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(2K20/ITE/73)

Group A (Assignment I)

Calculus And Analytical Geometry

Assignment I

Q#1

let $f(x) = \frac{x - 3/2}{x^2 + 2}$ and $g(x) \frac{x^2+1}{x^2+2}$ At what values of x do the curves $y = f(x)$ and $y = g(x)$ have parallal tangent line.

Solution

$$F(x) = \frac{x - 3/2}{x^2+2}$$

To find x-intercept substitute $F(x) = 0$

$$0 = \frac{x - 3/2}{x^2+2}$$

$$= \frac{2x - 3/2}{x^2+2}$$

$$= \frac{2x - 3}{2} \times \frac{1}{x^2+2}$$

$$0 = \frac{2x-3}{2(x^2+2)}$$

Swap the sides of the equation.

$$\frac{2x-3}{2(x^2+2)} = 0$$

when the quotient equals 0 the numerator has to be 0. $2x - 3 = 0$

$$2x = 3$$

$$\boxed{x = \frac{3}{2}}$$

$$g(x) = \frac{x^2+1}{x^2+2}$$

To find x-intercept substitute $g(x) = 0$

$$0 = \frac{x^2+1}{x^2+2} \qquad x \in R$$

Since the Right hand side is always positive the statement is false for any value of $x \in R$.

Since there is no solution for $g(x)=0$ therer is

$\boxed{\text{No } x\text{-intercept / zero}}$

Answer

Q#2

The tangent line to the graph of a function f at $x=2$ passes through the points $(0,-20)$ & $(5,40)$. Then $F(2) = ?$ and $f'(2) = ?$

Solution,

$F(x) = ?$

$F(2) =$

$F(x) = 2$

$\boxed{F(2) = 2}$

Now we find $F'(2)$.

we use slope formula.

$\because x = \frac{y_1 - y_2}{x_1 - x_2}$

$$F'(2) = \frac{-20-40}{0-5}$$

$$= \frac{\cancel{+60}^{12}}{\cancel{+5}_{1}}$$

$\boxed{F'(2) = 12}$

Answer

Question No# 3.

let $f(x) = \begin{cases} x^2 & \text{for } x \leq 1 \\ 1 & \text{for } 1 < x \leq 3 \\ 5-2x & \text{for } x > 3 \end{cases}$. Then $f'(0) =$ ___, and

$f'(2) = ?$ and $f'(6) = ?$ suppose that g is a differentiable function and that $f(x) = g(x+5)$ for all x. If $g'(1) = 3$ then $f'(a) = 3$ where $a =$ ___ .

Solution:

Given that : $f(x) = \begin{cases} x^2 & \text{for } x \leq 1 \\ 1 & \text{for } 1 < x \leq 3 \\ 5-2x & \text{for } x > 3 \end{cases}$

Now, we diff w.r.t x on b/s

$$f(x) = \frac{df(x)}{dx} = \frac{d}{dx}(x^2) \Rightarrow \boxed{f'(2) = 1}$$

at $x = 6$: $f(x) = 5-2x$

$$f'(x) = -2$$

$$\boxed{f'(6) = -2}$$

Now,

$f(x) = g(x+5)$ and

$f'(a) = 3$

$f'(x) = 2x$

$f'(a) = 2a$

$3 = 2a$

$$\boxed{a = \frac{3}{2}}$$

$\because x = a$

$\because f'(a) = 3$

Answer

Question No 4:- Part I

• Let $f(x) = [2-(x-1)] - 1$ for every Real Number x. Then. $f'(2) = ?$, $f'(0) = ?$ $f'(-2) = ?$ $f'(4) = ?$

Solution:-

$$f(x) = [2-(x-1)] - 1$$

To find x-intercept Subtitute $f(x) = 0$

$$0 = [2-(x-1)] - 1$$

$$[2-(x-1)] = 1$$

using the absolute value as two seperate equations.

$\boxed{2-(x-1) = 1}$ The eq have 4 solutions.

Solve the equation for x

$x = 2$, $x = 0$, $x = 4$, $x = -2$

$\boxed{f'(2) = 2}$, $\boxed{f'(0) = 0}$, $\boxed{f'(4) = 4}$, $\boxed{f'(-2) = -2}$

Answer

Part II

Let $f(x) = \tan^3 x$. Then $Df(\pi/3) = ?$

Solution:-

$$f(x) = \tan^3 x$$

To find x-intercept substitute $f(x) = 0$

$0 = \tan^3 x$ determined the range $x = \frac{\pi}{2} + k\pi$

$\boxed{\tan(x)^3 = 0}$

The only way exponentiation can be 0. when the base is equal 0.

$$\tan(x) = 0$$

Since $\tan(t) = 0$ for $t = k\pi$ then $x = k\pi$

$$x = k\pi \quad k \in Z, \quad x \neq \pi/2 + k\pi, \quad k \in Z$$

$\boxed{x = k\pi \ , \ k \in Z}$

Answer

Part III

Let $f(x) = 1/x \cdot \csc^2 1/x$. Then $Df(6/\pi) = \frac{\pi^2}{a}\left(\frac{\pi}{\sqrt{b}} - 1\right)$ where a & b = ?

Solution:

$$f(x) = \frac{1}{x} \cdot \csc^2 \frac{1}{x}$$

Taking derivative b/s

$$F'(x) = \frac{d}{dx}\left\{\frac{1}{x} \cdot \csc\left(\frac{1}{x}\right)^2\right.$$

$$F'(x) = \frac{d}{dx}\left\{\frac{\csc(1/x)^2}{x}\right\}$$

use differentiate Rule.

$$F'(x) = \frac{\frac{d}{dx}\left\{\csc\left(\frac{1}{x}\right)^2\right\} x - \csc\left(\frac{1}{x}\right)^2 \cdot \frac{d}{dx}(x)}{x^2}$$

$$= \frac{2\csc\left(\frac{1}{x}\right)\left\{-\cot\left(\frac{1}{x}\right)\csc\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\right\} - \csc\left(\frac{1}{x}\right)^2 \cdot \frac{d}{dx}(x)}{x^2}$$

$$F'(x) = \frac{2\csc\left(\frac{1}{x}\right)\left(-\cot\left(\frac{1}{x}\right)\csc\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) - \csc\left(\frac{1}{x}\right)^2 \times 1\right.}{x^2}$$

Simplify the expressions

$$\boxed{F'(x) = \frac{2\cos\left(\frac{1}{x}\right) - x \cdot \sin\left(\frac{1}{x}\right)}{x^3 \cdot \sin\left(\frac{1}{x}\right)^3}}$$

Answer

Part IV

Let $F(x) = \sin^2(3x^5+7)$. Then $F'(x) = ax^4 \sin(3x^5+7)\, f(x)$ where a and $f(x) = ?$

Solution:

$F(x) = \sin(3x^5+7)$

Taking the derivative of b/s

$$F'(x) = \frac{d}{dx}\{\sin(3x^5+7)^2\}$$

Using the Chain Rule

$$F'(x) = \frac{d}{dg}(g^2) \times \frac{d}{dx}\{\sin(3x^5+7)\}$$

$$= 2g \cdot \frac{d}{dx}\{\sin(3x^5+7)\}$$

$= 2g \cdot \cos(3x^5+7)(5x^4 \cdot 3)$ ∵ $(\sin)' = \cos$

= Substitute back $g = \sin(3x^5+7)$

$= 2\{\sin(3x^5+7) \cdot \cos(3x^5+7)(5x^4)(3)$

= Simplify the expressions.

$= 15x^4 \cdot \sin(6x^5+14)$

$$\boxed{F'(x) = 15x^4 \cdot \sin(6x^5+14)}$$

Answer

Question 5:

Using L'Hospitals Rule Solve:-

① $$\lim_{x \to 0} \frac{\frac{1}{3}x^3 + 2x - 2\sin x}{4x^3} = \frac{1}{a} \text{ where } a = ?$$

Solution:

$$\lim_{x \to 0} \frac{\frac{d}{dx}\{\frac{1}{3}x^3 + 2x - 2\sin(x)\}}{\frac{d}{dx}(4x^3)}$$

Now, we calculate derivative.

$$\lim_{x \to 0} \left\{ \frac{x+2-2\cos(x)}{\frac{d}{dx}(4x)^3} \right\}$$

$$\boxed{\lim_{x \to 0} \left\{ \frac{x+2-2\cos(x)}{12x^2} \right\}}$$

Since Evaluating limits of the numerator & denominator would Result in an intermediate form use the L,Hospital's Rule.

$$\lim_{x \to 0} \left\{ \frac{\frac{d}{dx}(x^2+2-2\cos(x))}{d/dx\,(12x^2)} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{2x+2\sin(x)}{24x} \right\}$$ || Taking Common 2.

$$\lim_{x \to 0} \left\{ \frac{\cancel{2}(x+\sin(x))}{\cancel{24}\,x} \right\}$$

$$\boxed{\lim_{x \to 0} \left\{ \frac{x+\sin(x)}{12x} \right\}}$$

Since Evaluating limit of the numerator & denominator would Result in an intermediate form use L'Hospital Rule.

$$\lim_{x \to 0} \left\{ \frac{\frac{d}{dx}(x+\sin x)}{d/dx\,(12x)} \right\}$$

$$\boxed{\lim_{x \to 0} \left\{ \frac{1+\cos x}{12} \right\}}$$

Evaluate the limit

$$= \frac{1+\cos(0)}{12} \Rightarrow \frac{1+1}{12} \Rightarrow \frac{\cancel{2}}{\cancel{12}} \quad \because \cos(0)=1$$

$$= \boxed{\frac{1}{6}}$$

Answer

2) $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \_\_\_\_\_$

Solution:-

$$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

Taking L.C.M

$$\lim_{x \to 0} \left(\frac{x - \sin x}{x \cdot \sin x} \right)$$

Since Evaluating limits of the N & D would be intermediate. use Hospital Rule.

$$\lim_{x \to 0} \left\{ \frac{\frac{d}{dx}(x - \sin x)}{\frac{d}{dx}(x \cdot \sin x)} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{1 - \cos x}{\sin x + x\cos x} \right\} \quad \because \text{Taking 2nd } F''(x) =$$

$$\lim_{x \to 0} \left\{ \frac{\frac{d}{dx}(1 - \cos x)}{\frac{d}{dx}(\sin x + x\cos x)} \right\}$$

$$\boxed{\lim_{x \to 0} \left\{ \frac{\sin x}{2\cos(x) - x\sin(x)} \right\}}$$

Evaluate the limit

$$\frac{\sin(0)}{2\cos(0)-(0)(\sin(0))}$$

$\because \sin(0)=0$
$\because \cos(0)=1$

$\boxed{0}$

Answer

(3) $\lim_{t\to 1} \frac{nt^{n+1}-(n+1)t^n+1}{(t-1)^2} = \_\_ \quad n\geq 2$

Solution:

$\lim t\to 1 \quad (nt^{n+1}-(n+1)t^n+1)$

$= n\cdot t^{n+1}-(n+1)\cdot 1^n+1$

$= 1^{n+1}n-1\cdot(n+1)+1$

$= 1^{n+1}\; n-(n+1)+1$

$= 1\cdot n-(n+1)+1$

$= \cancel{n}-\cancel{n}-\cancel{1}+\cancel{1}$

$\boxed{\lim_{t\to 1} = 0}$

$\lim_{t\to 1} \quad \{(t-1)^2\}$

$= (1-1)^2$

$= 0$

$= \lim_{t\to 1} \left\{\frac{nt^n(n+1)-nt^{n-1}(n+1)}{\{2(t-1)\}'}\right\}$

$\frac{d}{dt}(nt^n(n+1)-nt^{n-1}(n+1))$

$\frac{d}{dt}\{nt^n(n+1)\}-\frac{d}{dt}\{nt^{n-1}(n+1)\}$

$\frac{d}{dt}\{nt^n(n+1)\}$

$= n(n+1)\,{}^{d}/_{dt}(t^n)$

$= n(n+1)\,nt^{n-1}$

$= (n+1)n^{1+1}t^{n-1}$
$= (n+1)n^{2}t^{n-1}$
$= n^{2}t^{n-1}(n+1)$

$\frac{d}{dt}\{nt^{n-1}(n+1)\}$
$= n(n+1)\,d/dt\,(t^{n-1})$
$= n(n+1)(n-1)t^{n-1-1}$
$= nt^{n-2}(n+1)(n-1)$
$= n^{2}t^{n-1}(n+1) - nt^{n-2}(n+1)(n-1)$

$\frac{d}{dt}\{2(t-1)\}$
$= 2\frac{d}{dt}(t-1)$
$= 2\frac{d}{dt}(t) - \frac{d}{dt}(1)$ $\quad \because t' = 1, \; 1' = 0$
$= 2\{1-0\}$
$= 2$

$\lim_{t\to 1}\left\{\frac{n^{2}t^{n-1}(n+1) - nt^{n-2}(n+1)(n-1)}{2}\right\}$

Put Value $t=1$

$= \frac{n^{2}.1^{n-1}(n+1) - n.1^{n-2}(n+1)(n-1)}{2}$

$= \frac{n^{2}(n+1) - n(n+1)(n-1)}{2}$ $\quad \because n^{2}.1^{n-1} = n^{2}.1, \; n^{2}.1 = n^{2}$

Simplify the equation $\quad \because n.1^{n-2} = n.1, \; n.1 = n$

$= \frac{n^{3}+n^{2} - (n^{2}+n)(n-1)}{2}$

$$= \frac{n^3 + n^2 - (n^3 - n^2 + n^2 - n)}{2}$$

$$= \frac{\cancel{n^3} + n^2 - \cancel{n^3} + \cancel{n^2} - \cancel{n^2} + n}{2}$$

$$\boxed{= \frac{n^2 + n}{2}}$$

Answer

(4) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x} =$ ______

Solution

$$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$$

Since Evaluating limits of the N & D would Results in an intermediate use L'Hospital Rule

$$\lim_{x \to 0} \left\{ \frac{\frac{d}{dx}(\tan(x) - x)}{\frac{d}{dx}(x - \sin(x))} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{\tan(x)^2}{1 - \cos(x)} \right\}$$

$$\lim_{x \to 0} \left\{ \frac{\left(\frac{\sin(x)}{\cos(x)}\right)^2}{1 - \cos(x)} \right\}$$

$\because \tan(t) = \frac{\sin(t)}{\cos(t)}$

$$\lim_{x \to 0} \left\{ \frac{\frac{\sin(x)^2}{\cos(x)^2}}{1 - \cos(x)} \right\}$$

$\because \sin(t)^2 = 1 - \cos(t)^2$

$$\lim_{x \to} \left\{ \frac{1 - \cos(x)^2}{\cos(x)^2 \cdot (1 - \cos(x))} \right\}$$

$$\lim_{x\to 0} \left\{ \frac{(1-\cos(x))(1+\cos(x))}{\cos(x)^2(1-\cos(x))} \right\}$$ factorizing the expression

$$\lim_{x\to 0} \left\{ \frac{1+\cos(x)}{\cos(x)^2} \right\}$$

Evaluate the limit

$$= \frac{1+\cos(0)}{\cos(0)^2} \Rightarrow \frac{1+1}{(1)^2} \Rightarrow \frac{2}{1} \quad \| \because \cos(0)=1$$

$$\boxed{=2}$$

Answer

(5) $$\lim_{x\to 2} \frac{x^4-4x^3+5x^2-4x+4}{x^4-4x^3+6x^2-8x+8} = \frac{a}{6}$$

where $a = ?$

Solution

$$\lim_{x\to 2} \frac{x^4-4x^3+5x^2-4x+4}{x^4-4x^3+6x^2-8x+8} = \frac{a}{6}$$

Since the expression is an intermediate form try transforming the expression.

$$\lim_{x\to 2} \frac{x^4-4x^3+5x^2-4x+4}{x^4-4x^3+6x^2-8x+8}$$

$$\lim_{x\to 2} \frac{x^4-2x^3-2x^3+4x^2+x-2x-2x+4}{x^4-2x^3-2x^3+4x^2+2x^2-4x-4x+8}$$

$$\lim_{x\to 2} \frac{x^3(x-2)-2x^2(x-2)+x(x-2)-2(x-2)}{x^3(x-2)-2x^2(x-2)+2x(x-2)-4(x-2)}$$

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x^3-2x^2+x-2)}{\cancel{(x-2)}(x^3-2x^2+2x-4)}$$

$$\lim_{x\to 2} \frac{x^3-2x^2+x-2}{x^3-2x^2+2x-4}$$

$$\lim_{x \to 2} \frac{x^2(x-2)+x-2}{x^2(x-2)+2x-4}$$

$$\lim_{x \to 2} \frac{x^2\cancel{(x-2)}+\cancel{(x-2)}1}{x^2\cancel{(x-2)}+2\cancel{(x-2)}}$$

$$\lim_{x \to 2} \left\{\frac{x^2+1}{x^2+2}\right\}$$

Evaluate the limit

$$\frac{x^2+1}{x^2+2} = \frac{a}{6}$$

$$\frac{(2)^2+1}{(2)^2+2} = \frac{a}{6}$$

$$\frac{4+1}{4+2} = \frac{a}{6}$$

$$\frac{5}{6} = \frac{a}{6}$$

Now, Cross Multiplication.

$$30 = a6$$

$$\cancel{6}a = \cancel{30}\,5$$

$$\boxed{a = 5}$$

Answer