MRI Signal Processing and Image Quality Modeling

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Introduction

A giant magnet and radio waves. One way to think of Magnetic Resonance Imaging (MRI) is as a giant magnet that creates a strong magnetic field around your body. This magnetic field aligns the protons in your body's atoms. Radio waves are then used to excite these protons, causing them to emit a signal that can be detected by the MRI machine (National Institute of Biomedical Imaging and Bioengineering, 2018). The strength of the signal depends on the density and arrangement of the protons in different tissues, which allows the MRI to create detailed images of the body. This paper will first explore the radio waves and the importance of the Larmor equation, go into further detail on magnetic field strength and its effects on imaging quality as well as the general application of magnetic and electric fields.

Radio Waves and Larmor Equation

MRI uses radio waves to create images of the body. The patient is placed in a strong magnetic field, causing the atomic nuclei to align. Radio waves are then applied to the body, causing the atomic nuclei to emit energy. The emitted energy is detected and used to create images.

Resonance in MRI refers to the phenomenon of nuclear magnetic resonance (NMR), which is the basis for MRI imaging. NMR occurs when the nuclei of certain atoms, such as hydrogen, absorb and re-emit electromagnetic radiation in the presence of a strong magnetic field. The frequency of the electromagnetic radiation that is absorbed and re-emitted is determined by the Larmor frequency, which is calculated using the Larmor equation. (Jones, 2021).

The Larmor equation is used to calculate the Larmor frequency, which is the rate of precession of the magnetic moment of a particle around an external magnetic field, B_0. By adjusting the strength of the magnetic field and the frequency of the radio waves, it is possible to selectively excite specific types of nuclei (magnetize them) and produce images with different contrasts.

Equation 1. Larmor Equation.

$$\omega = \gamma * B_0$$

ω	Larmor frequency (angular precessional frequency of a hydrogen atom (proton))
B_0	Strength of the magnetic field from the MRI in tesla (T)
γ	Gyromagnetic ratio, in the context of MRI, refers to the gyromagnetic ratio of the hydrogen nucleus (proton). The description of a static magnetic field of 1 T (Tesla), will make the hydrogen nucleus process at a frequency of 42.58 MHz.

The interpretation of Fig. 1 indicates how the magnetization of hydrogen atoms in a magnetic field changes over time. We are considering two components of magnetization by Larmor frequency, defined as the net magnetic moment per unit volume of a sample, such as one that is in the direction of the magnetic field (M_z) and the other one that is perpendicular to it (M_{xy}) (Haacke et al., 2015; Hoult & Lauterbur, 1979). If we imagine a 3D plane we can see how the z-axis is perpendicular to the xy plane.

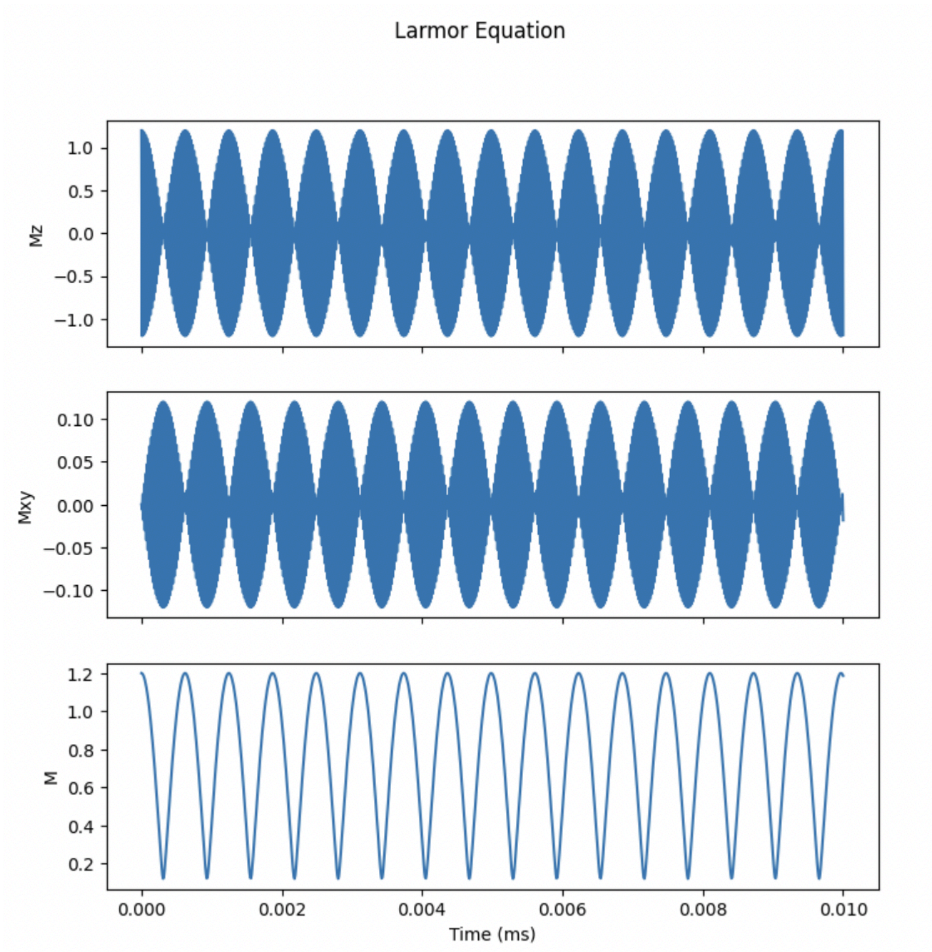

Fig 1: Visualization of how different components of the magnetization of hydrogen atoms change over time in a magnetic field (see code in Appendix). The equations used are Mz = Mz0 * cos(w0 * t) and Mxy = Mxy0 * sin(w0 * t), where w0 is the Larmor frequency and Mz0 or Mxy0 the initial value of magnetization.

What we further observe is that the M_z component and M_{xy} component oscillate together and result in the total 3D magnetization M ($\sqrt{M_{xy}^2 + M_z^2}$) which we call the Larmor frequency.

We can further observe that the M component decreases again after the climax (see figure 1, bottom graph) and we call this free induction decay (FID, see figure 2). This process over time indicates the spins of the hydrogen atoms that become unsynchronized from each other (signal gets weaker as it dephases).

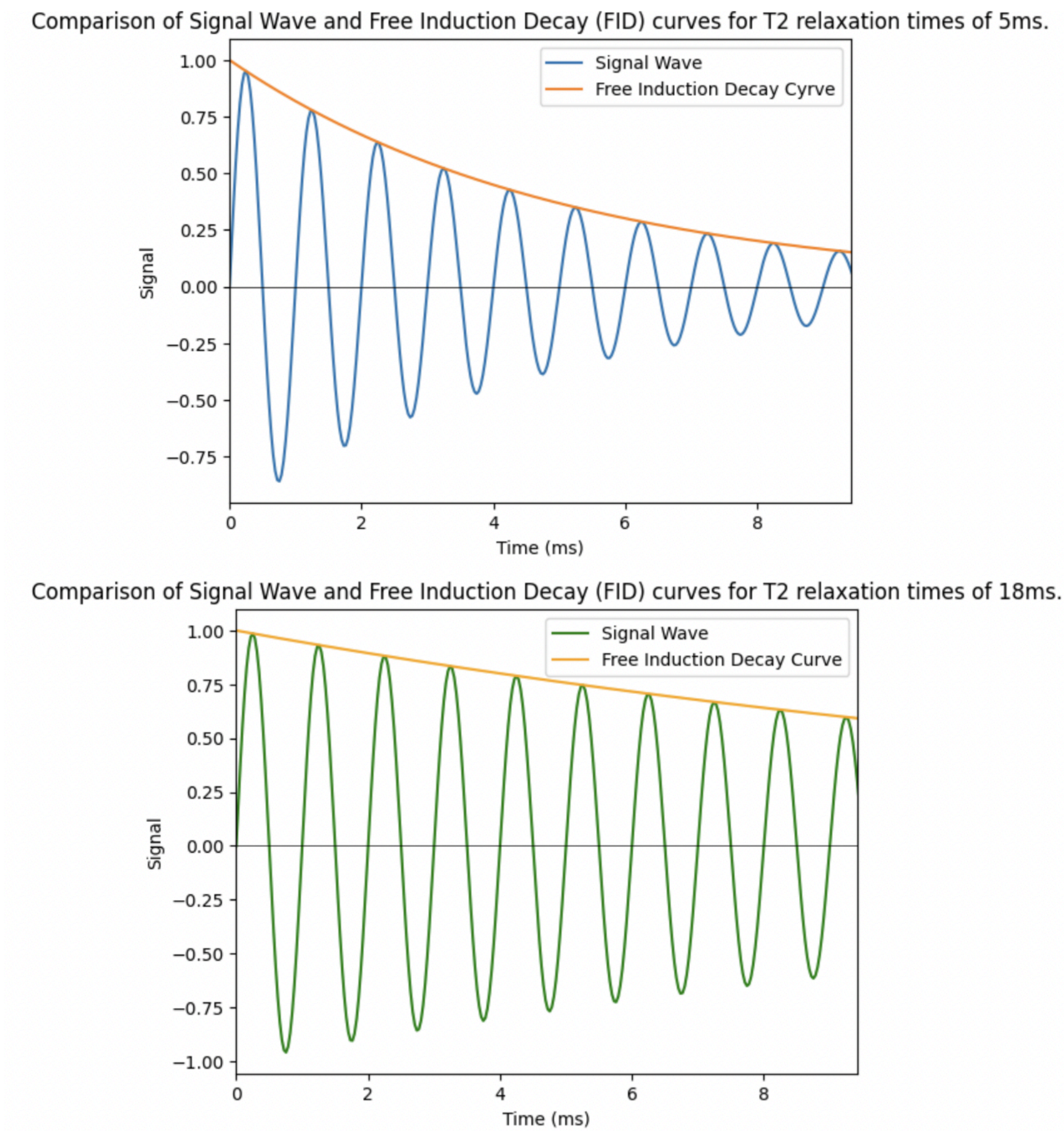

Fig. 2: Visualization of S(t) wave and the free induction decay curve. The top has A = 1. A common tendency we can see is that both of them are decreasing in amplitude over time. The x-axis represents time and the y-axis represents the signal. The billions of Hydrogen-atoms go out of sync as time passes and the signal decreases (see Code in Appendix).

This visualization in Fig. 2 aims to observe the decay behavior of the MRI signal over time. The signal's amplitude decreases over time as the billions of magnetic moments go out of phase. By analyzing the decay behavior of the MRI signal, researchers and scientists can differentiate between different types of tissue. For example, the top of figure 2 has a lower T2 (see equation 3) than the bottom of figure 2 and thus, the signal waves decays slower over time, the amplitude takes longer to decrease and the FID curve has a stronger gradient. For example, fat tissue has a higher T2 value than muscle (Lakrimi et al., 2011).

The amplitude, A, of the detected signal is made up of crucial MRI parameters, such as the number of spins, the magnetic field strength and gyromagnetic ratio. The magnetic moments of billions of hydrogen atoms aligned will add up to a signal that is large enough to detect.

Metrics	Description	Interpretation
f (frequency)	Frequency is the number of cycles of the signal that occur in one second and is directly related to the magnetic field strength and the gyromagnetic ratio of the spins contributing to the signal.	In Fig. 2 the green and blue curves have constant frequency of 1 Hz
λ **(wavelength)**	Wavelength is the distance between two successive peaks or troughs of the signal. The wavelength of the MRI signal depends on the frequency of the electromagnetic radiation used in the imaging process.	$\lambda = \frac{c}{f\ (frequency)}$ - c is the speed of light and - f is the frequency (constant) λ = c / f λ = 3 x 10^8 m/s / 1 Hz **λ = 3 x 10^8 m (constant for both green and blue curves)**

Time Period (T)	Time takes for a full oscillation.	1 ms/cycle for both the plots.
A (amplitude)	Amplitude is the maximum displacement of the signal from its equilibrium position. Additionally, the wavelength is the distance between two successive peaks or troughs of the signal.	The amplitude decreases over time (until 0) as the signal decreases and the hydrogen atoms dephase.

Table 1 shows the wave properties of FID in Fig. 2

Equation 2. Parameters make up the amplitude of the detected waves in MRI.

$$A = N * sin(\phi) * \gamma * B$$

N	The amount of hydrogen atom spins in the imaged tissue.
$sin(\phi)$	The pulse angle which refers to the angle that the magnetic moment vector of the hydrogen atom is tipped by the radiofrequency pulse (note that its max is at $\phi = 90°$). It is directly proportional to the magnitude of the pulse.
γ	Gyromagnetic ratio.
B	Magnetic field strength of the MRI [T].

Equation 3. Free Induction Decay.

$$S(t) = A * sin(\omega * t) * e^{-t/T_2}$$

The wave function for a decaying sine curve, representing the signal generated from the initially aligned magnetic moments of Hydrogen atoms, dephasing over time (adapted from Long & McLauchlan, 2021).

A	The maximum amplitude of a wave over a single cycle of oscillation, as described in equation 2.
$sin(\omega * t)$	Models the sinusoidal wave, where ω the angular frequency, $\omega * t$ is the phase of the wave.
e^{-t/T_2}	Euler's constant is used to show the decay (negative -t), as the magnetic moments of the hydrogen atoms dephase. T_2 is a parameter in MRI used to differentiate between tissues; it relates to the different signal strengths obtained from different tissues after a set amount of time on the FID curve.

However, in Fig. 2, the blue wave represents the signal generated by the MRI machine, which is initially high but decreases over time due to the *loss of coherence* (spins become dephased) between the hydrogen atoms in the body being imaged (the magnetic moments facing with different vector directions, will cancel each other out). The orange curve represents the MRI signal's free induction decay (FID), which also decreases over time due to the same loss of coherence. From the values of parameters generated from the FID curve (e.g. T_2 in the equation above), the different body tissues can be differentiated and further processed as an image.

Magnetic Field in MRI

The operation of MRI depends heavily on the magnetic field. Superconducting magnets are the main magnets used in MRI scans because they provide a powerful, uniform magnetic field that organizes the body's protons (Johansen-Berg & Behrens, 2013). Fig. 3 shows the connection between magnetic fields generated via superconducting magnets on two sides and the connection with signals produced and processed due to radio frequency and electric field processing.

Electric Field in MRI

The electric field serves a crucial supporting role in producing the radiofrequency pulse that activates the protons, even though the magnetic field is the main source of power for MRI. By running an electric current through coils of wire, the body produces a momentary magnetic field that is used to generate the electric field (Radiology Masterclass, 2017). When the electric field is turned on and off, radio waves interact with the protons to form a radiofrequency pulse (Johansen-Berg & Behrens, 2013). The signal that the protons release as they realign with the magnetic field is likewise picked up by the electric field (Radiology Masterclass, 2017). A coil that converts the magnetic fluctuations into an electric current, which is subsequently analyzed by a computer, detects the signal (National Institute of Biomedical Imaging and Bioengineering, 2018).

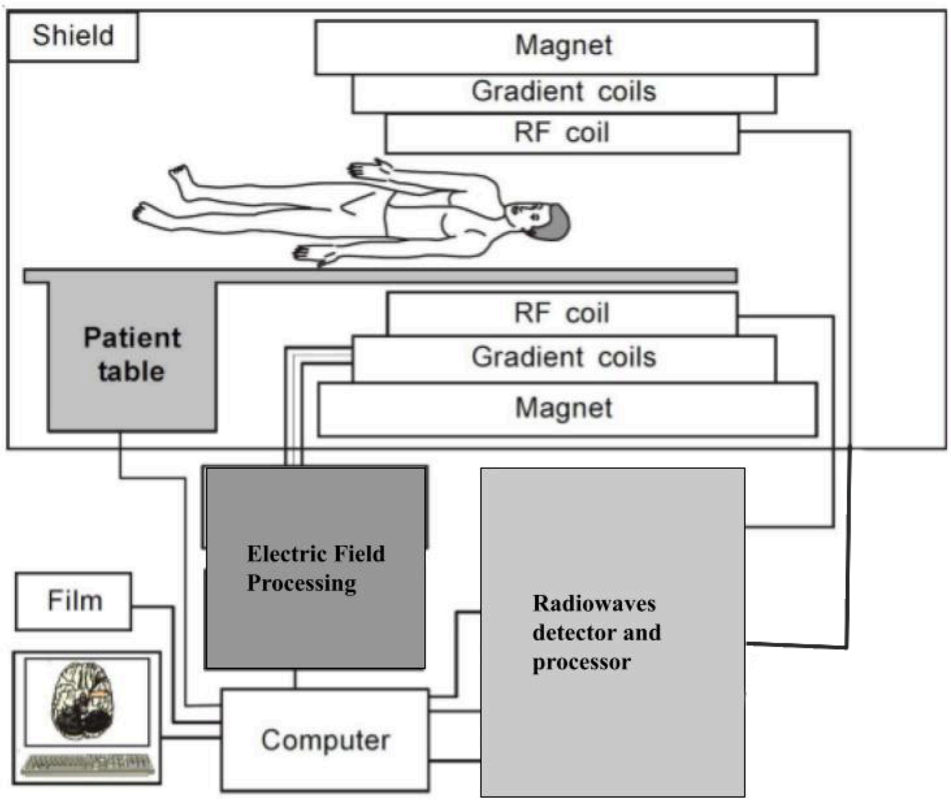

Fig. 3: A schematic diagram of an MRI machine with its main constituents (radiofrequency (RF) coils, gradient coils, magnet). The bottom indicates the data detection and processing procedure (Electric field and radio waves processor) (Image from Mulindi, 2020).

Role of Magnetic Field Strength

One way to improve MRI images is to increase the strength of the magnetic field. Higher field strengths can show more details and improve the contrast between different tissues (Chen &

Ni, 2014; Duyn, 2012). This is because higher field strengths increase the signal-to-noise ratio (SNR), which is a measure of how well the signal from the tissue can be distinguished from the background noise. A higher SNR means that the image has more clarity and less distortion. Higher field strengths also increase the chemical shift effect, which is a phenomenon that causes different nuclei to resonate at slightly different frequencies depending on their chemical environment. This effect can be used to enhance the contrast between different metabolites or molecules in the tissue, such as water and fat (Chen & Ni, 2014).

However, higher field strengths also pose some challenges and limitations for MRI imaging. For example, higher field strengths increase the susceptibility artifacts, which are distortions or signal loss caused by variations in the magnetic field due to air-tissue interfaces or magnetic materials in the body. A higher SAR can cause thermal effects or damage to the tissue or devices implanted in the body (Duyn, 2012). Therefore, increasing the magnetic field strength is not always the best option for improving MRI images, and other factors such as pulse sequences, coil design, and image processing techniques should also be considered.

Image Quality in MRI

Before examining the relationship between magnetic field intensity and image quality, it is important to understand why MRIs require good image quality.

Enhancing image quality can make tissues more visible and distinct, making it possible to identify anomalies like tumors, infections, inflammation, and injuries more precisely. Moreover, with higher quality images, we will observe lower signal noise that leads to confusion and misinterpretation in readings (Kruskal et al., 2011).

By improving accuracy, with improved image quality the number of scans or invasive treatments can be decreased, diagnostic accuracy and confidence can be increased, and patient satisfaction can all be improved.

Magnetic Field Strength and Image Quality

As stated above, by increasing magnetic field strength, we can improve image quality. In order to model MRI images, we need more parameters and complex data sets. However, we can use simpler sigmoid models to understand their relationship in more depth.

The sigmoid model employs a sigmoid function to articulate the connection between an input variable and an output variable (Fig. 4). Essentially, a sigmoid function is a mathematical function that maps any real number to a value between 0 and 1 through an S-shaped curve as in our case, increase in magnetic field strength leads to an increase in image quality, but with diminishing returns as the magnetic field strength approaches its maximum value. Code of Figure 4, Appendix) shows a model using the following equation:

Equation 4. Sigmoid Model to simplify relationship between magnetic field strength and image quality

$$f(B) \; = \frac{1}{1 + e^{(-B+1)}}$$

$f(B)$	Output of the function for a given magnetic field strength B that represents
B	Magnetic field strength in tesla (T).
e	Mathematical constant that is approximately equal to 2.71828.
e^{-B+1}	Exponential term that represents the inverse of the natural logarithm of B−1.

We have modeled an ideal scenario without noise as well as added noise in our data through noise standard deviation(0.5 that is 50% of the signal intensity) in order to map the real-world conditions (Code of Fig. 4, Appendix). Lastly, we have utilized Monte Carlo simulation to compare scaling of image quality with magnetic field strength.

Monte Carlo simulation is a mathematical technique for estimating the likely outcomes of an uncertain event (IBM, n.d.). It is a computational approach that employs repeated random sampling to determine the likelihood of a variety of outcomes occurring (IBM, n.d.). In this case, the simulation generates a signal with a linearly increasing intensity and adds Gaussian noise (noise_std = 0.5) to it. The image quality is then calculated as the correlation between the signal and the noisy signal. This process is repeated for a range of magnetic field strengths and the results are plotted.

Lastly, we can analyze Table 2, which shows that by adding noise, the correlation decreases between the two sigmoid models. By comparing the correlation coefficients of SMN and MCS from Table 2, we can observe that the result is similar which confirms the relationship of increasing magnetic field strength to improve image quality. Connecting this back to the free induction decay (see Fig. 2, equation 2,3), we can see that as we increase the magnetic field strength parameter in the wave's amplitude, the amplitude will increase and thus, the signal will increase as well, improving image quality.

Moreover, from Fig. 4, we can observe that after 5T, the image quality becomes constant; therefore, it raises questions for the purpose of 7T magnets in modern day MRIs.

Sigmoid Model (SM)	Sigmoid Model with Noise (SMN)	Monte Carlo Simulation (MCS)

`Correlation: 0.90`
`Mean: 0.85`
`Standard Deviation: 0.00`
	`Correlation: 0.87`
	`Mean: 0.85`
	`Standard Deviation: 0.49`
		`Correlation: 0.85`
		`Mean: 0.84`
		`Standard deviation: 0.17`

Table 2 shows the correlation, mean and standard deviation for the plots of the three models shown in the figure below.

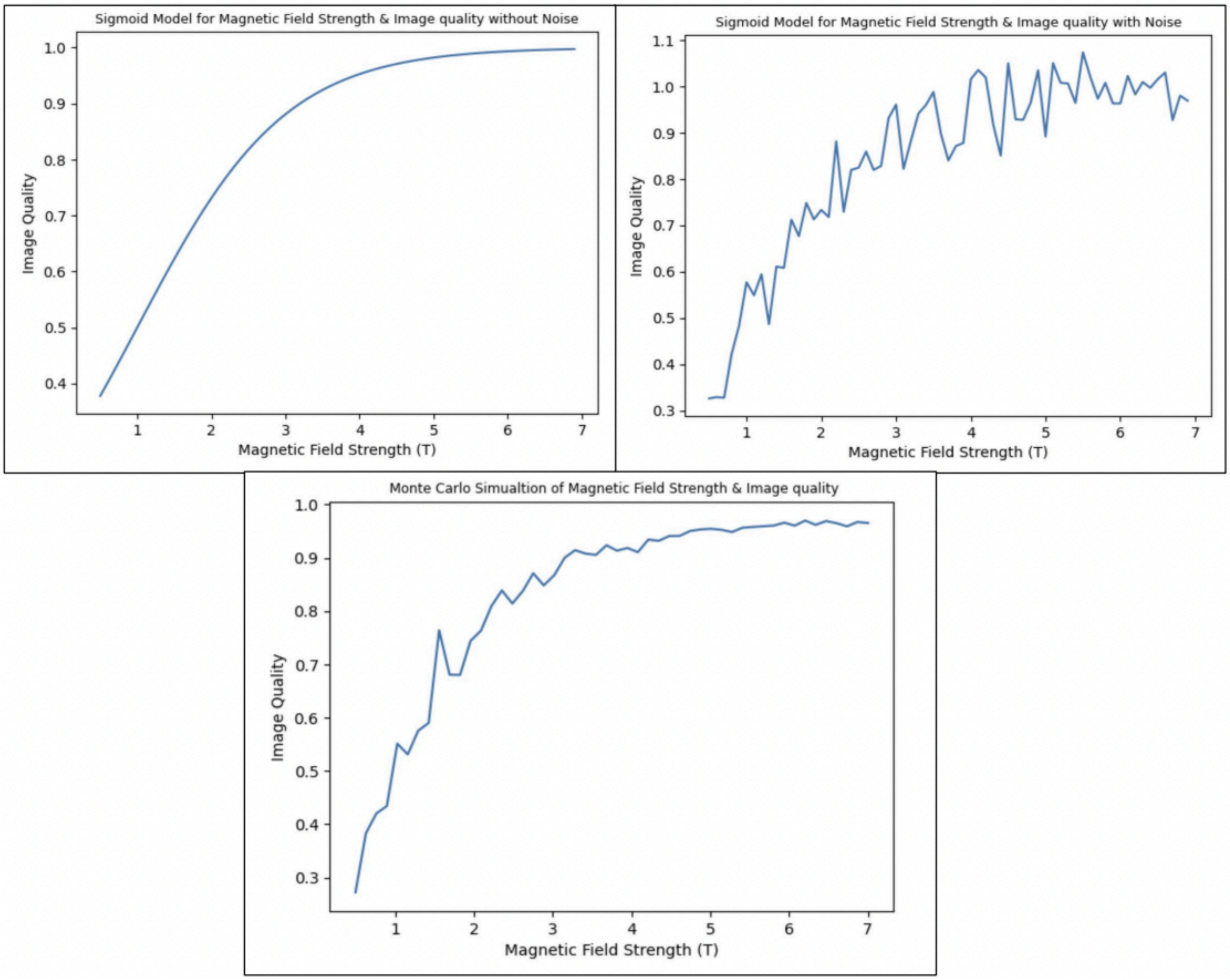

Fig. 4: Shows three plots (Sigmoid model, top left; Sigmoid model with noise, top right; Monte Carlo Simulation, bottom) that model how image quality scales with increasing magnetic field strength up to 7T (maximum magnetic field strength of modern day MRIs), nicely illustrating how changes in parameters (and thus, of the strength of fields involved) of the MRI can influence the final output.

Conclusion

The Larmor equation provides a value for the radio wave frequency in order to result in the resonance of the hydrogen atoms. As the magnetic moment of billions of hydrogen atoms goes out of phase, the signal (and amplitude) decreases over time. By connecting the amplitude of free induction decay to image quality, we were able to connect wave properties and a change in the magnetic field (resulting in a change in the amplitude) to the resulting image quality. The latter has real-world applications, such as detecting tumors.

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