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Demand-side energy flexibility estimation for day-ahead models☆

Marcelo Salgado-Bravo [a,c,*], Matias Negrete-Pincetic [a,b], Aristides Kiprakis [c]

[a] Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Santiago, Chile
[b] Institute Complex Engineering Systems (ISCI), República 695, Santiago, Chile
[c] University of Edinburgh, Institute for Energy Systems, Edinburgh EH9 3DW, UK

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ABSTRACT

A flexibility estimation model is proposed to evaluate the immediate aggregate flexibility response in a day-ahead scheme. The proposed model schedules a set of appliances and calculates the aggregated flexibility according to the energy and flexibility prices. The temporal granularity of the problem can be adjusted to evaluate the flexibility for the next 15 min or 4 h, using an *alternative flexibility scenario* approach to evaluate immediate flexibility available. The alternative flexibility scenario is a limited number of time step used to evaluate whether the estimated flexibility is available for a defined time window in each time step. New flexibility constraints were introduced to evaluate the flexibility and the rebound effect for air conditioners, pool pumps, and electric water heater, according to the alternative flexibility scenario. The model was tested under different energy price schemes and flexibility requirement durations to observe how the price signals influenced the flexibility offered. The results show how a model that considers extended flexibility, even if it does not consider the total duration requirement, can offer a more accurate flexibility response than a flexibility estimation without an extended flexibility response.

1. Introduction

1.1. Motivation

Two key trends have gained relevance regarding power systems in recent years. First, the rise of renewable energy generation (solar and wind power plants) introduces challenges associated with the uncertainty and high variability in their generation [1]. Second, decarbonization plans and the planned retirement of thermal plants has reduced the number of power plants providing inertia to the grid. Consequently, power systems have reduced their capacity to respond to unexpected fluctuations in generation or transmission [2], increasing the need for flexibility elsewhere in the grid. In the renewable-energy scenario, demand-side flexibility could maintain the balance between generation and demand in electrical grids [3,4].

Numerous studies propose the use of flexibility from demand-side appliances to prevent imbalances between generation and demand [5–7], for example, by offering ancillary services using air conditioning appliances [8–11], electric vehicles [12–15], an aggregator model for various appliances [16–18], and energy community management systems where the prosumers can exchange energy and use flexibility from controllable appliances [19]. However, the ability of appliances to modify their demand over continuous time intervals has not been extensively studied. This work addresses the problem considering that the duration required may vary from 15 min to four hours, according to the flexibility and ancillary services requirements.

1.2. Flexibility literature

For power systems, flexibility can be defined as the power system's capacity to balance supply with demand [20], while demand-side response can be defined as the regulation of the consumer energy patterns [21], usually motivated by financial incentives. Flexibility can therefore be redefined as the device's ability to shift its operation to maintain the balance between supply and demand without affecting the power system stability or user satisfaction. This definition covers both the power-system and demand-side aspects of flexibility.

The flexibility quantification can be explored under two approaches: top-down and bottom-up. The top-down approach explores the demand profiles of users and buildings to determine patrons associated with the demand flexibility. Data-driven models of behaviour-based electric-vehicle usage [22], occupancy behaviour models of buildings cluster [23], data-disaggregation methods to evaluate demand response

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* Corresponding author at: Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Santiago, Chile.
E-mail address: mjsalgado@ing.puc.cl (M. Salgado-Bravo).

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capacity for air conditioners [24], long short-term memory and gated recurrent unit methods to predict heat pumps flexibility [25] and autoregressive moving average (ARMA) modelling of Heating, Ventilation and Air Conditioning (HVAC) appliance Demand Response [26] can all be considered under the top-down approach. In contrast, the bottom-up approach evaluates the appliance's properties to shift its demand under an optimal control model and optimize its consumption according to price signals, CO2 minimization, or other incentives. An exception to the differentiation between bottom-up or top-down can be found in [27], which proposes the use of a bottom-up demand simulation software to train a top-down flexibility model.

Under the bottom-up approach, two methodologies have been identified in the literature to evaluate flexibility [28]:

- Ex-ante flexibility: Evaluate flexibility and appliance operation under the same model.
- Ex-post flexibility: Evaluate the flexibility as the difference between a demand curve that offers flexibility and a reference demand trajectory.

The ex-ante approach is used to evaluate changes in the control strategies, optimizing one or more attributes (for example, CO2 emissions, energy prices, demand response requests) and considering how flexibility can help to achieve these objectives [28]. This approach compares innovative control strategies with flexibility indicators to evaluate their performance, as seen in the work of Six [29], which introduces the 'delayed flexibility' and 'forced flexibility' indicators to estimate the time that a residential heat pump can be delayed or anticipated This is similar to D'hulst's [30] considering delayed flexibility, and increased and decreased power consumption as flexibility indicators. Stinner [31] proposes 'energy flexibility' and 'power flexibility' indicators to quantify the flexibility available for a water storage tank, while Klein [32,33] proposes the 'grid support coefficients' indicator to quantify the energy used from non-renewable sources. Marotta [34] presents a complete flexibility indicator review, analyzing 27 indicators and using the flexibility factor to compare the energy used in high and low penalty hours. Other ex-ante flexibility models do not use flexibility indicators, for example Wen's model [35], which evaluates the flexibility of various appliances under an aggregated feasible region approach, using the Fourier–Motzkin elimination process to reduce constraint redundancy, while Diaz-Londono [36] introduces a upward or downward flexibility variable to maximize in a model predictive control (MPC) for electric vehicles. Iria [16,17] formulates a demand aggregator model that includes flexibility constraints in the appliance formulation, offering flexibility according to market price and evaluating performance using an MPC.

The ex-post approach requires directly quantifying flexibility based on the difference between a predefined demand profile and the appliance's response to a flexibility requirement. This approach is employed by De Coninck [37], to calculate a flexibility cost curve in proportion to the energy required by the flexibility request and is related to the Oldewurtel [38] approach, using price signals to evaluate the power shifting efficiency and potential. Alternatively, Junker [39] proposes quantifying flexibility using a dynamic flexibility function based on the impulse response function for thermal systems, tested with two flexibility indicators, while in [40] is proposed a new flexibility indicator for energy consumption and CO2 emission reduction using the energy shifting, prebound and rebound effect. Other ex-post works evaluate the flexibility costs including the cost of unplanned use of storage system's and EV's [41]; estimate the demand response capacity from a building based on its thermostat set points [42]; consider the flexibility as a virtual energy storage system to determine the available storage capacity, the storage efficiency, and the power shifting capability in HVAC systems [28]; and estimate the savings on the operation cost using flexible appliances in a microgrid [43].

1.3. Contributions

Based on these approaches, the flexibility models in demand response scenarios can evaluate the available flexibility for each time interval or the demand deviation when a demand response request is received. A gap in the flexibility literature appears whether the frequency reserve problem is evaluated: a flexibility request duration can vary between 15 min and 2 h, according to the Independent System Operator (ISO) requirements. For example, CAISO secondary and tertiary response has a 30 min duration (the activation time changes between secondary and tertiary response) [44], the RTE (*Réseau de Transport d'Électricité*) considers the replacement reserve volume and the manual frequency restoration service with 90-minutes and 120-minute durations, respectively [45]. In comparison, the non-spinning reserve in ERCOT requests 30 min of interrupted demand or the capability to manage demand for 1 h [46].

State-of-the-art flexibility models can solve the flexibility estimation problem when the flexibility requirement is known in advance. Still, whether the flexibility can be requested and dispatched with a 15-minute notice, a day-ahead model cannot offer an adequate flexibility response for a requirement longer than the used time interval. The MPC models for microgrids [47,48] can tackle this problem for real-time operation. However, a day-ahead flexibility model designed for frequency reserve markets presents a research opportunity if the flexibility can be offered for continuous time intervals.

This study focused on developing an Aggregated Flexibility Estimation model with a defined flexibility request duration, which is the number of time blocks needed to offer the estimated flexibility to the grid. For example, in the case of a flexibility request with a three-block duration, where each block represents 15 min, is equivalent to a 45-minute flexibility request. To offer flexibility for a defined duration, an alternative flexibility scenario is proposed, which evaluates how the offered flexibility by the appliances in one time block affect to the next time block until the flexibility duration is satisfied. The model quantifies and maximizes the flexibility in the alternative scenario, where the minimum flexibility value from the alternative time blocks will be used as the appliance's aggregated flexibility, ensuring a constant flexibility offer for the expected duration. The main contributions of this paper are:

- A flexibility estimation model for centralized energy management systems that includes the effects of offering flexibility on the appliance's state.
- A new set of constraints is presented to quantify the flexible appliances' flexibility under a day-ahead model.
- The flexibility rebound effect is integrated into the appliance's constraint.
- The model estimates aggregated flexibility as the maximum value offered for continuous time blocks.

1.4. Document structure

The publication is organized as follows: Section 2 presents the proposed approach and introduces the mathematical formulation of the aggregated flexibility estimation model. Section 3 presents the results and discussion of various case studies related to this model. Finally, Section 4 provides the concluding remarks.

2. Proposed model and methodology

2.1. Aggregated flexibility estimation approach

The proposed aggregated flexibility estimation model aimed to minimize operational costs and estimate the available flexibility from a set of appliances, assuming that the flexibility offered must be available for

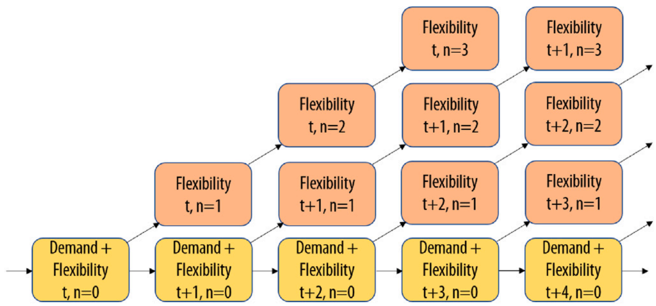

Fig. 1. Graphical representation of the alternative flexibility scenario. The aggregated flexibility at t is the minimal flexibility value found in the n time steps.

a defined flexibility service duration. This model introduces the alternative flexibility scenario to determine whether the expected flexibility could be offered for continuous time intervals. The appliance's schedule in the alternative scenario considered the flexibility delivered in each time step to guarantee that the required flexibility can be sustained over the requested time.

The model was introduced as an aggregated flexibility approach because it considered the coordinated response of the appliances being evaluated for the continuous time steps. For example, if two appliances are respond to a 30-minute flexibility request, appliance 1 provides 2 kW of flexibility for the first 15 min whilst appliance 2 offers 2 kW of flexibility in the last 15 min, the aggregated flexibility response considers that both appliances have provided 2 kW of flexibility for 30 min. This approach allows to appliances with short-time flexibility to offer services for a longer request. The mathematical formulation for this approach is presented in expression (1), where the flexibility estimated for the time step t must be lower or equal to the sum of the flexibility offered in each time block from the alternative scenario.

In the model formulation, the index n introduced the time steps for the alternative scenario. In this notation, the appliance's properties were evaluated assuming that the flexibility estimated in $n = 0$ only considers the scheduling for t, as shown in Fig. 1. If $n > 0$ the flexibility is evaluated based on the scheduling decisions in $t+n$ and the flexibility offered in $n-1$. The aggregated flexibility offered in t correspond to the minimal flexibility value found in the n time steps.

2.1.1. Time blocks

The proposed model can be used with different time intervals according to the energy management system preferences. The constraints expressed have been tested considering a 15-minute approach (96 time blocks per day), 30-minute approach (48 time blocks), and 1-hour approach (24 time blocks). The flexibility estimation uses the notation '1-block' to indicate over how many time blocks the aggregated flexibility response was evaluated. For example, a 30-minute approach with a 2-block flexibility estimation evaluates the flexibility for 1 h in each time block, while a 1-hour approach with a 3-block estimation considers the flexibility response for 3 h.

2.2. Flexibility model formulation

The flexibility estimation model scheduled a set of flexible appliances such as pool pumps, electric water heaters, and air conditioning units, considering the operational and comfort constraints for each appliance. The model formulation aimed to provide flexibility for continuous time steps, while considering variable time steps (24, 48, and 96 time steps per day). The model accounted energy prices to minimize its operation and estimate its flexibility according to the price signals (energy and flexibility prices), presenting a deterministic model. There was no penalty if a demand deviation occurs.

2.2.1. Objective function and flexibility estimation

The proposed objective function aimed to minimize the operational costs considering energy and flexibility prices for a daily operation. The model optimized the appliance's schedule and the available flexibility for each time step, where the flexibility must be provided for an N number of time steps defined by the flexibility service duration, according to the expression (1).

$$\min \sum_{t} \left(C_t^E * D_t - C_t^{UF} * UF_t - C_t^{DF} * DF_t\right) \tag{1a}$$

$$\text{s.t.} \quad D_t = \sum_{EWH} P_t^{ewh} + \sum_{PP} P_t^{pp} \tag{1b}$$
$$+ \sum_{HVAC} \left(P_t^{hvac,H} + P_t^{hvac,C}\right) \qquad \forall t \in T$$

$$UF_t \leq \sum_{EWH} UF_{t,n}^{ewh} + \sum_{PP} UF_{t,n}^{pp} \tag{1c}$$
$$+ \sum_{HVAC} \left(UF_{t,n}^{hvac,H} + UF_{t,n}^{hvac,C}\right) \qquad \forall t \in T, n \in N$$

$$DF_t \leq \sum_{EWH} DF_{t,n}^{ewh} + \sum_{PP} DF_{t,n}^{pp} \tag{1d}$$
$$+ \sum_{HVAC} \left(DF_{t,n}^{hvac,H} + DF_{t,n}^{hvac,C}\right) \qquad \forall t \in T, n \in N$$

Expression (1b) introduces the total energy consumption at time t as the sum of the consumption of the appliances. The expressions ((1c), (1d)) introduced the flexibility estimation for continuous time steps for upward and downward flexibility. The flexibility estimated for t time step must be lower than the flexibility provided by the flexible appliances for the n time steps associated with the alternative flexibility scenario, maximizing the UF_t and DF_t values that are limited by the lowest aggregated flexibility in n time steps.

2.2.2. Pool Pumps (Deferrable appliances)

Deferrable appliances are electrical loads that can be stopped or shifted without any penalization related to their operation. They need a certain amount of energy in a given period. The pool pump (PP) can be considered the appliance that exemplifies the deferrable appliance definition. Numerous publications [49–51] promote the active usage of PP to provide flexibility in power systems.

Flexibility estimation constraints were introduced into the PP operational model. The expression (2) presents the PP operational constraints, including the total energy required per day as the sum of the energy supplied per time step (2a), the maximum power and the pool pump availability ((2b), (2c)).

$$\sum_{t} P_t^{pp} = E^{pp} \tag{2a}$$

$$0 \leq P_t^{pp} \leq P_{max}^{pp} \quad \forall t \in T \tag{2b}$$

$$P_t^{pp} = 0 \qquad if\ t \notin [T_{start}, T_{end}] \tag{2c}$$

In expression (3) the required constraints are shown for estimating the pool pump flexibility per time step. Constraint (3a) limited the flexibility to the time steps when the PP was available. Expressions (3b) and (3c) limited the upward and downward flexibility in time step t and alternative time step n to the maximum displacement from the scheduled demand. The previous expressions needed additional constraints to ensure that the PP could offer flexibility without breaking the constraints presented in (2). Therefore, to reinforce constraint (2a), the expression (3d) evaluated in each time step n whether the upward flexibility offered exceeded the PP's daily power. Also, the expression (3e) verified that the PP had enough upward flexibility in the next time steps to compensate for the downward flexibility needed at time step t. Both expressions estimated the rebound effect in the PP model, ensuring for future compensation.

$$UF_{t,n}^{pp} = DF_{t,n}^{pp} = 0 \qquad if\ t+n \notin [T_{start}, T_{end}] \tag{3a}$$

$$UF_{t,n}^{pp} \leq P_{max}^{pp} - P_{t+n}^{pp} \qquad \forall t \in T, \forall n \in N \tag{3b}$$

$$DF_{t,n}^{pp} \leq P_{t+n}^{pp} \quad \forall t \in T, \forall n \in N \tag{3c}$$

$$\sum_{l=0}^{t+n} P_l^{pp} + \sum_{s=0}^{n} UF_{t,s}^{pp} \leq E^{pp} \quad \forall t \in T, \forall n \in N \tag{3d}$$

$$\sum_{s=0}^{n} DF_{t,s}^{pp} \leq \sum_{l=t+1+s}^{T} UF_{l,0}^{pp} \quad \forall t \in T, \forall n \in N \tag{3e}$$

2.2.3. Heating, Ventilation, and Air Conditioning appliances

The high energy consumption of the HVAC appliances and the thermal inertia associated with a building's heat capacity allows HVAC appliances to shift their consumption and modify their demand curve. HVAC appliances and their flexibility have been extensively studied [51–53] and included in various energy management system approach such as virtual power plants (VPP), demand Aggregators (DA), and home energy management systems (HEMS).

HVAC appliances depend on the room temperature, the building's thermal inertia, and the outdoor temperature to schedule their operation without exceeding the user-defined comfort temperature. The base HVAC formulation is presented in expression (4). Eqs. (4a) and (4b) evaluate the room temperature depending on its previous state, the forecast outdoor temperature, and the power used to heat or cool the room. In both expressions the terms α and β are used as constant values related to the effect of the relevant variables (room temperature in the previous time step, outdoor temperature, heating, and cooling power) relating to room temperature. The different approaches to estimating α and β are presented on Appendix. Expressions (4c) and (4d) introduce the room temperature constraints and the HVAC maximum power consumption.

$$T_t = T_{start} + \alpha_{hvac} * (T_{t-1}^{Forecast} - T_{start}) + \beta_{hvac,H} * P_t^{hvac,H} - \beta_{hvac,C} * P_t^{hvac,C} \quad t = 0, \forall hvac \in HVAC \tag{4a}$$

$$T_t = T_{t-1} + \alpha_{hvac} * (T_{t-1}^{Forecast} - T_{t-1}) + \beta_{hvac,H} * P_t^{hvac,H} - \beta_{hvac,C} * P_t^{hvac,C} \quad t \neq 0, \forall hvac \in HVAC \tag{4b}$$

$$T_{min} \leq T_t \leq T_{max} \quad \forall t \in T \tag{4c}$$

$$0 \leq P_t^{hvac,H} + P_t^{hvac,C} \leq P_{max} \quad \forall t \in T, \forall hvac \in HVAC \tag{4d}$$

In order to calculate the demand and the flexibility provided by a HVAC appliance in a unique problem, the expression (5) evaluates whether the room temperature surpasses the room temperature constraints when offering flexibility to the power system. The expressions (5a) and (5b) calculate the room temperature if the flexibility exceeds the expected consumption, while expressions (5c) and (5d) estimate the downward flexibility effects on the room temperature for the n time steps. The expression (5e) limits the room temperature under flexibility requirements.

$$T_{t,n}^{UF} = T_{t-1} + \alpha_{hvac} * (T_{t-1}^{Forecast} - T_{t-1}) + \beta_{hvac,H} * (P_t^{hvac,H} + UF_{t,n}^{hvac,H}) - \beta_{hvac,C} * (P_t^{hvac,C} + UF_{t,n}^{hvac,C}) \quad n = 0, \forall hvac \in HVAC \tag{5a}$$

$$T_{t,n}^{UF} = T_{t,n-1}^{UF} + \alpha_{hvac} * (T_{t-1+n}^{Forecast} - T_{t,n-1}^{UF}) + \beta_{hvac,H} * (P_{t+n}^{hvac,H} + UF_{t,n}^{hvac,H}) - \beta_{hvac,C} * (P_{t+n}^{hvac,C} + UF_{t,n}^{hvac,C}) \quad n \neq 0, \forall hvac \in HVAC \tag{5b}$$

$$T_{t,n}^{DF} = T_{t-1} + \alpha_{hvac} * (T_{t-1}^{Forecast} - T_{t-1}) + \beta_{hvac,H} * (P_t^{hvac,H} - DF_{t,n}^{hvac,H}) - \beta_{hvac,C} * (P_t^{hvac,C} - DF_{t,n}^{hvac,C}) \quad n = 0, \forall hvac \in HVAC \tag{5c}$$

$$T_{t,n}^{DF} = T_{t,n-1}^{DF} + \alpha_{hvac} * (T_{t-1+n}^{Forecast} - T_{t,n-1}^{UF}) + \beta_{hvac,H} * (P_{t+n}^{hvac,H} - DF_{t,n}^{hvac,H}) - \beta_{hvac,C} * (P_{t+n}^{hvac,C} - DF_{t,n}^{hvac,C}) \quad n \neq 0, \forall hvac \in HVAC \tag{5d}$$

$$T_{min} \leq (T_{t,n}^{UF}, T_{t,n}^{DF}) \leq T_{max} \quad \forall n \in N \tag{5e}$$

HVAC appliances can provide flexibility independently of their operational mode (heating or cooling). Having two or more variables to quantify flexibility can produce discrepancies in the appliance scheduling. To avoid this problem, the expression (6) unifies the flexibility variables per operational mode into one variable for the upward and another for the downward flexibility. The expressions (6a) and (6b) limit the downward flexibility according to the expected power for the heating and cooling process, while the expression (6c) limits the upward flexibility according to the HVAC maximum power consumption. Finally, expressions (6d) and (6e) add the flexibility from both operational modes together resulting in a single variable.

$$DF_{t,n}^{hvac,H} \leq P_{t+n}^{hvac,H} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \tag{6a}$$

$$DF_{t,n}^{hvac,C} \leq P_{t+n}^{hvac,C} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \tag{6b}$$

$$P_{t+n}^{hvac,H} + P_{t+n}^{hvac,C} + UF_{t,n}^{hvac,H} + UF_{t,n}^{hvac,C} \leq P_{max} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \tag{6c}$$

$$UF_{t,n}^{hvac} = UF_{t,n}^{hvac,H} + UF_{t,n}^{hvac,C} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \tag{6d}$$

$$DF_{t,n}^{hvac} = DF_{t,n}^{hvac,H} + DF_{t,n}^{hvac,C} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \tag{6e}$$

The expression (6) considers flexibility for heating and cooling in the same equations [(6c), (6d), and (6e)], but HVAC appliances cannot heat and cool simultaneously. Hence, the HVAC heating and cooling operations are limited by the expression (7) using a BigM notation to couple the expected demand to a heating or cooling binary variable.

$$P_t^{hvac,H} \leq P_{max} * bP_t^{hvac,H} \quad \forall t \in T, \forall hvac \in HVAC \tag{7a}$$

$$P_t^{hvac,C} \leq P_{max} * bP_t^{hvac,C} \quad \forall t \in T, \forall hvac \in HVAC \tag{7b}$$

The inclusion of binary variables associated with the two HVAC operational modes requires a decision between a free binary model (first approach) or an artificial limit on the binary decisions (second approach). The first approach (expression (8)) allows the HVAC to select between both operation modes without limitations, approach that has two weaknesses. First, allowing the model to select between heating and cooling in each time step increases the computational power required to determine the schedule problem. Second, the scheduling model can alternate between heat and cooling to artificially increase flexibility. The associated energy cost can diminish the artificial flexibility increase if that scheduling strategy is used, but systems with low or zero energy cost will increase their energy consumption unnecessarily. This approach is useful in models with a longer time step (1-hour), but it can be unfeasible for daily appliance scheduling using a 15-minute or a 30-minute time step.

First Approach

$$bP_t^{hvac,H} + bP_t^{hvac,C} \leq 1 \quad \forall t \in T, \forall hvac \in HVAC \tag{8a}$$

Consequently, the operational decision mode used is the second approach (expression (9)), which limited heating or cooling depending on the outdoor temperature forecast. For example, the HVAC appliance could not heat the room if the predicted outside temperature is 28 °C and the maximum room temperature is 24 °C. The appliance could choose without restriction its operational mode when the forecast temperature does not surpass the temperature comfort range (expression (9c)). The temperature limits in the expressions (9a) and (9b) can be changed in line with the requirements.

Second Approach

$$bP_t^{hvac,H} = 0, \ if \ T_t^{Forecast} \geq T_{max} \quad \forall t \in T, \forall hvac \in HVAC \tag{9a}$$

$$bP_t^{hvac,C} = 0, \ if \ T_t^{Forecast} \leq T_{min} \quad \forall t \in T, \forall hvac \in HVAC \tag{9b}$$

$$bP_t^{hvac,H} + bP_t^{hvac,C} \leq 1 \quad if \ T_t^{Forecast} \in [T_{min}, T_{max}] \quad \forall t \in T, \forall hvac \in HVAC \tag{9c}$$

Finally, the expression (10) limited the HVAC flexibility according to the operational mode selected.

$$UF_{t,n}^{hvac,H} \le P_{max} * bP_{t+n}^{hvac,H} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \quad (10a)$$

$$UF_{t,n}^{hvac,C} \le P_{max} * bP_{t+n}^{hvac,C} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \quad (10b)$$

$$DF_{t,n}^{hvac,H} \le P_{max} * bP_{t+n}^{hvac,H} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \quad (10c)$$

$$DF_{t,n}^{hvac,C} \le P_{max} * bP_{t+n}^{hvac,C} \quad \forall t \in T, \forall n \in N, \forall hvac \in HVAC \quad (10d)$$

2.2.4. Electric Water Heater

The electric water heater (EWH) is present in most homes and other buildings. These appliances are used in 39% [54] of US residential houses, and their energy consumption can be 30% of a household's energy bills [55]. The EWH uses a resistor as a heating element, relying on the water temperature and the user's water usage to offer flexibility.

The EWH flexibility model used in this work is based on the one-node approach, where the water temperature is considered homogeneous throughout the tank. Other approaches consider two or more water compartments with different temperatures [56]. The basic EWH model is presented in the expression (11). The temperature evolution per time step is presented in (11a), including water usage and thermal losses related to thermal transmittance. Expression (11b) introduces the temperature limitation in the model. The term eP_t^{ewh} is used to exceed the minimum temperature constraint if the water temperature is lower than the expected value, enabling water heating at maximum power (expression (11a)).

$$T_t = T_{t-1} * \frac{V_{tank} - WU_t}{V_{tank}} + T_{water} * \frac{WU_t}{V_{tank}} + \frac{P_t^{ewh} + P_{max}^{ewh} * eP_t - W_{tank} * (T_{t-1} - T_{room})}{V_{tank} * \rho_{water} * \frac{Cp_{water}}{3600}} \quad if\ t \ne 0 \quad (11a)$$

$$T_{min} - bigM * eP_t^{ewh} \le T_t^{ewh} \le T_{max} \quad \forall t \in T \quad (11b)$$

$$0 \le P_t^{ewh} + P_{max}^{ewh} * eP_t^{ewh} \le P_{max}^{ewh} \quad \forall t \in T \quad (11c)$$

The flexibility evaluation used the same approach as presented in the HVAC model. The available flexibility was estimated based on the demand deviation without surpassing the thermal constraints, as shown in expression (12). The upward flexibility temperature $T_{t,n}^{UF}$ was introduced in expressions (12a) and (12b), while the downward flexibility temperature $T_{t,n}^{DF}$ is shown in (12c) and (12d). Expressions (12a) and (12c) help the transition between the appliance scheduling and the alternative flexibility scenario, using T_{t-1} as a starting point and changing to $T_{t,n-1}^{UF}$ and $T_{t,n-1}^{DF}$ for the alternative time step. The temperature limits shown in (12e) did not consider the emergency heating variable because the appliance cannot provide flexibility if the emergency heating is active.

$$T_{t,n}^{UF} = T_{t-1} * \frac{V_{tank} - WU_{t+n}}{V_{tank}} + T_{water} * \frac{WU_{t+n}}{V_{tank}} + \frac{P_{t+n}^{ewh} + P_{max}^{ewh} * eP_{t+n}^{ewh} + UF_{t,n}^{ewh} - W_{tank} * (T_{t-1} - T_{room})}{V_{tank} * \rho_{water} * \frac{Cp_{water}}{3600}} \quad if\ n = 0 \quad (12a)$$

$$T_{t,n}^{UF} = T_{t,n-1}^{UF} * \frac{V_{tank} - WU_{t+n}}{V_{tank}} + T_{water} * \frac{WU_{t+n}}{V_{tank}} + \frac{P_{t+n}^{ewh} + P_{max}^{ewh} * eP_{t+n}^{ewh} + UF_{t,n}^{ewh} - W_{tank} * (T_{t,n-1}^{UF} - T_{room})}{V_{tank} * \rho_{water} * \frac{Cp_{water}}{3600}} \quad if\ n \ne 0 \quad (12b)$$

$$T_{t,n}^{DF} = T_{t-1} * \frac{V_{tank} - WU_{t+n}}{V_{tank}} + T_{water} * \frac{WU_{t+n}}{V_{tank}} + \frac{P_{t+n}^{ewh} + P_{max}^{ewh} * eP_{t+n}^{ewh} - DF_{t,n}^{ewh} - W_{tank} * (T_{t-1} - T_{room})}{V_{tank} * \rho_{water} * \frac{Cp_{water}}{3600}} \quad if\ n = 0 \quad (12c)$$

$$T_{t,n}^{DF} = T_{t,n-1}^{DF} * \frac{V_{tank} - WU_{t+n}}{V_{tank}} + T_{water} * \frac{WU_{t+n}}{V_{tank}} + \frac{P_{t+n}^{ewh} + P_{max}^{ewh} * eP_{t+n}^{ewh} - DF_{t,n}^{ewh} - W_{tank} * (T_{t,n-1}^{DF} - T_{room})}{V_{tank} * \rho_{water} * \frac{Cp_{water}}{3600}} \quad if\ n \ne 0 \quad (12d)$$

$$T_{min} \le T_{t,n}^{UF}, T_{t,n}^{DF} \le T_{max} \quad \forall t \in T, \forall n \in N \quad (12e)$$

The expression (13) limited EWH flexibility according to the appliance demand and maximum demand. The downward flexibility depends on the energy consumption (expression (13a)), and its limited by the emergency heating variable (expression (13b)). The upward flexibility depends on the difference between the appliance consumption and the EWH maximum demand (expression (13c)).

$$DF_{t,n}^{ewh} \le P_{t+n}^{ewh} \quad \forall t \in T, \forall n \in N \quad (13a)$$

$$DF_{t,n}^{ewh} \le (1 - eP_{t+n}^{ewh}) * P_{max}^{ewh} \quad \forall t \in T, \forall n \in N \quad (13b)$$

$$P_{t+n}^{ewh} + P_{max}^{ewh} * eP_{t+n}^{ewh} + UF_{t,n}^{ewh} \le P_{max}^{ewh} \quad \forall t \in T, \forall n \in N \quad (13c)$$

2.3. Case study

In order to validate the proposed model, it is necessary to specify the energy resources, databases, case generation process, and other inputs used in the model formulation. First, two sets of appliances were used to evaluate its flexibility:

- 300 appliances — 100 HVAC, 100 PP, and 100 EWH
- 3000 appliances — 1000 HVAC, 1000 PP, and 1000 EWH

The HVAC formulation considered a building with five floors and ten apartments per floor, totalling 50 HVAC appliances per building. The apartment's roof area is 80 square metres. Each apartment had one EWH with a 120L tank and maximum power consumption of 2200 W. The PP were randomized from a set of 46 different models, assuming that the PP needed to operate for a minimum required time to filter the water at least twice daily.

Specific data were obtained from various databases. Hot water consumption profiles for the EWH were generated with the software *Load Profile Generator* [57]. Historical outdoor air temperature from NOAA NCEI Climate Normals [58] is used for the temperature forecast, data station: San Francisco International Airport (ID GHCND:USW00023234). Building material properties and wall-building design were extracted from *EnergyPlus* software [59]. Energy prices were downloaded from the CAISO OASIS Platform [60]. The Energy Component in the HASP Locational Marginal Prices and the Regulation Up and Down from the Interval AS Clearing Prices were used as Energy and Flexibility prices. The energy and flexibility prices data have a 15-minute granularity, and the mean price value is used for the 30-minute and 1-hour cases.

Finally, to test the flexibility changes in a predefined set of appliances, we used 5 case studies to evaluate how the flexibility changed according to the duration of the flexibility requirement and the energy prices:

- Case 1: Flexibility estimation without considering energy prices for the different time intervals
- Case 2: Response to a 1-hour flexibility requirement for the different time intervals
- Case 3: Flexibility under different tariff schemes
- Case 4: Flexibility response for real-time prices (one per season)
- Case 5: Appliance response to a 4-block flexibility requirement

The first case aimed to evaluate the appliance response for various flexibility requirements with different duration. The objective was to show how the flexibility offered decreased if the flexibility duration request was increased. This case did not consider energy prices, and the flexibility prices are fixed at 1 USD/MWh. The second case evaluated

Table 1
Energy and flexibility prices for the different tariff schemes.

	Energy [USD/MWh]		Upward flexibility [USD/MWh]		Downward flexibility [USD/MWh]	
No price	0		1		1	
Fixed Tariff	80		24		24	
Real-time	CAISO clearance prices at May 10th, 2022					
	Energy		Upward flexibility		Downward flexibility	
	Non-peak	Peak	Non-peak	Peak	Non-peak	Peak
Time-of-Use (ToU)	70	100	24	30	24	30
ToU — Case 1	70	100	24	30	24	0
ToU — Case 2	70	100	24	0	24	30

Table 2
Daily aggregated flexibility for the evaluated time steps for different duration requirements. The Flexibility percentage uses the 1-block case as reference.

Time step		Requirement duration			
		1-block	2-block	3-block	4-block
15-minute	Up. Flex. [MWh]	14.27	14.62	13.14	12.06
	% Up. Flex.	100%	102%	92%	84%
	Down. Flex. [MWh]	6.65	5.34	5.23	5.26
	% Down. Flex.	100%	80%	79%	79%
30-minute	Up. Flex. [MWh]	13.93	11.95	10.82	10.17
	% Up. Flex.	100%	86%	78%	73%
	Down. Flex. [MWh]	6.18	5.18	4.64	4.28
	% Down. Flex.	100%	84%	75%	69%
1-hour	Up. Flex. [MWh]	11.22	10.06	9.29	8.50
	% Up. Flex.	100%	90%	83%	76%
	Down. Flex. [MWh]	5.39	3.89	3.16	2.78
	% Down. Flex.	100%	72%	59%	52%

how the time interval used to estimate the flexibility can affect the flexibility itself. The third flexibility case looked at how flexibility could be affected by the price schemes indicated in Table 1, where the peak time was between 16:00 and 21:00 h. The fourth case examines how the appliances modified their flexibility for four different real-time price profiles, considering one day per season (and their respective temperature forecast). Temperature values from May and November considered 50-percentile temperatures, February use 10-percentile temperatures and August, 90-percentile temperatures. Finally, the last case validated the use of extended flexibility estimation models. The flexibility estimated for the 4-block approach is introduced as a flexibility requirement for the 1-block, 2-block, and 3-block appliance scheduling calculated by the model. The objective was to quantify the demand deviation if a lower (in power) flexibility requirement but higher in time was received.

The first four cases used the 300 appliances set, while the last used the 3000 appliances set to evaluate the aggregated response to the flexibility requirements. The first two cases evaluated three time intervals (15 min, 30 min, and 1 h), while the last three use a 15-minute approach with a 2-block flexibility requirement to standardize results. Energy prices used in the flexibility estimation were from 10 May, 2022, unless otherwise indicated. The proposed methodology to evaluate day-ahead flexibility is summarized in Fig. 2. The case studies were coded in Python using Pyomo Libraries [61,62]. The solver GUROBI [63] with a 0.02 MIP GAP was chosen to solve the proposed problems. Cases 1 to 4 were run on a Ryzen 9 5900HX processor with 16 GB RAM. The last case was run on an Intel i7 10700 processor with 32 GB RAM. The 15-minute approach with a high number of appliances requires large amounts of RAM, and this requirement will be higher if the model considers more than 4-blocks. In contrast, a commercial laptop can easily calculate the 30-minute and 1-hour approaches with a huge set of appliances.

3. Results and discussion

The five case studies proposed in Section 2.3 to evaluate the alternative flexibility scenario and the prices effects on flexibility are discussed below.

3.1. Flexibility under different approaches

Fig. 3 shows the flexibility estimated for the 15-minute approach considering flexibility requirements from 15 min (1-block) until 1 h (4-block), assuming energy prices equal to zero. The flexibility results for 1-block present higher flexibility and more variability on the scheduled demand compared with the extended flexibility requirements. Table 2 shows the total daily flexibility and the flexibility percentage related to the 1-block case. The flexibility values indicated are the total response capacity, without consider rebound effects. For the 15-minute approach, the downward flexibility showed at least a 20% decrease if the flexibility requested had a greater than 1-block duration, while the upward flexibility showed that flexibility for the 2-block requirement in the 15-minute approach was higher than the base case, indicating that the flexibility was restricted by the power limitations and the appliance's scheduling rather than the thermal storage capacities in the EWH and the HVAC. This does not apply to the 30-minute and 1-hour approach, which shows decreases in the upward flexibility as expected.

The demand variability observed for the 1-block case in Fig. 3 is repeated in each evaluated time step. As seen in Fig. 4, the upward and downward flexibility estimated for the 1-block scenario present a high temporal variability when compared to the flexibility for multiple blocks. This behaviour can be explained based on the non-temporal flexibility dependence in the 1-block case. In contrast, the flexibility temporal dependence for the 2-block case and others, forces the model to choose an smooth demand scheduling to maximize the flexibility. For example, if turning on an appliance in $t+1$ reduces the possible flexibility for t (flexibility evaluated with $n=1$ for the time step t), the model will prefer to postpone the appliance activation.

Fig. 4 compares the flexibility proposed for the different time interval approaches with different flexibility requirements (from 1 to 4-block requirements). In each hour, the estimated flexibility for an n-block flexibility requirement decreases according to the increase in n, if it is considered only the cases where $n \geq 1$. This statement is not true if $n=1$ because of the high variability in the flexibility estimated.

A flexibility reduction is observed at the end of the day, which is directly increased by the flexibility requirement duration. The flexibility reduction can be explained by the energy displaced to increase the flexibility when the day starts, and the rebound constraints introduced in the appliances, offering less flexibility if the displaced flexibility cannot be compensated. Both effects exhibit its maximum observed decrease in the 4-block 1-hour case, where the flexibility reduction that starts at hour 16. A rolling horizon approach to estimate flexibility could be helpful for the HVAC and EWH appliances, which evaluates their flexibility according to the Room/Water temperature. However, it can be an issue for the PP appliances that need a specific amount of energy daily. If a limited number of hours are evaluated, the proposed flexibility estimation model cannot be enough to capture the PP behaviour.

3.2. Aggregated response for a 1-hour flexibility requirement

Fig. 5 shows the estimated flexibility offered in the three time interval approaches if a 1-hour flexibility requirement is received. On the left side can be found the estimated flexibility without considering energy prices, while the right side shows the flexibility considering real-time pricing. The first difference between no-price and real-time pricing is the appearance of zero flexibility hours, conditioned by a zero

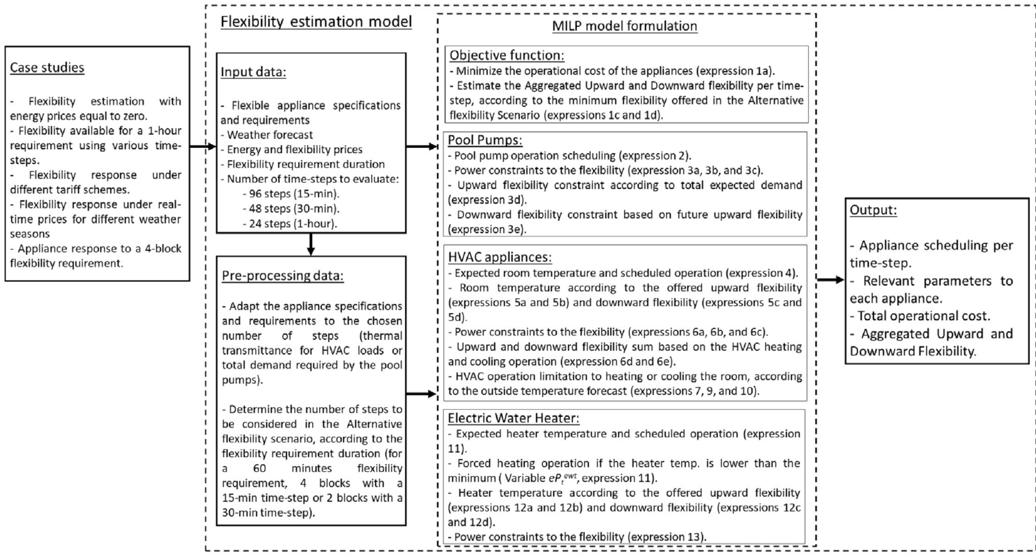

Fig. 2. Resume flowchart for the proposed methodology.

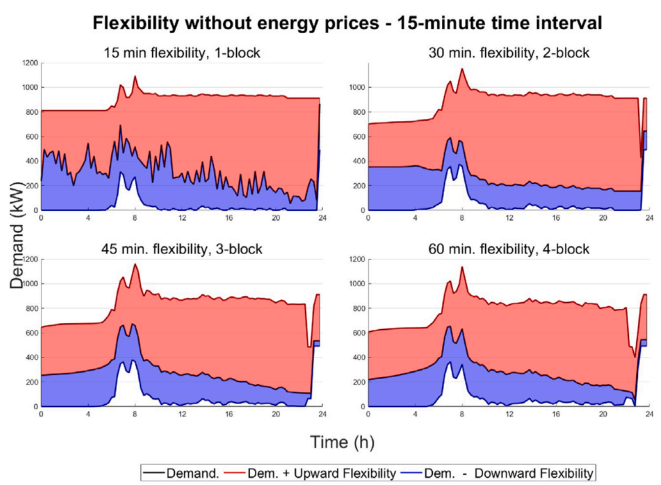

Fig. 3. Flexibility estimation for various time blocks under a 15-minute time interval.

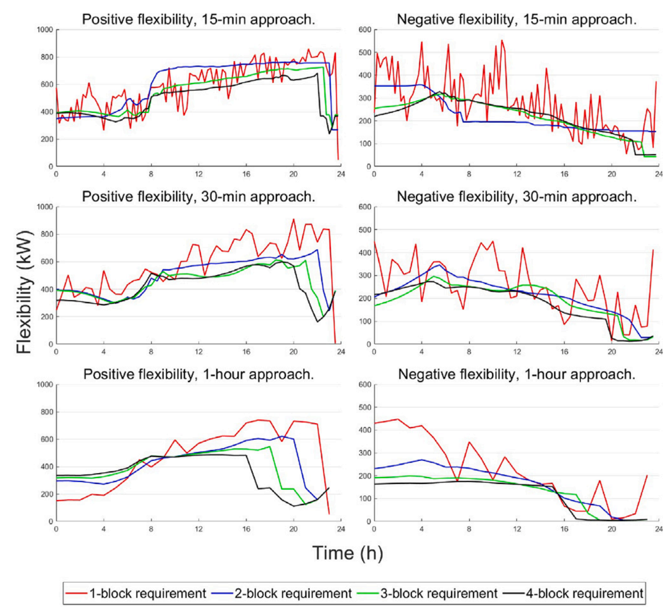

Fig. 4. Flexibility estimation results for the studied time blocks and time intervals.

Table 3
Daily aggregated flexibility considering a 1 h flexibility requirement.

	No price		Real-time pricing	
	Up. flex. [MWh]	Down. flex. [MWh]	Up. flex. [MWh]	Down. flex. [MWh]
15-minute	12.05	5.26	8.38	3.48
30-minute	11.95	5.18	8.79	3.77
1-hour	11.22	5.39	10.26	4.30

price in specific time intervals. This effect cannot be shown in the 1-hour approach because the mean flexibility price diminish the impact of the 15-minute intervals with zero flexibility prices. A second difference between both input signals is the downward flexibility shape. In the no-price signal, the downward flexibility is distributed throughout the day with an expected reduction at the end of the day, while the real-time price approach concentrates the downward flexibility between 8:00 h and 18:00 h.

An unexpected result is an increase in the upward flexibility in no-price schemes if the time-interval is reduced (for example, from 1-hour to 30-minutes), while in real-time pricing, the flexibility decreases, as seen in Table 3. The expected result was a flexibility increase if the time interval considered more time blocks in a day, because specific constraints such as temperature limitation are evaluated four times per hour in the 15-minute time interval and one time per hour in the 1-hour approach. This result can be an unexpected effect of the energy price that concentrates the demand and flexibility in specific hours.

3.3. Tariff schemes response

Fig. 6 shows the estimated flexibility for the 300 appliances under different tariff schemes. The tariffs scheme used are indicated in Table 1. The difference between a no-price flexibility estimation and a fixed tariff is the downward flexibility reduction at the end of the day, limited by the lack of demand to shift its consumption. A similar problem occurs with the Time-of-Use (ToU) tariff. In this case, the energy price in the peak hour (16:00 to 21:00 h) directly affects the downward flexibility, reducing the demand because of the increase on the energy cost. This behaviour, plus the flexibility reduction in the last hours of the day, significantly reduce the appliance's capacity to change

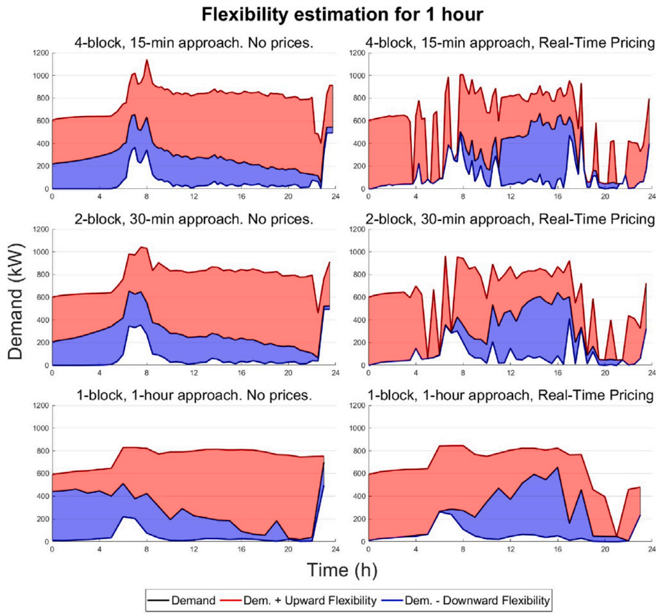

Fig. 5. Flexibility estimation for 1-hour requirement under various time intervals. The 15-minute flexibility approach shows a high variability in the demand and flexibility estimation. In contrast, the 1-hour approach presents smoothed values and a reduction in the available flexibility.

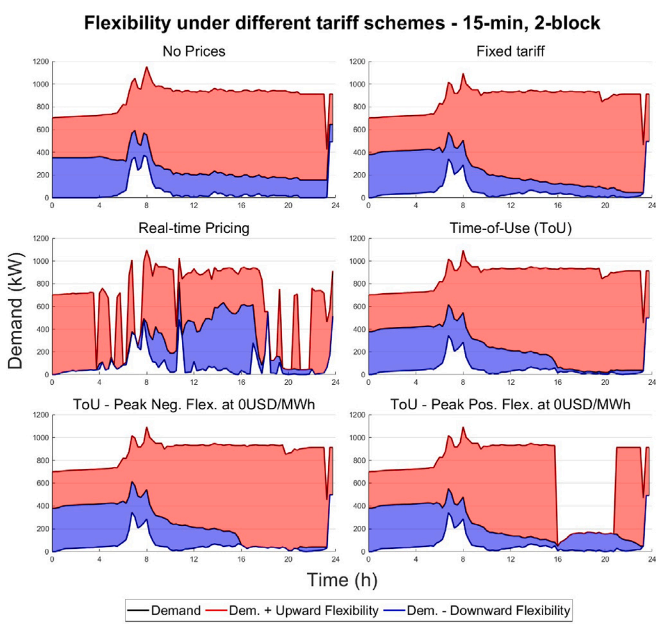

Fig. 6. Flexibility estimation for 30 min under a 15-minute time interval. The energy and flexibility prices are indicated in Table 1.

its consumption during critical hours. The Time-of-Use tariff considers that upward and downward flexibility have the same value in the peak hour, generating an artificial incentive to increase upward flexibility because the downward flexibility depends on the energy consumption, which is penalized during the peak hours. To visualize how the input signal (in this case, flexibility prices) can affect the flexibility, two additional cases were considered: ToU tariffs plus downward flexibility with no value and ToU Tariffs plus upward flexibility with no value. The reduction in the flexibility prices is only considered for peak hours. The first case shows similar results to the ToU case, reducing demand consumption to nearly zero in peak hours. In contrast, the demand in the second case increases in the peak hours because if the downward

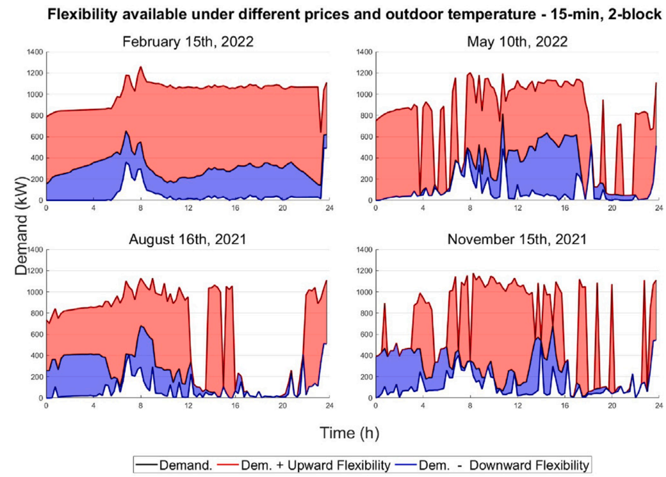

Fig. 7. Flexibility estimation values for 30 min under a 15-minute time interval, considering temperature and price values for one day per season.

flexibility prices are higher, the model will increase its demand to offer more flexibility, negatively affecting the ToU peak hours aim of decreasing energy consumption in the power systems.

In comparison, the upward flexibility can increase capacity during the day regardless of the scheduled demand (and if the flexibility prices are higher than zero), being limited only by the appliance's maximum power and rebound constraints. It must be highlighted that upward flexibility considers capacity to increase consumption if a flexibility requirement is received without considering the energy price in the alternative scenario to prioritize the maximum energy the appliances can shift in time. The reason to use this approach is because the EMS does not try to increase consumption without reason, only if it receives an external request. A new formulation that considers the EMS cost to provide flexibility if a new request is received and defines how many energy shifts in time according to the received signal can be explored in the future.

3.4. Flexibility estimation per season

The energy and flexibility prices (or other incentives) define the demand consumption and how much flexibility will be offered during the day, based on the scheduled demand. To assess the price effect of flexibility, Fig. 7 shows the proposed demand consumption and the flexibility offered under four scenarios, one scenario per season. Demand consumption on February 15th depends on the HVAC operation because of the low outdoor temperatures (the 10-percentile temperature was used in this case). The constant energy consumption allows to the scheduled appliances to offer downward flexibility throughout the day. In contrast, August 16th shows a prohibitive energy price between 16:00 and 22:00 h, limiting the downward flexibility to the night and the first hours in the morning, while the upward flexibility price was 0 USD/MWh in the indicated time window. The scheduled demand consumption for May 10th and November 15th presents similar patterns to August demand, a considerable demand decrease in the peak hours and some hours in the day where there are no incentives to offer upward flexibility to the grid. An unexpected conclusion from the results is the lack of downward flexibility in peak hours; unless the day-ahead flexibility incentives surpass the benefits of not consuming energy in peak hours, the flexible appliances prefers to use flexibility to decrease total cost instead of offering it to the system.

As a deterministic model, modifying the objective function (expression (1a)) to return the energy price if a downward flexibility offer is delivered will generate an artificial increase in energy consumption. The downward flexibility will not have a cost, so we can shift all the

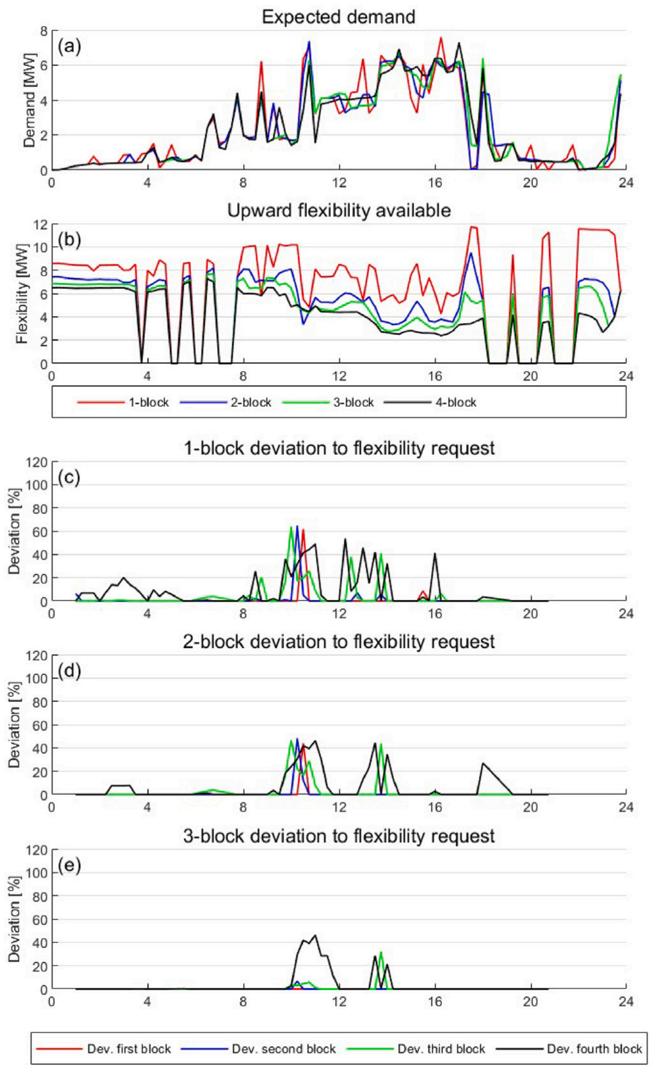

Fig. 8. 4-block upward flexibility requirement for the 1, 2, and 3-block approach. Plots (a) and (b) shows the scheduled demand and appliance flexibility. Plots (c) to (e) present the percentage of the not supplied flexibility per case. The "Dev. first block" legend indicates the flexibility deviation in the first block, according to the 4-block flexibility requirement.

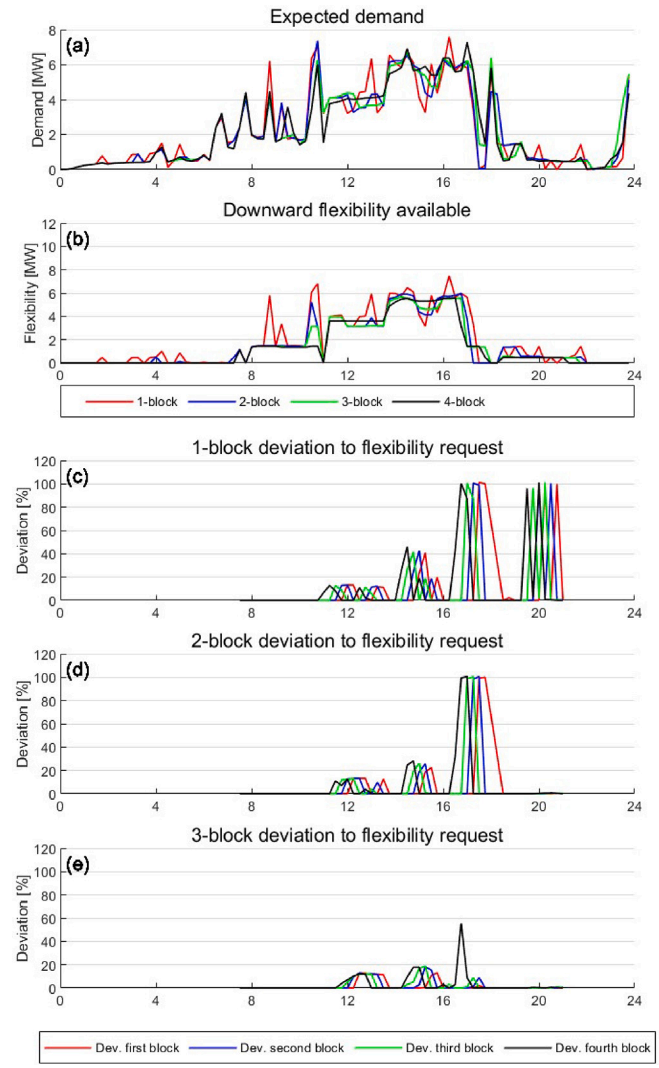

Fig. 9. 4-block downward flexibility requirement for the 1, 2, and 3 block approach. Plots (a) and (b) shows the scheduled demand and appliance flexibility. Plots (c) to (e) present the percentage of the not supplied flexibility per case.

demand to the hours where the downward flexibility has high prices. To tackle this problem, a stochastic approach must be developed to include the possibility of offering (or not offering) flexibility according to the possible energy prices and non-controllable demand, including the profitability and possible losses if the flexibility offered is not adequate.

3.5. Aggregated appliance response for a 4-block flexibility requirement

One of the principal advantages of the proposed model is its capacity to consider flexibility requirements that can be extended over various time steps. In order to validate the extended flexibility response advantage, the flexibility estimated by the 4-block approach will be introduced as a flexibility requirement for the appliance scheduling proposed by the 1-block, 2-block, and 3-block flexibility approaches. The model assumes that the potential flexibility estimated by the 4-block problem is lower than the potential flexibility offered by the other approaches. However, considering a higher flexibility duration in their estimation is decisive to the energy flexibility offered by the appliances. The model will evaluate if the 1-block to 3-block cases can follow their initial scheduling and offer the same (and lower in power) flexibility as the 4-block case. To clarify, flexibility or response deviation will be considered as the flexibility requirements that the proposed appliance's scheduling cannot fulfil.

Figs. 8 and 9 compare the upward and downward flexibility response to the indicated 4-block flexibility requirement. The upward flexibility in Fig. 8 shows the flexibility reduction when the requirement duration increases, except for the flexibility at hour 10, which shows lower flexibility values for the 1-block case. The low energy price can explain the flexibility reduction between hours 10 to 17 compared to the rest of the day, which increases the demand scheduled in the indicated time window. The mean price difference between the 10 to 17 h and the rest of the day is over 40 USD/MWh, which is decisive in the appliance scheduling proposed by all the cases.

The flexibility requirements are compared per block to evaluate if the observed deviations can be correlated to a specific time block. For the upward case, the flexibility estimated by the 1-block approach presents as much as 60% difference at hour 10, influenced by the scheduled demand at 10:30 am. The demand scheduled by the 1-block approach surpasses the demand by the 4-block approach by 86%, while the demand peak at 10:45 am generates flexibility deviations at 10:45 (1-block deviation), 10:30 (2-block), and 10:15 (3-block).

Flexibility deviations between 12:00 to 16:00 were produced by the high variability in demand scheduled by the 1-block approach.

Flexibility deviations for the 2-block and 3-block approaches are diminished by the reduced demand variability, especially for the 3-block approach. Flexibility deviations were observed at hour 10 and hour 14, which are associated with an increase in the scheduled demand. If the flexibility deviation obtained is compared with the 1-block deviation, the extended flexibility estimation demonstrates that including temporal dependence in flexibility reduces the demand variability and increases the response capacity to offer flexibility services.

The downward flexibility case (Fig. 9) shows higher differences between the estimated flexibility per block approach and their final implementation, reaching a 100% flexibility deviation in some hours. For the 1-block case, the high variability in the scheduled demand produces energy reductions from 6 MW to 0 MW between 17:00 h to 18:00 h (generated by an energy price increase), decreasing the appliance's response and denying possible flexibility services to the grid. A similar episode occurs between 19:00 and 21:00 h, where the demand differences in time diminish the appliance's response. The principal difference between upward and downward flexibility is the downward dependence on the scheduled demand and the energy cost. The proposed model can offer a demand schedule that diminishes or nullifies the possibility of offering flexibility in specific hours if the energy price drastically changes between two successive time steps. Unless the flexibility prices are high enough to consume energy in low energy cost hours, the proposed model will use their own flexibility to decrease its cost instead of offer flexibility to the grid.

To conclude, the extended flexibility estimation can ensure the energy to offer in comparison with an instantaneous flexibility offer for multiple time steps, using fast flexibility appliances to offer an aggregated and longer in time flexibility response. A negative effect of the proposed approach is the variability reduction on the scheduled demand, reducing its capacity to catch the price differences if the flexibility price is high enough to generate this effect.

4. Conclusions

A flexibility estimation model is proposed to evaluate the aggregated flexibility response from a set of appliances. The model offers an aggregated response for a defined flexibility duration that can change between 15 min and 4 h, according to the evaluated time interval and flexibility service duration based on the ISO reserve requirements. New flexibility constraints were introduced to evaluate the flexibility and the rebound effect for air conditioners, pool pumps, and electric water heater appliances. The aggregated flexibility has been evaluated based on maximizing the minimum flexibility offered by the appliances, using a unique variable for the flexibility estimation in the subsequent n time blocks.

The flexibility model was tested under different energy price schemes and scenarios to observe how the flexibility offered changes over time. The appliances depend directly on energy prices to offer downward flexibility. If the energy prices are low; the demand and downward flexibility increase, while a lack of demand increases the upward flexibility. In contrast, the limited downward flexibility in the hours with high energy costs are counterproductive to the high downward flexibility prices. Unless the potential benefits from offering flexibility could outweigh the energy cost, the estimated flexibility will be minimal because the model prefers to use its flexibility to reduce the operational cost.

The extended flexibility requirement approach was tested with a flexibility request equal to the values estimated by the 4-block estimated flexibility. Even if the potential estimated flexibility was lower than in the other cases, the 1-block to 3-block approaches could not fulfil the flexibility requirement without surpass the appliance's constraints. A lower deviation in the flexibility was observed when the number of blocks considered in the flexibility estimation was higher.

Future work includes designing a virtual power plant formulation for flexible appliances to coordinate local flexibility with renewable generation and non-controllable demand under a stochastic approach. The proposed second work is to develop a heuristic to determine the number of blocks to consider in the flexibility formulation according to the scheduled demand, energy prices, and historical flexibility duration requirements. The alternative flexibility scenario can be used to estimate the aggregated flexibility whether a demand response requirements is received, where the model could decide to provide (or not) the required flexibility. The estimated aggregated flexibility can be used to mitigate the effects of the uncertainty and variability if a generation profile must be followed. Finally, the inclusion of new appliances such as electric vehicles and an economical approach based on a competitive flexibility market with multiple flexibility providers could improve the economical evaluation of the flexibility in distribution networks.

CRediT authorship contribution statement

Marcelo Salgado-Bravo: Conceptualization, Formal analysis, Investigation, Methodology, Software, Visualization, Writing – original draft, Writing – review & editing. **Matias Negrete-Pincetic:** Methodology, Resources, Supervision, Writing – review & editing, Conceptualization. **Aristides Kiprakis:** Conceptualization, Methodology, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix. Thermal models

The literature on thermal building modelling can be divided into three categories based on the approach used:

- White-box models: Mathematical formulations where each one of their expressions, constants, and variables is known can be considered white-box models; these are usually based on physical and first-principle models [64]. Various formulations can be considered white-box models, starting with the static linear approach until complex dynamic formulations evaluate the heat balance between the thermal zones. This approach requires detailed physical knowledge to build a proper model (e.g. building geometry, material properties) to reduce discrepancies between the simulated building and temperature measurements. The software applications EnergyPlus [59,65], TRNSYS [66], and Modelica [33,67] use models to simulate the building's thermal behaviour that can be considered white-box.
- Black-box models: Mathematical formulations that ignore the physics related to the problem and rely on on-site measurements to train data-driven models are considered [68]. The model training requires HVAC consumption, operational points, and internal and outdoor temperature measurements to cover any possible relevant variable to the model, but the resulting predictions outperform other thermal approaches [69]. The dataset must be as long and exhaustive as possible, covering all the seasons and taking at least one year to increase the accuracy of the training process. Artificial Neural Networks [70], Ensemble Learning models [71], and Support Vector Machines [72] are approaches used in black-box models.

- Grey-Box models: Mixed mathematical formulations that considers both empirical and physical components in the model can be considered grey-box model. They depend on on-site measurements to adjust the parameters introduced in the physical model [73]. Compared to a black-box model, the required training dataset is smaller, with a few weeks of measurements instead of years of data. The resistive–capacitive thermal network model [74] is the most used approach that can be considered a grey-box model.

In the proposed approach, it is challenging to use black-box models because of the high number of variables affecting the computational power required by the model formulation [69]. Consequently, the white-box and grey-box approaches are preferred to formulate building optimization models. The white-box approach is usually used in formulation that considered numerous buildings with minimal information [75]. Its simplicity in defining the household properties in a static linear formulation allows it to include a hundred or thousand households in demand aggregation or virtual power plant models. In addition, more complex models as first-principle approaches presented by EnergyPlus and TRNSYS offer highly detailed thermal modelling but are computationally expensive for an optimization approach. It is possible to use high-level white-box models together with a grey-box model, using the data generated by the white-box model as measurements to calibrate the grey-box [65], simplifying the white-box model so that is can be effectively used in an optimization model.

Independently of the white-box or grey-box approach, it is possible to present the parameters in the thermal linear model as a constant value (in this work is used α and β for the parameters) associated with the thermal losses, energy efficiency, and thermal storage in the model. The thermal model presented in this work uses this notation as a generalization independently of a white-box or grey-box approach.

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