

Φ_2 ,

$$\left[\begin{array}{ccc} 3 & 4 & 2 \\ 2 & -3 & 1 \\ 5 & 7 & 1 \end{array} \right] \quad \left[\begin{array}{ccc} 1 & 7 & 1 \\ 0 & 17 & 1 \\ 0 & 7 & 1 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{ccc} 1 & 7 & 1 \\ 0 & -17 & -1 \\ 0 & -28 & -4 \end{array} \right]$$

$R_1 - R_2$

$R_2 - 2R_1$

$R_3 - 8R_1$

$$\left[\begin{array}{ccc} 1 & 7 & 1 \\ 0 & 17 & 1 \\ 0 & 7 & 1 \end{array} \right]$$

$-R_2$

$R_3 / -4$

Yes! they are now equivalent.

as their elements become identical
after the ERO's above.

columns.

Q2-

(i)

$$\left[\begin{array}{ccccc|c} x & y & z & w & b \\ 1 & -7 & 0 & 6 & -5 \\ 0 & 0 & 1 & -2 & -3 \\ 1 & -7 & 4 & -2 & -6 \end{array} \right]$$

(iii)

$$\left[\begin{array}{ccccc|c} 1 & -7 & 0 & 6 & -5 \\ 1 & -7 & 4 & -2 & -6 \\ 0 & 0 & 1 & -2 & -3 \end{array} \right]$$

$R_2 - R_1$

$$\left[\begin{array}{ccccc|c} 1 & -7 & 0 & 6 & -5 \\ 0 & 0 & 4 & -8 & -1 \\ 0 & 0 & 1 & -2 & -3 \end{array} \right]$$

$$R_2 - 4R_3 \rightarrow \left[\begin{array}{ccccc} 1 & -7 & 0 & 6 & -5 \\ 0 & 0 & 0 & 0 & 11 \\ 0 & 0 & 1 & -2 & -3 \end{array} \right]$$

\Rightarrow No solution as second leading 1 is zero. or $0 \neq 11$
inconsistent.

(ii)

$$\begin{array}{cccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & b \\ \hline 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{array}$$

$$\left[\begin{array}{ccccc|c} 3 & -7 & 8 & -8 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & -12 & 6 & -15 & 2 & 20 \end{array} \right] \xrightarrow{R_3 + R_1}$$

$R_3 - R_1$
updated

$$R_2 - R_1 \rightarrow \left[\begin{array}{ccccc|c} 3 & -7 & 8 & -8 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{array} \right]$$

$$\left[\begin{array}{cccccc} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{array} \right]$$

$$\begin{array}{l} R_1/3 \\ R_2 - R_3 \\ R_3 + 3R_2 \end{array} \left[\begin{array}{cccccc} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & -1 & 2 & -2 & -2 & -1 \\ 0 & 0 & 0 & 0 & -2 & -8 \end{array} \right]$$

$R_3 + 3R_2$
updated

$$\begin{array}{l} R_1 + 3R_2 \\ -R_2 \end{array} \left[\begin{array}{cccccc} 1 & 0 & -2 & 3 & 8 & 8 \\ 0 & 1 & -2 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & -2 & -8 \end{array} \right]$$

\Rightarrow No solution, in R_3 no leading 1 left \Rightarrow but a constant value.

$$x_1 - 2x_3 + 3x_4 + 8x_5 = 8$$

$$x_2 - 2x_3 + 2x_4 + 2x_5 = 1$$

$$-2x_5 = -8$$

$$x_5 = 4$$

Inconsistency unique solution.

(iii)

[Incomplete]

$$\left[\begin{array}{ccc|c} x & y & z & b \\ 35 & 6 & -7 & 18 \\ 90 & 15 & -21 & 40 \\ 25 & 3 & -7 & 11 \end{array} \right]$$

$$\begin{array}{l} R_1 - R_3 \\ \Rightarrow \\ R_2/3 \end{array} \left[\begin{array}{ccc|c} 10 & 3 & 0 & 4 \\ 30 & 5 & -7 & 40/3 \\ 25 & 3 & -7 & 11 \end{array} \right]$$

$$\begin{array}{l} R_1/10 \\ \Rightarrow \\ R_2 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 3/10 & 0 & 4/10 \\ 0 & -10 & -7 & 4/3 \\ 5 & -3 & -7 & 3 \end{array} \right]$$

$$R_3 - 2R_1$$

$$\begin{array}{l} +10R_2/-10 \\ \Rightarrow \\ R_3 - 5R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 3/10 & 0 & 4/10 \\ 0 & 1 & 7/10 & 2/15 \\ 0 & -9/2 & -7 & 1 \end{array} \right]$$

$$\begin{array}{l} 10R_1 \\ 2R_3 \end{array} \left[\begin{array}{ccc|c} 10 & 3 & 0 & 4 \\ 0 & 1 & 7/10 & 2/15 \\ 0 & -9 & -14 & 2 \end{array} \right]$$

$$\left[\begin{array}{cccc} 10 & 3 & 6 & 4 \\ 0 & 1 & 7/10 & 2/15 \\ 0 & -9 & -14 & 2 \end{array} \right]$$

$$R_1 - 3R_2 \rightarrow \left[\begin{array}{cccc} 10 & 0 & -21/10 & 18/5 \\ 0 & 1 & 7/10 & 2/15 \\ 0 & 0 & -77/10 & 16/5 \end{array} \right]$$

$$R_3 + 9R_2 \rightarrow \left[\begin{array}{cccc} 10 & 0 & -21/10 & 18/5 \\ 0 & 1 & 7/10 & 2/15 \\ 0 & 0 & 1 & -32/77 \end{array} \right]$$

Incomplete

$$R_1/10 \rightarrow \left[\begin{array}{cccc} 1 & 0 & -21/100 & 9/25 \\ 0 & 1 & 7/10 & 2/15 \\ 0 & 0 & 1 & -32/77 \end{array} \right]$$

$$R_3 / (-77/10)$$

$$R_1 + \frac{21}{100} R_3 \rightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 3/11 \\ 0 & 1 & 0 & 14/33 \\ 0 & 0 & 1 & -32/77 \end{array} \right]$$

$$R_2 - \frac{7}{10} R_3 \rightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 3/11 \\ 0 & 1 & 0 & 14/33 \\ 0 & 0 & 1 & -32/77 \end{array} \right]$$

$$x = 3/11$$

$$y = 14/33$$

$$z = -32/77$$

Q3.

(i)

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

$R_2 + R_3$ updated

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 0 & 0 & 2g+h+k \\ 0 & -3 & 5 & k+2g \end{bmatrix}$$

$R_3 + 2R_1$

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & -3 & 5 & k+2g \\ 0 & 0 & 0 & 2g+k+h \end{bmatrix}$$

$$2g + k + h = 0$$

(ii)

$$\begin{bmatrix} 1 & 3 & : & a \\ c & d & : & b \end{bmatrix}$$

$c \neq -3$ & $d \neq -3$

if c & d become -3
then the solution would be
infinite or no-solution.

- For a consistent and unique solution $c = 0$ & $d = 1$ should be the values.

Q4

(i)

$$\left[\begin{array}{ccc|c} x & y & z & b \\ 3 & 1 & 0 & 6 \\ 5 & 1 & 1 & 6 \\ 3 & 0 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 5 \\ 2 & 1 & 0 & 5 \\ 1 & -1 & 1 & -4 \end{array} \right] \quad \begin{matrix} R_1 - R_3 \\ R_2 - R_3 \\ R_3 - R_2 \text{ update} \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 3 & -2 & 13 \\ 0 & 1 & -1 & 5 \end{array} \right] \quad \begin{matrix} R_2 - 2R_1 \text{ update} \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & 2 \end{array} \right] \quad \begin{matrix} R_1 - R_3 \\ R_2 - 2R_3 \\ R_3 - R_2 \text{ update} \end{matrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right] \quad \begin{matrix} -R_3 \end{matrix}$$

$$x = -9, \quad y = 3, \quad z = -2$$

Point in common is,

$$(-9, 3, -2)$$

- (ii)

$$\begin{bmatrix} x & y & z \\ 1 & a & 2 \\ 4 & 8 & B \end{bmatrix}$$

$$R_2 - 4R_1 \quad \begin{bmatrix} 1 & a & 2 \\ 0 & 8-4a & B-8 \end{bmatrix}$$

* Remaining

Q4.

(ii)

$$\begin{bmatrix} x & y & B \\ 1 & a & 2 \\ 4 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & a & 2 \\ 0 & 8 - 4a & B \\ R_2 - 4R_1 \end{bmatrix}$$

(a) No Solution

$$8 - 4a = 0$$
$$8 = 4a$$
$$a = 2$$

$$a = 2, b \neq 2$$

(b) Unique solution

$$8 - 4a = 1$$

$$8 - 1 = 4a$$

$$\frac{7}{4} = a$$

$$a = 7/4 \quad , \quad B = \mathbb{Z}$$

/

any

integer

or Real number

(c) Infinitely many Solution

$$a = 2 \quad , \quad b = 2$$

as,

$$\boxed{\begin{array}{l} 8 - 4a = 0 \\ a = 2 \end{array}}$$

Q5,

(i)

$$\begin{bmatrix} x & y & z \\ 1 & 2 & -3 \\ 3 & -1 & 3 \\ 4 & 1 & a^2 - 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ -7 & 12 & -11 \\ 0 & -7 & a^2 - 10 \end{bmatrix}$$

$R_2 - 3R_1$

$R_3 - 4 \cdot R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -7 & 12 \\ 0 & 0 & a^2 - 22 \end{bmatrix}$$

$$\begin{aligned}
 (ax+1) - 4(a-1) &= 0 \\
 a+4 - 4a + 4 &= 0 \\
 a - 12 &= 0
 \end{aligned}$$

(ii)

no solution

$$a^2 - 22 = 0$$

$$a^2 = 22$$

$$\sqrt{a} = \sqrt{22} \Rightarrow a = \sqrt{22}$$

(iii)

'unique' solution

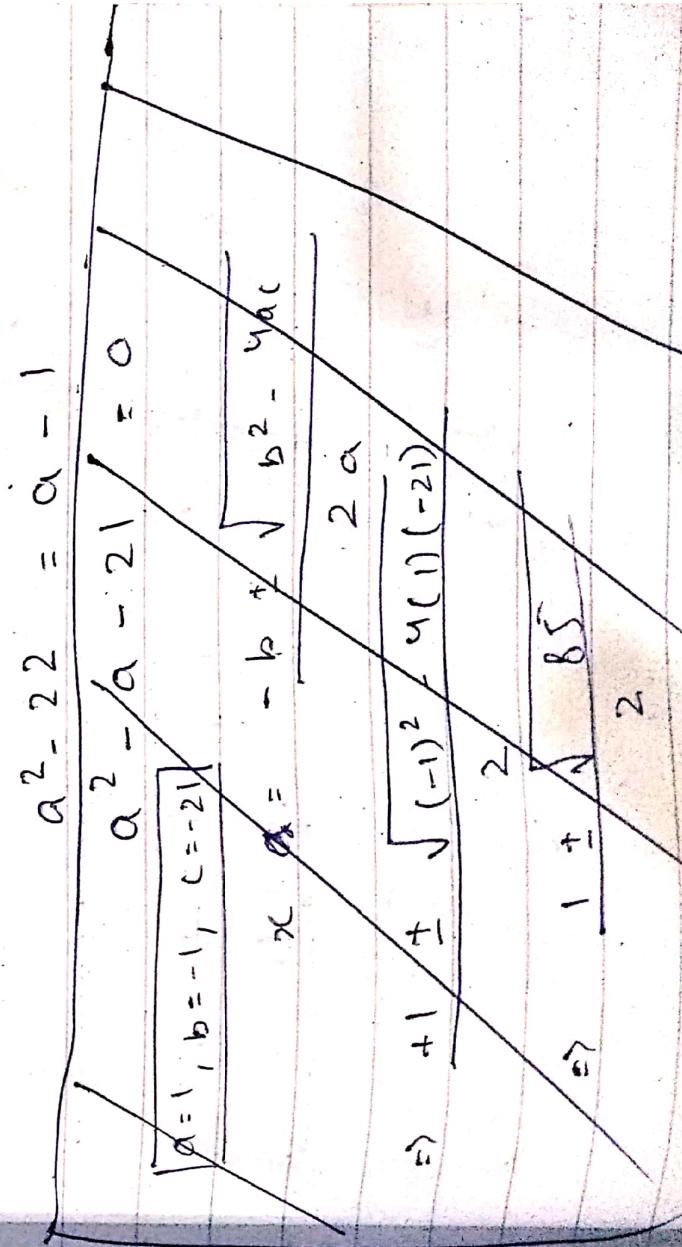
$$a^2 - 22 = 1$$

$$a^2 = 23$$

$$a = \sqrt{23}$$

(iv)

infinitely many solutions



$$\alpha = -1$$

$$\begin{aligned}\alpha^2 - 2\alpha &= 1 \\ \alpha^2 &= \alpha + 21 \\ \alpha &= \sqrt{\alpha + 21}\end{aligned}$$

(ii)

$$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \alpha - \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} \alpha = \begin{bmatrix} 2 & -2 \\ 5 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3-1 & 1-4 \\ -1-2 & 2-0 \end{bmatrix} \alpha = \begin{bmatrix} 2 & -2 \\ 5 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix} \alpha = \begin{bmatrix} 2 & -2 \\ 5 & 4 \end{bmatrix}$$



inverse:

$$R_1 - R_2 \rightarrow \begin{bmatrix} -1 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned}-R_1 &\rightarrow \begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -3 & 4 \end{bmatrix} \\ R_2 + 3R_1 &\end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & -1 & 1 \\ 0 & 5 & 1 & -3 & 4 \end{bmatrix}$$

$$R_1 - R_2 \rightarrow \begin{bmatrix} 1 & 0 & -2/5 & 1/5 \\ 0 & 1 & -3/5 & 4/5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2/5 & 1/5 \\ -3/5 & 4/5 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} -2/5 & 1/5 \\ -3/5 & 4/5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 5 & 4 \end{bmatrix}$$

$$\begin{aligned} & (-3/5 \times 2) + (4) \\ \Rightarrow & \begin{bmatrix} (-2/5 \times 2) + 1 & (-3/5 \times 2) + (4/5 \times 4) \\ (+2/5 \times 2) + (\frac{1}{5} \times 4) & (+3/5 \times 2) + (4/5 \times 4) \end{bmatrix} \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 14/5 & 14/5 \\ 8/5 & 22/5 \end{bmatrix} \text{ Ans.}$$

(Q6)

i) $\begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix}$

$$\begin{array}{l} -R_1 \\ R_2 - 3R_1 \\ R_3 + 2R_1 \\ R_4 - R_1 \end{array} \rightarrow \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & -4 & 8 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix}$$

$$\begin{array}{l} R_1 + 3R_4 \\ -R_2 \\ R_3 \\ R_4 + R_2 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 6 & 2 \\ 0 & 1 & -8 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -6 & 2 \end{bmatrix}$$

update

$$\begin{array}{l} R_1 - GR_3 \\ R_2 + 8R_3 \\ R_3 / 3 \\ R_4 + 6R_3 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 2R_4 \\ R_2 - R_4 \\ R_3 - R_4 \\ R_4/2 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

\Rightarrow Matrix is invertible as it have a Reduced echelon form with unique solutions.

(ii)

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} : \begin{bmatrix} 5 & 3 & 2 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 - R_2 \rightarrow \begin{bmatrix} 2 & 0 & 0 & 1 & -1 & 0 \\ 1 & 3 & 2 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{update}$$

$$\begin{bmatrix} 1 & 3 & 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ 2 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$R_1 - 3R_2 \rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & -3 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$R_3 - 2R_1 \rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & -3 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & -1 & 1 & 6 \end{bmatrix}$$

$$\begin{aligned} R_1 + R_3 &\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1/2 & 1/2 & 0 \end{bmatrix} \\ (Up) R_2 - R_3 &\rightarrow \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \\ R_3/2 &\rightarrow \begin{bmatrix} 0 & 0 & 2 & -1 & 1 & 6 \end{bmatrix} \end{aligned}$$

inverse :

$$\begin{bmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 3 \\ -\frac{1}{2} & \frac{1}{2} & 3 \end{bmatrix}$$

SOL:

$$\left[\begin{array}{ccc|c} \frac{1}{2} & \frac{1}{2} & 0 & 4 \\ \frac{1}{2} & -\frac{1}{2} & -2 & 2 \\ -\frac{1}{2} & \frac{1}{2} & 3 & 5 \end{array} \right]$$

$$3 \times \begin{bmatrix} 3 & 3 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 16 \\ 2 & -1 & -16 \\ -2 & 1 & 15 \end{array} \right]$$

$$\Rightarrow \begin{bmatrix} 3 & x_1 \\ -9 & x_2 \\ 14 & x_3 \end{bmatrix}$$

(ii)

$$\left(\begin{array}{ccccc|c} z_1 & z_2 & z_3 & z_4 & z_5 & b \\ 0 & 0 & 1 & 1 & 1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & -1 & -2 & 0 & -1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \end{array} \right)$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 0 & -1 & 0 \\ R_2 + R_1 & 0 & 0 & 0 & -3 & 0 \\ R_3 - R_1 & 0 & 0 & 0 & 0 & 0 \\ R_4 - 2R_1 & 0 & 0 & 3 & 0 & 0 \end{array} \right]$$

⇒ This matrix have no solution.

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

⇒ The matrix have no solution.
as there is no second leading 1.

(ii)

$$(i) \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 3 & 1 & -2 & 0 & 1 & 0 \\ -5 & -2 & 3 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & -2 & -7 & 5 & 0 & 1 \end{bmatrix}$$

$R_2 - 3R_1$
 $R_3 + 5R_1$

$$\begin{array}{l} R_1 + 2R_3 \\ R_2 - 4R_3 \\ R_3 + 2R_2 \end{array} \begin{bmatrix} 1 & 0 & 0 & -1 & 4 & 2 \\ 0 & 1 & 0 & 1 & -7 & -4 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{bmatrix}$$

inverse :

$$\begin{bmatrix} -1 & 4 & 2 \\ 1 & -7 & -4 \\ -1 & 2 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} -2 & -7 & -9 & 1 & 1 & 0 & 0 \\ 2 & 5 & 6 & 0 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{array}{l} R_1 + R_2 \\ R_1 - R_3 \\ R_2 - 2R_3 \end{array} \begin{bmatrix} 0 & -2 & -3 & 1 & 1 & 0 & 5 \\ 0 & -1 & -2 & 0 & 1 & -2 & 5 \\ 1 & 3 & 4 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 4 & 0 & 0 & 1 & 5 \\ 0 & -2 & -3 & 1 & 1 & 0 & 5 \\ 0 & -1 & -2 & 0 & 1 & -2 & 1 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 3R_2 \\ -R_2 \\ R_3 + 2R_2 \end{array} \begin{bmatrix} 1 & 0 & -2 & 0 & 3 & -5 \\ 0 & 1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & -1 & 4 \end{bmatrix}$$

$$\begin{array}{l} R_1 + 2R_3 \\ R_2 - 2R_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 2 & 1 & 3 \\ 0 & 1 & 0 & -2 & -1 & -6 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{bmatrix}$$

inverse:

$$\begin{bmatrix} 2 & 1 & 3 \\ -2 & 1 & -6 \\ 1 & -1 & 4 \end{bmatrix}$$

Q 8.

$$A \Rightarrow \begin{bmatrix} x & y & z & b_1 & b_2 & b_3 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 4 & -1 & 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

\Rightarrow

$$\begin{bmatrix} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & -5 & -4 & 1 & 0 & 1 \\ 0 & -3 & -2 & 0 & 1 & 1 \\ 0 & -2 & -4 & -2 & 0 & 1 \end{bmatrix}$$

$$R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & -2 \\ 0 & -2 & -4 & -2 & 0 & 1 \\ 0 & -2 & -4 & -2 & 0 & 1 \end{bmatrix}$$

$$R_3 - R_2$$

update

$$\begin{bmatrix} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & -2 \\ 0 & -2 & -4 & -2 & 0 & 1 \\ 0 & -2 & -4 & -2 & 0 & 1 \end{bmatrix}$$

$$R_1 - 3R_2$$

$$R_2 - R_3$$

$$R_3$$

$$\begin{bmatrix} 1 & 0 & 17 & 7 & 6 & 3 \\ 0 & 1 & -5 & -2 & -2 & -1 \\ 0 & 0 & 14 & 2 & 1 & 3 \end{bmatrix}$$

$$-2 + \frac{5}{14} \left(\frac{6}{14} \right)$$

$$R_1 - 17R_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & -2/14 & -1/14 & -43/14 \\ 0 & 1 & 0 & 1/14 & -3/14 & 11/14 \\ 0 & 0 & 1 & 6/14 & 5/14 & R_3/14 \end{bmatrix}$$

$$R_2 + 5R_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & -2/14 & -1/14 & -43/14 \\ 0 & 1 & 0 & 1/14 & -3/14 & 11/14 \\ 0 & 0 & 1 & 6/14 & 5/14 & R_3/14 \end{bmatrix}$$

$$x_1 = -2/14, \quad -1/14, \quad -43/14$$

$$x_2 = 1/14, \quad -3/14, \quad 11/14$$

$$x_3 = \frac{6/14}{14} = \frac{3}{14}, \quad \frac{5}{14}, \quad \frac{5}{14}$$