Linear Algebra

Assignment #2

Syed Yousha

K226007

$\boxed{Ex : 4.2}$

Q2. (a) All vectors of the form (a,b,c) where $b = a + c + 1$

$b = a + c + 1$

let $U_1 = a + c + 1$, $U_2 = a_2 + c_2 + 1$

$(a + c) + (a_2 + c_2) \pm (2)$ => Not subspace

(b) $(a, b, 0)$

addition : $(a_1, b_1, 0) + (a_2, b_2, 0)$

$= [(a_1 + a_2), (b_1 + b_2), 0]$

=> closed, is a subspace

(c) (a, b, c) , $a + b = 7$

(a_1, b_1, c_1) (a_2, b_2, c_2)

$a_1 + b_1 = 7$, $a_2 + b_2 = 7$

add: $(a_1 + a_2) + (b_1 + b_2) = 14$

Not closed, Not a subspace

Q7 (a) All function f in $F(-\infty, \infty)$
for which $f(0) = 0$

if $f(0) = g(0) = 0$

so, $f+g \rightarrow F(-\infty, \infty)$

$f+g\ (0) = f(0) + g(0) = 0+0 = 0$

(add)

(Mul): $(kf)(0) = k(f(0)) = 0$

(b) f in $F(-\infty, \infty)$, $f(0) = 1$

$\Rightarrow f(0) = 1$

$f(x) = 1$, $g(x) = \cos x$

$\Rightarrow f+g\ (0) = f(0) + g(0) = 1+1 = 2$

not a subspace.

Q10 (a) Seq in R^∞

$v = (v, 2v, 4v, 8v, 16v \ldots)$

* Closed Addition

$u = (u, 2v, 4v, 8v, 16v \ldots)$

$w = (w, 2w, 4w, 8w, 16w \ldots)$

$(u+w) = (u+w), 2(u+w), 4(u+w) \ldots$

Scalar Multiplication

* $\Rightarrow cv, 2(cv), 4(cv), 8(cv), 16(cv) \ldots$

$\Rightarrow$ is a subspace of R^∞

(b)

add: $u + w$

Mul : $k(uw)$

$\Rightarrow$ it is subspace of R^{∞}

Q11.

(a) add: $\begin{bmatrix} a_1 & 0 \\ b_1 & 0 \end{bmatrix} + \begin{bmatrix} a_2 & 0 \\ b_2 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} (a_1+a_2) & 0 \\ (b_1+b_2) & 0 \end{bmatrix}$ (closed)

Mul : $c \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} ca & 0 \\ cb & 0 \end{bmatrix}$

(close) , is a ~~subset~~ space of M_{22}

(b) $\begin{bmatrix} a_1 & 1 \\ b_1 & 1 \end{bmatrix} + \begin{bmatrix} a_2 & 1 \\ b_2 & 1 \end{bmatrix} = \begin{bmatrix} (a_1+a_2) & 2 \\ (b_1+b_2 & 2 \end{bmatrix}$

not close , not a ~~subset~~ space of M_{22}

(c) $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $B\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$(A+B) \Rightarrow A\begin{bmatrix} 1 \\ -1 \end{bmatrix} + B\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\Rightarrow (A+B)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$, A+B is a subspace

Q12 (a) $A\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$,

Add $\Rightarrow$ $A_1\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} + A_2\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\Rightarrow$ $(A_1 + A_2)\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ closed

Mul $\Rightarrow$ $KA\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$, closed

is a subset

(b)

Add: $A_1\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A_1 + A_2\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}A_2$

$\Rightarrow$ $(A_1 + A_2)\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}(A_1 + A_2)$ (closed)

Mul: $KA\begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 1\end{bmatrix}KA$, ~~cotes~~ (closed) is a subspace

(c) $\det(A) = 0$

Add : $\det(A) = 0$ + $\det(B) = 0$ $\Rightarrow$ $0 + 0 = 0$ [closed]

Mul : $\det(KA) = 0$, [closed] , is a subspace

Q 13 (a) (a, a^2, a^3, a^4)

Addition: $A = 1$, $b = 1$

$\Rightarrow (1,1,1,1) + \cancel{(4,4,4,4)}\ (1,1,1,1)$

$\Rightarrow (2,2,2,2)$, Not a subspace

(b)

Add: $(a_1, 0, b_1, 0) + (a_2, 0, b_2, 0)$

$\Rightarrow (a_1 + a_2, 0, b_1 + b_2, 0)$, closed

Mul: $k(a, 0, b, 0) = (ka, 0, kb, 0)$, closed

is a ~~sup~~ subspace for R^4

Q 14. (a) $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

Add: $Ax + Ay = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, not closed

not a R^4 subspace

(b) $Ax = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Add: $\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, not closed

not a subspace

Q19. (a)

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & -1 & 0 \\ 2 & -4 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{array}{l} -R_1 \\ R_2+3R_1 \\ R_3+2R_1 \end{array} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{bmatrix} \Rightarrow \begin{array}{l} R_1+R_2 \\ R_2/2 \; \cancel{R_2+2R_1} \\ R_3+R_2 \end{array} \begin{bmatrix} \cancel{1} \; 1 & \cancel{1} \; 0 & \cancel{1} \; 1/2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix} \quad z = t, \quad y = -\tfrac{3}{2}t, \quad x = -\tfrac{1}{2}t$$

(b)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \vdots \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 6 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

ERO's: $R_2 - \frac{1}{2}R_1$, $R_3 - R_1$, R_{23}, $R_1 + R_2$, $R_3 + R_1$

$$\begin{array}{l} R_1-3R_2 \\ R_2+3R_3 \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right], \text{ unique sol}$$

$x = 0$, $y = 0$, $z = 0$

(c)

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix} \Rightarrow \begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \end{array} \begin{bmatrix} 1 & -3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vdots \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x - 3y + z = 0$$

(solution space)

Qn (d)

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 0 \\ 2 & -1 & 4 & 0 \\ 3 & 1 & 11 & 0 \end{array}\right]$$

ERO's : $R_2 - 2R_1$, $R_3 - 3R_1$, $R_3 - 4R_2$, $R_1 + R_2$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x = -3t$, $y = -2t$, $z = t$

(paramatic equation)

Q_{22} .

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$S = \{ x \in R^n \mid Ax = b \}$

$$A(x_1 + x_2) = Ax_1 + Ax_2$$
$$b + b$$
$$= 2b$$

$A(x_1 + x_2) = 2b$

so, $x_1 + x_2 \notin S$

not closed under addition

not a subspace of R^n

~~Ex 4.5~~ Ex : 4.3

Q8 :

(a) $(2, 3, -7, 3)$

$a_1(2,1,0,3) + a_2(3,-1,5,2) + a_3(-1,0,2,1)$

$\cancel{2a}$ $2a_1 + 3a_2 - a_3 = 2$

$a_1 - a_2 = 3$

$5a + 2a_3 = -7 \Rightarrow 3a_1 + 2a_2 + a_3 = 3$

$a_1 = a_2 + 3$

$a_3 = -\frac{5}{2}a_2 - \frac{7}{2}$

$2(a_2 + 3) + 3a_2 - \left(-\frac{5}{2}a_2 - \frac{7}{2}\right) = 2$

$2a_2 + 6 + 3a_2 + \frac{5}{2}a_2 + \frac{7}{2} = 2$

$\frac{15a_2}{2} = -\frac{15}{2}$ $\boxed{a_2 = -1}$

$a_1 = 2$, $a_3 = -1$

$\Rightarrow 3.2 + 2(-1) + (-1) = 3$

$6 - 2 - 1 = 3$, $3 = 3$ $\Rightarrow (2,3,-7,3)$

No span

(b) $(0, 0, 0, 0)$

$a_1(2,1,0,3) + a_2(3,-1,5,2) + a_3(-1,0,2,1)$

$a_1 = a_2 = a_3$ in span

CamScanner

5

c)

$$(1,1,1,1) = a_1(2,1,0,3) + a_2(3,-1,5,2) + a_3(-1,0,2,1)$$

$$2a_1 + 3a_2 - a_3 = 1 \quad -(1)$$

$$a_1 - a_2 = 1 \quad -(2)$$

$$5a_2 + 2a_3 = 1 \quad -(3)$$

$$3a_1 + 2a_2 + a_3 = 1 \quad -(4)$$

$(2) \Rightarrow a_1 = a_2 + 1$

$(3) \Rightarrow a_3 = -\frac{5}{2} a_2 + \frac{1}{2}$

$$2(a_2+1) + 3a_2 - \left(-\frac{5a_2}{2} + \frac{1}{2}\right) = 1$$

$$\frac{15a_2}{2} = -\frac{1}{2}, \quad \boxed{a_2 = -1/15}$$

$a_1 = 14/15$, $a_3 = 2/3$

$\xrightarrow{eq\ 4}$ $3\left(\frac{14}{15}\right) + 2\left(\frac{-1}{15}\right) + \frac{2}{3} = 1$

$$\frac{14}{15} - \frac{2}{15} + \frac{2}{3} = 1, \quad \frac{10}{3} = 1$$

not in span

(d) $(-4, 6, -13, 4)$

$2a + 3a_2 - a_3 = -4$

$a_1 - a_2 = 6$

$5a_2 + 2a_3 = -13$

$3a_1 + 2a_2 + a_3 = 4$

$a_1 = a_2 + 6$ — (2)

$a_3 = \frac{-5}{2} a_2 - \frac{13}{2}$ — (3)

$a_2 = -3$

$a_1 = 3$

$a_3 = 1$

$(-4, 6, -13, 4) = 3v_1 - 3v_2 + v_3$

$\Rightarrow$ is in span

Q9

$P_1 = 1 - x + 2x_2$, $P_2 = 3 + x$

$P_3 = 5 - x + 4x^2$, $P_4 = -2 - 2x + 2x^2$

span $\{P_1, P_2, P_3, P_4\} \neq P_2$

$5 - x + 4x^2 = a_1(1 - x + 2x^2) + a_2(3 + x)$

$5 - x + 4x^2 = a_1 - a_1 x + 2a_1 x^2 + 3a_2 + a_2 x$

$5 - x + 4x^2 = (a_1 + 3a_2)(-a + a_2)x + 2a_1 x_2$

$5 = a_1 + 3a_2$

$-1 = a_1 + a_2$

$4 = 2a_1$

$a_1 = 2$

$5 = 2 + 3.1$

$2 + 3$

$\boxed{5 = 5}$

do not span P_4

Q10

$P_1 = 1+x, \quad P_2 = 1-x$

$P_3 = 1+x+x^2, \quad P_4 = 2-x^2$

Sol:-

$$k_1P_1 + k_2P_2 + k_3P_3 + k_4P_4 = a + bx + cx^2$$

$$(k_1+k_2+k_3+2k_4) + (k_1-k_2+k_3)x + (k_3-k_4)x^2 = a+bx+cx^2$$

$$k_1+k_2+k_3+2k_4 = a$$
$$k_1 - k_2 + k_3 = b$$
$$k_3 - k_4 = c$$

$$A = \left[\begin{array}{ccccc} 1 & 1 & 1 & 2 & a \\ 1 & -1 & 1 & 0 & b \\ 0 & 0 & 1 & -1 & c \end{array}\right]$$

$$R_2 - R_1 = \left[\begin{array}{ccccc} 1 & 1 & 1 & 2 & a \\ 0 & -2 & 0 & -2 & b-a \\ 0 & 0 & 1 & -1 & c \end{array}\right]$$

$-\frac{1}{2}R_2, \quad R_3 - R_1, \quad R_2 - R_1$

$$\Rightarrow \left[\begin{array}{ccccc} 1 & 0 & 0 & 2 & \frac{a+b-2c}{2} \\ 0 & 1 & 0 & 1 & -\frac{b-a}{2} \\ 0 & 0 & 1 & -1 & c \end{array}\right]$$

$\Rightarrow P_1, P_2, P_3$ span P_2

Q11

(a)

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$k_1 \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + k_2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$+ k_4 \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} k_1 + k_2 & k_2 + k_3 \\ k_1 + k_4 & k_3 + k_4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & a \\ 0 & 1 & 1 & 0 & b \\ 1 & 0 & 0 & 1 & c \\ 0 & 0 & 1 & 1 & d \end{bmatrix}$$

$$\det = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} - 1 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} + 0 - 0$$

$$1(1(0-1) - 1(0-0) + 0) - 1(1-0) + 0)$$

$$1(-1) - 1(-1)$$

$$-1 + 1$$

does not span

Q11

7

(b)

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix};$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} k_1 + k_3 + k_4 & -k_1 + k_2 + k_3 \\ k_3 & k_1 + k_4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$k_1 + k_3 + k_4 = a$$
$$-k_1 + k_2 + k_3 = b$$
$$k_3 = c$$
$$k_1 + k_4 = d$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & a \\ 1 & 1 & 1 & 0 & b \\ 0 & 0 & 1 & 0 & c \\ 1 & 0 & 0 & 1 & d \end{bmatrix}$$ Coefficient Matrix

$$\Rightarrow 1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 0 + 1 \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow -1 \cdot \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow 1\,[1\,(1-0) - 1(0-0) + 1\,[-1(0-0) - 1\,(0-0) + 0)]$$

$$1(1) + 1(0) - 1(1)$$

$$1 - 1 = 0$$

does not span

(c)

$k_1 + k_2 + k_3 + k_4 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$k_1 + k_2 + k_3 + k_4 = a$

$k_2 + k_3 + k_4 = b$

$k_3 + k_4 = c$

$k_4 = d \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

$-0 + 0 - 0 + 1 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow 1 [0 - 0 + 1(1-0)]$

$1(1) \Rightarrow 1$ (spanning)

Q4 (a)

$\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(2x) = f - g$

$\cos(2x)$ is spanned by f and g

Q14) (b) $3+x^2$

$a\sin^2 x + b\cos^2 x = 3+x^2$

$3+0^2$

$3+0$

$\boxed{b=3}$

$3+(\pi/2)^2$

$3+\frac{\pi^2}{4}$

$a = 3+\frac{\pi^2}{4}$

$\Rightarrow \quad a\sin^2\left(\frac{\pi}{4}\right) + b\cos\left(\frac{\pi}{4}\right) = 3+\left(\frac{\pi}{4}\right)^2$

$a \times \frac{1}{2} + b \times \frac{1}{2} = 3+\frac{\pi^2}{16}$

$\Rightarrow$ is not spanning

(c)

$1 = \cos^2 x + \sin^2 x$

$1 = f+g$

1 is spanned by f and g

(d) $\sin x$

$\sin x(-x) = -\sin x$

are even function

$\sin x$ is not spanning

(e)

$0 = 0 \cdot \cos^2 x + 0 \cdot \sin^2 x$

$0 = 0 \cdot f + 0 \cdot g$

0 is spanned by f and g

Q16 (a)

$a = (1,1,1,0)$, $v = (0,-1,0,1)$

$R_2 - 2R_1$ $\begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 3 \end{bmatrix}$

$R_3 - 3R_1$ $\begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$y - z + w = 0$

$0 = 0$

$0 = 0$

$(k_3, k_2 - k_1, k_2, k_1)$

$(k_3, 0, 0, 0) + (0, -k, 0, k) + (0, k_2, k_2, 0)$

$x = k_3$

$k_3(1,0,0,0) + k_2(0,-1,0,1)\ k_3(0,1,1,0)$

$k_3 = 1$, $k_2 = 1$

$(x, y, z, w) = (1,1,1,0) + (0,-1,0,1)$

the set $\{u, v\}$ spans w

Q16 (b) $U = (0, 1, 1, 0)$, $V = (1, 0, 1, 1)$ (9)

$$\begin{bmatrix} 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$R_2 - 2R_1$, $R_3 - 3R_1$

$$\begin{bmatrix} 0 & 1 & -1 & +1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$y - z + w = 0$

$0 = 0$

$w = k_1$, $z = k_2$

$y = k_2 + k_1 = 0$

$y = k_2 - k_1$

$(x, y, v, w) = (k_3, k_2, -k_1, k_2, k_1)$

$\Rightarrow k_3(1, 0, 0, 0) + k_1(0, -1, 0, 1) + k_2(0, 1, 1, 0)$

does not span

Ex : 4.4

Q7.

(a) $v_1 = (2, -2, 0)$, $v_2 = (6, 1, 4)$, $v_3 = (2, 0, -4)$

$$\begin{bmatrix} 2 & 6 & 2 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 4 & -4 & 0 \end{bmatrix}$$

$R_2 + R_1$, $R_3 - \frac{4}{7} R_1$, $\frac{-7}{52} R_3$

$R_2 - 2R_3$, $R_1 - 2R_3$, $\frac{1}{7} R_2$, $R_1 - 6R_2$, $\frac{1}{2} R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$a = 0$, $b = 0$, $c = 0$

linearly dependent

does not span

(b) $a(-6, 7, 2)$, $b(3, 2, 4)$, $c(4, -1, 2)$

$$\begin{bmatrix} -6 & 3 & 4 & 0 \\ 7 & 2 & -1 & 0 \\ 2 & 4 & 2 & 0 \end{bmatrix}$$

ERO's: R_{12}, $R_2 + \frac{6}{7} R_1$, $R_3 - \frac{2}{3} R_1$, $R_3 - \frac{8}{11} R_2$, $\frac{7}{33} R_3$,

$R_1 - 2R_2$, $\frac{1}{7} R_1$

$$\begin{bmatrix} 1 & 0 & -1/3 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$a = \frac{1}{3} c$, $b = \frac{-2}{3} c$,

$c = c$

linearly dependent

lie in span

10

Q8

(a) since $v_2 = -2v$

v_1 and v_2 lies on same plane

$\Rightarrow V_3$ is not multiple of v_1 and v_2

$\Rightarrow$ vector does not lie on same line

(b) None of them are multiple of each other

the three vector don't lie on same plane.

(c) $v_1(4,6,8)$, $v_2\ (2,3,4)$ $v_3(-2,-3,-4)$

since $v_1 = 2v_2 = -2v_3$

~~tip~~ lie on same line through origin.

Q13 a) $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

$u(1,2)$ $\quad u_2(-1,1)$

$TA(u_1) = Au_1 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

$TA(u_2) = Au_2 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$\{(-1,4), (-2,2)\}$

linearly independent in R^2

(b)

$\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

$= \{(-1,2), (-2,4)\}$

linearly dependent set in R_2

$\underline{Q_{14}}$ (a)

$$Au_1 \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$Au_2 \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$Au_3 \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

$\{ (1,1,2)\ (3,-1,2)\ (3,-3,2) \}$

$\rightarrow$ linearly Independent

(b)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix}$$

$$Au_1 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$Au_2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

$$Au_3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

$\{ (1,1,2)\ (2,-2,2)\ (2,-2,2) \}$

linearly dependent in R_3

Q.19

(a) $1, x, e^x$

$f_1(1) = 1 \quad , \quad f_1'(1) = 0 \quad , \quad f''(0) = 0$

$f_2(x) = x \quad , \quad f_2'(1) = 1 \quad , \quad f_2''(1) = 0$

$f_3(e^x) = e^x \quad , \quad f_3'(e^x) = e^x \quad , \quad f_3''(e^x) = e^x$

$$\begin{bmatrix} 1 & x & e^x \\ 0 & 1 & e^x \\ 0 & 0 & e^x \end{bmatrix}$$

$$\begin{bmatrix} 1 & e^x \\ x & e^x \end{bmatrix} \quad e^x - xe^x = e^x(1-x)$$

$$w\left(\frac{\pi}{2}\right) = e^{\pi/2}\left(1 - \frac{\pi}{2}\right)$$

$$\frac{\pi}{2} e^{\pi/2} \neq 0$$, linearly Independent

(b) $1, x, x^2$

$$\begin{bmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2x \\ 0 & 2 \end{bmatrix} = 2 \qquad (-\infty, \infty)$$

$$w\left(\frac{\pi}{2}\right) = 2 \neq 0$$

linearly Independent

Ex 4.5

Q7 (a)

$a = 2, -3, 1$

$b = 4, 1, 1$

$c = 0, -7, 1$

$2a + 4b = 0$

$-3a + b - 7c = 0$

$a + b + c = 0$

$$\begin{bmatrix} 2 & 4 & 0 \\ -3 & 1 & -7 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ERO's: R_{12}, R_2, $2R_1$, $R_3 + \frac{1}{3} R_1$, $R_3 - \frac{2}{7} R_2$, $\frac{3}{14} R_2$, $R_1 - R_2$, $\frac{1}{3} R_3$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$a + 2c = 0 \Rightarrow a = -2c$

$b - c = 0 \Rightarrow b = c$

$c = c$

set of vector is not a basis.

(12)

Q7

(b)

$a + 2b - c = 0$

$6a + 4b + 2c = 0$

$4a - b + 5 = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 6 & 4 & 2 \\ 4 & -1 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ERO's: R_{12}, $R_2 + \frac{1}{6} R_1$, $R_3 - \frac{2}{3} R_1$, R_{23}, $R_3 + \frac{4}{11} R_2$

$-\frac{3}{11} R_2$, $R_1 - 4R_2$, $\frac{1}{6} R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$a + c = 0$ $\quad$ $a = -c$

$b - c = 0$ $\quad \Rightarrow \quad$ $b = c$

$c = c$

not a basis.

Q12

(a) $u_1 = (1, -1)$ $\quad u_2\ (1, 1)$ $\quad w(1, 0)$

$(a + b, \ -a + b) = (1, 0)$

$a + b = 1$

$-a + b = 0$ $\qquad\qquad -a + b = 0$

$a + b - a + b = 1 + 0$ $\quad \Rightarrow \quad$ $a = \cancel{2}b$, $a = \frac{1}{2}$

$2b = 1$

$b = \frac{1}{2}$ $\qquad\qquad w = \left(\frac{1}{2}, \frac{1}{2}\right)$

(b) Considering Basis $av_1 + bv_2$ $w = (0, 1)$

$a + b = 0$ $\Rightarrow$ $a + b = 0$

$-a + b = 1$ $\quad a = -b$

Same case $\quad a = -\frac{1}{2}$

$w = \left(-\frac{1}{2}, \frac{1}{2}\right)$

Q20

$S = \{0, v_1, v_2, v_n\}$

$(1.0 + 0.v_1 + 0.v_2 \ldots .0.v_n)$

equals to 0

hence therom 4.3.1,

is linearly dependent set in V

Ex: 4.8

Q6.

$x_1 = -1, \quad x_2 = 2, \quad x_3 = 4, \quad x_4 = -3$

$Ax = b, \quad Ax = 0$

$x_1 = -3r + 4s, \quad x_2 = r - s, \quad x_3 = r, \quad x_4 = s$

(a) $Ax = 0$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3r + 4s \\ r - s \\ r \\ s \end{bmatrix} = r\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Ans.

(b) $Ax = b$

$$A = \begin{bmatrix} -1 \\ 2 \\ 4 \\ -3 \end{bmatrix} + r\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

Q7.

(a) $x_1 - 3x_2 = 1$

$2x_1 - 6x_2 = 2$

$x_2 = v$

$x_1 - 3v = 1$

$\Rightarrow x_1 = 1 + 3v$

$Ax = b$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + v\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = v\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(b) $x_1 + x_2 + 2x_3 = 5$

$x_2 + x_3 = -2$

$2x + x_2 + 3x_3 = 3$

$\Rightarrow$ $x_3 = r$, $x_1 + x_3 = -2$

$x_1 + r = -2$, $x_1 = -2 - r$

$x_1 = -2 - r$, $x_3 = r$ in $2x_1 + x_2 + 3x_3 = 3$

$2(-2-r) + x_2 + 3r = 3$

$-4 - 2r + x_2 + 3r = 3$

$x_2 = 7 - r$

So, $\begin{bmatrix} -2 \\ 7 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

Q8 (a) ~~$x_1 - 2x$~~ $x_1 - 2x_2 + x_3 + 2x_4 = -1$

~~$x_2 = t$~~ $2x_1 - 4x_2 + 2x_3 + 4x_4 = -2$

$-x_1 + 2x_2 - x_3 - 2x_4 = 1$

$3x_1 - 6x_2 + 3x_3 + 6x_4 = -3$

$\Rightarrow$

$x_2 = r$, $x_3 = s$, $x_4 = t$

$x_1 = -2r + s + 2t = -1$

$x_1 = -1 + 2r - s - 2t$

$Ax = b$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$Ax = 0$

$2\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ Ans.

Q 8

(b) $x_1 + 2x_2 - 3x_3 + x_4 = 4$

$-2x_1 + x_2 + 2x_3 + x_4 = -1$

$-x_1 + 3x_2 - x_3 + 2x_4 = 3$

$4x_1 - 7x^2 - x_4 = -5$

$\boxed{Ax = b}$

Reduced:

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 4 \\ -2 & 1 & 2 & 1 & -1 \\ -1 & 3 & -1 & 2 & 3 \\ 4 & -7 & 0 & -5 & -5 \end{bmatrix}$$

R_{12}, $R_2 + \frac{1}{2}R_1$, $R_3 + \frac{1}{4}R_1$, R_{24}, $R_3 - \frac{1}{3}R_2$,

$R_4 + \frac{2}{3}R_3$, $\frac{4}{5}R_2$, $R_1 + 7R_2$, $\frac{1}{4}R_1$

$$\begin{bmatrix} 1 & 0 & -7/5 & -1/5 & 6/5 \\ 0 & 1 & -4/5 & 3/5 & 7/5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\cancel{x_1 - 7/5}$ $x_1 - \frac{7}{5}x_3 - \frac{1}{5}x_4 = \frac{6}{5}$

$x_1 = \frac{6}{5} + \frac{7}{5}x_3 + \frac{1}{5}x_4$

$x_2 = \frac{7}{5} + \frac{1}{5}x_3 - \frac{6}{5}x_4$

let $r = x_3$, $x_4 = s$

$$\begin{bmatrix} 4k + 7/5 r + 1/5 s \\ 7k + 1/5 r - 6/5 s \\ r \end{bmatrix} = \begin{bmatrix} 6/5 \\ 7/5 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} 7/5 \\ 1/5 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1/5 \\ -6/5 \\ 0 \\ 1 \end{bmatrix}$$

$$= r \begin{bmatrix} 7/5 \\ 1/5 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1/5 \\ -6/5 \\ 0 \\ 1 \end{bmatrix} \text{ Ans.}$$

Q9: $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$, Applying REF, R_{13}, $R_2 - \frac{5}{7} R_1$, $R_3 - \frac{1}{7} R_1$, $R_3 + \frac{1}{2} R_2$, $\frac{7}{2} R_2$, $\frac{1}{7} R_1$

$$\begin{bmatrix} 1 & 0 & -16 \\ 0 & 1 & -19 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 16x_3 = 0 \Rightarrow x_1 = 16x_3$

$x_2 = 19x_3$

$x_3 = x_2$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16t \\ 19t \\ t \end{bmatrix} \Rightarrow t \begin{bmatrix} 16 \\ 19 \\ t \end{bmatrix} \Rightarrow \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix}$$

$r_1 = 1, 0, 16$

$r_2 = 0, 1, -19$

So, it a Basis for Row space

(15)

Q9 (b) $A = \begin{bmatrix} 2 & 0 & -2 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ Reduce: R_{12}, $R_2 - \frac{1}{2}R_1$, $\frac{1}{4}R_1$

$$\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = \frac{1}{2}x_3$, $x_2 = x_2$, $x_3 = x_3$

$x_2 = s$, $x_3 = t$, $x_1 = t\,1/2$

$$\begin{bmatrix} t\,1/2 \\ s \\ t \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$r_1 = \{1, 0, 1/2\}$, Basis for Rowspace.

Q12 (a) $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Basis for Rowspace :- $(1, 2, 4, 5)$ $(0, 1, -3, 0)$
$(0, 0, 1, -3)$ $(0, 0, 0, 1)$ $(0, 0, 0, 0)$

Basis for column space :-
$(1, 0, 0, 0, 0)$ $(2, 1, 0, 0, 0)$ $(4, -3, 1, 0, 0)$
$(5, 0, -3, 1, 0)$

$\Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$

Q12

(b)
$$\begin{bmatrix} 1 & 2 & -1 & 5 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ Basis for Row space

$(1, 2, -1, 5)$ $(0, 1, 4, 3)$ $(0, 0, 1, -7)$
$(0, 0, 0, 1)$

Basis for column space

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 7 \\ 1 \end{bmatrix}$$

Q 20

(a) $v = \begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 4 \end{bmatrix}$

$\Rightarrow$ $v = c_1 v + c_2 v_2$

$$c_1 \begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 0 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} c_1 + 2c_2 \\ -c_1 \\ 3c_1 - 2c_2 \\ 2c_1 + 4c_2 \end{bmatrix}$$

$x_1 = c_1 + 2c_2$

$x_2 = -c_1$

$x_3 = 3c_1 - 2c_2$

$x_4 = 2c_1 + 4c_2$

$x_1 + x_2 - \frac{1}{2} x_4 = 0$

$2x_2 + x_3 + \frac{1}{2} x_4 = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & -1/2 \\ 0 & 2 & 1 & 1/2 \end{bmatrix}$$ Ans.

Ex : 4.9

Q11. $A = \begin{bmatrix} 1 & 4 \\ 0 & 3 \\ -9 & 0 \end{bmatrix}$

$A^T \begin{bmatrix} 1 & 0 & -9 \\ 4 & 3 & 0 \end{bmatrix}$

$R_3 + 9R_1 = \begin{bmatrix} 1 & 4 \\ 0 & 3 \\ 0 & 36 \end{bmatrix}$

$\frac{1}{3} R_2 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \\ 0 & 36 \end{bmatrix}$

$R_1 - 4R_2 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

2) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} \right\}$

dim [col(A) = 2]

Rank $(A^T) = 2$

$\{(1, 0), (0, 1)\}$

dim [row(A)] = 2

rank (A^T) + Nullity $(A^T) = 3$

rank $(A^T) : 2$

rank + Nullity = n

2 + Nullity = 2

2 − 2

Nullity : 0

$A^T = \begin{bmatrix} 1 & 0 & -9 \\ 4 & 3 & 0 \end{bmatrix}$

$R_2 - 4R_1 \begin{bmatrix} 1 & 0 & -9 \\ 0 & 3 & 36 \end{bmatrix}$

$\frac{1}{3} R_2 \begin{bmatrix} 1 & 0 & -9 \\ 0 & 1 & 12 \end{bmatrix}$

$x_1 - 9x_3 = 0$

$x_2 + 12x_3 = 0$

$x_3 = t$

$x_1 - 9x_3 = 0$

$x_1 - 9(t) = 0$

$x_1 = 9t$

$x_2 + 12x_3 = 0$

$x_2 + 12t = 0$

$x_2 = -12t$

General solution :-

$x_1 = 9t$, $x_2 = -12t$, $x_3 = t$

$x = t(9, -12, 1)$

Null A^T $\begin{bmatrix} 9 \\ -12 \\ 1 \end{bmatrix}$

$\dim[\text{null}(A^T)] = 1$

Basis for Null(A) : $\emptyset$

Basis for Null $(A^T) = \begin{bmatrix} 9 \\ -12 \\ 1 \end{bmatrix}$

Ans.

Q15-18

Matrix in Ex: 11

$A = \begin{bmatrix} 1 & 4 \\ 0 & 3 \\ -9 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 0 & -9 \\ 4 & 3 & 0 \end{bmatrix}$

$R_3 + 9R_1$ $\begin{bmatrix} 1 & 4 \\ 0 & 3 \\ 0 & 36 \end{bmatrix}$

$\frac{1}{3}R_2$, $R_3 - 36R_2$, $R_1 - 4R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \{(1,0), (0,1)\}$

$r + n = N$
$2 + n = 2$
$n = 2 - 2$

$\begin{bmatrix} 1 & 0 & -9 \\ 4 & 3 & 0 \end{bmatrix} = A^T$

$R_2 - 4R_1$ $\begin{bmatrix} 1 & 0 & -9 \\ 0 & 3 & 36 \end{bmatrix}$

$\frac{1}{3}R_2$ $\begin{bmatrix} 1 & 0 & -9 \\ 0 & 1 & 12 \end{bmatrix}$

RREF $\begin{bmatrix} 1 & 0 & -9 \\ 0 & 1 & 12 \end{bmatrix}$

$x_1 - 9x_3 = 0$, $x_2 + 12x_3 = 0$

$x_1 - 9(t) = 0$, $x_2 = 7t$

$x_2 = -12t$, $x_3 = t$

$x = t(9, -12, 1)$

$\Rightarrow \left\{ \begin{bmatrix} 9 \\ -12 \\ 1 \end{bmatrix} \right\}$

$\det(A) = \begin{bmatrix} 1 & 4 & 9 \\ 0 & 3 & -12 \\ -9 & 0 & 1 \end{bmatrix}$

$678 \neq 0$

$\begin{bmatrix} 1 \\ 0 \\ -9 \end{bmatrix} \begin{bmatrix} 9 \\ -12 \\ 1 \end{bmatrix}$

$(1 \times 9) + (0 \times (-12)) + (-9 \times 1)$

$9 - 9 = 0$

$\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 9 \\ -12 \\ 0 \end{bmatrix}$

$(4 \times 9) + (3 \times (-12)) + 0 \times 1$

$36 - 36 = 0$

$\Rightarrow$ Orthogonal

Ans.

17

Q19-2c

$$A = \begin{bmatrix} 0 & 2 & 8 & -7 \\ 2 & -2 & 4 & 0 \\ -3 & 4 & -2 & 5 \end{bmatrix}$$

$$\left[\begin{array}{cccc|ccc} 0 & 2 & 8 & -7 & 1 & 0 & 0 \\ 2 & -2 & 4 & 0 & 0 & 1 & 0 \\ -3 & 4 & -2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$R_{12} \begin{bmatrix} 2 & -2 & 4 & 0 & 0 & 1 & 0 \\ 0 & 2 & 8 & -7 & 1 & 0 & 0 \\ -3 & 4 & -2 & 5 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{2}R_1 \begin{bmatrix} 1 & -1 & 2 & 0 & 0 & 1/2 & 0 \\ 0 & 2 & 8 & -7 & 1 & 0 & 0 \\ -3 & 4 & -2 & 5 & 0 & 0 & 1 \end{bmatrix}$$

$$R_3 + 3R_1 \begin{bmatrix} 1 & -1 & 2 & 0 & 0 & 1/2 & 0 \\ 0 & 2 & 8 & -7 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 3/2 & 1 \end{bmatrix}$$

$1/2\, R_2$, $R_3 - R_2$, $\frac{2}{17} R_3$,
$R_2 + \frac{7}{2} R_3$, $R_1 + R_2$

$$\left[\begin{array}{cccc|ccc} 1 & 0 & 6 & 0 & 5/17 & 19/17 & 7/17 \\ 0 & 1 & 4 & 0 & 5/17 & 21/34 & 7/17 \\ 0 & 0 & 0 & 1 & -1/17 & 3/17 & 2/17 \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} -6x_3 \\ -4x_3 \\ x_3 \\ 0 \end{bmatrix}$$

Null Space Basis: $\left\{ \begin{bmatrix} -6 \\ -4 \\ 1 \\ 0 \end{bmatrix} \right\}$

Row space: Basis:-

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Column Space Basis:-

$$\left\{ \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 5 \end{bmatrix} \right\}$$

Left Nullspace Basis:- ϕ

Ans.

$\boxed{5.1}$

Ex 8.1

Q12,

(a)
$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

$\Rightarrow \lambda I - A = 0$

$$\begin{bmatrix} \lambda-1 & 3 & -3 \\ 3 & \lambda+5 & 3 \\ 6 & -6 & \lambda-4 \end{bmatrix}$$

$$\boxed{\lambda^3 - 12\lambda^2 - 16 = 0}$$

$\lambda_1 = \cancel{4}\ -2$

$\lambda_2 = -2$

$\lambda_3 = 4$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3 & 3 & -3 \\ -3 & 3 & -3 \\ -6 & 6 & -6 \end{bmatrix}$$

$x_1 = s, \quad x_2 = t, \quad x_3 = s$

$$\begin{bmatrix} s-t \\ s \\ t \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Now $\lambda_3 = 4$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} = 0$$

$$\approx \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} t/2 \\ t/2 \\ t \end{bmatrix} = \boxed{t = 1} \quad \text{Basis} \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

Q16. $T(x, y, z) = (2x - y - z,\ x - z,\ -x + y + 2z)$

$$\Rightarrow A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda-2 & 1 & 1 \\ -1 & \lambda & 1 \\ 1 & -1 & \lambda-2 \end{bmatrix}$$

$\lambda^3 - 4\lambda^2 + 5\lambda^2 - 2 = 0$

$\lambda_1 = 1, \quad \lambda_2 = 1, \quad \lambda_3 = 2$

For $\lambda = 1$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} t+s \\ s \\ t \end{bmatrix}$$

Basis for eigen space

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda_3 = 2$

$$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -t \\ -t \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$ Basis

Ans.