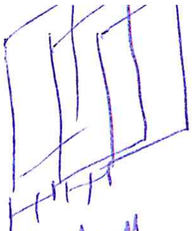

Lec 3

$$A \cos\left[\frac{\omega}{c} \cdot ct - k^x x - k^y y - k^z z\right]$$

10

$$A \cos\left(\eta_{tt} k^t ct + \eta_{xx} k^x_x \ldots\right)$$

$$A \cos\left[k_\sigma x^\sigma\right] \to \mathrm{Re}\left[A e^{i k_\sigma x^\sigma}\right]$$

$$\boxed{\bar{h}_{\mu\nu} = \mathrm{Re}\left[A_{\mu\nu} e^{i k_\sigma x^\sigma}\right]}$$

* Derivative of Plane wave: $\partial_\alpha(\bar{h}_{\mu\nu})$

$$\partial_\alpha\left(A_{\mu\nu} e^{i k_\sigma x^\sigma}\right) \xrightarrow{\text{"PLAY"}} \bar{h}_{\mu\nu} i k_\alpha$$

* $\bar{h}_{\mu\nu} = A_{\mu\nu} e^{i k_\sigma x^\sigma} \longrightarrow$ null $\boxed{k^\alpha k_\alpha = 0}$

$$\left\{ \text{Proof} : \Box \bar{h}_{\mu\nu} = g^{\alpha\beta} \partial_\alpha \partial_\beta \bar{h}_{\mu\nu} \;\ldots\ldots\; k^\alpha k_\alpha = 0 \right\}$$

* $A_{\mu\nu} \to \underline{16 \text{ components}} \to$ 2?

(i) $A_{\mu\nu} = A_{\nu\mu} \Longrightarrow$

$$\begin{bmatrix} A_{tt} & A_{tx} & A_{ty} & A_{tz} \\ \cancel{A_{xt}} & A_{xx} & A_{xy} & A_{xz} \\ \cancel{A_{yt}} & \cancel{A_{yx}} & A_{yy} & A_{yz} \\ \cancel{A_{zt}} & \cancel{A_{zx}} & \cancel{A_{zy}} & A_{zz} \end{bmatrix}$$

10 independent dof $\Longleftarrow$

(ii) Lorenz Gauge $\partial_\beta \bar{h}^{\alpha\beta} = 0 \to i k_\beta \bar{h}^{\alpha\beta} = 0$

$$i\,\boxed{k_\beta A^{\alpha\beta}}\, e^{i k_\sigma x^\sigma} = 0 \to \boxed{k_\beta A^{\alpha\beta} = 0}$$

$$\begin{pmatrix} k_t & k_x & k_y & k_z \end{pmatrix} \begin{bmatrix} A^{tt} & \ldots \\ A^{xt} & \ldots \\ A^{yt} & \ldots \\ A^{zt} & \ldots \end{bmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$$

$\hookrightarrow K_\beta A^{\alpha\beta} = 0 \rightarrow$ gives 4 constraints

$10 - 4 \rightarrow 6$ dof

(iii) Recall : L.G is class of coordinate system

$\boxed{\partial_\beta \bar{h}^{\alpha\beta} = 0}$ $\rightarrow$ Displacement Field $\tilde{x}^\alpha = x^\alpha + \chi^\alpha \rightarrow \partial_\beta \bar{\tilde{h}}^{\alpha\beta} = \partial_\beta \bar{h}^{\alpha\beta} - \cancel{\Box \chi^\alpha}$

L.G with x^α

$\partial_\beta \bar{\tilde{h}}^{\alpha\beta} = \partial_\beta \bar{h}^{\alpha\beta}$

$\Box \chi^\alpha = 0$ still in L.G

$\Downarrow$

So we have freedom to pick one L.G from all L.G's

$\Downarrow$

get to $\boxed{2 \text{ dof}}$

$\Downarrow$

Our specific choice is $\boxed{\text{TT Gauge}}$

$$\Box \chi^\alpha = 0 \rightarrow \begin{bmatrix} \chi^t \\ \chi^x \\ \chi^y \\ \chi^z \end{bmatrix} \leftarrow 4 \text{ dof}$$

4 dof on $\chi^\alpha \rightarrow x^\alpha \rightarrow x^\alpha + \chi^\alpha$

$\downarrow$

4 dof on $\bar{h}^{\alpha\beta} \rightarrow \bar{h}^{\alpha\beta} \rightarrow \bar{h}_{\alpha\beta} - \chi_{\alpha,\beta} - \chi_{\beta,\alpha} - \eta_{\alpha\beta}\chi^\sigma_{\ ,\sigma}$

$\downarrow$

4 dof on $A_{\alpha\beta} \rightarrow \bar{h}^{\alpha\beta} = A_{\alpha\beta} e^{ik_\sigma x^\sigma}$

$\Downarrow$

$\therefore$ define 4 more constrains on $A_{\mu\nu}$

$\Downarrow$

3 transverse constraint + 1 traceless constrains

3 Transverse constraint

↳ Consider an observer with 4-vel $\vec{U} = U^\mu \vec{e}_\mu$ who is experiencing GWs

$\Rightarrow \vec{U}$ In theory GWs can pass by at any and

- Want GW amplitude $A_{\mu\nu}$ to be at right-angle " ortho/transver" to observe

$$\boxed{A_{\mu\nu} U^\mu = 0}$$

$A_{\mu\nu} U^\mu = 0$

$K^\mu A_{\mu\nu} = 0$

4 constraints, 4 cons $\Rightarrow$ 7 total constraint

1 shared constraint

To see why:

$$\begin{pmatrix} A_{tt} & A_{tx} & - & - \\ - & & - & \\ - & & - & \\ - & & & \end{pmatrix} \begin{pmatrix} U^t \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\boxed{A_{tt} U^t + A_{xt} U^t + A_{yt} U^t + A_{zt} U^t = 0}$$

$\Downarrow$

$A_{\mu t} = 0 = A_{t\nu}$

$\therefore K^\mu A_{\mu\nu} = 0 \rightarrow$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & - & \\ 0 & - & & \\ 0 & - & & \end{bmatrix} \begin{bmatrix} K^t \\ K^x \\ | \\ | \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$k^t A_{t\nu} = 0$ already satisfied since $A_{t\nu} = 0$.

So $k^\mu A_{\mu\nu} = 0$ → only gives us 3 extra constraints

↓

∴ Lorenz Gauge additional constraints on $A_{\mu\nu}$:

↳ $A_{\mu\nu} U^\mu = 0$ → gives 3 "transverse" component

1 traceless constraint

↳ Taking $A_{\mu\nu}$ to be traceless

$0 = A_\mu{}^\mu = A^t{}_t + A^x{}_x \ldots$

∴ Starting with any L.G coordinate system we pick out a specific L.G system aka TT-gauge by defining 4 constraints on wave amplitude $A_{\mu\nu}$

~~3 traceles~~

3 "transverse"

$A_{\mu\nu} U^\mu = 0$

⇓

$h^{\mu\nu} U^\mu = 0$

1 traceless con

$A_\mu{}^\mu = 0$

↓

$h_\mu{}^\mu = 0$

$$\boxed{6-4} \Rightarrow \boxed{2 \text{ dof}}$$

(12)

TT- Gauge : Specific Lorenz Gauge

$\partial_\mu \bar{h}^{\mu\nu} = 0$ where

$\bar{h}_{\mu\nu} U^\mu = 0 = \bar{h}_\mu{}^\mu$

↑
4 vel of person observing GW

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

$\times$

$$\bar{h}_{\mu\nu}\eta^{\mu\nu} = h_{\mu\nu}\eta^{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\mu\nu} h$$

$$\bar{h}_\mu{}^\mu = h_\mu{}^\mu - \frac{1}{2}\delta_\mu{}^\mu h$$

$$\bar{h} = h \cancel{h_\mu{}^\mu} - \frac{1}{2} \times 4 \times h$$

$$\boxed{\bar{h} = -h}$$

We know in T.T gauge $\bar{h}_\mu{}^\mu = \bar{h} = 0$

So $h = 0$ $\therefore$ if $h \Rightarrow$ $\boxed{\bar{h}_{\mu\nu} = h_{\mu\nu}}$

⇓
In TT-gauge

→ Components of $A_{\mu\nu}$ ⟹ 16 components

↳ Sym $A_{\mu\nu} = A_{\nu\mu}$ → -6 ~~comp~~ 0 constrain

↳ L.G $A_{\mu\nu}k^\mu = 0$ → -4 constraints

↳ TT $\boxed{A_{\mu\nu}U^\mu = A_\mu{}^\mu = 0}$ → -4 constrain

2 dof

To make use of the constraints we need to
(I) Pick 4-vel U^μ
(II) Wave vec k^μ

***

Take $\vec{U} = c\vec{e}_t \rightarrow U^\mu \Rightarrow [c,0,0,0]$

$k^\mu \rightarrow \left(\frac{\omega}{c}, 0, 0, \frac{\omega}{c}\right) \rightarrow$ wave is z-direct"

(I) (*) $A_{\mu\nu} U^\mu = 0$ gives

$A_{t\nu} U^t = A_{t\nu} c = 0$

$A_{t\nu} = A_{\nu t} = 0$

$$\begin{bmatrix} \cancel{A_{tt}} & \cancel{A_{tx}} & \cancel{A_{ty}} & \cancel{A_{tz}} \\ A_{xt} & A_{xx} & A_{xy} & A_{xz} \\ A_{yt} & A_{yx} & A_{yy} & A_{yz} \\ A_{zt} & \cancel{A_{zx}} & \cancel{A_{zy}} & A_{zz} \end{bmatrix}$$

(II) $A_{\mu\nu} k^\mu = 0$ gives

$\cancel{A_{t\nu} k^t}^{\,0} + A_{z\nu} k^z = 0$

$\boxed{A_{z\nu} = A_{\nu z} = 0}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{yx} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(III) $A^\mu{}_\mu = 0 \rightarrow \eta^{\mu\nu} A_{\mu\nu} = 0$

$\eta^{xx} A_{xx} + \eta^{yy} A_{yy} = 0$

$-A_{xx} - A_{yy} = 0$

$\boxed{A_{xx} = -A_{yy}}$

Two indepent components $\rightarrow$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_t & A_x & 0 \\ 0 & A_x & -A_t & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

13

$$h = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_t & 0 & 0 \\ 0 & 0 & -A_t & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{"Plus"}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & A_x & 0 \\ 0 & A_x & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{"Cross"}}$$

$$\Rightarrow \boxed{\begin{aligned} h_{tt} = h_{ti} = 0 \\ h_t{}^t = 0 \end{aligned}}$$

$$$

P.T.O

X ——— 0 ——— X

CKM

- YT
  - Videos
  - Thumbnail → Alipriyo
- BM → Certificate

M	T	W	T	F	S	S
~~17~~	18	19	20	21	22	23
24	25	26	27	28	29	30
31	1	2	3	4	5	6

ICTS-CMI

return

*) Kaushik LAL Tut !

*) End Sem → $\boxed{\text{May 14}}$

4- Lec Outline

Lec 1 : Intro + Linearized Gravity

$\bar{D}h_{\mu\nu} = 0$

Lec 2 : Lorenz Gauge

Lec 3 : Transverse-Traceless $\begin{cases} + \\ \times \end{cases}$

Lec 4 : How GW's affects Free Particles

CKM

Relative seperat^n : Geo-Deviat^n

$$\frac{D^2(\delta x^\alpha)}{D\tau^2} = R^\alpha{}_{\mu\nu\kappa}\left(\frac{dx^\mu}{d\tau}\right)\left(\frac{dx^\nu}{d\tau}\right)\delta x^\kappa$$

$\downarrow$ slow + weak gravity

$$\frac{d^2}{dt^2}(\delta x^i) = -R^i{}_{0j0}\,\delta x^j$$

(verify)

$\downarrow$ $\forall$ a wave in z-dir $\boxed{x^3 = z}$

$$R^i{}_{0j0} = -\frac{\partial \Gamma^i{}_{0j}}{\partial x^0} \xrightarrow{h_{0j}=0} \Gamma^i{}_{0j} = \frac{1}{2}\frac{\partial h^i_j}{\partial x^0}$$

$\Downarrow$

$$\frac{d^2}{dt^2}(\delta x_i) = \frac{1}{2}\frac{\partial^2 h_{ij}}{\partial t^2}\delta x^j \xrightarrow{sol} \delta x_i = \delta x_{i,0} + \frac{1}{2}h_{ij}\delta x^j$$

$\downarrow$

$$\delta x_1 = \delta x_{1,0} + \frac{1}{2}A_+ e^{ikt}\delta x$$

- - - - - - - -

$\bar{h}_{\mu\nu}U^\mu = 0$; $\bar{h}_\mu{}^\mu = 0$

$\downarrow$

$\boxed{\$\$\$}$

Relat^n b/w metric perturbat^n & linearized Riem

$$\boxed{R_{0i0j0} = -\frac{1}{2}\ddot{h}_{ij}^{TT}}$$

$\downarrow$

Interact^n of GW with a detector

$\downarrow$

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

$\downarrow$ τ $\quad \mu = 1$

$$\frac{d^2x^t}{dt^2} + \frac{dx^{i2}}{dt^2} + \Gamma^t$$

$\downarrow$

$$\boxed{\frac{d^2x^i}{dt^2} = 0}$$

$\Downarrow$

Need G.D Eq^n