K226007

1

Q1.

(i)
- Undirected edges [UE]
- Multiple edges [ME]
- no loops
- Undirected Multigraph

(ii)
- UE
- No ME
- no loops
- Undirected simple graph

(iii)
- Undirected edges
- Multi. edges
- have 3 loops

(iv)
- directed graph
- Multi edges
- 2 loops
- directed Multi graph

Q2.

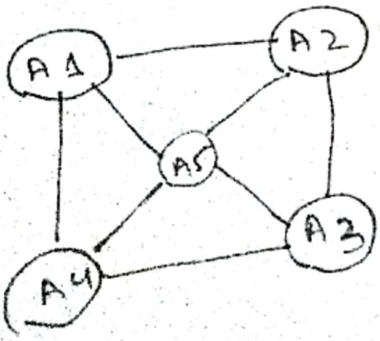
i. → $A1 \cap A2 = \{2, 4\}$

→ $A2 \cap A3 = \{1, 3\}$

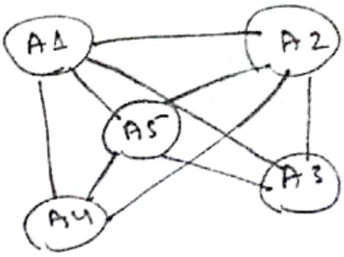
→ $A3 \cap A4 = \{7, 9\}$

→ $A4 \cap A5 = \{8, 9\}$

→ $A1 \cap A4 = \{6, 8\}$



(ii)



Q3 (a). (i)

- vertices : 5
- edges : 13
- deg(a, b, c) = 6

deg(d) = 5

deg(e) = 3

Neighbours

a = a, b, e
b = a, e, d, c
c = c, d, b
d = e, b, c
e = a, b, d

(ii)
- vertices : 9
- edges : 12

- **Neighbours**

a = c, e, i
b = h, e
c = a, e, i, g
d = none
e = a, b, c, g
f = none
g = c, e
h = b, i
i = a, c, h

- Degrees,

a = 3
b = 2
c = 4
d = 0
e = 6
f = 0
g = 4
h = 2
i = 3

Q3

(b) (i)

vertices : 5

edges : 13

In-degree

$a^- = 6$, $b^- = 1$

$c^- = 2$, $d^- = 4$

$e^- = 0$

Out-degree

$a^+ = 1$, $b^+ = 5$

$c^+ = 5$, $d^+ = 2$

$e^+ = 0$

(ii) vertices : 4

edges : 8

In-degree

$a^- = 2$, $b^- = 3$

$c^- = 2$, $d^- = 1$

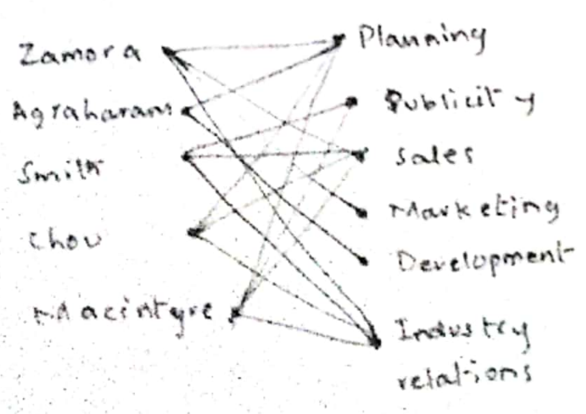
Out-degree

$a^+ = 2$, $b^+ = 4$

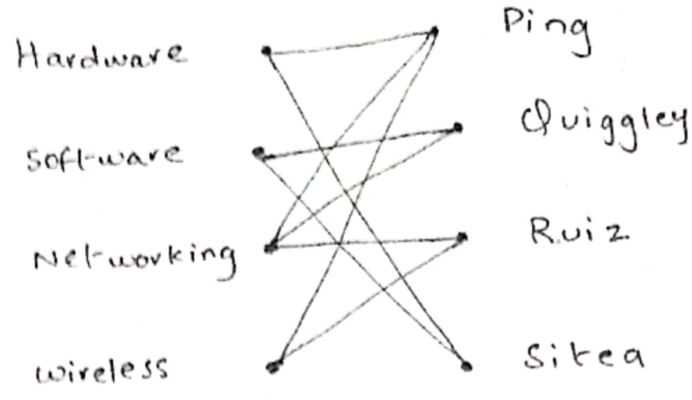
$c^+ = 1$, $d^+ = 1$

Q4 (a)

~~Planning Publicity Sales~~



(b) Bipartite Graph



Q5 (i) Not bipartite

(b and f are of same color).

(ii) Bipartite

$A(v_1, v_3, v_5)$

$B(v_2, v_4, v_6)$

(iii) Not Bipartite

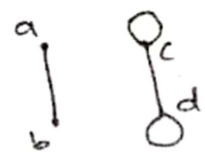
(v_5 & v_4 are adjacent)

(iv) Not Bipartite

(b and d are adjacent)

Q6. (a) Not possible.

(b)



(c) Not possible

Q7.

a. No, it is not possible,

as, $15 * 3 \neq 2e$

Handshaking

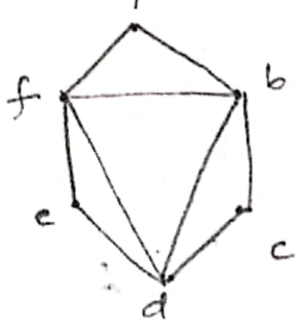
b. Yes it is possible

as, $4 * 3 = 2e$

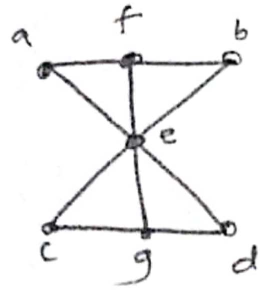
(even)

Q8.

(i)



(ii)



b. regular graph,

$E = \frac{n \times d}{2}$

edges

$10 = \frac{n \times 4}{2}$

$n = 5$

Q9, (i)

vertices => 5 : 5

deg req ✓

Map
$v_1 = w_2$, $v_3 = w_1$
$v_2 = w_3$, $v_4 = w_5$
$v_5 = w_4$ (isomorphic)

(ii)

vertex = 6 : 6

Map: $U_1 = V_5$, $U_2 = V_2$

$U_3 = V_4$, $U_5 = V_1$

$U_4 = V_3$, $U_6 = V_6$

(isomorphic)

(iii) vertex = 7 : 7

Map: $V_1 = U_5$, $V_2 = U_4$, $V_3 = U_3$,

$V_4 = U_2$, $V_5 = U_7$, $V_6 = U_1$,

$V_7 = U_6$

(iv) vertex: 5 : 5

graph a => 3, 3, 3, 3, 2

grap b => 4, 3, 3, 2, 2

→ not same degree sequence

(not isomorphic)

(b)

Z = S — Graphs

X = K

V = Z = S = M

T = F

A = R

∞ = gap

Q10.

Source a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	2	4	1	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
ad	2	4			6	5																			
adb		4		3	6	5																			
adbe		4			6	5	6																		
adbec					6	5	6																		
adbeg					6		6			7															
// gf							6	8	10	7															
// gfh								8	10	7	7				14										
// hk								8	10		7			11	14			9							
// hkl								8	10				10	11	13			9							
// klj									10				10	11	13			9							
// iv									10				10	10	13		17		14						
													10	10	13		17		14						
														10	13	12	17		14						
															13	12	12		14						
															13		12		14	13					
															13				14	13					
																			14	13					
																			14	t					

21

16 z

(ii)

Source (a)	D (b)	c	d	e	f	g	z
a	4	3	∞	∞	∞	∞	∞
ac	4	3	6	9	∞	∞	∞
acb	4	3	6	9	∞	∞	∞
acbd	4	3	6	7	11	∞	∞
acbde	4	3	6	7	11	12	∞
acbdef						12	18
acbdefg							16

Q11 (i)

A + B + C + D + A

30 + 30 + 25 + 40

=> 125

(ii)

A + B + C + D + A

20 + 30 + 12 + 35

= 97

Q12 (a)

$$\begin{tikzcd}
A \arrow[d] & B \arrow[r] & C \arrow[d] \\
H \arrow[r] & G \arrow[u] \arrow[d] & D \arrow[l] \\
& F \arrow[r] & E
\end{tikzcd}$$

(b)

$$\begin{tikzcd}
A \arrow[r] & G \arrow[r] & H \arrow[r] & I \arrow[d] \\
& F \arrow[d] & H \arrow[l] & E \arrow[l] \\
B & E \arrow[r] & K \arrow[d] & \\
C \arrow[u] & D \arrow[l] & J \arrow[l] &
\end{tikzcd}$$

(Euler path)

Q13 (i) Hamiltonion circuit exist,

Path : $v_0, v_1, v_2, v_6, v_5, v_4, v_7, v_3$

Circuit : all above and, v_0

(ii) Hamitoniom circuit does'nt exist.

(iii) Hamiltonion circuit exist,

Path : d, c, b, a, g, f, e

Circuit : all above, d

Q14 (a)(i)

→ vertices have even degree.

euler circuit exist,

$v_1, v_3, v_5, v_4, v_5, v_2, v_3, v_4, v_1$

(ii) Some vertices have odd degree's euler circuit does not exist.

(b)(i) vertices f, e, h, w have odd degrees, euler path does not exist.

(ii) Path exist:

$u, v_1, v_0, v_7, u, v_2, v_3, v_4, v_2, v_6, v_5, w, v_6, v_4, w$

→ Two vertices have odd degree. not more than two.

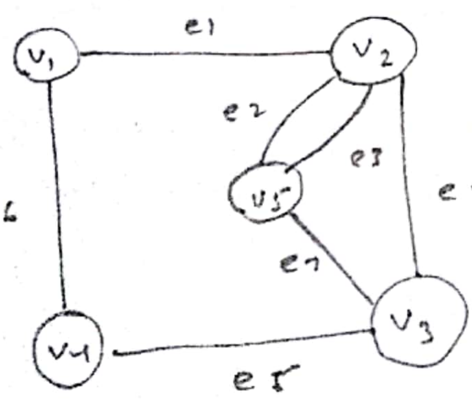
Q15 (a) (i)

$$
\begin{array}{c|ccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\
\hline
v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
v_2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
v_3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
v_5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
v_6 & 1 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

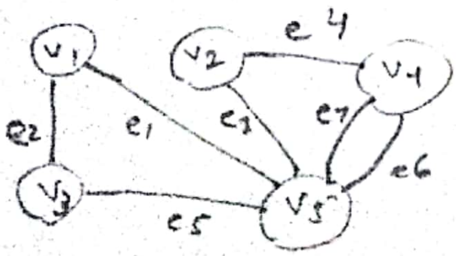
(ii)

$$
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
$$

(b) (i)



(ii)



Q16 (i)

$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
a & 1 & 1 & 1 & 1 \\
b & 0 & 0 & 0 & 1 \\
c & 1 & 1 & 0 & 0 \\
d & 0 & 1 & 1 & 1
\end{array}
$$

(ii)

Vertices		adjacents
a	:	b, d
b	:	a, c, d, e
c	:	b, c
d	:	a, e
e	:	c, e

(iii)

Vertex	adjacents
a	b, d
b	a, c
c	b, d
d	a, c

(iv)

Vertex	adjacents
a	a, c, d
b	b, c, d
c	a, c, b
d	a, b, d

Q17 (i) 3

(ii) 0

(iii) 5

(iv) u, v .

(v) d

(vi) k, L

(vii) m, s, n, r, y

(ix) v, n, h, d, a

(X) j, q, v, L, s, ,
x, y, g, u, z, w, p

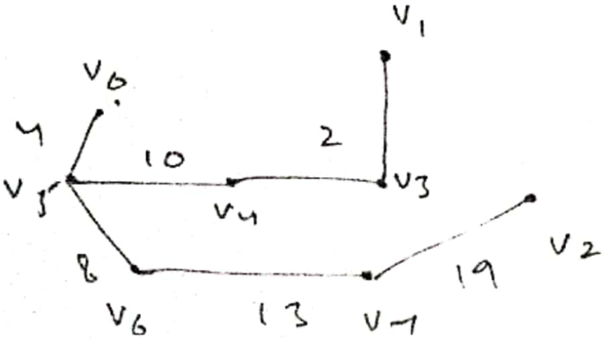
Q18.

(i) ~~$v_0, v_5 = 4$~~

~~$v_0, v_1 = 12$~~

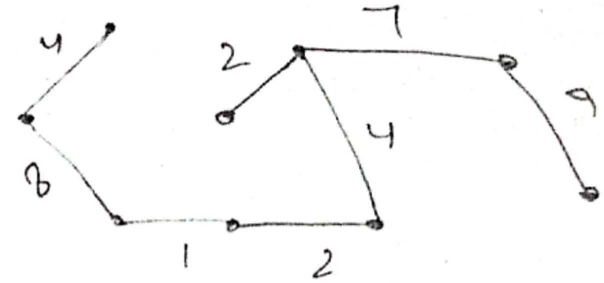
~~$v_1, v_2 = 20$~~

~~$v_2, v_7 = 19$~~



cost = 61

(ii)



Q19

(i) → edges weight ascending,

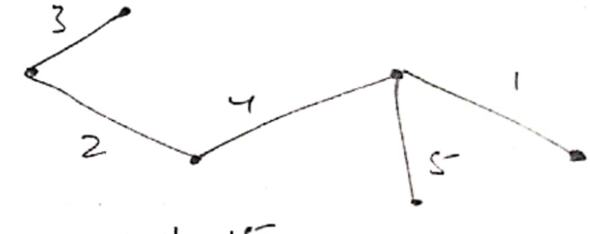
d, f = 1, ~~a,c~~ d, e = 5

a, c = 2   f, e = 6.

a, b = 3   ~~e, f = 6~~

~~b, c = 3~~

c, d = 4



cost = 15

(ii)

g, f = 1   ~~e, f = 4~~

a, b = 3   ~~g, e = 5~~

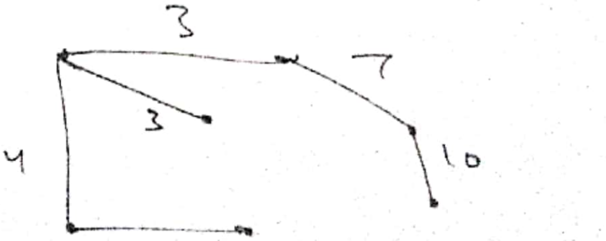
a, c = 3   b, c = 7

a, g = 4   ~~f, e = 7~~

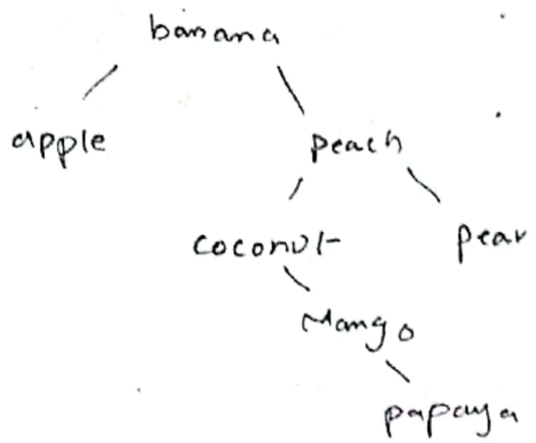
~~b, e = 4~~   c, d = 10

~~d, e = 11~~

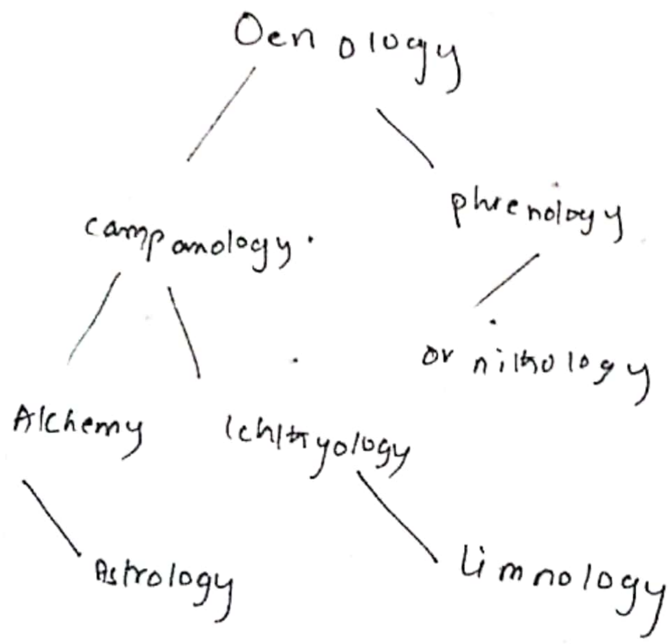
~~d, f = 12~~



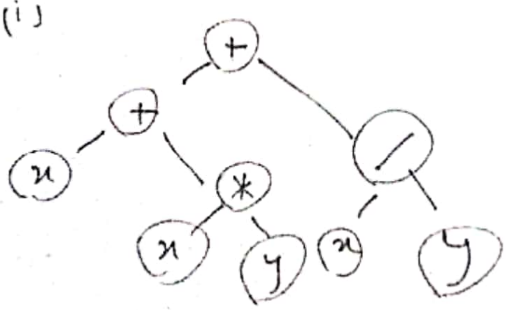
Q20, (i)



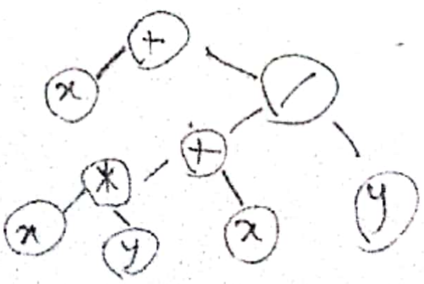
(ii)



(b) (i)



(ii)



Q21

(i) Preorder: abeklmfgnrscdholjpq

Inorder: kelmbfrnsgacohdipjq

Postorder: klmfrsngbcohlpqjda

(ii)

Pre: abdeijmnocfghklp

In: dbiemjnoafcgkhpl

Post: dimnojebfgkplhca

Q22

(a) edges = |v-1|

e = 1000-1 = 999

(b)

1000 * 2 = 2000

(c) m-ary tree

formula ⇒ $n = mi + 1$

⇒ $m = 5$, $i = 100$

$n = 500 + 1 = 501$ vertices

Q23 (a) (i)

Prefix: ++x*xy/xy

Postfix: xxy*+xy/+

(ii) Prefix: +x/+*xyxy

Postfix: xxy*x+y/+

(b) (i) ⇒ 4

(ii) ⇒ 3

Q24

(i) it is not a full m-ary

a), some nodes two childs

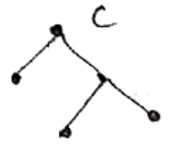
and some have three.

(ii)

it is not balanced.

(iii)

(a) c



b) f

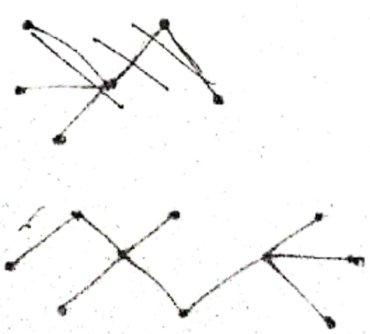
c) - q

Q25.

(i)

(ii)

(iii)



~~Q26~~

~~(a)~~

~~=>~~

~~(b)~~

~~Q27 (a)~~

Q27 (a) reflexive ✓ (a,a) (b,b) (c,c)

(b) symmetric X (a,c) (c,a)

(c) antisymetric X (b,c) $\in R$

(c,b) $\in R$

(d) Transitive X (a,c) $\in R$

(c,b) $\in R$

(a,b) $\notin R$

(e) irreflexive X

(f) Assymetric X

(Q28)

(a) {(0,0), (1,1), (2,2), (3,3), (4,4)}

(b) {(1,3), (2,2), (3,1), (4,0)}

(c) {(1,0), (2,0), (3,0), (4,0), (2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}

(d) {(1,0), (2,0), (3,0), (4,0), (1,1), (1,2), (2,2), (1,3), (3,3)}

(e) {(1,0), (0,1), (1,1), (1,2), (1,3), (2,1), (3,1), (4,1), (2,3), (3,2), (4,3)}

(f) {(1,2), (2,1), (2,2)}

Q29

$R = \{(1,1), (1,2), (1,3)$
$(1,4), (1,5), (1,6),$
$(2,1), (3,3), (3,6),$
$(4,4), (5,5), (6,6)$
$\}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Q30 (a)

$\{(2,2), (2,3), (2,4), (3,2)$
$(3,3), (3,4)\}$

1. R is not reflexive
not have (1,1) (4,4)
2. R_4 is not symmetric
3. R is not anti-symtric
4. R_4 is Transitive.

(b) is symtric, reflexive
and Transitive.

(c) R is symetric
only.

(d) R is anti-symtric.

(e) R is reflexive, symtric,
anti-symtric and Transitive

(f) R is not reflexive,
symetric, anti-symmetric and
transitive.

Q31 (a) – not symmetric
– is anti symtric
– is Transitive

(b)
– is reflexive
– is symtric
– not anti-symtric
– is Transitive

(c) → is reflexive, symmetric
and transitive

(d) relation is reflexive
and symmetric.

Q32) (a) $\{(1,1), (2,2), (3,3),$
$(4,4)\}$

(b) $\{(1,2), (2,1), (3,4)\}$

Q34) (a)(i) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) (i) $R = \{(1,1), (1,3), (2,2), (3,1)$
$(3,3)\}$

(ii) $R = \{(1,2), (2,2), (3,2)\}$

(iii) $R = \{(1,1), (1,2), (1,3), (2,1)$
$(2,2), (3,1), (3,2), (3,3)\}$

Q36 (a)

i) $T_1 = 2^n - 1$ $\quad T_2 = 2^2 - 1$ $\quad T_3 = 2^3 - 1$, $\quad T_4 = 2^4 - 1$, $\quad T_5 = 2^5 - 1$

$n = 1$ $\quad T_2 = 3$ $\quad T_3 = 7$ $\quad T_4 = 15$ $\quad T_5 = 31$

$\Rightarrow 1, 3, 7, 15, 31, \dots$

(ii) $8.5, 7, 5.5, 4, 2.5$

(iii) $-1, \frac{1}{4}, \frac{-1}{9}, \frac{1}{16}, \frac{-1}{25}$

(iv) $7, \frac{10}{3}, \frac{13}{5}, \frac{16}{7}, \frac{19}{9}$

Q40)

(a) Propositional logic: Digital circuit, Logical Reasoning

(b) Prediacte Quantifiers: Database SQL Queries, Mathematic Proof

(c) Sets: Social Networks, Data Representation.

(d) functions: Economic Modeling, Signal Processing,

(e) Relations: Database Managment & Social Network analysis.

(f) Sequences: computer Algorithms.

(g) graph ~~Sequence and Series~~: Machine learning, Data Representation.

(h) Trees: Machine learning, Hierchical structure.