{h_{kj,j}=0} \qquad \boxed{h_{kk}=0}$$

$\rightarrow$ only $h_{ij}\neq 0$

$\therefore$ $h=h^{\mu}{}_{\mu}=h_{kk}=0$ $\rightarrow$ $\exists$ No distinction b/w $h_{\mu\nu}$ & $\bar{h}_{\mu\nu}$

{ In general Most general purely spatial tensor S_{ij} }

can be decomposed:

S_{ij}^{TT} $\downarrow$ "transverse ; traceless"	$S_{ij}^{T}=\frac{1}{2}(\delta_{ij}f_{,kk}-f_{,ij})$ $\downarrow$ "transverse	$S_{ij}^{L}=S^{L}_{i,j}+S^{L}_{j,i}$ $\downarrow$ Longitudinal

In Linearized Theory: h^{L}_{ij} $\rightarrow$ is purely gauge part of $h_{\mu\nu}$ }

$h^{T}_{ij},\ h^{TT}_{ij}$ $\rightarrow$ are gauge-invariant parts of $h_{\mu\nu}$

Special gauge which reduces $h_{\mu\nu}$ to its transverse-traceless = TT

$$\rightarrow \boxed{h_{\mu\nu}=h_{\mu\nu}^{TT}}$$

In TT: $R_{j0k0} = R_{0j0k} = -R_{j00k} = -R_{0jk0}$

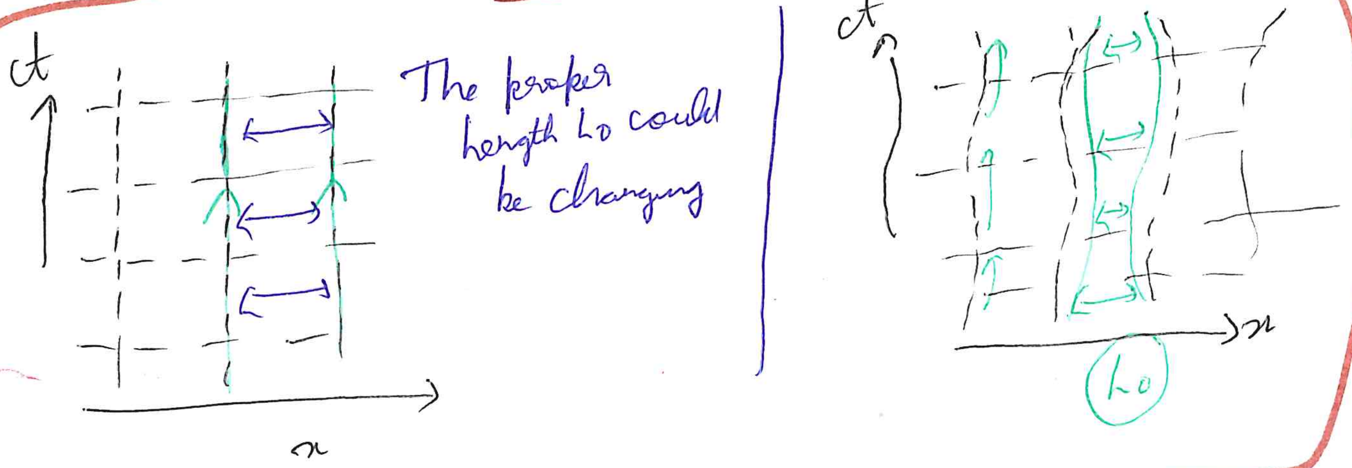
$\downarrow$ Ex

$$\boxed{R_{j0k0} = -\frac{1}{2} h^{TT}_{jk,00}}$$

Extra-3

What we have $\Box \bar{h}_{\mu\nu} = 0$ $[h_{\cdots} \; h_+, h_\times]$

Picture



Need Affect particle $\downarrow$ $d^2x^\mu/d\tau^2$

$\downarrow$

Will need Geo-Deviat^n

$$\frac{D^2(\xi^\mu)}{D\tau^2}$$