K226007

NC Assignment 2

Q1. Langrange Interpolation

	x_0	x_1	x_2	x_3	x_4	x_5
x	0	20	40	60	80	100
Y	26	-48.6	61.6	-71.2	74.8	-75.2

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)}$$

$$\Rightarrow \frac{(x-20)(x-40)(x-60)(x-80)(x-100)}{(0-20)(0-40)(0-60)(0-80)(0-100)}$$

$$\Rightarrow \boxed{x = 50}$$

$$\Rightarrow \frac{(50-20)(50-40)(50-60)(50-80)(50-100)}{(-20)(-40)(-60)(-80)(-100)}$$

$$L_0(50) \Rightarrow \frac{-4500000}{} = 0.011718 \Rightarrow \frac{3}{256}$$

$$P_x = f(x_0)L_0(x) + f(x_1)L_1(x) + f(x_2)L_2(x) + f(x_3)L_3(x) + f(x_4)L_4(x) + f(x_5)L_5(x)$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}$$

$$= \frac{(x-0)(x-40)(x-60)(x-80)(x-100)}{(20-0)(20-40)(20-60)(20-80)(20-100)}$$

$$L_1(50) = \frac{(50-0)(50-40)(50-60)(50-80)(50-100)}{(20)(-20)(-40)(-60)(-80)}$$

$$= \frac{-25}{256} \approx -0.097656$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}$$

$$L_2(x) = \frac{(x-0)(x-20)(x-60)(x-80)(x-100)}{(40-0)(40-20)(40-60)(40-80)(40-100)}$$

$$L_2(50) = \frac{(50-0)(50-20)(50-60)(50-80)(50-100)}{(40)(20)(-20)(-40)(-60)}$$

$$\frac{75}{128} \Rightarrow 0.58593$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_5-x_4)(x_3-x_5)}$$

$$= \frac{(x-0)(x-20)(x-40)(x-80)(x-100)}{(60-0)(60-20)(60-40)(60-80)(60-100)}$$

$$L_3(50) = \frac{(50-0)(50-20)(50-40)(50-80)(50-100)}{(60)(40)(20)(-20)(-40)}$$

$$\frac{75}{128} = 0.58593$$

$$L_4(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)}$$

$$= \frac{(x-0)(x-20)(x-40)(x-60)(x-100)}{(80-0)(80-20)(80-40)(80-60)(80-100)}$$

$$L_4(50) = \frac{(50)(50-20)(50-40)(50-60)(50-100)}{(80)(60)(40)(20)(-20)}$$

$$= \frac{-25}{256} \approx -0.097656$$

$$L_5(50) = \frac{(50)(50-20)(50-40)(50-60)(50-80)}{(100)(100-20)(100-40)(100-60)(100-80)}$$

$$= \frac{3}{256} \approx 0.011718$$

$$P(50) = (2.6)\left(\frac{3}{256}\right) + (-48.6)\left(\frac{-25}{256}\right) + (61.6)\left(\frac{75}{128}\right)$$
$$+ (-71.2)\left(\frac{75}{128}\right) + (74.8)\left(\frac{-25}{256}\right) + (-75.2)\left(\frac{3}{256}\right)$$

$$\Rightarrow \frac{-11215}{1280} \approx -8.76016$$

* Newton forward diff

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots\dots$$

x	f(x)	1st	2nd	3rd	4th	5th
0	2.6					
20	48.6	1.13				
40	61.6	0.65	+0.012			
60	71.2	0.48	-4.25×10^{-3}	1.29×10^{-4}		
80	74.8	0.18	-7.5×10^{-3}	-5.42×10^{-5}	-2.29×10^{-6}	
100	75.2	0.02	-4×10^{-3}	5.83×10^{-5}	1.41×10^{-6}	3.7×10^{-8}

$x = 50$

$$P_n = 26 + 1.13(x-0) + (-0.012)(x)(x-20) + (1.29\times10^{-4})(x)(x-20)(x-40) + (-2.29\times10^{-6})(x)(x-20)(x-40)(x-60) + (3.7\times10^{-8})(x)(x-20)(x-40)(x-60)(x-80)$$

$\Rightarrow 28.78024$