MRI Signal Processing and Image Quality Modeling

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Introduction

A giant magnet and radio waves. One way to think of Magnetic Resonance Imaging (MRI) is as a giant magnet that creates a strong magnetic field around your body. This magnetic field aligns the protons in your body's atoms. Radio waves are then used to excite these protons, causing them to emit a signal that can be detected by the MRI machine (National Institute of Biomedical Imaging and Bioengineering, 2018). The strength of the signal depends on the density and arrangement of the protons in different tissues, which allows the MRI to create detailed images of the body. This paper will first explore the radio waves and the importance of the Larmor equation, go into further detail on magnetic field strength and its effects on imaging quality as well as the general application of magnetic and electric fields.

Radio Waves and Larmor Equation

MRI uses radio waves to create images of the body. The patient is placed in a strong magnetic field, causing the atomic nuclei to align. Radio waves are then applied to the body, causing the atomic nuclei to emit energy. The emitted energy is detected and used to create images.

Resonance in MRI refers to the phenomenon of nuclear magnetic resonance (NMR), which is the basis for MRI imaging. NMR occurs when the nuclei of certain atoms, such as hydrogen, absorb and re-emit electromagnetic radiation in the presence of a strong magnetic field. The frequency of the electromagnetic radiation that is absorbed and re-emitted is determined by the Larmor frequency, which is calculated using the Larmor equation. (Jones, 2021).

The Larmor equation is used to calculate the Larmor frequency, which is the rate of precession of the magnetic moment of a particle around an external magnetic field, B_0. By adjusting the strength of the magnetic field and the frequency of the radio waves, it is possible to selectively excite specific types of nuclei (magnetize them) and produce images with different contrasts.

Equation 1. Larmor Equation.

$$\omega = \gamma * B_0$$

ω	Larmor frequency (angular precessional frequency of a hydrogen atom (proton))
B_0	Strength of the magnetic field from the MRI in tesla (T)
γ	Gyromagnetic ratio, in the context of MRI, refers to the gyromagnetic ratio of the hydrogen nucleus (proton). The description of a static magnetic field of 1 T (Tesla), will make the hydrogen nucleus process at a frequency of 42.58 MHz.

The interpretation of Fig. 1 indicates how the magnetization of hydrogen atoms in a magnetic field changes over time. We are considering two components of magnetization by Larmor frequency, defined as the net magnetic moment per unit volume of a sample, such as one that is in the direction of the magnetic field (M_z) and the other one that is perpendicular to it (M_{xy}) (Haacke et al., 2015; Hoult & Lauterbur, 1979). If we imagine a 3D plane we can see how the z-axis is perpendicular to the xy plane.

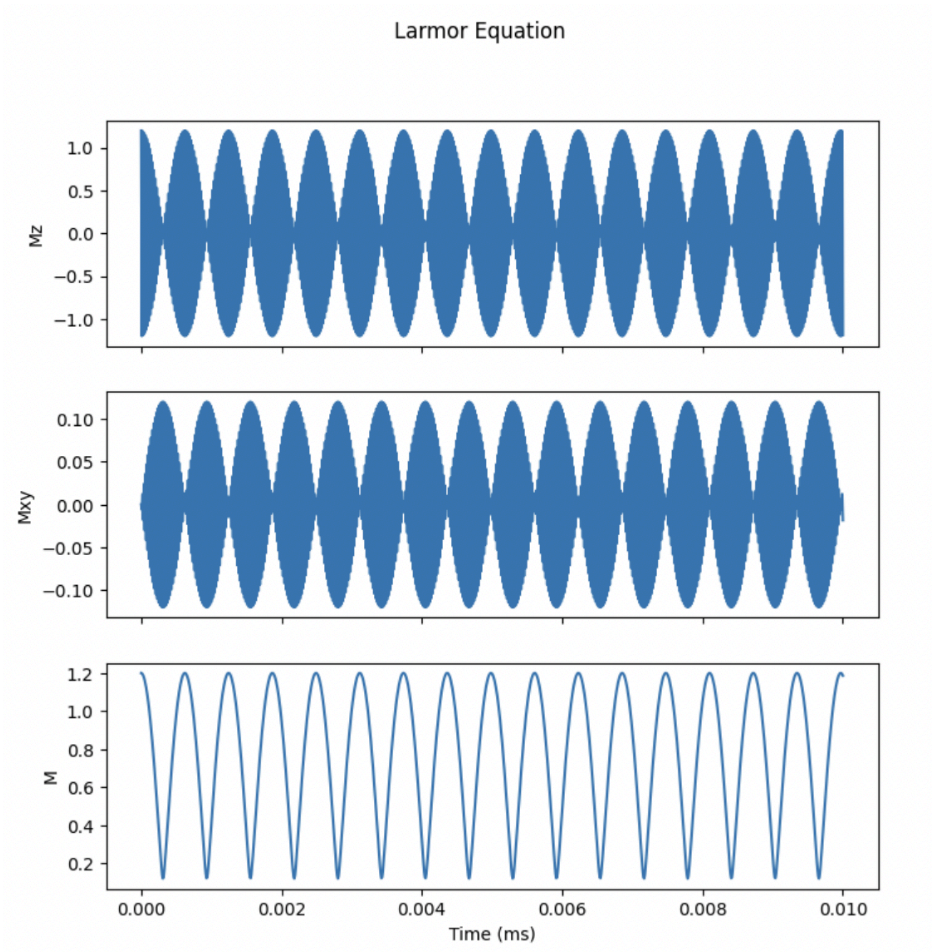


Fig 1: Visualization of how different components of the magnetization of hydrogen atoms change over time in a magnetic field (see code in Appendix). The equations used are Mz = Mz0 * cos(w0 * t) and Mxy = Mxy0 * sin(w0 * t), where w0 is the Larmor frequency and Mz0 or Mxy0 the initial value of magnetization.

What we further observe is that the M_z component and M_{xy} component oscillate together and result in the total 3D magnetization M ($\sqrt{M_{xy}^2 + M_z^2}$) which we call the Larmor frequency.

We can further observe that the M component decreases again after the climax (see figure 1, bottom graph) and we call this free induction decay (FID, see figure 2). This process over time indicates the spins of the hydrogen atoms that become unsynchronized from each other (signal gets weaker as it dephases).

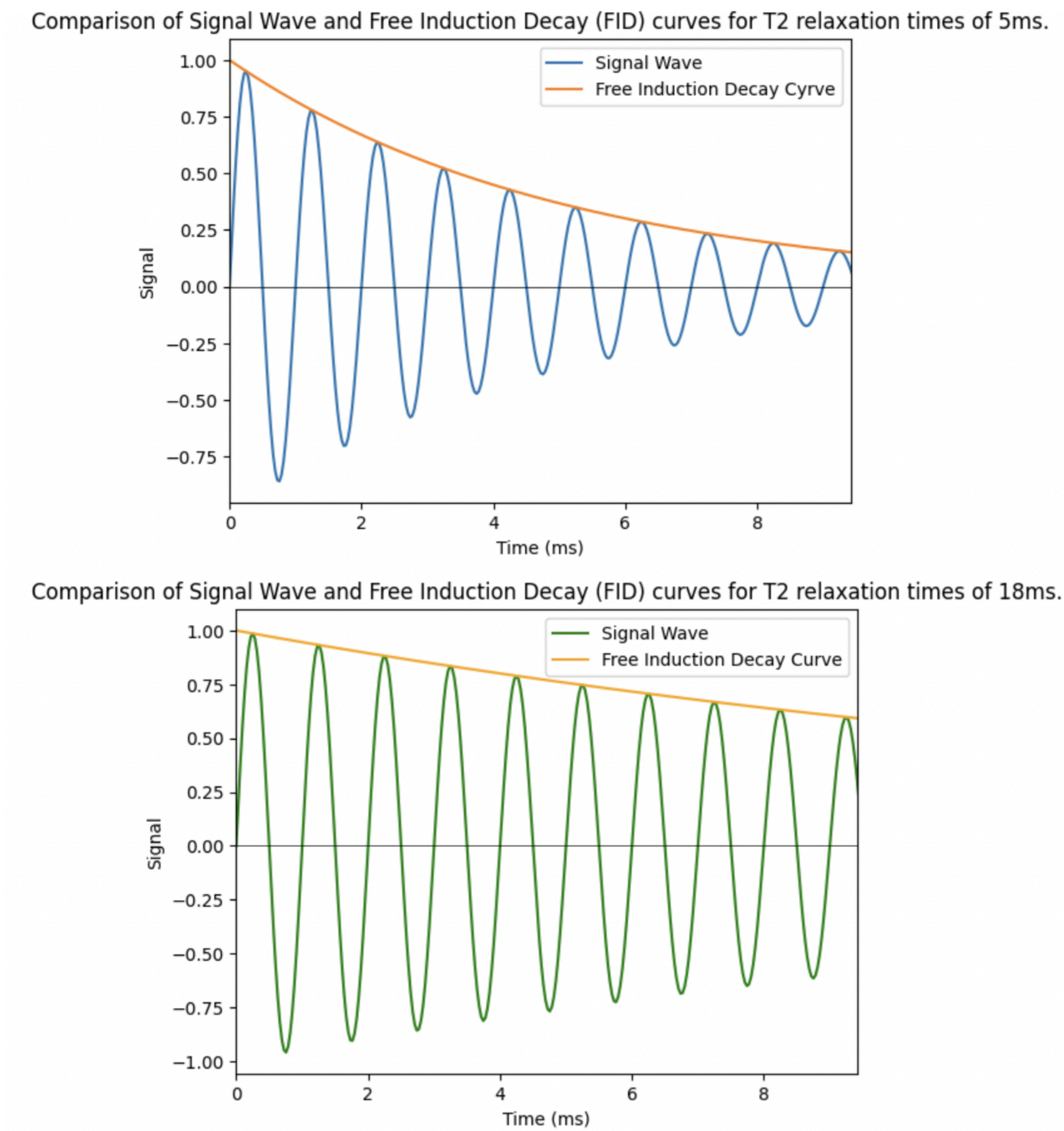


Fig. 2: Visualization of S(t) wave and the free induction decay curve. The top has A = 1. A common tendency we can see is that both of them are decreasing in amplitude over time. The x-axis represents time and the y-axis represents the signal. The billions of Hydrogen-atoms go out of sync as time passes and the signal decreases (see Code in Appendix).

This visualization in Fig. 2 aims to observe the decay behavior of the MRI signal over time. The signal's amplitude decreases over time as the billions of magnetic moments go out of phase. By analyzing the decay behavior of the MRI signal, researchers and scientists can differentiate between different types of tissue. For example, the top of figure 2 has a lower T2 (see equation 3) than the bottom of figure 2 and thus, the signal waves decays slower over time, the amplitude takes longer to decrease and the FID curve has a stronger gradient. For example, fat tissue has a higher T2 value than muscle (Lakrimi et al., 2011).

The amplitude, A, of the detected signal is made up of crucial MRI parameters, such as the number of spins, the magnetic field strength and gyromagnetic ratio. The magnetic moments of billions of hydrogen atoms aligned will add up to a signal that is large enough to detect.

Metrics	Description	Interpretation
f (frequency)	Frequency is the number of cycles of the signal that occur in one second and is directly related to the magnetic field strength and the gyromagnetic ratio of the spins contributing to the signal.	In Fig. 2 the green and blue curves have constant frequency of 1 Hz
λ **(wavelength)**	Wavelength is the distance between two successive peaks or troughs of the signal. The wavelength of the MRI signal depends on the frequency of the electromagnetic radiation used in the imaging process.	$\lambda = \frac{c}{f\ (frequency)}$ - c is the speed of light and - f is the frequency (constant) λ = c / f λ = 3 x 10^8 m/s / 1 Hz **λ = 3 x 10^8 m (constant for both green and blue curves)**

Time Period (T)	Time takes for a full oscillation.	1 ms/cycle for both the plots.
A (amplitude)	Amplitude is the maximum displacement of the signal from its equilibrium position. Additionally, the wavelength is the distance between two successive peaks or troughs of the signal.	The amplitude decreases over time (until 0) as the signal decreases and the hydrogen atoms dephase.

Table 1 shows wave properties of FID in Fig. 2

Equation 2. Parameters making up the amplitude of the detected waves in MRI.

$$A = N * sin(\phi) * \gamma * B$$

N	The amount of hydrogen atom spins in the imaged tissue.
$sin(\phi)$	The pulse angle which refers to the angle that the magnetic moment vector of the hydrogen atom is tipped by the radiofrequency pulse (note that its max is at $\phi = 90°$). It is directly proportional to the magnitude of the pulse.
γ	Gyromagnetic ratio.
B	Magnetic field strength of the MRI [T].

Equation 3. Free Induction Decay.

$$S(t) = A * sin(\omega * t) * e^{-t/T_2}$$

Wave function for a decaying sine curve, representing the signal generated from the initially aligned magnetic moments of Hydrogen atoms, dephasing over time (adapted from Long & McLauchlan, 2021).

A	The maximum amplitude of a wave over a single cycle of oscillation, as described in equation 2.
$sin(\omega * t)$	Models the sinusoidal wave, where ω is the angular frequency, $\omega * t$ is the phase of the wave.
e^{-t/T_2}	Euler's constant used to show the decay (negative -t), as the magnetic moments of the hydrogen atoms dephase. T_2 is a parameter in MRI used to differentiate between tissues; it relates to the different signal strengths obtained from different tissues after a set amount of time on the FID curve.

However, in Fig. 2, the blue wave represents the signal generated by the MRI machine, which is initially high but decreases over time due to the *loss of coherence* (spins become dephased) between the hydrogen atoms in the body being imaged (the magnetic moments facing with different vector directions, will cancel eachother out). The orange curve represents the MRI signal's free induction decay (FID), which also decreases over time due to the same loss of coherence. From the values of parameters generated from the FID curve (e.g. T_2 in the equation above), the different body tissues can be differentiated and further processed as an image.

Magnetic Field in MRI

The operation of MRI depends heavily on the magnetic field. Superconducting magnets are the main magnets used in MRI scans because they provide a powerful, uniform magnetic field that organizes the body's protons (Johansen-Berg & Behrens, 2013). Fig. 3 shows the connection between magnetic fields generated via superconducting magnets on two sides and the connection with signals produced and processed due to radio frequency and electric field processing.

Electric Field in MRI

The electric field serves a crucial supporting role in producing the radiofrequency pulse that activates the protons, even though the magnetic field is the main source of power for MRI. By running an electric current through coils of wire, the body produces a momentary magnetic field that is used to generate the electric field (Radiology Masterclass, 2017). When the electric field is turned on and off, radio waves interact with the protons to form a radiofrequency pulse (Johansen-Berg & Behrens, 2013). The signal that the protons release as they realign with the magnetic field is likewise picked up by the electric field (Radiology Masterclass, 2017). A coil that converts the magnetic fluctuations into an electric current, which is subsequently analyzed by a computer, detects the signal (National Institute of Biomedical Imaging and Bioengineering, 2018).

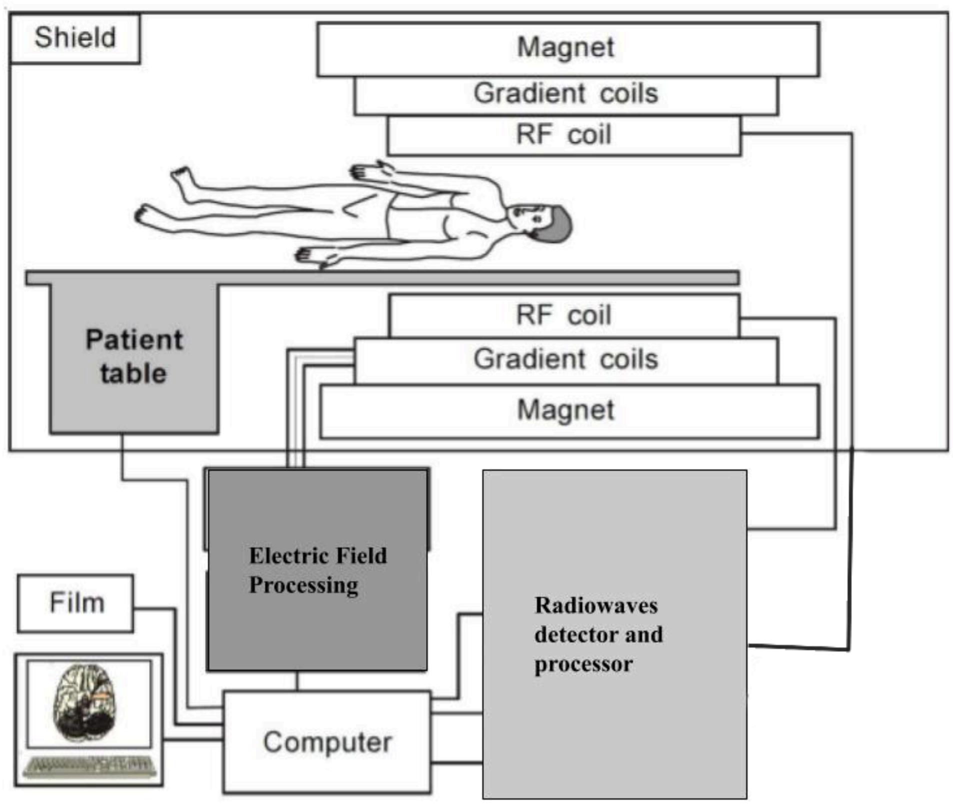


Fig. 3: A schematic diagram of an MRI machine with its main constituents (radiofrequency (RF) coils, gradient coils, magnet). The bottom indicates the data detection and processing procedure (Electric field and radio waves processor) (Image from Mulindi, 2020).

Role of Magnetic Field Strength

One way to improve MRI images is to increase the strength of the magnetic field. Higher field strengths can show more details and improve the contrast between different tissues (Chen &

Ni, 2014; Duyn, 2012). This is because higher field strengths increase the signal-to-noise ratio (SNR), which is a measure of how well the signal from the tissue can be distinguished from the background noise. A higher SNR means that the image has more clarity and less distortion. Higher field strengths also increase the chemical shift effect, which is a phenomenon that causes different nuclei to resonate at slightly different frequencies depending on their chemical environment. This effect can be used to enhance the contrast between different metabolites or molecules in the tissue, such as water and fat (Chen & Ni, 2014).

However, higher field strengths also pose some challenges and limitations for MRI imaging. For example, higher field strengths increase the susceptibility artifacts, which are distortions or signal loss caused by variations in the magnetic field due to air-tissue interfaces or magnetic materials in the body. A higher SAR can cause thermal effects or damage to the tissue or devices implanted in the body (Duyn, 2012). Therefore, increasing the magnetic field strength is not always the best option for improving MRI images, and other factors such as pulse sequences, coil design, and image processing techniques should also be considered.

Image Quality in MRI

Before examining the relationship between magnetic field intensity and image quality, it is important to understand why MRIs require good image quality.

Enhancing image quality can make tissues more visible and distinct, making it possible to identify anomalies like tumors, infections, inflammation, and injuries more precisely. Moreover, with higher quality images, we will observe lower signal noise that leads to confusion and misinterpretation in readings (Kruskal et al., 2011).

By improving accuracy, with improved image quality the number of scans or invasive treatments can be decreased, diagnostic accuracy and confidence can be increased, and patient satisfaction can all be improved.

Magnetic Field Strength and Image Quality

As stated above, by increasing magnetic field strength, we can improve image quality. In order to model MRI images, we need more parameters and complex data sets. However, we can use simpler sigmoid models to understand their relationship in more depth.

The sigmoid model employs a sigmoid function to articulate the connection between an input variable and an output variable (Fig. 4). Essentially, a sigmoid function is a mathematical function that maps any real number to a value between 0 and 1 through an S-shaped curve as in our case, increase in magnetic field strength leads to an increase in image quality, but with diminishing returns as the magnetic field strength approaches its maximum value. Code of Figure 4, Appendix) shows a model using the following equation:

Equation 4. Sigmoid Model to simplify relationship between magnetic field strength and image quality

$$f(B) \ = \frac{1}{1 + e^{(-B+1)}}$$

$f(B)$	Output of the function for a given magnetic field strength B that represents
B	Magnetic field strength in tesla (T).
e	Mathematical constant that is approximately equal to 2.71828.
e^{-B+1}	Exponential term that represents the inverse of the natural logarithm of B−1.

We have modeled an ideal scenario without noise as well as added noise in our data through noise standard deviation(0.5 that is 50% of the signal intensity) in order to map the real-world conditions (Code of Fig. 4, Appendix). Lastly, we have utilized Monte Carlo simulation to compare scaling of image quality with magnetic field strength.

Monte Carlo simulation is a mathematical technique for estimating the likely outcomes of an uncertain event (IBM, n.d.). It is a computational approach that employs repeated random sampling to determine the likelihood of a variety of outcomes occurring (IBM, n.d.). In this case, the simulation generates a signal with a linearly increasing intensity and adds Gaussian noise (noise_std = 0.5) to it. The image quality is then calculated as the correlation between the signal and the noisy signal. This process is repeated for a range of magnetic field strengths and the results are plotted.

Lastly, we can analyze Table 2, which shows that by adding noise, the correlation decreases between the two sigmoid models. By comparing the correlation coefficients of SMN and MCS from Table 2, we can observe that the result is similar which confirms the relationship of increasing magnetic field strength to improve image quality. Connecting this back to the free induction decay (see Fig. 2, equation 2,3), we can see that as we increase the magnetic field strength parameter in the wave's amplitude, the amplitude will increase and thus, the signal will increase as well, improving image quality.

Moreover, from Fig. 4, we can observe that after 5T, the image quality becomes constant; therefore, it raises questions for the purpose of 7T magnets in modern day MRIs.

Sigmoid Model (SM)	**Sigmoid Model with Noise (SMN)**	**Monte Carlo Simulation (MCS)**

`Correlation: 0.90`
`Mean: 0.85`
`Standard Deviation: 0.00`
	`Correlation: 0.87`
	`Mean: 0.85`
	`Standard Deviation: 0.49`
		`Correlation: 0.85`
		`Mean: 0.84`
		`Standard deviation: 0.17`

Table 2 shows the correlation, mean and standard deviation for the plots of the three models shown in the figure below.

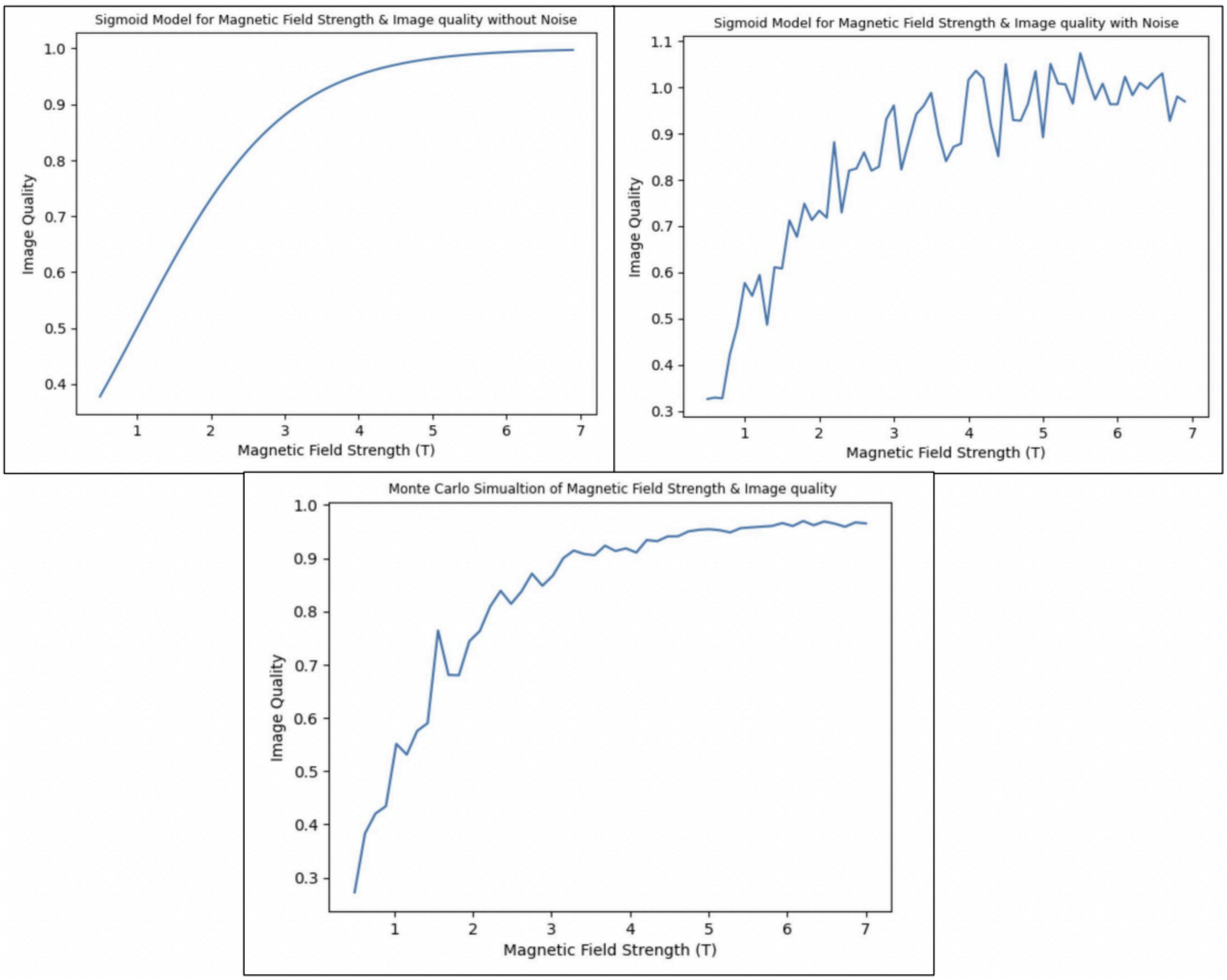


Fig. 4: Shows three plots (Sigmoid model, top left; Sigmoid model with noise, top right; Monte Carlo Simulation, bottom) that model how image quality scales with increasing magnetic field strength up to 7T (maximum magnetic field strength of modern day MRIs), nicely illustrating how changes in parameters (and thus, of the strength of fields involved) of the MRI can influence the final output.

Conclusion

The Larmor equation provides a value for the radio wave frequency in order to result in resonance of the hydrogen atoms. As the magnetic moment of billions of hydrogen atoms goes out of phase, the signal (and amplitude) decreases over time. Through connecting the amplitude of free induction decay to image quality, we were able to connect wave properties and a change in the magnetic field (resulting in a change in the amplitude) to the resulting image quality. The latter has real-world applications, such as detecting tumors.

Word Count: 1691 words (no in-text citations and table included)

LO and HC Appendix

LO	Applied	Description and Justification
#wavemechanics	Radio waves and Larmor equation	Magnetic signal is critical to understanding how radio waves are generated, transmitted, and received. In the introduction we elaborated on the radio waves and delved deeper into the formation of the magnetic signals. We elaborated on properties of the waves in detail and their quantitative measures. We analyzed Radio Frequencies and their applications. On top of this the role of the NMR and the connection of it with the Larmor equation was established. Majority of the concepts applied further lead to FID and Larmour equation proof/formation. - Properties of waves. - Radiofrequenties and their use in MRI. - NMR and its role in MRI. - Link of NMR to Larmor equation and Larmor frequency. - Analysis in the change of the amplitude of the FID curve and its components. - Free Induction decay and Larmor equation waves in figures.

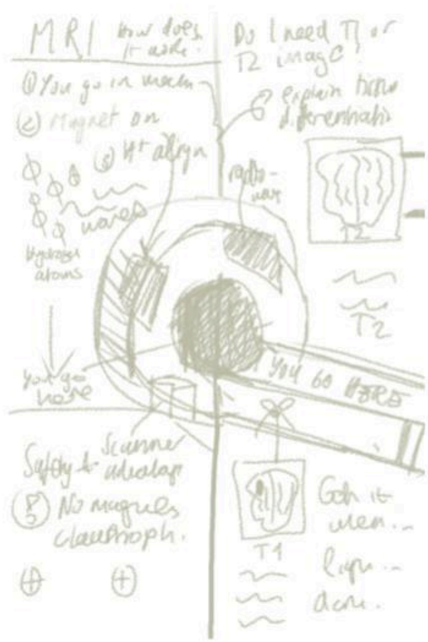
#fieldsforcesmotion	From Magnetic Field till the end.	- Firstly, we determined the fields that were involved (magnetic and electric fields) and described their purpose and application. - We expanded on magnetic field strength, its purpose and impact (challenges/limitations) on the physical system that is our body and the image generated. - We modeled the role of the magnetic field using the Larmor equation and sigmoid model and connected the relationship between magnetic field strength and image quality using Monte Carlo simulations to visualize and find correlations between them. - In our situation the simulations substituted free body diagram and field lines as there are billions of H (hydrogen) atoms impacted differently by a superconducting magnet so we cannot visualize these lines in a 2-d space without inaccuracies; therefore, we focused on connections between magnetic field and MRI and the output image.
#computationaltools	1. Larmor Equation. 2. Free Induction Decay (FID) 3. Magnetic Field Strength (MFS) and Image Quality (IQ).	**(1) Larmor Equation** - This was the an important anchor of our interpretations. Based on the long research on the MRI, we found the simplified equation of $\omega = \gamma * B_0$. Based on the interpretations we have we delved deeper into this and build the visualization in Fig. 1 of how different components of the magnetization of hydrogen atoms change over time in a magnetic field. Our intensive collaborated efforts resulted in representing and writing python code for the modeling of the magnetization on 3-d space. **(2) Free Induction Decay (FID):** - We modelded FID by initiating scenarios with fixed amplitude signals to test our assumptions. The Fig. 2 is an important application and illustration of our understanding of this LO. We further elaborated on the consequences of keeping the metrics constant and evaluated its impact on the T2 change as seen in Fig. 2 due to the rate of amplitude between blue and green plots.

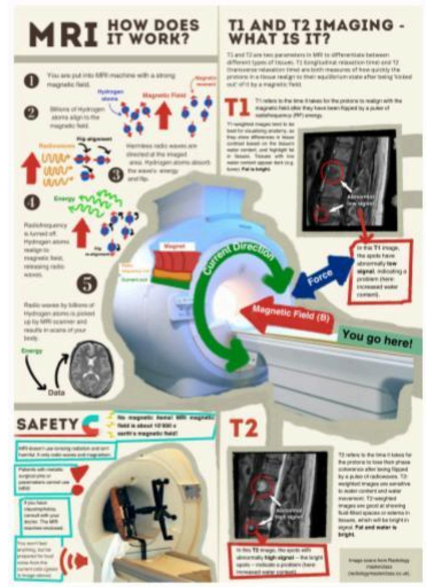
		(3) MFS and IQ: - We used #computationaltools to extract the correlation between magnetic field strength and image quality in order to justify the relationship between these two parameters. The purpose was to signify the role of magnetic fields, subsequently magnetic field strength in producing images and making MRI functional. - The role was also to identify the usage of specific magnets in MRI like 5T and we justified its usage beyond what is recommended as we saw in Fig. 4 how the image quality becomes constant around 5T. - We have explained the choice of our models/computational tools as well by explaining the general purpose of them. For instance, sigmoid models simplify relations where two variables are scaling. Whereas, Monte Carlo Simulations can be used to find likely outcomes for uncertain events. - The computational approach was novel and justifications strengthened their need.
#mathmethods	Equations 1 - 3	- The **FID** curve was set up mathematically through multiplying a sinusoidal curve with Euler's constant with a negative fraction in the power, which would make the sinusoidal curve decay. Further, the amplitude and sin(w*t) value was then set to 1 to clearly show the difference in the two curves through a simple function and its gradient (yellow line in figure 2). The results of modeling through this equation are interpreted. - The **amplitude** of the curve was shown to consist of multiple parameters. The analysis of the build-up of this equation was later used to connect changes in the amplitude's parameters to the image quality (e.g. magnetic strength). - **Dimensional analysis** was complex as we were dealing with multiple parameters and the context of waves was through MRI which changed the conception of signal and the wave properties. Therefore, we were cautious in ensuring dimensions were accurate in each equation and computation. - Our formulation of the **Larmor Equatio**n is interpreted in detail, and is well-explained, with

		clear steps and correct use of units and dimensions for the MRI. We take this equation and build up on it, with further theoretical application of magnetic signal formation.

HC	Applied	Description and Justification
#dataviz:	1. Larmor Equation. 2. Free Induction Decay 3. Magnetic Field Strength and Image Quality.	Graphs and visualizations were chosen to communicate the information to the reader effectively. The title, axes, captions, and design were carefully considered and implemented to ensure clarity and ease of interpretation. By selecting appropriate visualization techniques and presenting the data clearly and with visually appeal, the reader can easily understand the data/models. Python programming honed the process of making the visualizations and further comments on the code are provided in the appendix.
#modeling	1. Free Induction Decay 2. Sigmoid model 3. Monte Carlo Simulation	(1) **FID** was mathematically modeled through setting up a decaying sinusoidal curve by using Euler's constant with a negative fraction in the power. The T2 parameter of the model was then changed to show the effect on the curve. This change was then discussed in the context of MRI: a larger T2 value will mean that the FID curve decays slower. The real-world MRI application for differentiation of tissues (different T2 times) was discussed to contextualize the model. **(2) Sigmoid model** was used without and with noise to use computational modeling to determine the correlation between magnetic field strength and image quality in MRI. The model was created using sigmoid function as a constraint given that there are multiple parameters involved to produce an MRI image and so to simplify the system, we utilized sigmoid function so that it becomes constant as magnetic field strength approaches its maximum. (3) **Monte Carlo Simulation** was used with a repeated random sample to determine the likelihood of a variety of outcomes occurring as in our case image quality was scaled to determine the correlation between these two variables. (4) The correlation in models matched each other with minor

		differences which further confirmed the results. (5) Finally, the purpose of these models was to understand the relation between these two components and inferences we made about MRI in general from the plots like 5T is perfect for highest image quality.
#algorithms	1. Sigmoid model 2. Monte Carlo Simulation	- We utilized #algorithms to create step-by-step deconstruction of deriving equations in both Sigmoid models and Monte Carlo simulation. - In the Sigmoid model, we utilized a theoretical approach of algorithms where the equation to relate image quality and magnetic field strength was derived from clear and efficient steps so that it is easier for the audience to understand and reuse in comparison with other parameters like Larmor frequency. - Whereas in Monte Carlo Simulation, an algorithm was applied computationally by generating a signal with a linearly increasing intensity and adding Gaussian noise (noise_std = 0.5). We knew that we needed to add noise to make it closer to the real world scenario. - Furthermore, data visualization was produced by writing an algorithm to create plots to visualize the relation and add more context for the audience when comparing the models and their correlation coefficients.
#organization	Technical Report	(1) The paper includes a table of contents for easy navigation, and subheadings. Images are used in iteration with description to help the reader understand the content better, including a good organization of equation descriptions in tables. (2) The paper builds up on each section, as we were able to connect wave properties and a change in the magnetic field (resulting in a change in the amplitude), culminating in the resulting image quality. This is good organization, as it leads the reader through concepts while culminating in a real-life application. (3) In the final part, we linked back the concept of image quality back to amplitude, waves and magnetic field that were introduced earlier. This is good organization of a long paper, as it nicely ties everything together.
#audience	Infographic	The audience for the designed infographic are MRI patients with an average education that want to find out more about not only the procedure they will go through as a patient, but the simplified physics behind the MRI images they will see. How does the MRI machine work

		and what do my results mean (T1, T2)? What do I do during the MRI and what safety precautions will I have to follow? How long will it take? In order to understand our #audience, we imagined the leaflet to be in the waiting room of an MRI and imagined the patient to feel a bit nervous as to the upcoming procedure. The leaflet is supposed to give a holistic and basic overview over what the patient should expect (see bottom part of leaflet and 'you go here' text on the right) and calm the nerves through giving an insight into the components (middle image; the MRI machine) of the MRI machine and how the physics works. Going into a large and loud machine can be daunting, but with this knowledge at hand (e.g. explanation that the loud noise comes from the current coils), the patient will feel more prepared, knowledgeable and calm.
#designthinking	Infographic	The process of #designthinking regarding the design of the infographic involved the ideation in form a layout sketch (image 1, see below) that tailored towards our #audience and kept the design engaging and interactive. After the sketch was realized in the first draft of figure 2, Annabel asked the team for feedback. Upon asking multiple people outside the team as well for feedback on the design, we concluded to change the right side to be more spacious with a larger text size and reduce the size of the central MRI piece. **1** 

		2  After this iteration, we sent in the infographic to the professor for feedback. The feedback was largely based on the audience and questioned how relevant details on e.g. T1/ T2 imaging are, while other useful information for patients might be missing. Upon this iteration cycle, I changed the content on the right side and included the procedure the patient will go through in the MRI. This will make the patient feel prepared. The final design does not only incorporate ‘what MRI is’ from a simplified physical perspective, but also from a patient’s perspective. Lastly, it includes T1 and T2 information that is relevant to the patient (no unnecessary information). Thus, we focused on the iterative process of design, first brainstorming a design, then iterating through multiple drafts of the infographic upon each cycle of feedback (from different individuals). Through this iterative process we were able to create the final infographic in figure 4, which tailors best towards our #audience and is less overwhelming than the first draft in figure 2.

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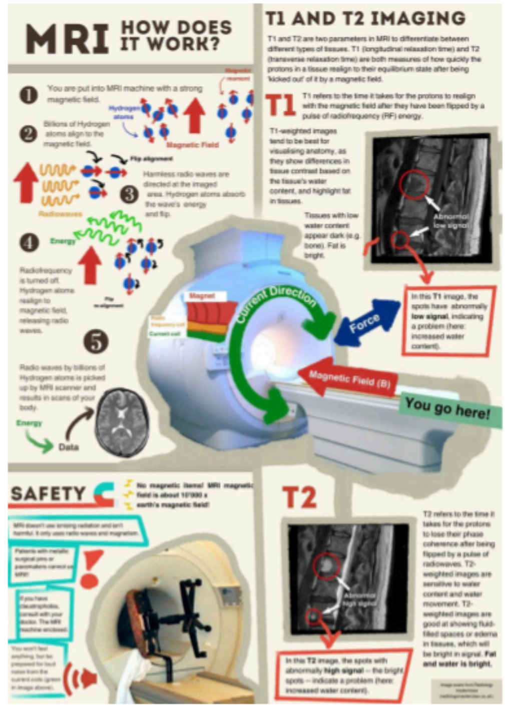

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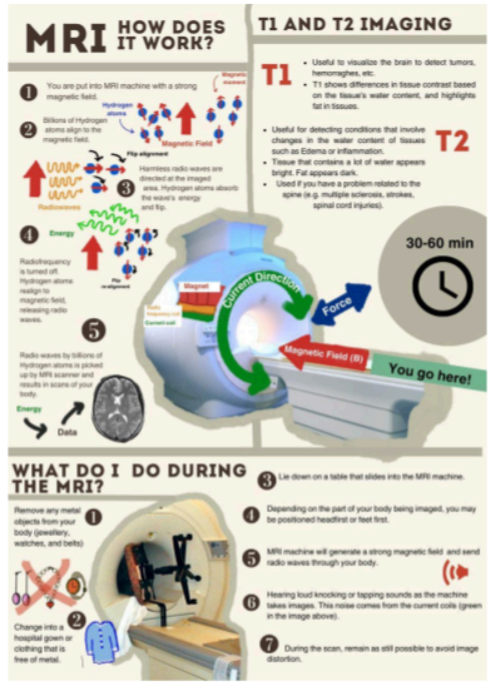

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Appendix

Code for Figure 1: Plotting Larmor Equation

```
import numpy as np
import matplotlib.pyplot as plt

# Define constants
gamma = 42.58e6  # Gyromagnetic ratio of hydrogen in MRI in Hz/T
B0 = 3.0  # Static magnetic field strength in T
w0 = gamma * B0  # Larmor frequency in Hz

# Define time range
t = np.linspace(0, 0.01, 1000)  # 10 ms with 1000 time points

# Define initial magnetization (Mz) and transverse magnetization (Mxy)
Mz0 = 1.2
Mxy0 = 0.12

# Calculate the Larmor equation solution for Mz and Mxy
Mz = Mz0 * np.cos(w0 * t)
Mxy = Mxy0 * np.sin(w0 * t)

# Calculate the total magnetization (M)
M = np.sqrt(Mz**2 + Mxy**2)

# Plotting results
fig, axs = plt.subplots(3, sharex=True, figsize=(8,8))

axs[0].plot(t, Mz, label='Mz')
axs[0].set_ylabel('Mz')

axs[1].plot(t, Mxy, label='Mxy')
axs[1].set_ylabel('Mxy')

axs[2].plot(t, M, label='M')
axs[2].set_ylabel('M')
axs[2].set_xlabel('Time (s)')

plt.suptitle('Larmor Equation')
plt.show()
```

Code for Figure 2: Visualization of how billions of H-atoms go out of sync as time passes.

```
import numpy as np
import matplotlib.pyplot as plt

# Define time range
t = np.linspace(0, 5*np.pi, 500) # 5 periods w/ 500 points

# Define amplitude and frequency of the wave
A = 1.0
f = 1.0

# Calculate the signal wave with decreasing amplitude over time
y_sin = A * np.sin(2*np.pi*f*t) * np.exp(-t/5)

# Calculate the induction decay
y_decay = np.exp(-t/5)
# CDenoting the  as 0 at the Amplitude is initially set to one

# Define the x-axis limit
x_min, x_max = 0, 3*np.pi

# Plotting results
plt.plot(t, y_sin, label='Signal Wave')
plt.plot(t, y_decay, label='Free Induction Decay Curve')
plt.axhline(y=0, color='black', lw=0.5)
plt.xlim(x_min, x_max)
plt.xlabel('Time')
plt.ylabel('Signal')
plt.legend()
plt.show()

import matplotlib.pyplot as plt

# Define time range
t = np.linspace(0, 5*np.pi, 500) # 5 periods of the sine wave with 500
points
```

```
# Define amplitude and frequency of the sine wave
A = 1.0
f = 1.0

# Calculate the sine wave with decreasing amplitude over time
y_sin = A * np.sin(2*np.pi*f*t) * np.exp(-t/18)

# Calculate the exponential decay
y_decay = A * np.exp(-t/18)

# Define the x-axis limit
x_min, x_max = 0, 3*np.pi

# Plot the results
plt.plot(t, y_sin, label='Signal Wave', color = 'green')
plt.plot(t, y_decay, label='Free Induction Decay Curve', color =
'orange')
plt.axhline(y=0, color='black', lw=0.5) # Add black x-axis
plt.xlim(x_min, x_max) # Set x-axis limit
plt.xlabel('Time')
plt.ylabel('Signal')
plt.legend()
plt.show()
```

Code for Figure 4: Sigmoid model without Noise

```
# Sigmoid model without noise
import matplotlib.pyplot as plt
import numpy as np

def sigmoid_model_with_noise(num_trials=100, magnetic_field_range=(0.5,
7.0, 0.1), noise_std=0.0, show_plot=True):
   """Simulate sigmoid model with noise and plot the results

   Args:
   - num_trials (int): The number of trials to run (default: 100)
   - magnetic_field_range (tuple): The range of magnetic field strength
values (default: (0.5, 7.0, 0.1))
```

```
   - noise_std (float): The standard deviation of the noise (default:
0.5)
   - show_plot (bool): Whether to show the resulting plot (default:
True)

   Returns:
   - tuple: A tuple containing the calculated mean, standard deviation
and correlation of the image quality values

   """
   # Unpacking the magnetic field range tuple
   magnetic_field_strength = np.arange(*magnetic_field_range)

   # Initializing an array to store the image quality values for each
trial
   image_quality = np.zeros((num_trials, len(magnetic_field_strength)))

   # Running the simulation for each trial
   for i in range(num_trials):
       # Setting the image quality values for this trial
       image_quality[i] = 1 / (1 + np.exp(-magnetic_field_strength + 1))

       # Adding noise to the image quality values
       image_quality[i] += np.random.normal(0, noise_std,
size=len(magnetic_field_strength))

   # Calculating the mean and standard deviation of the image quality
values across all trials
   image_quality_mean = np.mean(image_quality, axis=0)
   image_quality_std = np.std(image_quality, axis=0)

   # Calculating the correlation between magnetic field strength and
image quality
   correlation = np.corrcoef(magnetic_field_strength,
image_quality_mean)[0, 1]

   # Printing the results
   print(f'Correlation: {correlation:.2f}')
   print(f'Mean: {np.mean(image_quality_mean):.2f}')
```

```
    print
    (f'Standard Deviation: {np.mean(image_quality_std):.2f}')

    # Plotting the data
    if show_plot:
        plt.plot(magnetic_field_strength, image_quality_mean)
        plt.xlabel('Magnetic Field Strength (T)')
        plt.ylabel('Image Quality')
        plt.title('Sigmoid Model for Magnetic Field Strength & Image
quality with Noise', fontsize=9)
        plt.show()

    return image_quality_mean, image_quality_std, correlation
image_quality_mean, image_quality_std, correlation =
sigmoid_model_with_noise(num_trials=100, magnetic_field_range=(0.5, 7.0,
0.1), noise_std=0.0, show_plot=True)
```

Code for Figure 4: Sigmoid model with Noise

```
# Sigmoid model with noise
# Noise_std = 0.5 (50% of initial signal intensity)
image_quality_mean, image_quality_std, correlation =
sigmoid_model_with_noise(num_trials=100, magnetic_field_range=(0.5, 7.0,
0.1), noise_std=0.5, show_plot=True)
```

Code for Figure 4: Image Quality modeling using Monte Carlo Simulation

```
# Monte Carlo Simulation
import numpy as np
import matplotlib.pyplot as plt

# Defining the range of magnetic field strengths to simulate
min_field_strength = 0.5
max_field_strength = 7.0
num_field_strengths = 50
field_strengths = np.linspace(min_field_strength, max_field_strength,
num=num_field_strengths)

# Defining the parameters for the simulation
```

```
num_samples = 100
mean_signal = 10
std_noise = 0.5
correlation_coefficient = 0.8

# Creating an empty array to store the image qualities for each magnetic
field strength
image_qualities = np.zeros(num_field_strengths)

# Simulating the MRI for each magnetic field strength
for i, field_strength in enumerate(field_strengths):
   # Generating a signal with a linearly increasing intensity
   signal = np.linspace(0, field_strength, num_samples)
   # Adding Gaussian noise to the signal
   noise = np.random.normal(scale=std_noise, size=num_samples)
   noisy_signal = signal + noise
   # Calculating the image quality as the correlation between the signal and
the noisy signal
   image_quality = np.corrcoef(signal, noisy_signal)[0, 1]
   image_qualities[i] = image_quality

# Calculating the mean, correlation coefficient, and standard deviation
mean_image_quality = np.mean(image_qualities)
corr_coefficient = np.corrcoef(field_strengths, image_qualities)[0, 1]
std_image_quality = np.std(image_qualities)

print(f"Correlation: {corr_coefficient:.2f}")
print(f"Mean: {mean_image_quality:.2f}")
print(f"Standard deviation: {std_image_quality:.2f}")

# Plotting the results
plt.plot(field_strengths, image_qualities)
plt.xlabel('Magnetic Field Strength (T)')
plt.ylabel('Image Quality')
plt.title(' Monte Carlo Simulation of Magnetic Field Strength & Image
quality', fontsize=8.5)
plt.show()
```

AI Policy

AI was only used to brainstorm for ideas and break down the project to simpler aspects. For instance, it gives us a breakdown of what to include under magnetic fields like type of magnet, magnetic field strength and image quality etc.

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