①

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(2K20/ITE/73)

Group A

Calculus And Analytical Geometry

(Assignment III)

Assignment III

Question No: 1

Techniques Of Intigration

① $\int_0^1 \frac{x^{1/2}}{1+x^{3/4}} dx = \frac{a}{3}(1-\ln b)$ where a&b=?

Solution:

$\Rightarrow \int_0^1 \frac{x^{1/2}}{1+x^{3/4}} dx$

$\Rightarrow$ Evaluating using numerical Calculation methods.

$\Rightarrow 0.40913$

$= 0.40913\, dx = \frac{a}{3}\{1-\ln b\}$

= Multiply fractions; $\because a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{(0.40913)(a)(1-\ln b)}{3}$

Now,

we divide numbers

$\Rightarrow \frac{0.40913}{3} \Rightarrow \boxed{0.13637}$

$= 0.13637\{a(1-\ln b)\}$

$\boxed{= 0.13637a(1-\ln b)}$ Answer

② $\int_0^9 \frac{\sqrt{x}}{1+\sqrt{x}} \, dx = a + 4 \ln b$ where a & b = ?

Solution:-

$\int_0^9 \frac{\sqrt{x}}{1+\sqrt{x}} \, dx$

Substitute $u = 1+\sqrt{x}$

$\frac{d}{dx}(1+\sqrt{x})$

Apply the Sum difference Rules,

$\frac{d}{dx}(1) + \frac{d}{dx}(\sqrt{x})$

Now the derivative of constant will be 0.

$= 0 + \frac{1}{2\sqrt{x}}$

$\because \sqrt{x} = x^{1/2}$

$\frac{d}{ddx}\sqrt{x} = 1/2\sqrt{x}$

$= \frac{1}{2\sqrt{x}}$

$\Rightarrow du = \frac{1}{2\sqrt{x}} dx$

$\Rightarrow dx = 2\sqrt{x} \, du$

$= \int \frac{\sqrt{x}}{u} \cdot 2\sqrt{x} \, du$

$\because a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{\sqrt{x}}{u} \cdot 2\sqrt{x}$

$\because \sqrt{x} \times \sqrt{x} = x$

$= \frac{\sqrt{x} \cdot 2\sqrt{x}}{u} \Rightarrow \boxed{\frac{2x}{u}}$

$= \int \frac{2x}{u} \, du \longrightarrow ①$

Now,

$u = 1 + \sqrt{x}$

$x = u^2 - 2u + 1$

$U = 1 + \sqrt{x}$

Subtract 1 b/s

$U - 1 = \cancel{1} + \sqrt{x} - \cancel{1}$

$U - 1 = \sqrt{x}$ Squaring on b/s

$(U-1)^2 = (\sqrt{x})^2$

$U^2 - 2u + 1 = x$

$\boxed{x = U^2 - 2u + 1}$ put Value of x in eq 1.

$\int \frac{2x}{u} dU$

$\int \frac{2(U^2 - 2u + 1)}{u} du$

$\int_1^4 \frac{2 \cdot (U^2 - 2U + 1)}{u} du$

Taking the Constant out

$= 2 \int_1^4 \frac{U^2 - 2U + 1}{u} du$

$\because x = 0$

$\because u = 1 + \sqrt{x}$

$= 1 + \sqrt{0}$

$\boxed{u = 1}$

$u = 1, x = 0$

$\because x = 9$

$\because U = 1 + \sqrt{9}$

$= 1 + 3$

$\boxed{U = 4}$

$u = 4, x = 9$

Now,

$= \frac{U^2 - 2u + 1}{u}$

$= \frac{u^{\cancel{2}}}{\cancel{u}} - \frac{2\cancel{u}}{\cancel{u}} + \frac{1}{u}$

$\boxed{= u - 2 + \frac{1}{u}}$

$= 2 \int_1^4 U - 2 + \frac{1}{u} du$

$= \boxed{2\left(\int_1^4 u\, du - \int_1^4 2du + \int_1^4 \frac{1}{u} du\right)} \rightarrow 2$

Apply the Power Rule $\because dx = \frac{x^{a+1}}{a+1} = a=1$

$udu = \left[\frac{u^{1+1}}{1+1}\right]_1^4 = \frac{u^2}{2}$

$\boxed{\int_1^4 udu = \frac{u^2}{2}}$

$= 8 - \frac{1}{2}$

convert element to fraction

$= \frac{8 \cdot 2}{2} - \frac{1}{2}$

$= \frac{8 \cdot 2 - 1}{2}$

$\boxed{udu = \frac{15}{2}} \rightarrow 3$

$\because u = 1, u = 4$

$\frac{u^2}{2}, \frac{u^2}{2}$

$\frac{(1)^2}{2}, \frac{(4)^2}{2}$

$\frac{1}{2}, \frac{\cancel{16}^{8}}{\cancel{2}_{1}}$

$\boxed{1/2, 8}$

$2du = \left[2u\right]_1^4$

$= 8 - 2$

$\boxed{2du = 6} \rightarrow 4$

$\because u = 1, u = 4$

$2u, 2u$

$2(1), 2(4)$

$\boxed{2, 8}$

$\frac{1}{u} du = \left[\frac{1}{u}\right]_1^4$

$= \left[\ln u\right]_1^4$

$1/u\, du = 2\ln 2 - 0$

$\boxed{1/u\, du = 2\ln 2} \rightarrow 5$

$\because u = 1, u = 4$

$\ln u, \ln u$

$\ln(1), \ln(4) \Rightarrow \ln 2^2$

$\boxed{0, 2\ln 2}$

Now, put the value of eq 3, 4, 5 to eq 2

$= 2\left(\int_1^4 udu - \int_1^4 2du + \int_1^4 \frac{1}{u} du\right)$

$= 2\left(\frac{15}{2} - 6 + 2\ln 2\right)$

$= 2\left(\frac{15-12}{2} + 2\ln 2\right)$

$= 2\left(\frac{3}{2} + 2\ln 2\right)$

$= \cancel{2}\left(\frac{3+4\ln 2}{\cancel{2}}\right)$

$= 3 + 4\ln 2$

$\boxed{4\ln(2)+3} \Rightarrow (a + 4\ln(b))$

$\boxed{a = 3}$, $\boxed{b = 4\ln 2}$ Answer

(3) $\int_{1/3\sqrt{3}}^{3\sqrt{3}} \frac{1}{x^{4/3}+x^{2/3}}\, dx =$

Solution:-

$\int_{1/3\sqrt{3}}^{3\sqrt{3}} \frac{1}{x^{4/3}+x^{2/3}}$

Now derivative of Contant will be 0.

$\frac{d}{dx}(x^{4/3}+x^{2/3})$

$\frac{d}{dx}\left(\frac{4}{3}x^{1/3} + \frac{2}{3}x^{-1/3}\right)$

$\because \frac{d}{dx} = x^{4/3}+x^{2/3}$

$= \frac{4}{3}x^{4/3-1} + \frac{2}{3}x^{2/3-1}$

$\boxed{= \frac{4}{3}x^{1/3} + \frac{2}{3}x^{-1/3}}$

Convert the element fraction.

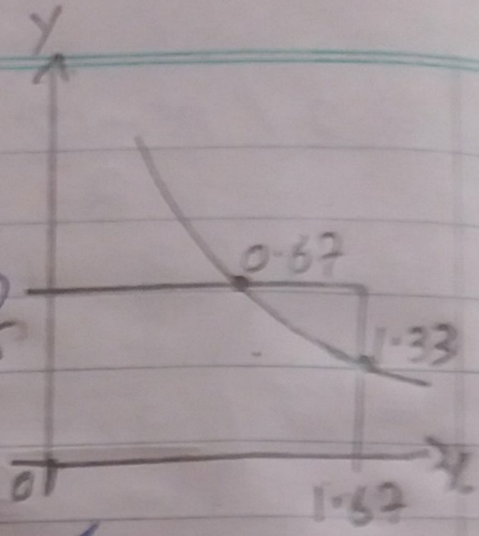
$\frac{d}{dx}\left(\frac{4x^{1/3}}{3} + \frac{2x^{-1/3}}{3}\right)$

$\frac{d}{dx}\left(\int_{1/3\sqrt{3}}^{3\sqrt{3}} \frac{4x^{1/3}}{3}\, dx + \int_{1/3\sqrt{3}}^{3\sqrt{3}} \frac{2x^{-1/3}}{3}\right)$

$$\frac{d}{dx}\left(\frac{4x^{1/3}}{3} + \frac{2x^{-1/3}}{3}\right)$$

$$\frac{d}{dx}\left(\frac{4x^{1/3}}{3} + \frac{2}{3x^{1/3}}\right) \rightarrow (1)$$

$$\frac{d}{dx}\left(\frac{12x^{1/3}+6}{9x^{1/3}}\right)$$



Evaluating using numerical calculation

Now put value

$$\int_{1/3\sqrt{3}}^{3\sqrt{3}} \frac{d}{dx}\frac{4x^{1/3}}{3}$$

$$\frac{d}{dx} = 2.3094 - 0.7698$$

$$\boxed{= 1.5396} \rightarrow (2)$$

$x = 3\sqrt{3}$, $x = 1/3\sqrt{3}$

$\frac{4x^{1/3}}{3}$, $\frac{4x^{1/3}}{3}$

$\frac{4(3\sqrt{3})^{1/3}}{3}$, $\frac{4(1/3\sqrt{3})^{1/3}}{3}$

$\boxed{2.3094, 0.7698}$

$$\int_{1/3\sqrt{3}}^{3\sqrt{3}} \frac{d}{dx}\frac{2}{3x^{1/3}}$$

$$\frac{d}{dx} = 0.3849 - 1.1547$$

$$\boxed{= 0.7698} \rightarrow (3)$$

$\frac{2x}{3x^{1/3}}$, $\frac{2}{3x^{1/3}}$

$\frac{2}{3(3\sqrt{3})^{1/3}}$, $\frac{2}{3(1/3\sqrt{3})^{1/3}}$

$\boxed{0.3849, 1.1547}$

put value of eq (2) & (3) in eq (1)

Add equation 2 & 3

$$1.5396 + 0.7698$$

$$\boxed{2.3094}$$

Answer

(4) $\int_0^{1/2} \frac{\arctan 2x}{1+4x^2} dx = \frac{\pi^2}{a}$ where $a = ?$

Solution:

$\int_0^{1/2} \frac{\arctan 2x}{1+4x^2} dx = \frac{\pi^2}{a}$

$\int_0^{1/2} \frac{\arctan 2x}{1+4x^2} dx$ || $\because$ Suppose $2x = u$

$= \int_0^{1/2} \frac{\arctan(u)}{1+2u^2} du$ || $\because$ Suppose $\arctan = v$

$= \int_0^{1/2} v\,dv$

$= \frac{1}{2} \cdot \int_0 \frac{1 \arctan(u)}{1+u^2} du$ || $\because$ Suppose $v = \arctan(u)$

$\frac{dv}{du} = \frac{1}{u^2+1}$

$dv = \frac{1}{u^2+1} du$

$\Rightarrow du = u^2+1 dv$

$= \int \frac{v}{1+u^2} (u^2+1) dv$

$= \frac{v(u^2+1)}{(1+u^2)}$

$\boxed{= \int v}$

Now table of $\arctan(1)$ is $\pi/4$.

$\boxed{v = 0}$, $\boxed{v = \pi/4}$

$= \int_0^{\pi/4} v\,dv$

$\because \frac{a \cdot b}{c} = \frac{a \cdot b}{c}$

$\because u = 0 \Rightarrow v = 0$

$v = \arctan(u)$

$= \arctan(0)$

$\boxed{v = 0}$

$\because u = 1$

$v = \arctan(u)$

$v = \arctan(1)$

$= \frac{1}{2}\int_0^{\pi/4} v\,dv$

Apply the power Rule;

$= \frac{1}{2}\left[\frac{v^{1+1}}{1+1}\right]_0^{\pi/4}$

$\because \frac{x^{a+1}}{a+1} = a \neq -1$

= Multiply fractions

$\because a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{\left[\frac{v^2}{2}\right]_0^{\pi/4}}{2}$

$= \frac{1}{2}\left[\frac{v^2}{2}\right]_0^{\pi/4}$

$= \frac{1}{2}\left(\frac{\pi^2}{32}\right)$

$\boxed{= \frac{\pi^2}{64}} \rightarrow (1)$

Let,

$\because V = 0 \quad , V = \pi/4$

$\frac{V^2}{2} \quad , \frac{V^2}{2}$

$\frac{(0)^2}{2} \quad , \frac{(\pi/4)^2}{2}$

$\frac{0}{2} \quad , \frac{\pi^2/16}{2}$

$\boxed{0 \quad , \pi^2/32}$

Now,

Equate equation (1) to π^2/a.

$\frac{\pi^2}{64} = \frac{\pi^2}{a}$

Now we Apply Cross Multiplication.

$\pi^2 a = 64\pi^2$

Now Divide π^2 b/s

$\frac{\cancel{\pi^2} a}{\cancel{\pi^2}} = \frac{64\cancel{\pi^2}}{\cancel{\pi^2}}$

$\boxed{a = 64}$

Answer

5 $\int_0^{\sqrt{2}} x\, 10^{1+x^2}\, dx = \frac{a}{\ln 10}$ where a = ?

Solution:

$$\int_0^{\sqrt{2}} x \cdot 10^{1+x^2} dx = \frac{a}{\ln(10)}$$

Suppose $u = 1+x^2$

$$\int_0^{\sqrt{2}} x \cdot 10^{u}\, dx$$

$$\frac{du}{dx}(1+x^2)$$

$$\frac{d(1)}{dx} + \frac{d(x^2)}{dx}$$

constant number derivative will be 0.

$$0 + \frac{d}{dx} 2x^{2-1}$$

$$0 + 2x$$

$$\boxed{2x}$$

$\Rightarrow du = 2x\, dx$

$$dx = \frac{1}{2x} du$$

$$= \int x \cdot 10^{u} \cdot \frac{1}{2x} du$$

Multiplied the factors // $\because a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$$= \frac{x \cdot 10^{u} \cdot 1}{2x}$$

$\because 10^{u} \times 1 = 10^{u}$

$$= \frac{10^{u}}{2}$$

Apply exponent rule.

$= \frac{2^u \cdot 5^u}{2}$ || $\because a^c \cdot b^c = (ab)^c$, $2^u \cdot 5^u = (10)^u$

$= 2^{u-1} \cdot 5^u$ || $\because \frac{x^a}{x^b} = x^{a-b}$

$= \int 5^u \cdot 2^{u-1} \, du$

Adjust integral boundaries;

$x = 0 \, , x = \sqrt{2}$

$U = 1 + x^2 \, , U = 1 + x^2$

$= 1 + (0)^2 \, , = 1 + (\sqrt{2})^2$

$\boxed{U = 1} \quad \boxed{U = 3}$

$= \int_1^3 5^u \cdot 2^{u-1} \, du$

$= \int_1^3 5^u \cdot 2^u \cdot 2^{-1} \, du$

$= 2^{-1} \int_1^3 5^u \cdot 2^u \, du$

$= \frac{1}{2} \int_1^3 5^u \cdot 2^u \, du$

Apply exponent rule, $\because a^m b^m = (ab)^m$

$5^u \cdot 2^u = (5 \cdot 2)^u$

$= 10^u$

$= \frac{1}{2} \int_1^3 10^u \, du$

$\because \int a^x dx = \frac{a^x}{\ln a}$

$= \frac{1}{2} \left[\frac{10^u}{\ln(10)} \right]_1^3$

$= \frac{1}{2} \left(\frac{990}{\ln(10)} \right)$

$= \frac{495}{\ln(10)}$ multiply $\ln(10)$ b/s.

$\frac{\ln(10) 495}{\ln(10)} = \frac{a(\ln(10))}{\ln(10)}$

$495 \ln(10) = a \ln(10)$

$\boxed{a = 495}$

$\because U = 1 \, , U = 3$

$U = \frac{10^u}{\ln(10)} \, , = \frac{10^u}{\ln(10)}$

$= \frac{10^{(1)}}{\ln(10)} \, , = \frac{10^3}{\ln(10)}$

$\boxed{U = \frac{10}{\ln(10)}} \, , \boxed{U = \frac{1000}{\ln 10}}$

Answer

$\frac{1000}{\ln(10)} - \frac{10}{\ln 10}$

$\boxed{\frac{990}{\ln 10}}$

⑥ $\int_1^{16} \frac{x-1}{x+\sqrt{x}}\, dx =$ ______.

Solution:

$\int_1^{16} \frac{x-1}{x+\sqrt{x}}\, dx$

Expand $\frac{x-1}{x+\sqrt{x}} : \frac{x}{x+\sqrt{x}} - \frac{1}{x+\sqrt{x}}$

Apply fraction Rule: $\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c}$

$= \int_1^{16} \frac{x}{x+\sqrt{x}}\, dx - \int_1^{16} \frac{1}{x+\sqrt{x}}\, dx$

$= \int_1^{16} \frac{x}{x+\sqrt{x}}\, dx$

factors $x+\sqrt{x} : \sqrt{x}(\sqrt{x}+1)$

$= \frac{x}{\sqrt{x}(\sqrt{x}+1)}$

$\because x+\sqrt{x}$
$x = \sqrt{x}\sqrt{x}$
$= \sqrt{x}\sqrt{x}+x$
$= \sqrt{x}(\sqrt{x}+1)$

$= \frac{x}{x^{1/2}(\sqrt{x}+1)}$

= Apply exponents Rule $\frac{x^a}{x^b} = x^{a-b}$

$= \frac{x^{1-1/2}}{\sqrt{x}+1}$

$= \frac{x^{1/2}}{\sqrt{x}+1}$

$\because x^{1/2} = \sqrt{x}$

$= \frac{\sqrt{x}}{\sqrt{x}+1}$

$$\boxed{= \int_1^{16} \frac{\sqrt{x}}{\sqrt{x}+1}\, dx}$$

$\because$ Suppose $u = \sqrt{x}+1$

$$\frac{d}{dx}(\sqrt{x}+1)$$

$$= \frac{d(\sqrt{x})}{dx} + \frac{d}{dx}(1)$$

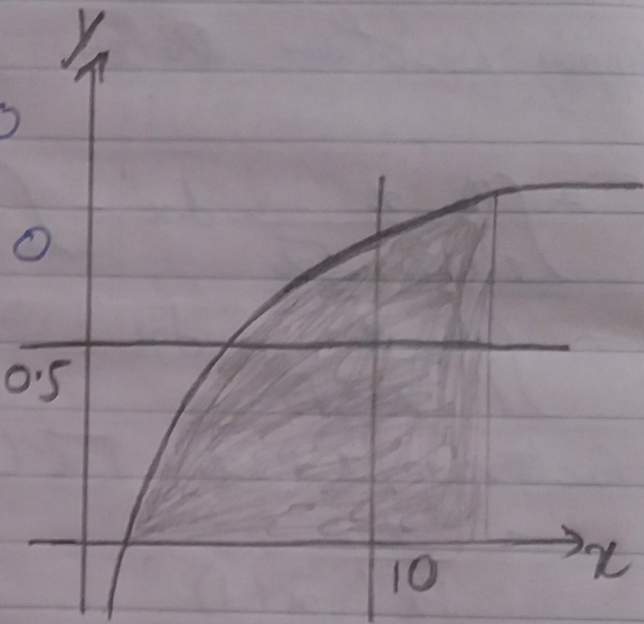
$$= \frac{d}{dx}x^{1/2} + 0 \qquad \| \because \sqrt{x} = x^{1/2}$$

$$= \frac{d}{dx}\tfrac{1}{2}x^{1/2-1} + 0$$

$$= \frac{d}{dx}\tfrac{1}{2}x^{-1/2} + 0$$

$$= \frac{d}{dx}\frac{1}{2}x^{-1/2} + 0$$

$$= \frac{1}{2x^{1/2}} + 0$$



$$\boxed{\frac{du}{dx} = \frac{1}{2\sqrt{x}}}$$

$$\Rightarrow du = \frac{1}{2\sqrt{x}}dx$$

$$dx = 2\sqrt{x}\,du$$

$$= \int \frac{\sqrt{x}}{u} \cdot 2\sqrt{x}\,du$$

$$dx = \frac{\sqrt{x}}{u} \cdot 2\sqrt{x} \qquad \Big\| \because a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$$

$$= \frac{\sqrt{x} \cdot 2\sqrt{x}}{u} \qquad \sqrt{x} \cdot \sqrt{x} = x$$

$$= \frac{2x}{u}$$

$$\boxed{= \int \frac{2x}{u}du} \longrightarrow (1)$$

$u = \sqrt{x} + 1$

$u = \sqrt{x} + 1 \Rightarrow \boxed{\sqrt{x} = u - 1}$

Apply squaring on b/s

$(\sqrt{x})^2 = (u-1)^2$ $\quad (a-b)^2 = a^2 - 2ab + b^2$

$\boxed{x = u^2 - 2u + 1}$ put value in eq (1)

$$= \int \frac{2(u^2 - 2u + 1)}{u}\, du$$

$$= \int_2^5 \frac{2(u^2 - 2u + 1)}{u}\, du$$

$\because x = 1$, $x = 16$

$u = \sqrt{x} + 1$, $u = \sqrt{x} + 1$

$= \sqrt{1} + 1$, $= \sqrt{16} + 1$

$= \sqrt{(1)^2} + 1$, $= \sqrt{(4)^2} + 1$

$= 1 + 1$, $= 4 + 1$

$\boxed{u = 2}$, $\boxed{u = 5}$

$$= 2 \cdot \int_2^5 \frac{u^2 - 2u + 1}{u}\, du$$

= Apply fraction Rule

$$= \frac{u^2 - 2u + 1}{u}$$

$$= \frac{u^{\cancel{2}}}{\cancel{u}} - \frac{2\cancel{u}}{\cancel{u}} + \frac{1}{u}$$

$$= u - 2 + \frac{1}{u}$$

$$= 2 \int_2^5 u - 2 + \frac{1}{u}\, du$$

Apply the sum Rule;

$$= 2\left(\int_2^5 u\,du - \int_2^5 2\,du + \int_2^5 \frac{1}{u}\, du\right)$$

$$= 2\left\{\left[\frac{u^{1+1}}{1+1}\right]_2^5 - \left[2u\right]_2^5 + \left[\ln(u)\right]_2^5\right\}$$

$$= 2\left\{\left[\frac{u^2}{2}\right]_2^5 - \left[2u\right]_2^5 + \left[\ln(u)\right]_2^5\right\}$$

Put $u = 2$

$$= 2\left\{\left[\frac{(2)^2}{2}\right] - 2(2) + \ln(2)\right\}$$

$$= 2\left\{\frac{4}{2} - 4 + \ln 2\right\}$$

$$= \cancel{2}\left\{\frac{4 - 8 + 2\ln 2}{\cancel{2}}\right\}$$

$$= \left\{4 - 8 + 2\ln 2\right\}$$

$$\boxed{= -4 + 2\ln 2} \longrightarrow (i)$$

put value $u = 5$

$$= 2\left\{\left[\frac{u^2}{2}\right]_2^5 - 2(u) + \ln u\right\}$$

$$= 2\left\{\left[\frac{(5)^2}{2}\right] - 2(5) + \ln(5)\right\}$$

$$= 2\left\{\frac{25}{2} - 10 + \ln(5)\right\}$$

$$= \cancel{2}\left\{\frac{25 - 20 + 2\ln(5)}{\cancel{2}}\right\}$$

$$\boxed{= \left\{5 + 2\ln(5)\right\}} \longrightarrow (ii)$$

Equate eq (i) & (ii)

$$5 + 2\ln(5) = -4 + 2\ln(2)$$

$$5 + 4 = 2\ln(2) - 2\ln(5)$$

$$9 = 2\ln(2) - 2\ln(5)$$

$$\boxed{= 2\ln(5) - 2\ln(2) + 9} \longrightarrow (5)$$

$$\int_1^{16} \frac{1}{x + \sqrt{x}}\, dx \qquad \because u = \sqrt{x}$$

$$\frac{d}{dx}(\sqrt{x})$$

$$\frac{d}{dx} x^{1/2} \Rightarrow \frac{d}{dx} \tfrac{1}{2} x^{-1/2} \Rightarrow \boxed{\frac{du}{dx} = \frac{1}{2\sqrt{x}}}$$

$$\Rightarrow du = \frac{1}{2\sqrt{x}} dx$$

$$dx = \cancel{2\sqrt{x}}\ 2\sqrt{x}\, du$$

$$= \int \frac{1}{x+u} \cdot 2\sqrt{x}\, du$$

$\because u = \sqrt{x}$

$$= \int \frac{1}{x+u} \cdot 2u\, du$$

$$= \int \frac{2u}{x+u} du$$

$\because u = \sqrt{x}$

sq on b/s

$U^2 = (\sqrt{x})^2$

$\boxed{U^2 = x}$

put the value of x

$$= \int \frac{2u}{U^2+u} du$$

$$= \int \frac{2\cancel{u}}{\cancel{U}(U+1)} du$$

$$= \int \frac{2}{U+1} du$$

$$= \int_1^4 \frac{2}{U+1} du$$

$\because U = 1 \Rightarrow x = 1$

$\because U = 4 \Rightarrow x = 16$

$U = \sqrt{x}$, $U = \sqrt{x}$

$= \sqrt{1}$, $= \sqrt{16}$

$= 1$ $\qquad = \sqrt{(4)^2}$

$\boxed{U = 1}$ $\qquad \boxed{U = 4}$

$\because v = U + 1$

Now taking constant outside.

$$= 2 \cdot \int_1^4 \frac{1}{U+1} du$$

$$= 2 \cdot \int_1^4 \frac{d(1)}{d(1)+1} du$$

$$= \cancel{2} \int_1^4 \frac{1}{\cancel{2}} du$$

$$= 1$$

$\because \frac{d}{du}(u) + \frac{d}{du}(1)$

$\frac{d}{du}(1) + 0$

$\boxed{1}$

$$\Rightarrow dv = 1\, du$$

$$du = 1\, dv$$

$= \int \frac{1}{v} \cdot 1\, dv$

$= \int \frac{1}{v}\, dv$

$= 2\int_2^5 \frac{1}{v}\, dv$

$u = 1$, $u = 4$
$v = u+1$, $v = u+1$
$= 1+1$, $v = 4+1$
$\boxed{v = 2}$, $\boxed{v = 5}$

$\because \frac{1}{v}\, dv = \ln(v)$

$= 2\int_2^5 \ln(v)$
$= 2\, [\ln(v)]_2^5$
$v = 2$, $v = 5$
$v = [\ln(v)]$, $= [\ln(v)]$
$\boxed{= \ln(2)}$, $\boxed{= \ln(5)}$

$\boxed{= 2\{\ln(5) - \ln(2)\}} \longrightarrow (6)$

Equate equation (5) & (6)

$2\{\ln(5) - 2\ln(2) + 9\} = 2\{\ln(5) - \ln(2)\}$

$\cancel{2\ln(5)} - \cancel{2\ln(2)} - \cancel{2\ln(5)} + \cancel{2\ln(2)} = 9$

$\boxed{9}$

<u>Answer</u>

(7) $\int_1^9 \frac{dx}{(x+1)\sqrt{x} + 2x} = \frac{1}{a}$ where $a = ?$

Solution

$\int_0^9 \frac{dx}{(x+1)\sqrt{x} + 2x} = \frac{1}{a}$

$\int_0^9 \frac{1}{(x+1)\sqrt{x} + 2x}\, dx$

Expand $(x+1)\sqrt{x} + 2x$

$= \sqrt{x}\,(x+1) + 2x$

Expand $\sqrt{x}(x+1)$ $\quad \because a=\sqrt{x},\ b=x,\ c=1$

Apply distribution law.

$\because a(b+c) = ab + ac$

$= \sqrt{x}\cdot x + \sqrt{x}\cdot 1$

$= x\sqrt{x} + 1\cdot\sqrt{x}$

$\boxed{= x^{3/2} + \sqrt{x} + 2x}$

$$= \int_1^9 \frac{1}{x^{3/2}+\sqrt{x}+2x}\,dx$$

$$= \int_1^9 \frac{1}{\frac{3\sqrt{x}}{2} + \frac{1}{2\sqrt{x}} + 2}$$

$$= \int \frac{1}{U+\sqrt{x}+2x}\cdot\frac{2}{3\sqrt{x}}\,du$$

$$\boxed{= \int \frac{2}{(u+\sqrt{x}+2x)(3\sqrt{x})}\,du}$$

$u = x^{3/2}$ sq on b/s

$U^2 = x(3/\cancel{2})^{2}$

$U^2 = x^3$

For $x^n = F(a)$ n is odd the solution of $x = \sqrt[n]{u^2}$

$x = \sqrt[3]{u^2}$

$$= \int \frac{2}{3u^{\frac{1/2}{3/2}}\left(u+u^{\frac{1/2}{3/2}}+2\sqrt[3]{u^2}\right)} \times du$$

$$= 3u^{\frac{1/2}{3/2}}\left(U+u^{\frac{1/2}{3/2}}+2\sqrt[3]{u^2}\right) = 3\sqrt[3]{u}$$

$$= u^{\frac{1/2}{3/2}} = 3\sqrt{u}$$

$\because x\sqrt{x}$

$x\cdot x^{1/2}$

$x^{1+1/2}$

$x^{3/2}$

$\because \frac{d}{dx}x^{3/2}$

$\frac{d}{dx}\frac{3}{2}x^{3/2-1}$

$\frac{d}{dx}\frac{3}{2}x^{1/2}$

$\boxed{= \frac{3\sqrt{x}}{2}}$

$\because \frac{d}{dx}\sqrt{x}$

$\frac{d}{dx}x^{1/2}$

$\frac{d}{dx}\frac{1}{2}x^{1/2-1}$

$\frac{d}{dx}\frac{1}{2}x^{-1/2}$

$\boxed{= 1/2\sqrt{x}}$

$\frac{d}{dx}(2x)$

$\boxed{\frac{d}{dx} = 2}$

$u \frac{1/2}{3/2}$

$\frac{\frac{1}{\cancel{2}}}{3/\cancel{2}} = \boxed{1/3}$

$= 3\sqrt[3]{u}\left(u + \sqrt[3]{u} + 2\sqrt[3]{u^2}\right)$

$= \sqrt[3]{u}^{1}\left(u + u^{1/3} + 2u^{2/3}\right)$

$= \int 3\sqrt[3]{u}\left(u + u^{1/3} + 2u^{2/3}\right)$

$\because 2\sqrt[3]{u^2}$

$u^{2\times 1/3}$

$\boxed{u^{2/3}}$

$$\boxed{= \int \frac{2}{3\sqrt[3]{u}\left(u + \sqrt[3]{u} + 2u^{2/3}\right.} du}$$

$$= \int_1^{27} \frac{2}{3\sqrt[3]{u}\left(u + \sqrt[3]{u} + 2u^{2/3}\right.} du$$

$$= \frac{2}{3}\int_1^{27} \frac{1}{\sqrt[3]{u}\left(u + \sqrt[3]{u} + 2u^{2/3}\right.} du$$

$x = 1$, $x = 9$

$u = x^{3/2}$, $= x^{3/2}$

$= (1)^{3/2}$, $= (9)^{3/2}$

$\boxed{u = 1}$, $u = \{(3)^2\}^{3/\cancel{2}}$

$\boxed{u = 27}$

Suppose $v = 3\sqrt{u}$

$\frac{dv}{du} = \frac{1}{3u^{2/3}}$

$\boxed{\frac{dv}{du} = 1/3u^{2/3}}$

$dv = 1/3u^{2/3}\, du$

$du = 3u^{2/3}\, dv$

$$= \int \frac{1}{v\left(u + v + 2u^{2/3}\right)} \cdot 3u^{2/3}\, dv$$

$$= \int \frac{3u^{2/3}}{v\left(u + v + 2u^{2/3}\right.} dv$$

$\because v = \sqrt[3]{u}$

Cube root on b/s

$v^3 = (u^{1/3})^3$

$v = \sqrt[3]{u} \Rightarrow \boxed{u = v^3}$

$= \int \frac{3v^{\cancel{2}}}{\cancel{v}(v^3 + v + 2v^2)} dv$

$= \int \frac{3\cancel{v}}{v^{\cancel{3}} + \cancel{v} + 2v^{\cancel{2}}}$

$= \int \frac{3}{v^2 + 1 + 2v} dv$

$\because u = 1 \quad , \quad u = 27$

$v = \sqrt[3]{u} \quad , v = \sqrt[3]{u}$

$= \sqrt[3]{1} \quad , \quad = \sqrt[3]{27}$

$= (1)^{1/3} \quad = \{(3)^{\cancel{3}}\}^{1/\cancel{3}}$

$\boxed{v = 1} \; , \; \boxed{v = 3}$

$\Rightarrow \int_1^3 \frac{3}{v^2 + 1 + 2v} dv$

$\Rightarrow \frac{2}{3} \int_1^3 \frac{3}{v^2 + 1 + 2v} dv$

$= \frac{2}{3} \cdot 3 \int_1^3 \frac{1}{v^2 + 1 + 2v} dv$

$\because v^2 + 1 + 2v = (v+1)^2$

$= \frac{2}{3} \cdot 3 \int_1^3 \frac{1}{(v+1)^2} dv$

$\frac{dw}{dv} = 1$

$\frac{d}{dv}(v+1) \Rightarrow \frac{d}{dv}(v) + \frac{d}{dv}(1) \Rightarrow 1 + 0$

$\boxed{= 1}$

$dw = 1 dv$

$dv = 1 dw$

$= \int \frac{1}{w^2} \cdot 1 dw$

$= \int_2^4 \frac{1}{w^2} dw$

$\because v = 1 \quad , \quad v = 3$

$w = v + 1 \quad , \quad w = v + 1$

$= 1 + 1 \quad , \quad = 3 + 1$

$\boxed{w = 2} \; , \; \boxed{w = 4}$

$\because 1/a^b = a^{-b}$

$= \frac{2}{3} \cdot 3 \int_2^4 \frac{1}{w^2} dw \Rightarrow 1/w^2 = w^{-2} \Rightarrow \frac{2}{3} \cdot 3 \int_2^4 w^{-2} dw$

Apply the power Rule

$$= \frac{2}{3} \cdot 3 \left[\frac{w^{-2+1}}{-2+1} \right]_2^4$$

$$= \frac{2 \cdot \cancel{3} \left[\frac{w^{-2+1}}{-2+1} \right]_2^4}{\cancel{3}}$$

$$= 2 \left[\frac{w^{-2+1}}{-2+1} \right] \Rightarrow \boxed{-w^{-1}}$$

$$= 2 \left[-w^{-1} \right]_2^4$$

$$\boxed{= 2 \left[-1/w \right]_2^4}$$

$\because w^{-1} = 1/w$

$\because w = 2 \quad , w = 4$

$= \frac{-1}{w} \quad , = \frac{-1}{w}$

$\boxed{= -\frac{1}{2}} \quad , \boxed{= -\frac{1}{4}}$

$$= -\frac{1}{4} - \left(-\frac{1}{2} \right)$$

$$= -\frac{1}{4} + \frac{1}{2}$$

$$= \frac{-2+4}{8} \Rightarrow \cancel{\frac{2}{8}} \Rightarrow \boxed{\frac{1}{4}}$$

$$= \cancel{2} \left(\frac{1}{\cancel{4}} \right)$$

$$\boxed{\Rightarrow \frac{1}{2}}$$

Now,

$$\frac{1}{2} = \frac{1}{a}$$

$$\boxed{a = 2}$$

<u>Answer</u>

(8) $\int_{27/8}^{8} \frac{2dx}{x^{5/3} - 3x^{4/3} + 3x - x^{2/3}}$

Solution:

$\int_{27/8}^{8} \frac{2}{x^{5/3} - 3x^{4/3} + 3x - x^{2/3}} dx$

$\frac{du}{dx} = \frac{1}{3x^{2/3}}$

$dx = 3x^{2/3}\, du$

$= \int \frac{1}{x^{5/3} - 3x^{4/3} + 3x - x^{2/3}} \cdot 3x^{2/3} du$

$= \int \frac{3x^{2/3}}{x^{5/3} - 3x^{4/3} + 3x - x^{2/3}} du$

$= \frac{3U^2}{U^5 - 3U^4 + 3U^3 - U^2} du$

$= \frac{3\cancel{U^2}}{\cancel{U^2}(U^3 - 3U^2 + 3U - 1)} du$

$\boxed{= \int \frac{3}{U^3 - 3U^2 + 3U - 1} du}$

$\because U' = \sqrt[3]{x}$

$U' = x^{1/3}$

$U' = \frac{1}{3} x^{-2/3}$

$\boxed{U' = \frac{1}{3x^{2/3}}}$

$\because U = \sqrt[3]{x}$

Taking cube root on b/s

$U^3 = \{(x)^{1/3}\}^3$

$\boxed{U^3 = x}$

$= \int_{3/2}^{2} \frac{3}{U^3 - 3U^2 + 3U - 1} du$

$= 2\int_{3/2}^{2} \frac{3}{U^3 - 3U^2 + 3U - 1} du$

$\boxed{= 2 \cdot 3 \int_{\frac{3}{2}}^{2} \frac{1}{U^3 - 3U^2 + 3U - 1} du}$

$\because = \int UV' = UV - \int U'V$

$U = \frac{1}{U^3 - 3U^2 + 3U - 1}$

$\because x = 27/8 \quad , \quad x = 8$

$U = \sqrt[3]{x} \quad , \quad U = \sqrt[3]{x}$

$= \sqrt[3]{\frac{27}{8}} \quad , \quad = \sqrt[3]{8}$

Taking cube root

$= \{(\frac{3}{2})^3\}^{1/3} \quad , \quad = \{2\}^{3 \times 1/3}$

$\boxed{U = \frac{3}{2}} \quad , \quad \boxed{U = 2}$

$v' = 1$

$$u' = \frac{d}{du}\left(\frac{1}{u^3-3u^2+3u-1}\right)$$

∵ Apply exponent rule. $1/a = a^{-1}$

$$= \frac{d}{du}\left\{(u^3-3u^2+3u-1)^{-1}\right\}$$

$$= \frac{d}{dv}(v^{-1})\frac{d}{du}(u^3-3u^2+3u-1)$$

$$= \frac{d}{dv}(v^{-1})\frac{d}{du}(3u^2-6u+3-0)$$

$$= \frac{d}{dv}(-v^{-2})\frac{d}{du}(3u^2-6u+3)$$

$$= \left(-\frac{1}{v^2}\right)(3u^2-6u+3)$$

Substitue $v = (u^3-3u^2+3u-1)$

$$= \left(-\frac{1}{(u^3-3u^2+3u-1)^2}\right)(3u^2-6u+3)$$

$$= -\frac{1}{(u^3-3u^2+3u-1)^2}(3u^2-6u+3)$$

$$= -\frac{3u^2-6u+3}{(u^3-3u^2+3u-1)^2}$$

∵ $(u-1)^2 = u^2-2u+1$

$$= -\frac{3(u^2-2u+1)}{(u^3-3u^2+3u-1)^2} \Rightarrow -\frac{3(u-1)^2}{(u^3-3u^2+3u-1)^2}$$

$$= -\frac{3(u-1)^2}{\{(u-1)^2\}^3}$$

∵ $u^3-3u^2+3u-1 = (u-1)^3$

$$= -\frac{3(u-1)^{\cancel{2}}}{(u-1)^6} \Rightarrow \boxed{-\frac{3}{(u-1)^4}}$$

$v = \int 1\,du$

$= 1 \cdot u$

Add a Constant Value

$= U + C$

$$= \left[\frac{1}{U^3 - 3U^2 + 3U - 1} u - \int \left(- \frac{3}{(U-1)^4} \right) U du \right]_{3/2}^{2}$$

$$= \left[\frac{1}{U^3 - 3U^2 + 3U - 1} \cdot U - \int - \frac{3}{(U-1)^4} U du \right]_{\frac{3}{2}}^{2}$$

$$= \left[\frac{U}{U^3 - 3U^2 + 3U - 1} - \int - \frac{3U}{(U-1)^4} U du \right]_{\frac{3}{2}}^{2}$$

$$= \left[\frac{U}{(U-1)^3} - \int -3 \cdot \frac{U}{(U-1)^4} du \right]_{3/2}^{2}$$

$= \quad dv = 1 du$

$$= \int \frac{U}{V^4} \cdot 1 dv \quad \Rightarrow \boxed{\int \frac{U}{V_4} dv} \because$$

$U = V + 1$

$V = U - 1$

$$= \int \frac{U}{V^4} dv$$

$$= \int \frac{V+1}{V^4} dv$$

$$= -3 \cdot \int \frac{V+1}{V^4} dv$$

$$= -3 \cdot \int \frac{\cancel{V}}{V^{\cancel{4}}} + \frac{1}{V^4} dv$$

$$= -3 \int \frac{1}{V^3} + \frac{1}{V^4} dv$$

Apply Sum Rule $\because$

$$= -3 \left(\int \frac{1}{V^3} dv + \int \frac{1}{V^4} dv \right)$$

$\because V^{-3} = 1/V^3$

$$= -3 \left(-\frac{1}{2V^2} - \frac{1}{3V^3} \right)$$

Put $V = U - 1$

$$= -3 \left\{ -\frac{1}{2(U-1)^2} - \frac{1}{3(U-1)^3} \right\}$$

$$= \left(\frac{3}{2(U-1)^2} + \frac{\cancel{3}}{\cancel{3}(U-1)^3} \right)$$

$$= \frac{3}{2(U-1)^2} + \frac{1}{(U-1)^3}$$

$$= 2\cdot 3\left[\frac{U}{U^3-3U^2+3U-1} - \left(\frac{3}{2(U-1)^2} + \frac{1}{(U-1)^3}\right)\right]_{\frac{3}{2}}^{2}$$

$$= 6\left[\frac{U}{U^3-3U^2+3U-1} - \left(\frac{3}{2(U-1)^2} + \frac{1}{(U-1)^3}\right)\right]_{3/2}^{2}$$

$$= 6\left[\frac{U}{U^3-3U^2+3U-1} - \frac{3}{2(U-1)^2} - \frac{1}{(U-1)^3}\right]_{3/2}^{2}$$

$$\because U^3-3U^2+3U-1 = (U-1)^3$$

$$= 6\left[\frac{U}{(U-1)^3} - \frac{3}{2(U-1)^2} - \frac{1}{(U-1)^3}\right]_{3/2}^{2}$$

$\times$ by 2:-

$$= 6\left[\frac{U\cdot 2}{(U-1)^3\cdot 2} - \frac{3(U-1)}{2(U-1)^3} - \frac{2}{2(U-1)^3}\right]_{3/2}^{2}$$

$$= 6\left[\frac{2U-3(U-1)-2}{2(U-1)^3}\right]_{3/2}^{2}$$

$$= 6\left[\frac{2U-3U+3-2}{2(U-1)^3}\right]_{3/2}^{2}$$

$$= 6\left[\frac{1-U}{2(U-1)^3}\right] \Rightarrow 6\left[\frac{-(1+\cancel{U})}{2(U-1)^{\cancel{3}}}\right]_{3/2}^{2}$$

$$\boxed{= 6\left[-\frac{1}{2(U-1)^2}\right]_{3/2}^{2}}$$

$U = 2$, $U = 3/2$

$$= \left[-\frac{1}{2(2-1)^2}\right] , \quad = \left[-\frac{1}{2(3/2-1)^2}\right]$$

$$= \left[-\frac{1}{2(1)^2}\right] , \quad = \left[-\frac{1}{2(1/2)^2}\right]$$

$$\boxed{U_1 = -\frac{1}{2}} , \quad \boxed{U_2 = -2}$$

Equating U_1 & U_2

$= \frac{-1}{2} - (-2)$

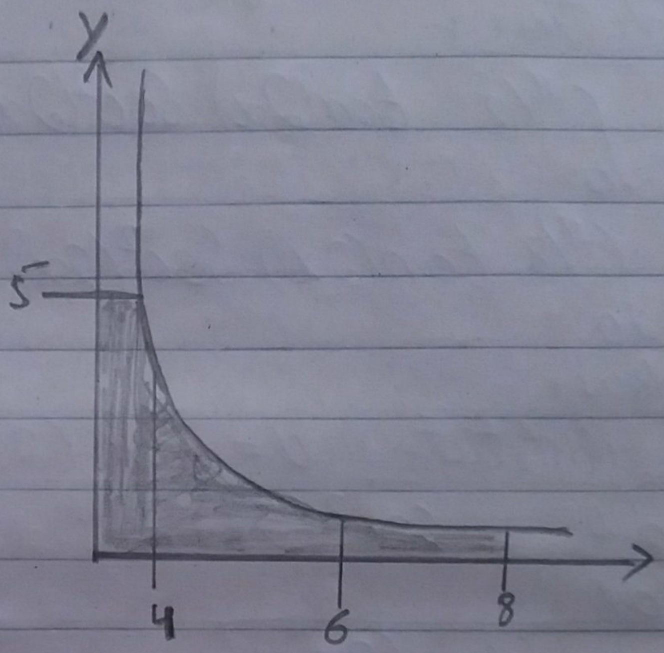
$= -\frac{1}{2} + 2$

$= \frac{-1+4}{2}$

$= \frac{3}{2} \times 6$

$= \frac{\cancel{18}}{\cancel{2}}$

$\boxed{= 9}$



Answer

(9) $\int_{\pi/6}^{\pi/2} \frac{\cos^3 x}{\sqrt{\sin x}} dx = \frac{a}{3} - \frac{b}{10\sqrt{2}}$ where a & b?

Solution:

$$\int_{\pi/6}^{\pi/2} \frac{\cos 3x}{\sqrt{\sin x}} = \frac{a}{3} - \frac{b}{10\sqrt{2}}$$

$$\int_{\pi/6}^{\pi/2} \frac{\cos(3x)}{\sqrt{\sin(x)}} dx$$

$$\boxed{= -0.73137}$$

Answer

(10) $\int_0^{\pi/8} \tan 2x \sec^2 2x \, dx = \frac{1}{a}$ where a?

Solution:-

$$\int_0^{\pi/8} \tan(2x) \sec^2(2x) \, dx = \frac{1}{a}$$

$$\int_0^{\pi/8} \tan(2x) \sec^2(2x) \, dx$$

Now,

Substitute $u = \tan(2x)$

$$\frac{du}{dx} = \sec^2(2x) 2$$

$$\boxed{\frac{d}{dx}(\tan(2x))}$$

Apply Chain Rule

$$\frac{d}{du}\{\tan(u)\}$$

$$\boxed{\frac{d}{du} = \sec^2(u)}$$

$$\frac{d(2x)}{dx}$$

$\because 2x = 2x^{1-1}$
$= 2$

$$\boxed{\frac{d}{dx} = 2}$$

Now,

Substitute $u = 2x$

$$= \sec^2(u) \cdot 2$$
$$= \sec^2(2x) \cdot 2$$

$$du = \sec^2(2x) \cdot 2 dx$$

$$dx = \frac{1}{\sec^2(2x) \cdot 2} du$$

$$= \int \sec^2(2x)\, u \cdot \frac{1}{\sec^2(2x)\cdot 2}\, du$$

$$= \int \frac{u}{2}\, du$$

Here

$x = 0$, $x = \pi/8$

$u = \tan(2x)$, $u = \tan(2x)$

$= \tan\{(2(0))\}$, $= \tan\{2(\pi/8)\}$

$= \tan(0)$, $= \tan(\pi/4)$

$\boxed{u = 0}$, $\boxed{u = 1}$

$\because \tan(0) = 0$, $\because \tan(\pi/4) = 1$

$$= \int_0^1 \frac{u}{2}\, du \Rightarrow \frac{1}{2}\cdot\int_0^1 u\, du$$

Apply power Rule

$$= \frac{1}{2}\left[\frac{u^{1+1}}{1+1}\right]_0^1 \qquad \Bigg\| \because \frac{x^{a+1}}{a+1} = x^a dx$$

$$= \frac{1}{2}\left[\frac{u^2}{2}\right]_0^1$$

Here

$u = 0$, $u = 1$

$= \frac{u^2}{2}$, $= \frac{u^2}{2}$

$= \frac{(0)^2}{2}$, $= \frac{(1)^2}{2}$

$= 0$, $= \frac{1}{2}$

Now, $\frac{1}{2} - 0$

$$= \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{1}{4}$$

$$\frac{1}{a} = \frac{1}{4}$$

Now,

we Apply Cross Multiplication.

$$1 \cdot 4 = 1 \cdot a$$

$$\boxed{a = 4}$$

Answer