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(2K20/ITE/73)

Group A

Calculus And Analytical Geometry

(Assignment)

(1) Calculate the limits $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5}$

Solution:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5} \quad \because \lim_{n\to\infty}(1+1/n)^n = e$$

$$\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)^5\right]$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \cdot \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^5$$

$$e \cdot 1$$

$$\boxed{= e}$$

Answer

(2) Find the limit $_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

Solution:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x} \quad \because e = \lim_{n\to\infty}(1+1/n)^n$$

By the product Rule of limits,
we obtain,

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \cdot \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \cdot \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

$$e \cdot e \cdot e$$

$$\boxed{e^3}$$

Answer

(3) Calculate the limit $\lim_{x \to \infty} \left(1 + \frac{6}{x}\right)^x$

Solution:-

$\lim_{x \to \infty} \left(1 + \frac{6}{x}\right)^x$

$\lim_{x \to \infty} \left(1 + \frac{6}{x}\right)^x$

Substituting $\frac{6}{x} = \frac{1}{y}$ so that $x = 6y$ and

$y \to \infty$ as $x \to \infty$ we obtain.

$\lim_{y \to \infty} \left(1 + \frac{1}{y}\right)^{6y}$

$\lim_{y \to \infty} \left\{\left(1 + \frac{1}{y}\right)^y\right\}^6$ $\| \because e = \lim_{n \to \infty} \left(1 + 1/n\right)^n$

$\boxed{e^6}$

Answer

(4) Find the limit $\lim_{x \to 0} \sqrt[x]{1 + 3x}$

Solution:-

$\lim_{x \to 0} \sqrt[x]{1 + 3x}$ $\| \because a^x = e^{\ln(a^x)} = e^{x \cdot \ln(a)}$

$\lim_{x \to 0} e^{1/x \ln(1+3x)}$

$\lim_{x \to 0} e^{\ln(1+3x) 1/x}$

$\lim_{x \to 0} e^{\ln(1+3x)/x}$

$e^{\ln(1+3x)} = 1 + 3x$ $\| \because a^{\log_a(b)} = b$

$= (1 + 3x)^{1/x}$

Now, we Applying Chain Rule.

$\lim_{x \to 0} \left(e^{1/x \ln(1+3x)}\right)$

$\lim_{x \to 0} \frac{1}{x} \ln(1 + 3x)$

$\lim_{x \to 0} \frac{\ln(1+3x)}{x}$

Now put $\lim_{x \to 0}$

$= \frac{\ln(1+3(0))}{0}$

$= \ln 1$ $\quad \| \because \log_a(1) = 0$

$\lim_{x \to 0} \left(\frac{\frac{3}{1+3x}}{1} \right)$

$\lim_{x \to 0} \frac{3}{1+3x} = \frac{3}{1+3(0)} = 3$

$\lim_{u \to 3} e^u$

then $u = 3$

$\boxed{e^3}$

Answer

(5) Find the $\lim_{x \to \infty} \left(\frac{x+a}{x-a} \right)^x$

Solution

$\lim_{x \to \infty} \left(\frac{x+a}{x-a} \right)^x$ $\quad \| \because e^{x \ln(a)} = e^{\ln(a^x)}$

$\lim_{x \to \infty} e^{x \ln\left(\frac{x+a}{x-a}\right)}$

$\lim_{x \to \infty} x \ln\left(\frac{x+a}{x-a}\right)$

$\lim_{x \to \infty} \frac{\ln\left(\frac{x+a}{x-a}\right)}{\frac{1}{x}} = \frac{\ln\left(\frac{x+a}{-(x+a)}\right)}{1/x}$

$$\lim_{x\to\infty} \cancel{\left(\frac{1}{x}\right)} \left(\frac{\ln\left(\frac{a+x}{-a+x}\right)}{1/x} \right) = 2a$$

$$\lim_{x\to\infty} 2a$$

$\| \because e^{\ln x \ln\left(\frac{x+a}{x-a}\right)}$

$$\boxed{e^{2a}}$$

Answer

⑥ Calculate the limit $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x$

Solution:

$$\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x$$

$$\lim_{x\to\infty}\left\{\left(\frac{x}{x+1}\right)^x\right\}$$

Apply the Common limit

$$\lim_{x\to\infty}\left\{\left(\frac{x}{x+k}\right)^x\right\} = e^{-k}$$

$$\boxed{= e^{-1} \quad \text{or} \quad = \frac{1}{e}}$$

Answer

⑦ Evaluate the limit $\lim_{x\to\infty}\left(\frac{x+3}{x-2}\right)^{x-1}$

Solution:

$$\lim_{x\to\infty}\left(\frac{x+3}{x-2}\right)^{x-1}$$

$$\lim_{x\to\infty}\left\{\left(\frac{x+3}{x-2}\right)^x\left(\frac{x+3}{x-2}\right)^{-1}\right\}$$

Apply exponent Rule $\| \because a^x = e^{\ln(a^x)} = e^{x\cdot\ln(a)}$

$$\lim_{x\to\infty} \left\{ e^{x\ln\left(\frac{x+3}{x-2}\right)} \right\}$$

limit chain Rule;

$\because a^{g(x)} = b$

$\because F(u) = e^u$

$$\lim_{x\to\infty} \left\{ x\ln\left(\frac{x+3}{x-2}\right) \right\}$$

$$\lim_{x\to\infty} \left\{ \frac{\ln\left(\frac{3+x}{-2+x}\right)}{\frac{1}{x}} \right\}$$

Now, Divide by denominator power

$$\lim_{x\to\infty} \ln\left(\frac{\frac{3}{x}+\cancel{\frac{x}{x}}}{-\frac{2}{x}+\frac{x}{x}}\right)$$

$$\lim_{x\to\infty} \frac{\ln\left(\frac{3}{x}+1\right)}{\left(\frac{-2}{x}+1\right)}$$

$\because \infty = 0$

Apply limit $x\to\infty$

$$\ln\left(\frac{\frac{3}{(\infty)}+1}{\frac{-2}{(\infty)}+1}\right)$$

$$\ln\left(\frac{0+1}{0+1}\right)$$

$$\boxed{\ln(1)}$$

$$\lim_{x\to\infty} \left\{ \frac{\ln\left(\frac{x+3}{-2+x}\right)'}{\left(\frac{1}{x}\right)'} \right\}$$

Apply Chain Rule:

$$= \frac{d}{du}(\ln(u))\,\frac{d}{dx}\left(\frac{3+x}{-2+x}\right)$$

$$= \frac{d}{du}(\ln(u))\,\frac{d}{dx}\left(\frac{3+x}{-2+x}\right) \quad \because \left(\frac{f}{g}\right)' = \frac{f'\cdot g - g'\cdot f}{g^2}$$

$$= \frac{1}{u}\,\frac{d}{dx}\,\frac{(-2+x)(3+x)}{(-2+x)^2}$$

$$= \frac{1}{u}\,\frac{\frac{d}{dx}(-2+x)(3+x) - \frac{d}{dx}(3+x)(-2+x)}{(-2+x)^2}$$

$$= \frac{1}{u}\cdot\frac{\{(0+(1)\}\{3+x\} - \{(0+1)(-2+x)\}}{(-2+x)^2}$$

$$= \frac{1}{u}\left\{\frac{-2+x-(x+3)}{(-2+x)^2}\right\}$$

$$= \frac{1}{u}\left\{\frac{-2+\cancel{x}-\cancel{x}-3}{(-2+x)^2}\right\}$$

$$= \frac{1}{u}\left\{-\frac{5}{(-2+x)^2}\right\} \quad \because u = \frac{3+x}{-2+x}$$

$$= \frac{1}{\frac{3+x}{-2+x}}\left\{-\frac{5}{(-2+x)^2}\right\}$$

$$= \frac{-2+x}{3+x}\left(-\frac{5}{(-2+x)^2}\right)$$

$$= -\frac{\cancel{(x-2)}(5)}{(3+x)(x-2)^2}$$

$$\boxed{= -\frac{5}{(x+3)(x-2)}}$$

$$\frac{d}{dx} = \left(\frac{1}{x}\right) = x^{-1} = -x^{-2} = \boxed{-\frac{1}{x^2}}$$

$$\lim_{x \to \infty} \frac{\left\{ + \frac{5}{(x+3)(x-2)} \right\}}{\left\{ + \frac{1}{x^2} \right\}}$$

$$\lim_{x \to \infty} \frac{5x^2}{(x+3)(x-2)} \quad \frac{U}{V}$$

$$\lim_{x \to \infty} \frac{10x}{2x+1}$$

$$\lim_{x \to \infty} \frac{10x}{2x+1} = \frac{\cancel{10}}{\cancel{2}}$$

$$\lim_{x \to \infty} 5$$

$$\lim_{U \to 5} (e^{u})$$

$$\boxed{e^5}$$

Answer

$\because U' = 5x^2$

$= 10x^{2-1}$

$\boxed{U' = 10x}$

$V' = (x+3)(x-2)$

$= x^2 - 2x + 3x - 6$

$= x^2 + x - 6$

$\boxed{V' = 2x+1}$

$U'' = 10x^{1-1}$

$\boxed{U'' = 10}$

$V'' = 2x+1$

$= 2x^{1-1} + 0$

$\boxed{V'' = 2}$

(8) Find the limit $\lim_{x \to a} \frac{\ln(x) - \ln(a)}{x - a}, (a > 0)$

Solution

$$\lim_{x \to a} \frac{\ln(x) - \ln(a)}{x - a} \quad \frac{U}{V}$$

$$\lim_{x \to a} \frac{\frac{1}{x}}{1}$$

$V' = (x-a)$

$V' = (x-0)$

$= 1-0$

$\boxed{V' = 1}$

$U' = \ln x - \ln a$

$= \frac{1}{x} - \frac{1}{a}$

$= \frac{1}{x} - \frac{1}{0}$

$\boxed{U' = \frac{1}{x}}$

$\lim_{x \to a} \frac{1}{x}$

apply $\lim_{x \to a}$

$= \frac{1}{x}$

$= \boxed{\frac{1}{a}}$

Answer

9 Calculate the $\text{limit}_{x \to 0} (1 + \sin x)^{1/x}$

solution:-

$\lim_{x \to 0} (1 + \sin x)^{1/x}$ $\quad \because a^x = e^{\ln(a^x)}$

$\quad \because e^{\ln(a^x)} = e^{x \cdot \ln(a)}$

$\lim_{x \to 0} e^{1/x \ln(1 + \sin x)}$

Suppose:-

$\lim_{x \to 0} \boxed{\frac{1}{x} \ln(1 + \sin x)} = e^{u}$

$\because e^{u}$

$\lim_{x \to 0} \frac{\ln(1 + \sin x)}{x} \frac{u'}{v'}$

Apply limit $x \to 0$

$\left\{ \frac{\ln(1 + \sin(0))}{x} \right\}' \Rightarrow$ $\quad \because \sin(0) = 0$

$\Rightarrow$ $\quad x = 0$

$\ln 1 + 0$

$\boxed{\ln 1}$ $\quad \because U = 1$

$\lim_{x \to 0} \left(\frac{\ln(1+\sin x)}{x} \right) \begin{matrix} U \\ V \end{matrix}$ || $U'' = 0 + \cos x$
$= \cos x$

$\lim_{x \to 0} \boxed{\frac{\cos x}{1+\sin(x)}}$

$\lim_{x \to 0} \ln(U)$

$\lim_{x \to 0} \frac{1}{u}$

Apply limit $x \to 0$

$\frac{\cos(0)}{1+\sin(0)}$ || $\because \cos(0) = 1$

$\frac{1}{1+0}$

1

$\lim_{U \to 1}$ Apply limit

e^{u}

e^{1}

$\boxed{e}$

Answer