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(2K20/ITE/73)
Group A
Calculus And Analytical Geometry
(Assignment II)

Applications of Derivative

Question No 1 (Part I)
One leg of a right triangle decreases at 1 in/min and the other leg increases at 2 in/min. At what rate is the area changing when the first leg is 8 inches and the second leg is 6 inches? Answer ____ in^2/min.

Solution:

Given

$$\frac{dz}{dt} = 1 \text{ in/sec} \quad , \quad \frac{dx}{dt} = 2 \text{ in/sec}$$

$$\boxed{z = 8} \quad , \quad \boxed{x = 6}$$

$$x^2 + z^2 + y^2$$

$$2x\frac{dx}{dt} + 2z\frac{dz}{dt} + 2y\frac{dy}{dt}$$

$$x\frac{dx}{dt} + z\frac{dz}{dt} + y\frac{dy}{dt}$$

$$\sqrt{(6)^2 + (8)^2} = y^2$$

$$y^2 = \sqrt{36 + 64}$$

$$y^{\cancel{2}} = \sqrt{100}$$

$$\boxed{y = 10 \text{ in}^2/\text{min}} \text{ Answer}$$

Part II
The volume of a sphere is increasing at the rate of 3 cubic feet per min. At what rate is the radius increasing when the radius is 8 feet? Answer $a/b\pi$ ft/min where a & b is.

Solution:

$$v = \frac{4}{3}\pi r^3$$

$$= \frac{dv}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$$= \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dv}{dt}$$

$$= \frac{dr}{dt} = \frac{3}{4\pi(8)2}$$

$$= \frac{dr}{dt} = \frac{3}{16\pi} \text{ cm/sec}$$

So,

$\boxed{a = 3 \text{ Ft/min}}$ & $\boxed{b = 256}$ Answer

Part III

A beacon on a light house 1 mile from shore revolves at the rate of 10π radius/min. Assuming that the short line is straight, calculate the speed at which the spot light is sweeping across the short line as it light up the sand 2 miles from the light?

Solution:

lets $x = 0x$, then $x = \tan\theta$ so x anywhere along the short line and $dx/d\theta = \sec^2\theta$, which can be written $dx = \sec^2\theta\, d\theta$. The rate of change in time t is $dx/dt = \sec^2\theta$.

$d\theta/dt$ is the angular speed of the becon 10π radius/min and dx/dt is the velocity of the becon along the short line, at $x=2$ $dx/dt = 10\pi$, and $\tan\theta = 2$, so $dx/dt = 50\pi$ miles per minute.

$\boxed{= 157 \text{ miles}}$ Answer

REMAIN'S INTEGRAL

Question No 2

Show that $\sum_{k=1}^{n} 2^{-k} = 1 - 2^{-n}$ for each n. Hint let $S_n = \sum_{k=1}^{n} 2^{-k}$ and consider the quantity $S_n - \frac{1}{2}S_n$. The number $L(n)$ is the lower sum associated with the partition P and $U(n)$ is the upper term sum associated with P.

Part A

let $n = 1$ (That is we do not subdivide $[0,2]$) Find $P_1, \Delta x_1, a_1, b_1, L(1)$ & $U(1)$ How good is $L(1)$ an an approximation to $\int_0^2 f$?

Solution:

$\sum_{k=1}^{1} 1 - 2^{-1}$ Now we use formula.

$\sum_{k=1}^{n} a = a \cdot n$

$a = 1 - 2^{-1}, n = 1$

$a = (1 - 2^{-1}) \cdot 1 : 1 \, 1/2$

$= (1 - 1/2)$

$\boxed{a = 1 - \frac{1}{2}}$ Answer

Part B

let $n=2$ find P_2 for $k=1,2$ find Δx_k, a_k & b_k. find $L(2)$ & $U(2)$ How good is $L(2)$ as an approximation to $\int_0^2 f$?

Solution:

$$\Sigma_k^2 = 1 \cdot 2^{1-2^{-2}}(1-1/4)$$
$$\Sigma_k^2 = 1 \cdot 2^{1-2^{-2}}$$
$$(1-2^{-2}) \cdot 2 : 2(1-1/4)$$
$$\boxed{= 2\left(1-\frac{1}{4}\right)}$$ Answer

Part C

let $n=3$ find P_3 for $k=1,2,3$ find Δx_k, a_k & b_k find $L(3)$ $U(3)$. How good is $L(3)$ as an approximation to $\int_0^2$?

Solution:

$$\Sigma_k^3 = 3 \cdot 2^{-3} = 1/8$$
$$\Sigma_k^3 = 3 \cdot 2^{-3}$$
$$\Sigma_k^3 = 1\ 2-3 = 3/8$$
$$\Sigma_k^3 = 1^{2-3} = 1/4$$
$$= 3/8 - 1/4$$
$$(1-2^{-2}) \cdot 2 : 2(1-1/4)$$
$$= 2(1-1/4)$$
$$\boxed{= 1/8}$$ Answer

Part D

let $n=4$ find P_4 for $k=1,2,3,4$ find Δx_k, a_k & b_k find $L(4)$ $U(4)$. How good is $L(4)$ as an approximation to $\int_0^2 f$?

Solution:

$\Sigma_k^4 = 4^{2-4} = 1/16$

$\Sigma_k^4 = 4^{2-4}$

$\Sigma_k^4 = m = \Sigma_k^n = 1 - \Sigma_{k=1}^{m-1}$

$\Sigma_k^4 = 1^{2-4} - \Sigma_k^3 = 1^{2-4}$

$\Sigma_k^4 = 1^{2-4} = 1/4$

$\Sigma_k^3 = 1^{2-4} = 3/16$

$= \frac{1}{4} - \frac{3}{16}$

$= \frac{1}{4} - \frac{3}{16} : \frac{1}{16}$

$\boxed{= \frac{1}{16}}$ Answer

Part E

Let $n=8$ find P_8 for $k=1,2\cdots 8$ find $\Delta x_k, a_k, b_k$ find $L(8)$ & $U(8)$ How good is $L(8)$ an approximation to $\int_0^2 f$?

Solution:

$\Sigma_k^8 = 8^{2-8} = 1/266$

$\Sigma_k^8 = 8^{2-8}$

$\Sigma_k^n = m = \Sigma_k^n = 1 - \Sigma_{k=1}^{m-1}$

$\Sigma_k^8 = 1^{2-8} - \Sigma_k^7 = 1^{2-8}$

$\Sigma_k^8 = 1^{2-8} = 1/32$

$\Sigma_k^7 = 1^{2-8} = 7/256$

$= \frac{1}{32} - \frac{7}{256} \Rightarrow \frac{1}{32} - \frac{7}{256} : \frac{1}{256}$

$\boxed{= \frac{1}{256}}$ Answer

Part E

Let $n=20$ find P_{20} for $K = 1,2 \cdots 20$ find $\Delta x_K, a_K, b_K$ find $L(20)$ $U(20)$. How good is $L(20)$ an approximation $\int_0^2$?

Solution:

$$\Sigma_K^{20} = 20^{2-20} = \frac{1}{1048576}$$

$$\Sigma_K^{20} = 20^{2-20}$$

$$\Sigma_K^{n} = m = \Sigma^{n}_{K} = 1 - \Sigma^{m-1}_{K=1}$$

$$\Sigma_K^{20} = 1^{2-20} = 5/262144$$

$$\Sigma_K^{19} = 1^{2-20} = 19/1048576$$

$$= \frac{5}{262144} - \frac{19}{1048576}$$

$$= \frac{5}{262144} - \frac{19}{1048576} : \frac{1}{1048576}$$

$$\boxed{= \frac{1}{1048576}}$$ Answer

Part II

Now let n be arbitrary natural number (Note arbitary means unspecified) for $K = 1,2 \cdots n$. find $\Delta x_K, a_K, b_K$ find $L(n)$ $U(n)$. Explain carefully?

Solution:

A Partition P of $[a:b]$ is a finite set of points $x_0, x_1, \cdots x_n$ such that $a = x_0, x_2 \cdots x_n$. $\angle \angle \angle$ $x_n/1 \cdot x_n = b$ we write $P = F x_0 ; x_1 ; x_2 ;;$; x_n. So the expression $L(n) \leq \int_0^2 \leq U(n)$ is a form of laplace transform. laplace is an integral transform that covers a function of real Variable to a function of a real Complex we define it. Answer

Part II

Suppose we wish to approximate $\int_0^2 f$ by $L(n)$ for Some n & have an error no greater than 10^{-5}. what is the Smallest value of n that our previous Calculations guarantee will do the Job?

Solution:

$$\int_b^a f(x)\,dx = \Delta x\left(f\frac{x_0+x_1}{2}\right)+f\left(\frac{x_1+x_2}{2}\right)+\cdots f\left(\frac{x_{n-1}+x_n}{2}\right)$$

Formula

$$= 1\cdot\left\{f\left(\frac{f_0+f_1}{2}\right)+f\left(\frac{f_1+f_2}{2}\right)\right\}$$

$$= 1\cdot\left(\frac{1}{2}+\frac{3}{2}\right)$$

$$\boxed{=2}$$ Answer

Part I

use the preceeding to Calculate $\int_0^2 f$ with an error of less than 10^{-5}?

Solution:

where $\Delta x = \frac{b-a}{n}$

Formula $= 1\cdot\left\{f\left(\frac{f_0+f_1}{2}\right)+f\left(\frac{f_1+f_2}{2}\right)\right\}$

$$= 1\cdot\left\{\left(\frac{1}{2}\right)\frac{1}{100000}+\left(\frac{3}{2}\right)\frac{1}{100000}\right\}$$

$$\boxed{= 1.99999}$$

Answer