Macroeconomic Forecasting Lec_01

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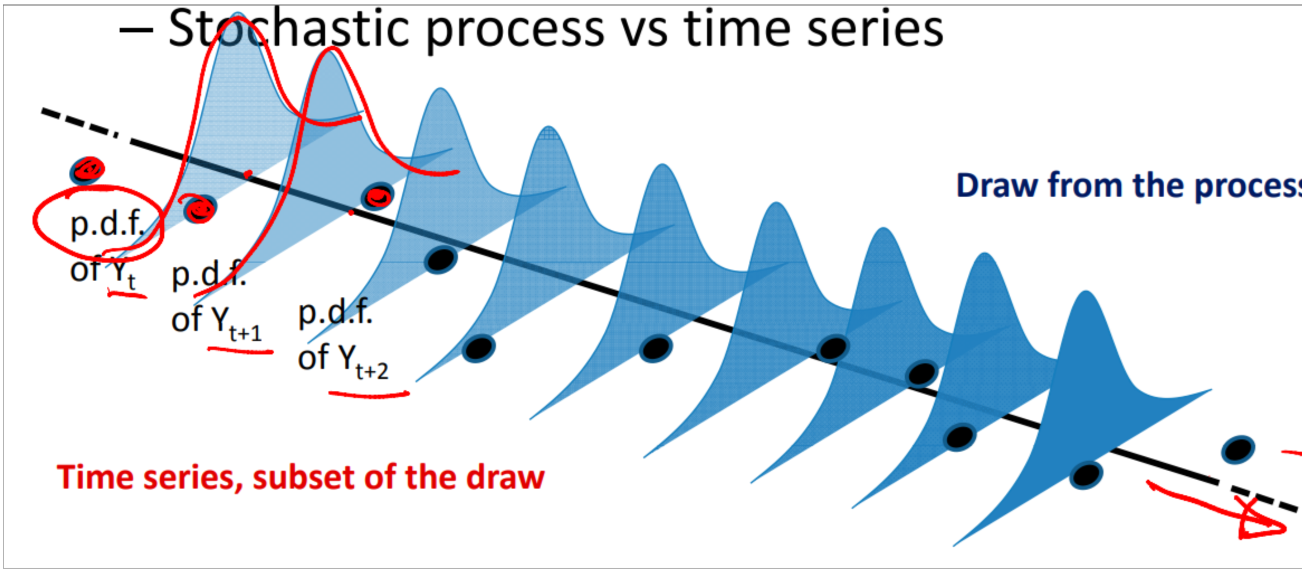
09-01-2025

Properties of Time Series

Introduction

- General comments
  - Univariate analysis
  - Two general “classes” of processes
  - Both science and art (judgement):
  - Understanding behavior and forecasting
  - Assessing/testing

Univariate Analysis

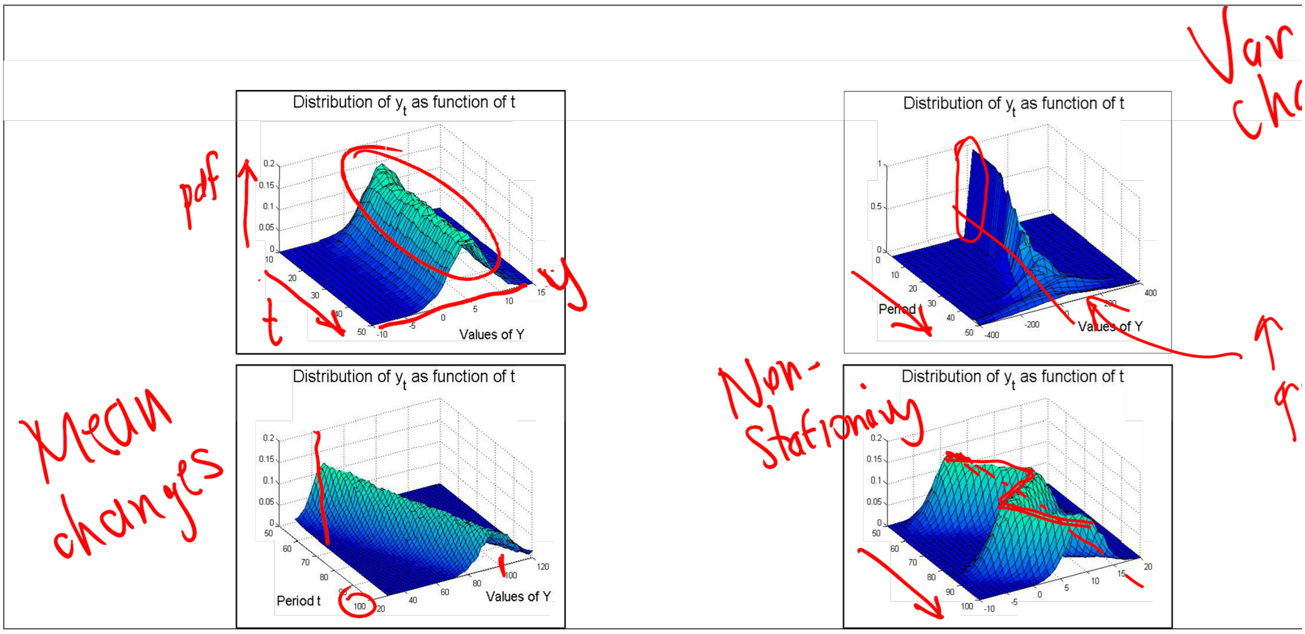


Stochastic Process vs time series

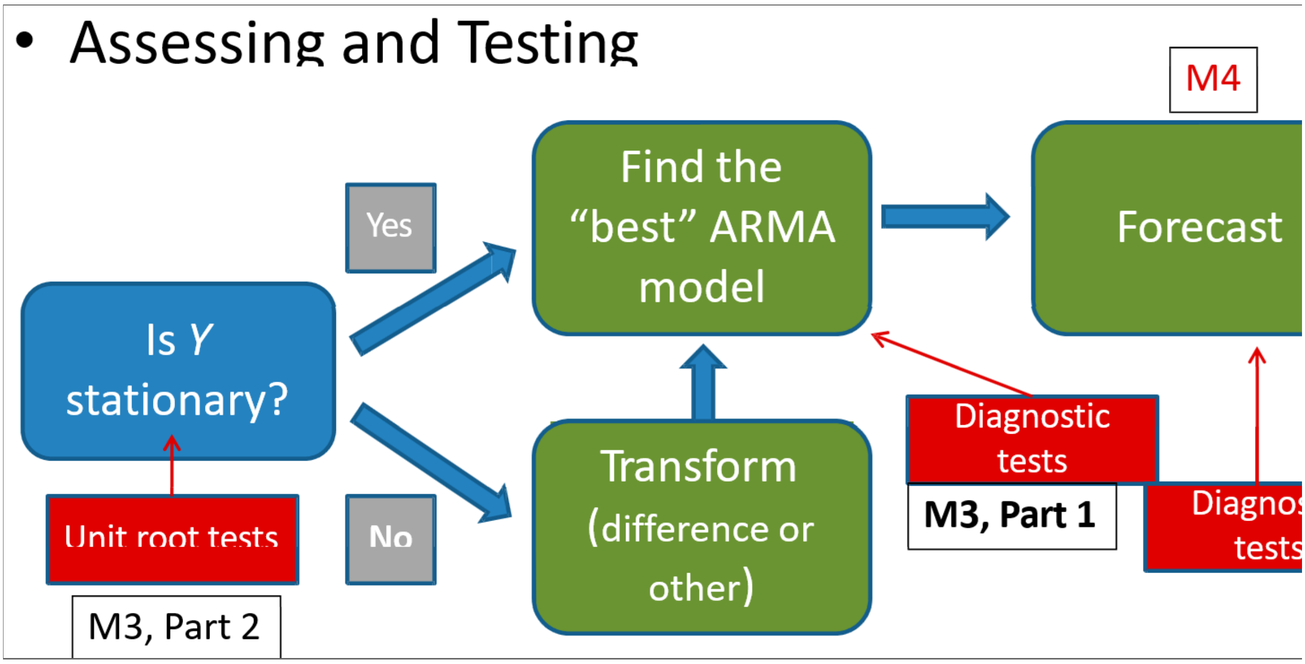
Each distribution is a draw from a **random process**.

Introduction

- Two general “classes” of processes
    - stationary vs nonstationary
    - Unchanged distribution (pdf) over time?
    - Covariance stationary:
        i. Unconditional mean $E(Y_t) = E(Y_{t+j}) = \mu$
        ii. variance constant $Var(Y_t) = Var(Y_{t+j}) = \sigma_y^2$ and
        iii. Covariance depnds on time j that has elapsed between observations, not on reference period $Cov(Y_t, Y_{t+j}) = Cov(Y_s, Y_{s+j}) = \gamma_j$



Stochastic Process vs time series



Diagnostic-ARMA

Outline

Part 1 : Stationary processes

- Identification
- Estimation & Model Selection
- Putting it all together

Part 2: Nonstationary processes

- Characterization
- Testing

Part1 : Stationary Process

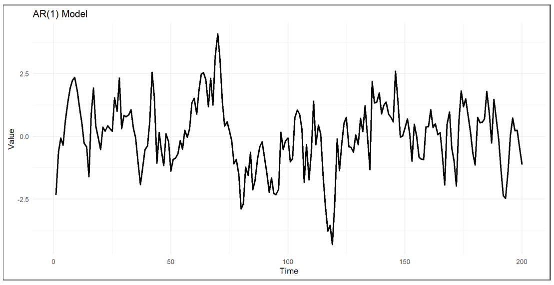
Just to remind you….

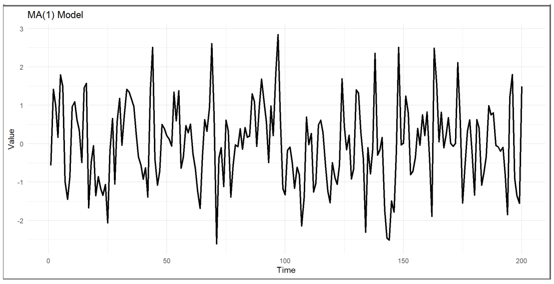
- Identification
- Estimation & Model Selection
- Putting it all together

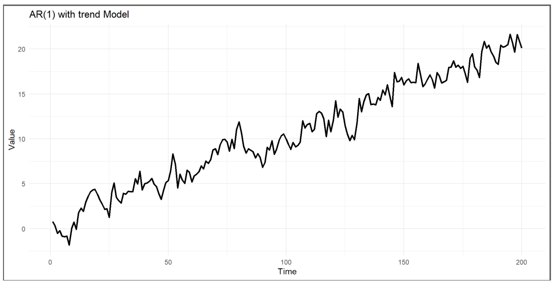
The first step is visual inspection: graph and observe your data.

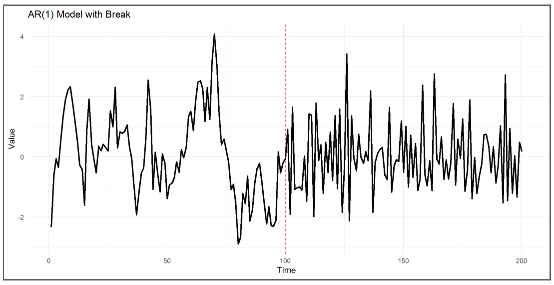
> “You can observe a lot just by watching” Yogi Berra

Plot plot and plot your data





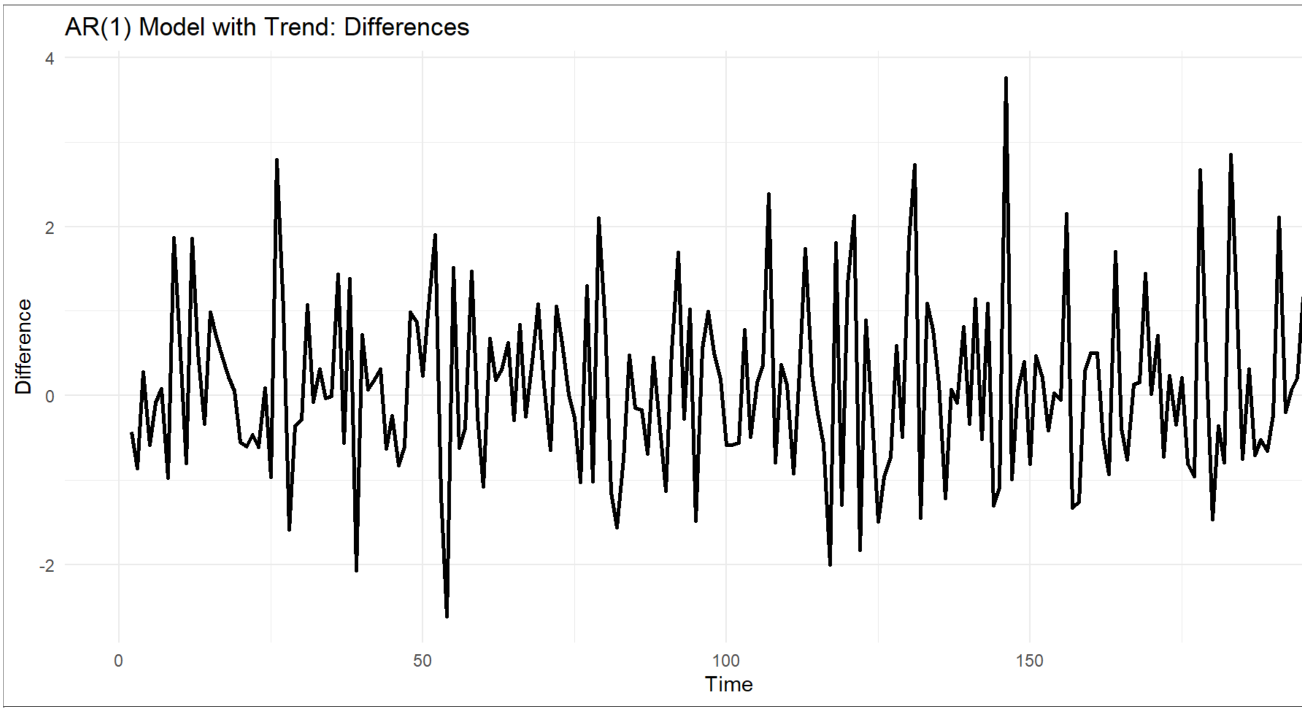




Differenced model of trend variable (third case)

Difference can remove the trend.

$y_t^* = y_t - y_{t-1}$



Identification

Assuming that the process is stationary, there are three basic types that interest us:
Autoregressive (process) $y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + \ldots + b_p y_{t-p} + \epsilon_t$
Moving Average : AR(process) :
$y_t = \mu + u_t + \phi_1 u_{t-1} + \phi_2 u_{t-2} + \ldots + \phi_q u_{t-q} + \epsilon_t$
Combined ARMA-process $y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + \ldots + b_p y_{t-p} + u_t + \phi_1 u_{t-1} + \phi_2 u_{t-2} + \ldots + \phi_q u_{t-q} + \epsilon_t$

Some notation: AR(p), MA(q), ARMA(p,q), where p,q refer to the order (maximum lag) of the process

ϵ_t is a white noise disturbance:

$E(\epsilon_t = 0), Var(\epsilon_t = \sigma^2)$, $Cov(\epsilon_t, \epsilon_s = 0, if\ t \neq s)$

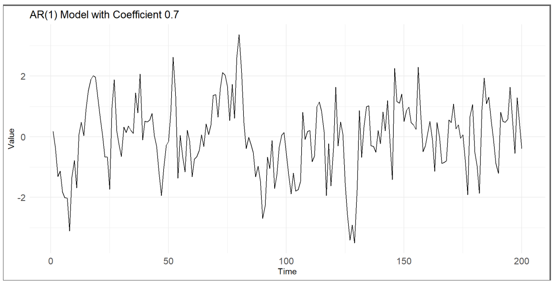
Tools for Identification

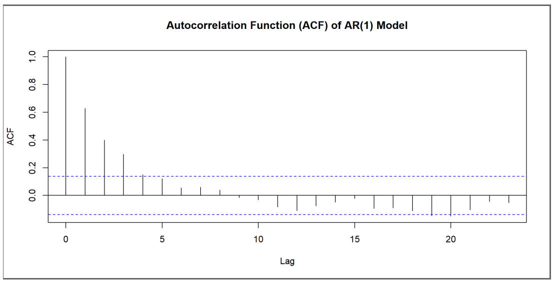
- Where are we? Where are we going?
- Stationary process (visual inspection) y
- Learned about possible processes for y
- Need to identify which one in order to understand, then eventually forecast y

> tools to help identify

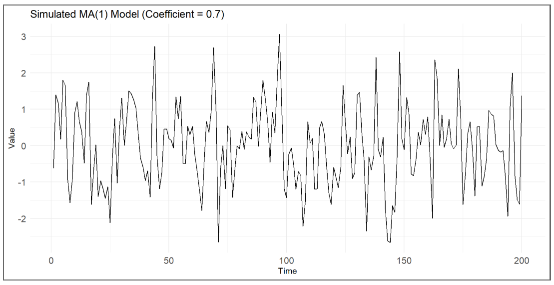
Autocovariance and autocorrelation Relations between observations at different lags:

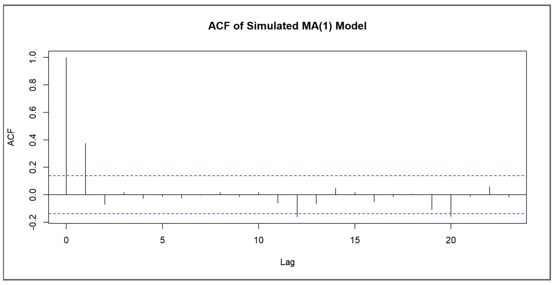
- Autocovariance $\gamma_j = E[(y_t - \mu)(y_{t-j} - \mu)]$
- Autocorrelation $\rho_j = \gamma_j / \gamma_0$
- ACF or Correlogram : graph of autocorrelations at each lag


AR(1) Model with Coefficient 0.7
Value
Time

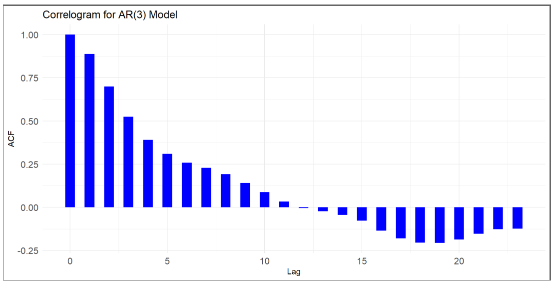

Autocorrelation Function (ACF) of AR(1) Model
ACF
Lag

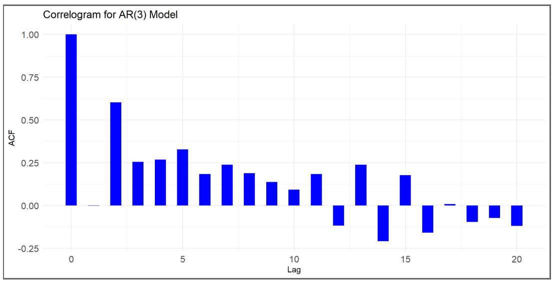
NULL


Simulated MA(1) Model (Coefficient = 0.7)
Value
Time

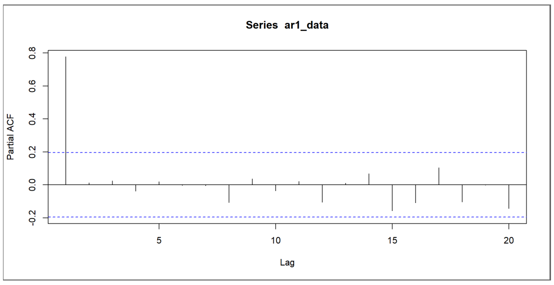

ACF of Simulated MA(1) Model
ACF
Lag

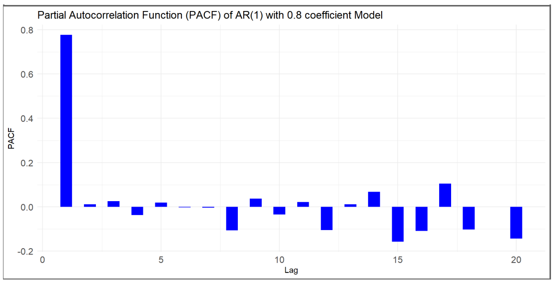
NULL

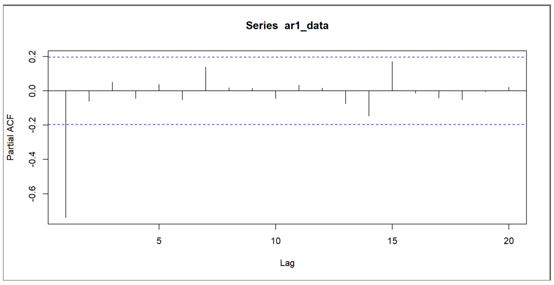

Correlogram for AR(3) Model
ACF
1.00
0.75
0.50
0.25
0.00
-0.25
0
5
10
15
20
Lag

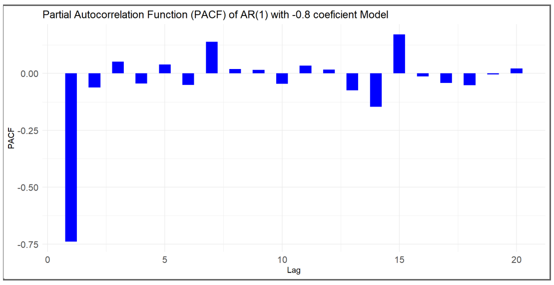

Correlogram for AR(3) Model
ACF
1.00
0.75
0.50
0.25
0.00
-0.25
0
5
10
15
20
Lag

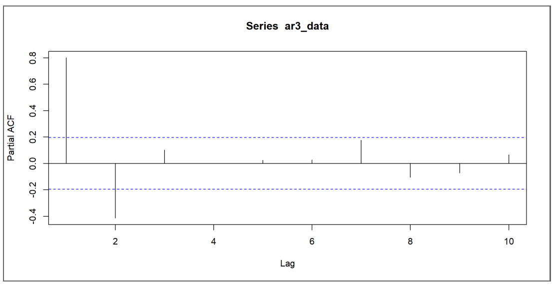
Patterns for PACF

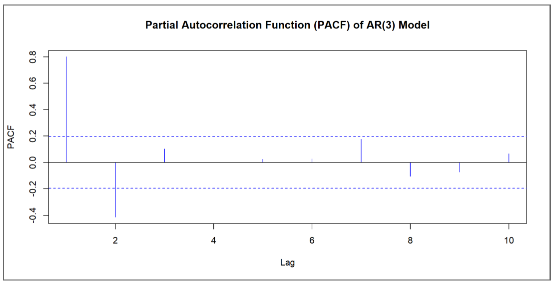


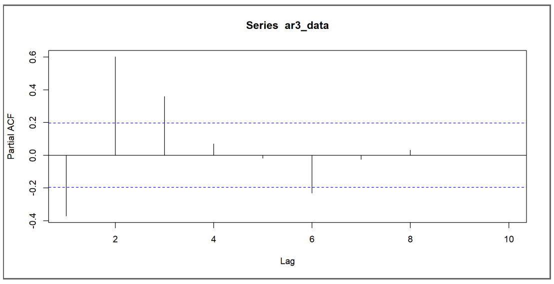


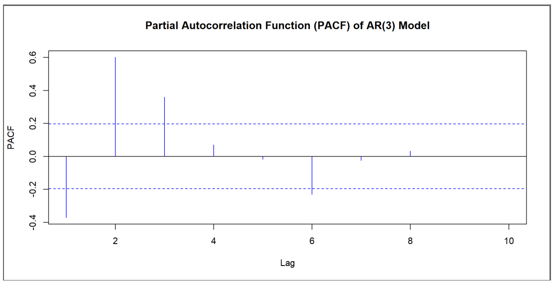


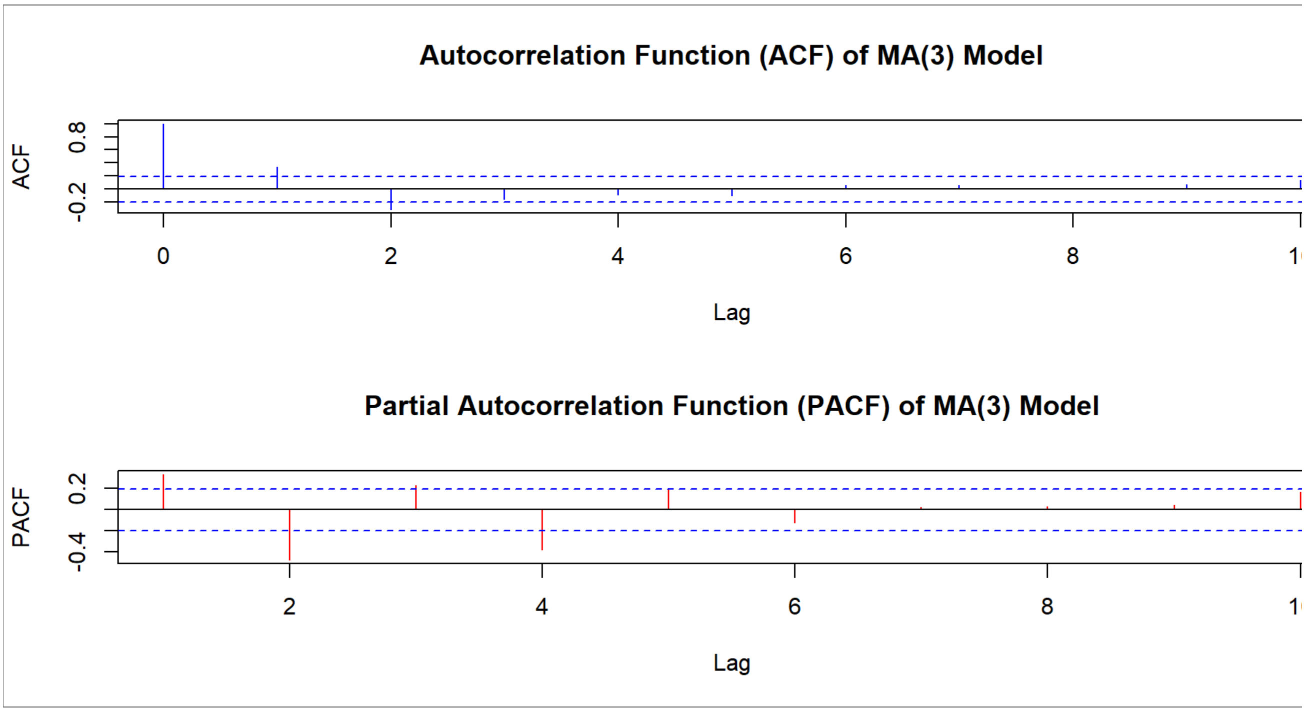





Partial Autocorrelation Function (PACF) of AR(3) Model
PACF
Lag


Series ar3_data
Partial ACF
Lag


Partial Autocorrelation Function (PACF) of AR(3) Model
PACF
Lag



Autocorrelation Function (ACF) of MA(3) Model
ACF
0.8
-0.2
0
2
4
6
8
Lag
Partial Autocorrelation Function (PACF) of MA(3) Model
PACF
0.2
-0.4
2
4
6
8
Lag

Stationary Time Series

We now have a tool (ACF, PACF) to help us identify the stochastic stochastic process underlying time series we are observing. Now we will: - Summarize the basic patterns to look for - Observe an actual data series and make an initial guess Observe an actual data series and make an initial guess - Next step: estimate (several alternatives) based on this guess

Summary Table Pattern

	ACF	PACF
White noise	All ρ's $= 0$	All b's $= 0$
AR(1)	**Geometric decay (oscillating if $b < 0$)**	**Cutoff after lag 1; $\rho_1 = b_1$**
AR(p)	Decays toward zero, may oscillate	**Cutoff after lag p.**
MA(1)	**Cutoff after lag 1.**	**Geometric decay (oscillating if $\phi < 0$)**
MA(q)	**Cutoff after lag q.**	Decay (oscillating if $\phi < 0$)
ARMA(1,1)	Geometric decay after lag 1 (oscillating if $b < 0$)	Geometric decay after lag 1 (oscillating if $b < 0$)
ARMA(p,q)	Decay (direct or oscillatory) after lag q	Decay (direct or oscillatory) after lag p

Pattern for model identification

Tips

- ACF's that do not go to zero could be sign of nonstationarity
- ACF of both AR, ARMA decay gradually, drops to 0 for MA
- PACF decays gradually for ARMA, MA, drops to 0 for AR

Possible approach: begin with parsimonious low order AR, check residuals to decide on possible MA terms.

When looking at ACF, PACF

- Box-Jenkins provide sampling variance of the observed ACF and PACFs (r_s and b_s)
- Permits one to construct confidence intervals around each $\rightarrow$ assess whether significantly $\neq 0$
- Computer packages provide this automatically

Estimation and Model Selection

- Decide on plausible alternative specifications (ARMA)
- Estimate each specification
- Choose “best” model, based on:
- Significance of coefficients
- Fit vs parsimony (criteria)
- White noise residuals
- Ability to forecast
- Account for possible structural breaks

Fit vs parsimony (information criteria):

- Additional parameters (lags) automatically improve fit but reduce forecast quality
- Tradeoff between fit and parsimony; widely used criteria:
- Akaike Information Criterion (AIC) $AIC = Tln(SST) + 2(p + q + 1)$
- Schwartz Bayesian Criterion (BIC) $SBC = Tln(SST) + (p + q + 1)ln(T)$
- SBC is considered to be preferable for having more parsimonious models than AIC

White noise errors :

- Aim to eliminate autocorrelation in the residuals (could indicate that model does not reflect the lag structure well)
- Plot "standardized residuals" (ϵ_{it}) No more than 5% of them should lie outside [-2,+2] over all periods
- Look at r_s , b_s (and significance) at different lags Box-Pierce Statistic: joint significance test up to lag s: ${x} =

  $Q = T \sum_{k=1}^{s} r_k^2$
  H_0 : all $r_k = 0$, H_1 : at least one $r_k \neq 0$

Forecastability

Can assess how well the model forecasts ” out of sample”:

- Estimate the model for a sub-sample (for example, the first 250 out of 300 observations).
- Use estimated parameters to forecast for the rest of the sample (last 50)
- Compute the ” forecast errors” and assess:
- Mean Squared Prediction Error
- Granger-Newbold Test
- Diebold-Mariano Test

Account for possible structural breaks:

- Does the same model apply equally well to the entire sample, or do parameters change (significantly) within the sample?

  How to approach:

- Own priors/suspicion : Chow test for parameter change
- If priors not strong, recursive estimation, tests for parameter stability over the sample, for example, CUSUM

Learning all above with simulated data

Lets simulate MA(1), AR(1) series in Excel, R and one can use STATA/EVIEWS as well

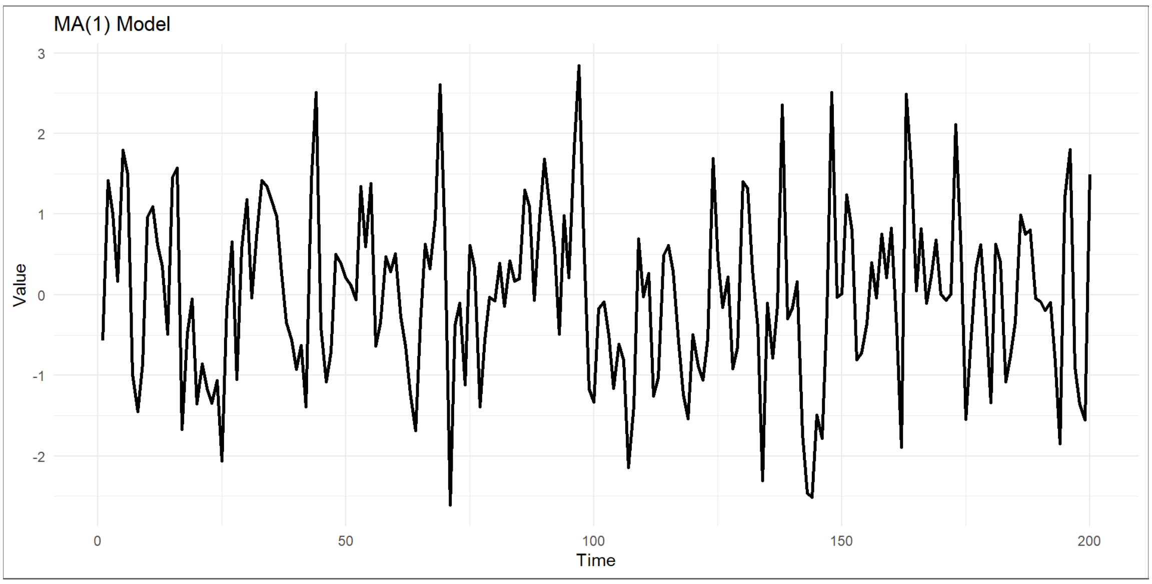
MA-1 code

MA1-output

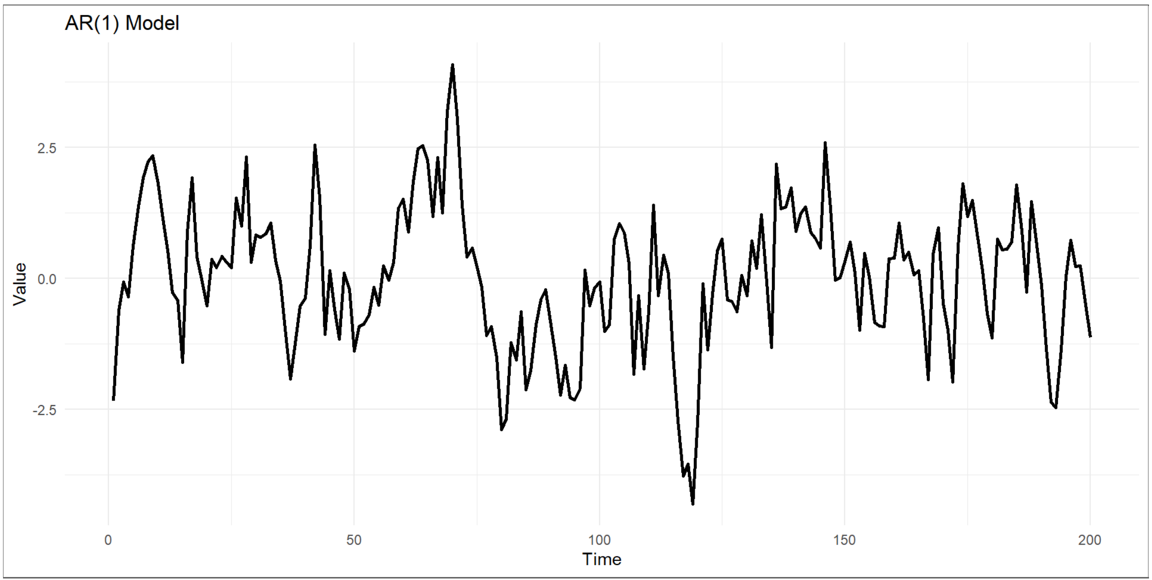
AR-1 code

AR-1 output

```
# Set the parameters
mu <- 0          # Mean of the series
theta <- 0.6     # Moving average coefficient
n <- 200         # Number of time points

# Simulate data from the MA(1) model
set.seed(123)   # For reproducibility
ma1_data <- arima.sim(model = list(ma = theta), n = n, me

# Create a data frame with time series data
time_series_data <- data.frame(Time = 1:n, Value = ma1_da

# Create a ggplot2 line plot
ggplot(time_series_data, aes(x = Time, y = Value)) +
  geom_line(linewidth=1) +labs(title = "MA(1) Model ", x
  theme_minimal()
```



```r
phi <- 0.8  # Autoregressive coefficient
n <- 200    # Number of time points

# Simulate data from the AR(1) model
set.seed(123)  # For reproducibility
ar1_data <- arima.sim(model = list(ar = phi), n = n)
# Create a data frame with time series data
time_series_data <- data.frame(Time = 1:n, Value = ar1_da
# Create a ggplot2 line plot
ggplot(time_series_data, aes(x = Time, y = Value)) +
  geom_line(linewidth=1) +labs(title = "AR(1) Model ", x 
  theme_minimal()
```



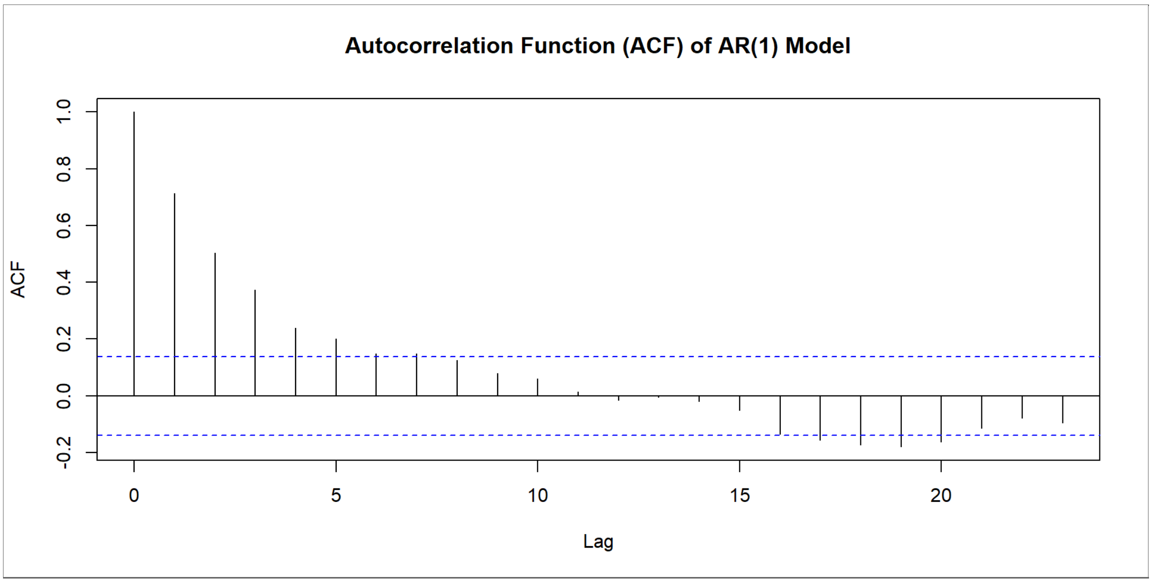
ACF and PACF of AR-1 and MA-`

ACF of AR-1

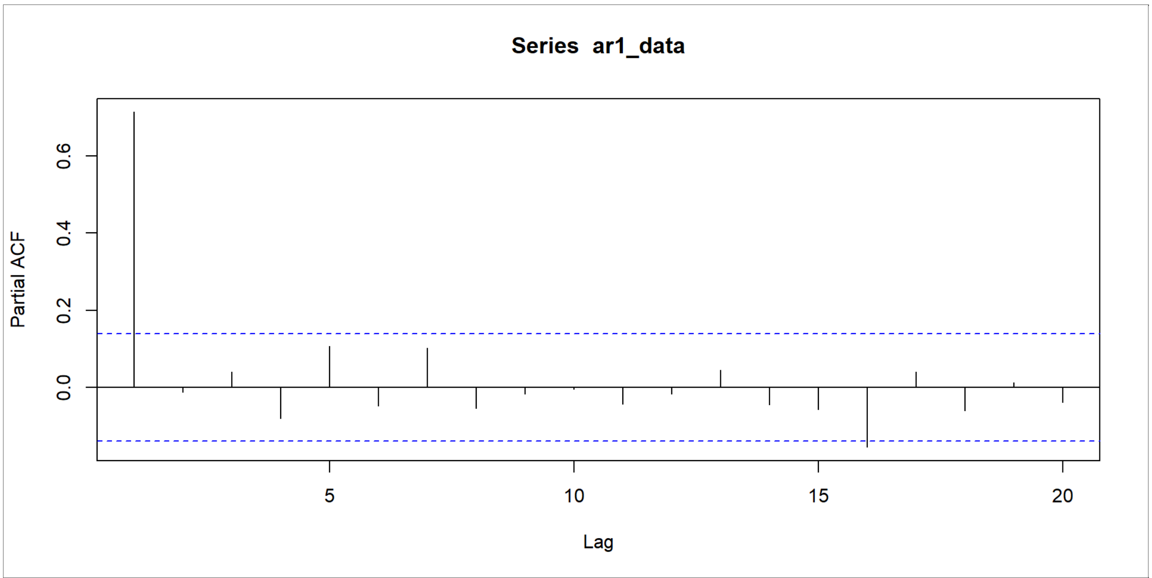
PACF of AR-1

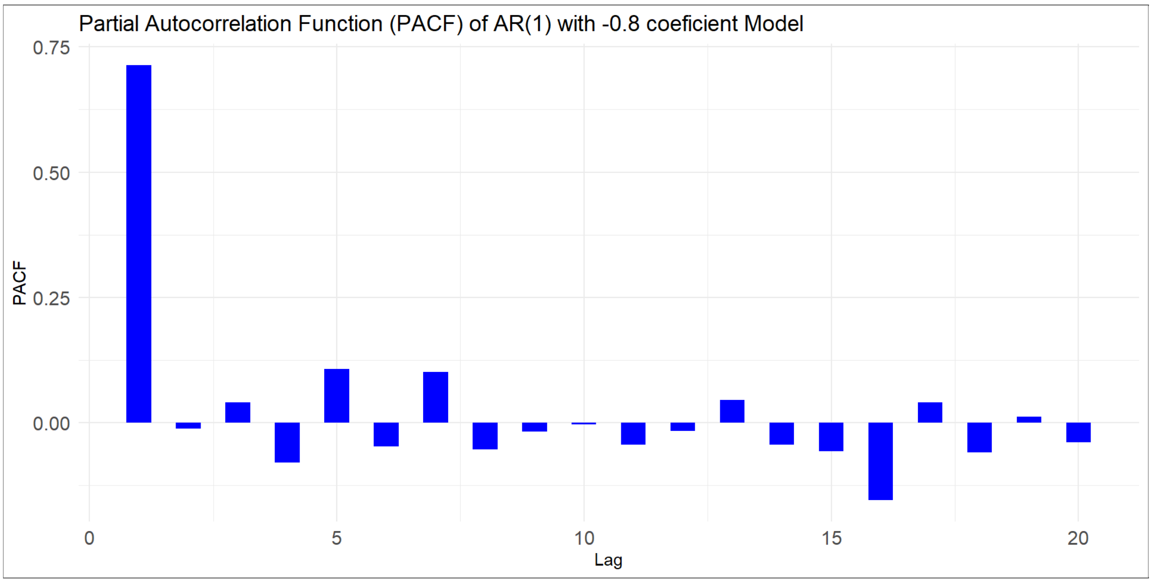
ACF-MA1

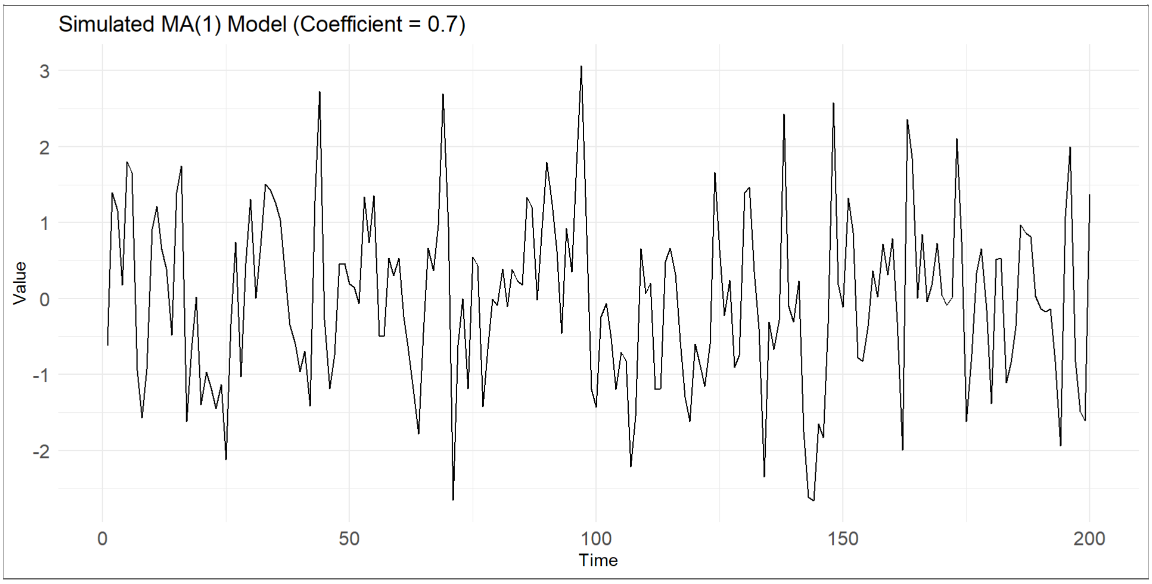
PACF-MA1

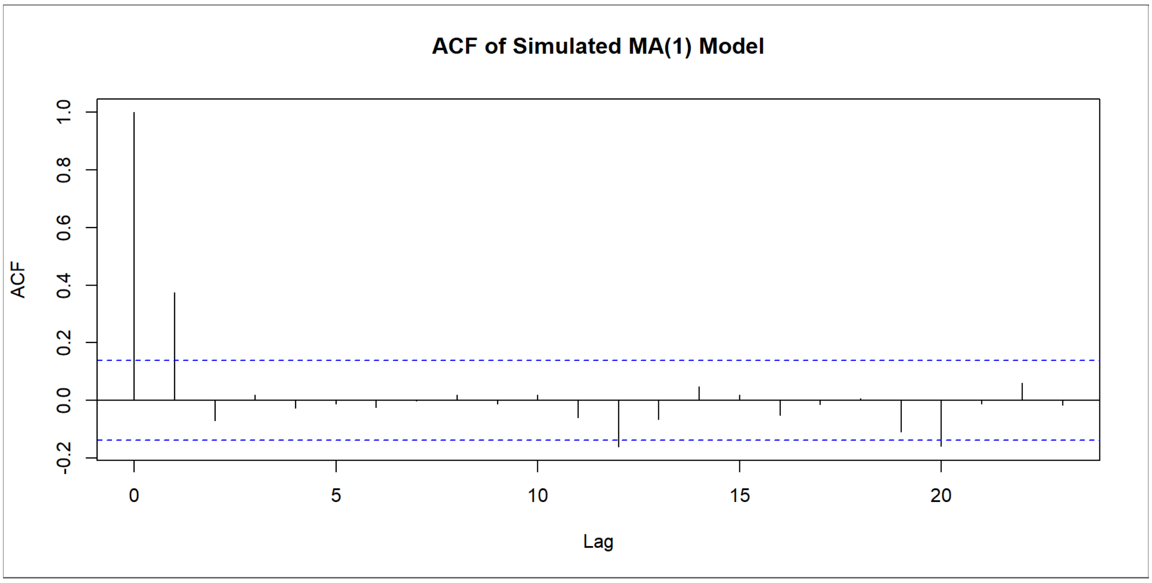


NULL

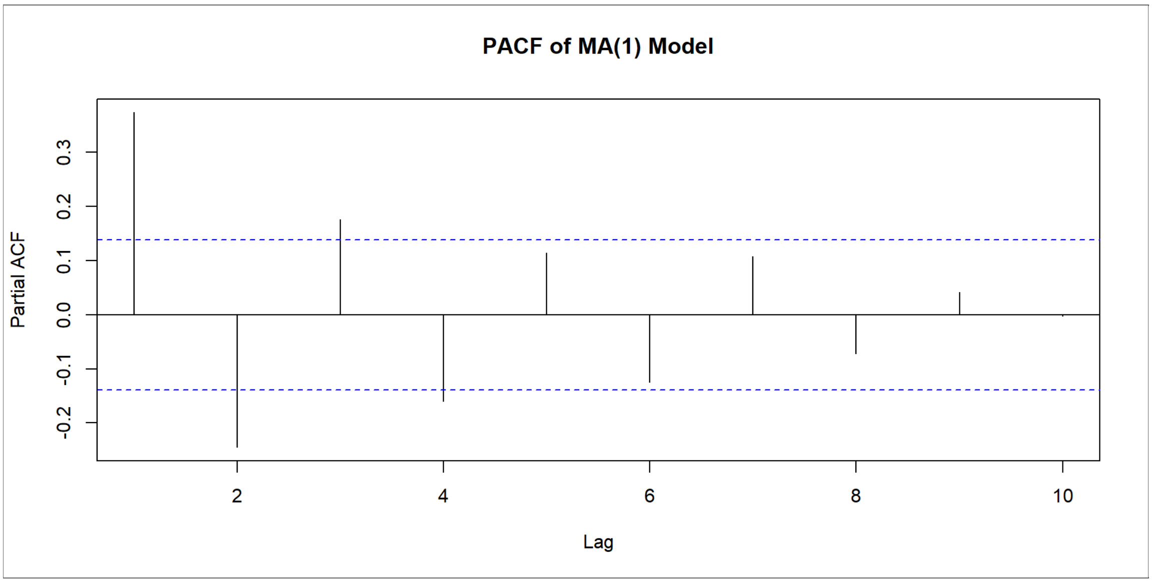



Partial Autocorrelation Function (PACF) of AR(1) with -0.8 coeficient Model
PACF
0.75
0.50
0.25
0.00
0
5
10
15
20
Lag


Simulated MA(1) Model (Coefficient = 0.7)
Value
3
2
1
0
-1
-2
0
50
100
150
200
Time


ACF of Simulated MA(1) Model
ACF
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0
5
10
15
20
Lag

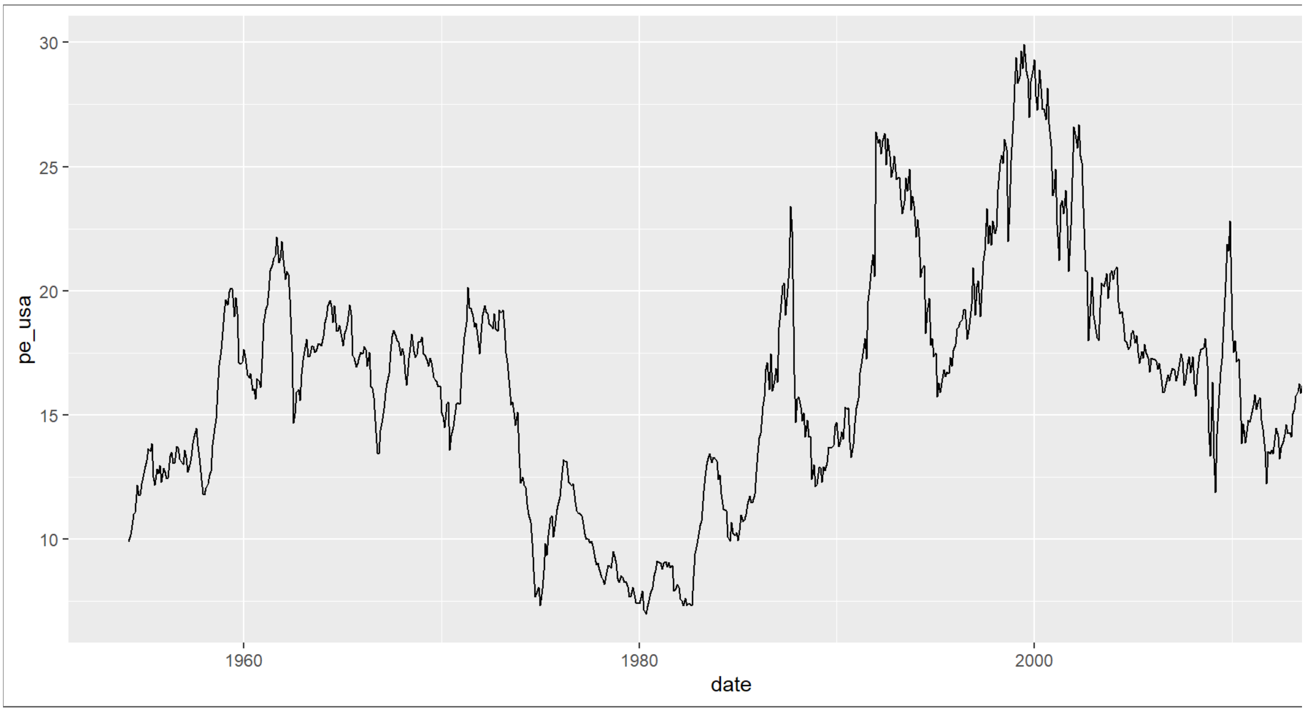
NULL


PACF of MA(1) Model
Partial ACF
-0.2
-0.1
0.0
0.1
0.2
0.3
2
4
6
8
10
Lag

Now let's work with real world data

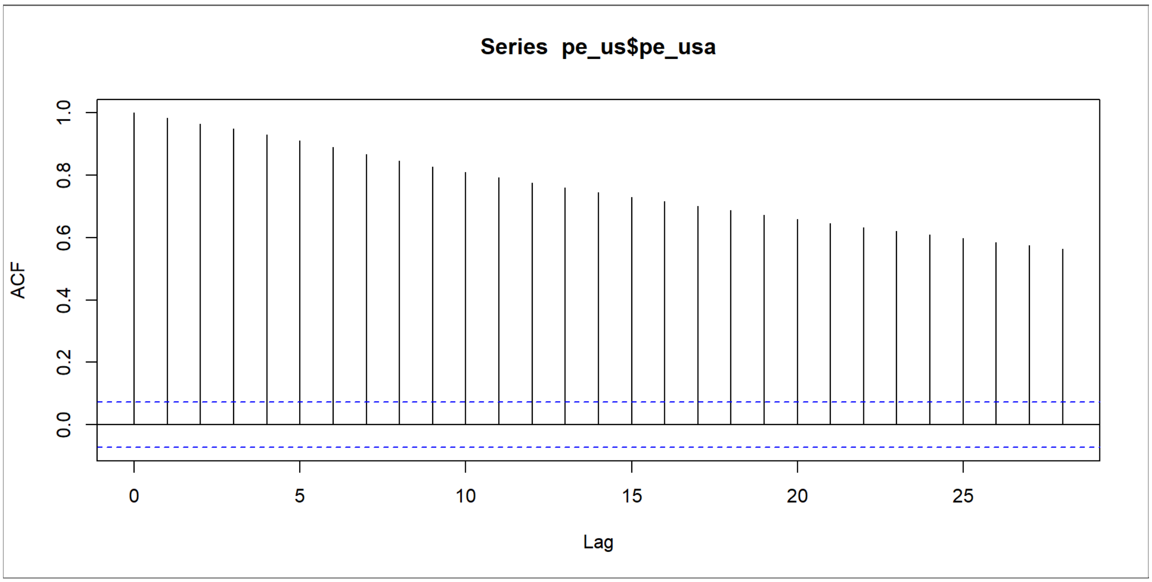
```
Rows: 733
Columns: 8
$ dateid01 <date> 1954-02-01, 1954-03-01, 1954-04-01, 1954-
05-01, 1954-06-01, …
$ dateid   <dttm> 1954-03-01 00:00:00, 1954-04-01 00:00:00,
1954-05-01 00:00:0…
$ date     <date> 1954-02-26, 1954-03-31, 1954-04-30, 1954-
05-31, 1954-06-30, …
$ pe_aus   <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,
NA, NA, NA, NA, NA, N…
$ pe_ind   <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,
NA, NA, NA, NA, NA, N…
$ pe_ndo   <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,
NA, NA, NA, NA, NA, N…
$ pe_saf   <dbl> NA, NA, NA, NA, NA, NA, NA, NA, NA, NA,
```

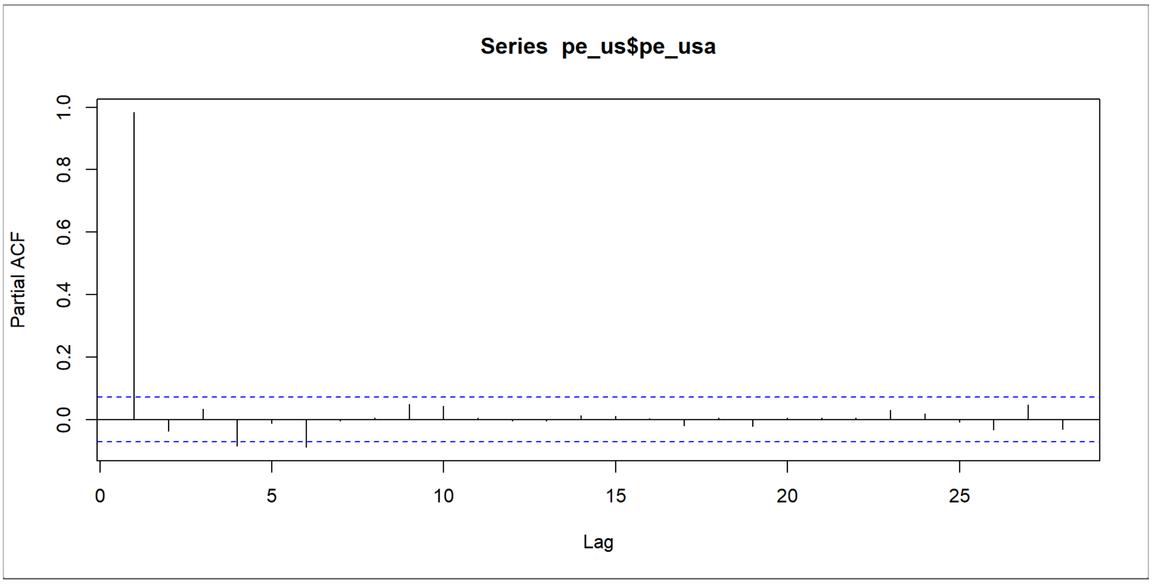
Pick pe_usa



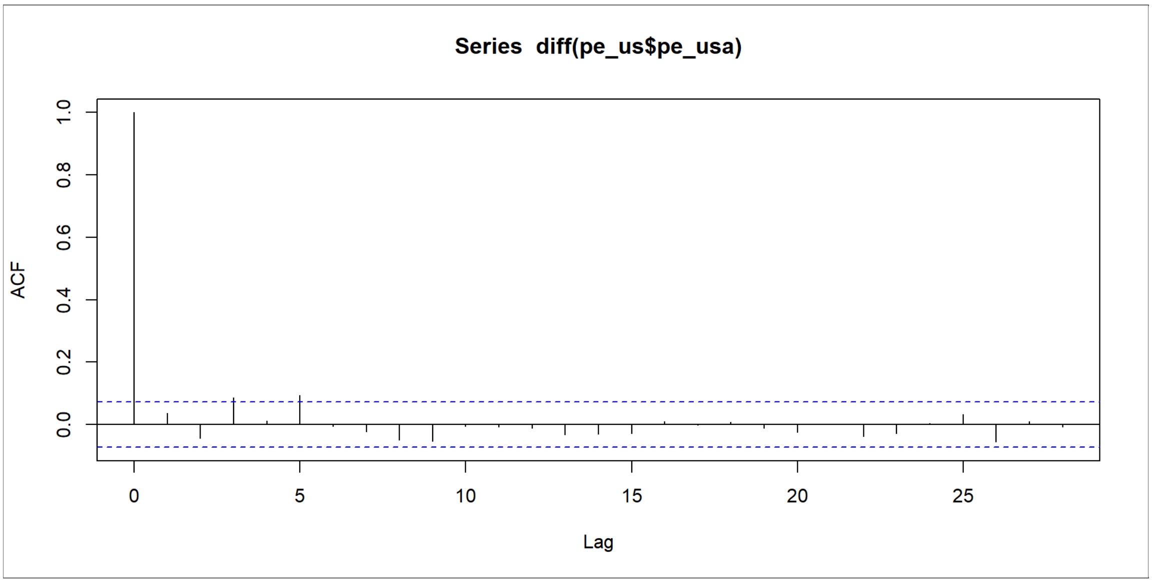
Identify model

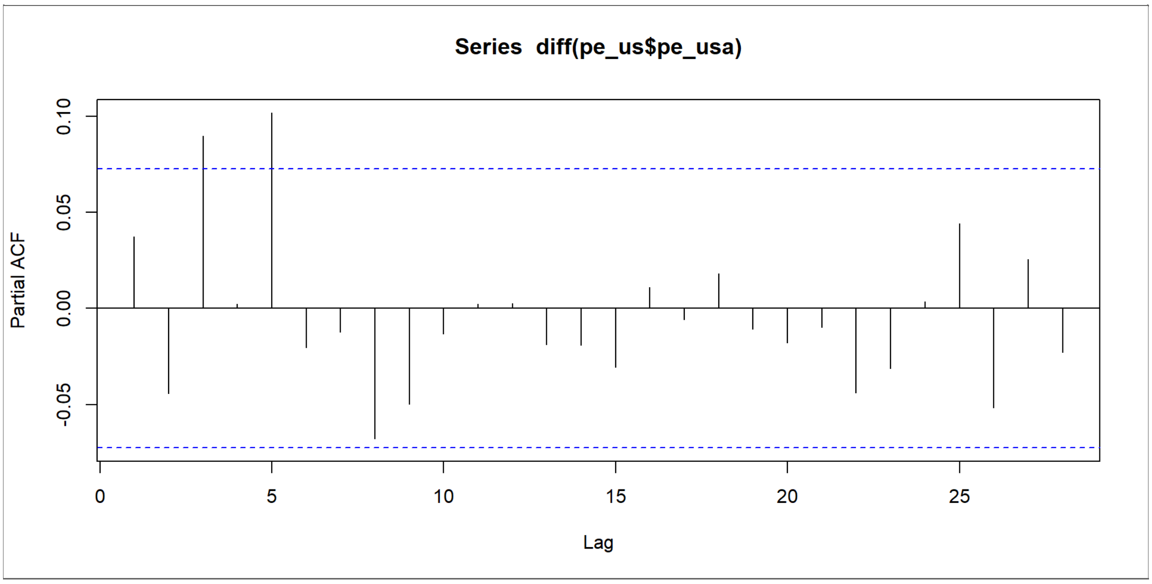
Use SACF and SPACF to choose model(s) and also use auto.arima and see which of these two wins: your judged model or auto.arima





acf and pacf patterns indicate series is non-stationary. So here we run acf and pacf of difference of the series.





From these two graphs it seems model is ARIMA(0,1,1)

```
Series: pe_us$pe_usa
ARIMA(1,1,1)

Coefficients:
          ar1     ma1
      -0.7554  0.8268
s.e.   0.1106  0.0944

sigma^2 = 0.7151:  log likelihood = -914.95
AIC=1835.91   AICc=1835.94   BIC=1849.69
```

So our model was ARIMA(0,1,1) while auto.ARIMA is ARIMA(1,1,1) not bad.

Nonstationary Series

Nonstationary Series

- Introduction:
- Key Questions:
- What is nonstationarity?
- Why is it important?
- How do we determine whether a time series is nonstationary?

What is nonstationarity?

Recall from earlier part on stationarity:

- Covariance stationarity of y implies that, over time, y has:
- Constant mean
- Constant variance
- Co-variance between different observations that do not depend on that time (t), only on the "distance" or "lag" between them (j):

$Cov(Y_t, Y_{tj}) = Cov(Y_s, Y_{s+j}) = \gamma_j$

What is nonstatinarity?

Thus, if any of these conditions does not hold, we say that y is nonstationary:
There is no long-run mean to which the series returns (economic concept of long-term equilibrium)
The variance is time-dependent. For example, could go to infinity as the number of observations goes to infinity
Theoretical autocorrelations do not decay, sample autocorrelations do so very slowly.

Nonstationary series can have a trend:

- Deterministic: nonrandom function of time:

$y_t = \mu + \beta t + u_t$, where u_t is "iid"

- Example $\beta = 0.45$