10/10/2022

Dynamic Macroeconomics

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→ Chapter 01:- Who is who in Macroeconomics?

1). The Aggregate labour Market:-

1.1). The demand for labour:-

- Production function under the perfect competition, has restriction that capital stock is given in the short Run.

Assumptions:-

1) $Y = F(N, \bar{K})$

Y = real output, $\bar{K}$ = given capital stock

N = labor employed

$F(\cdot)$ → Production function.

2) → Marginal Products of labor $= F_N = \dfrac{\partial F(N,\bar{K})}{\partial N}$

→ Marginal Product of Capital $= F_K = \dfrac{\partial F(N,\bar{K})}{\partial N}$

3) → Marginal Product of labour (capital) declines as employment (capital) is increased.

$$F_{NN} = \frac{\partial^2 F(N,\bar{K})}{\partial N^2} < 0$$

$$F_{KK} = \frac{\partial^2 F(N,\bar{K})}{\partial \bar{K}^2} < 0$$

4) → Factors are cooperative in sense that increasing one factor raises the MP marginal productivity of other factor.

$$\frac{\partial^2 F(N,\bar{K})}{\partial \bar{K}\partial N} = F_{KN} = F_{NK} = \frac{\partial^2 F(N,\bar{K})}{\partial N \partial K}$$

5) → Constant Returns to Scale

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- perfect competition
not any influence on the prices

· Short Run Profits

$$\pi = PY - WN$$

π = Profit, P is price charged by firm
W = Nominal wages,

$(PY) \rightarrow$ Total Revenue | $\pi = \overset{TR}{PQ} - TC$
$(WN) \rightarrow$ Total Cost

→ The only choice that is open to the firm (in short Run) is to determine the amount of production (Y) & employment (N) such that profit is maximized.

$$\max_{\{N\}} \pi = PF(N, \bar{K}) - WN.$$

$$\frac{d\pi}{dN} = PF_N(N, \bar{K}) - W$$

$$PF_N(N, \bar{K}) - W = 0 \longrightarrow (1.4)$$

· The firm should keep expanding its employment upto the point where the marginal ~~worker~~ unit of labour exactly breaks even.

↳ The additional output produced by the marginal worker yields a revenue that exactly covers the wage that is paid to the worker.

$$PF_N(N, \bar{K}) = W$$

$$F_N(N, \bar{K}) = W/P$$

$$dF_N(N^D, \bar{K}) = d(W/P)$$

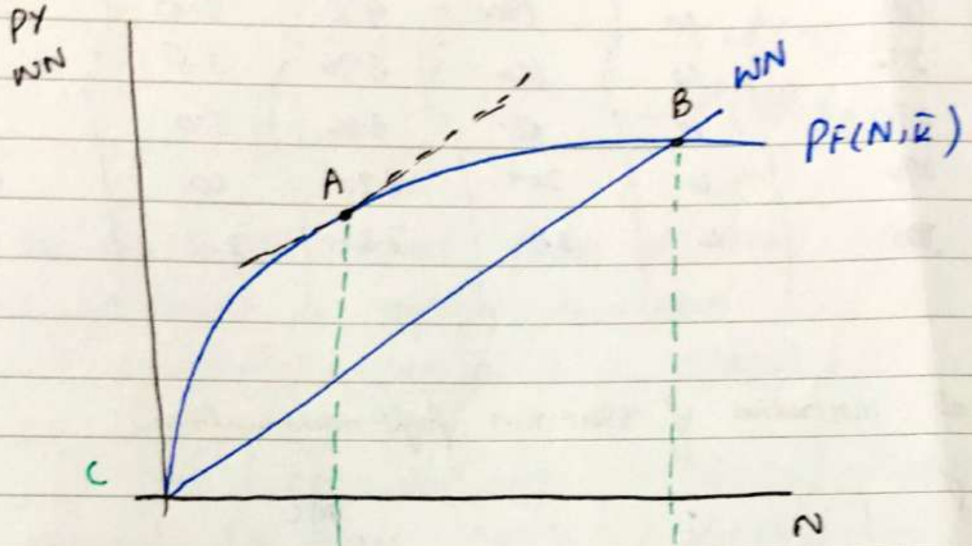
$$F_{NN} dN^D + F_{NK} d\bar{K} = d(W/P)$$

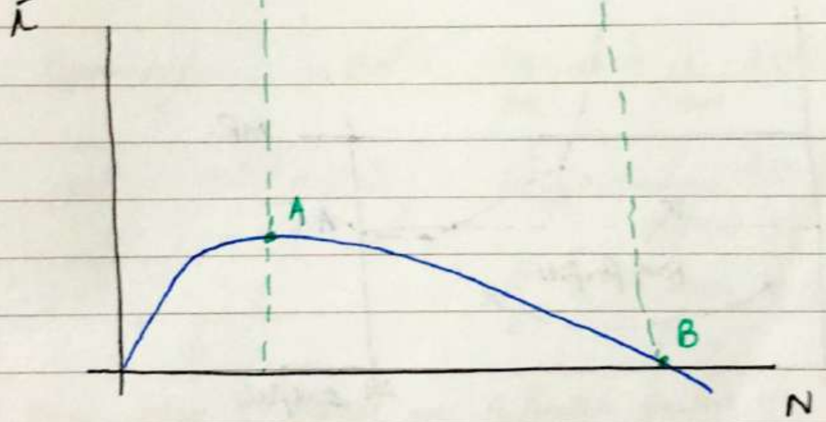
$\hookrightarrow (1.5)$

We can easily transform (1.4) into the demand for labour, a schedule which shows how much labour a firm wants to hire for a given real wage rate.

$$F_{NN} \, dN^D = -F_{NK} \, d\bar{K} + d(w/P)$$

$$dN^D = \frac{-F_{NK} \, d\bar{K}}{F_{NN}} + \frac{1}{F_{NN}} \, d(w/P) \rightarrow (1.6)$$





→ Short-Run Profit Maximization (1.1)

- A firm maximizes its profits by choosing to supply the level of output where its marginal revenue equals its marginal cost. when MR > MC, the firm can earn greater profits by increasing its output. & when MR < MC → loss & reduce output.

$\pi = PQ - WN$

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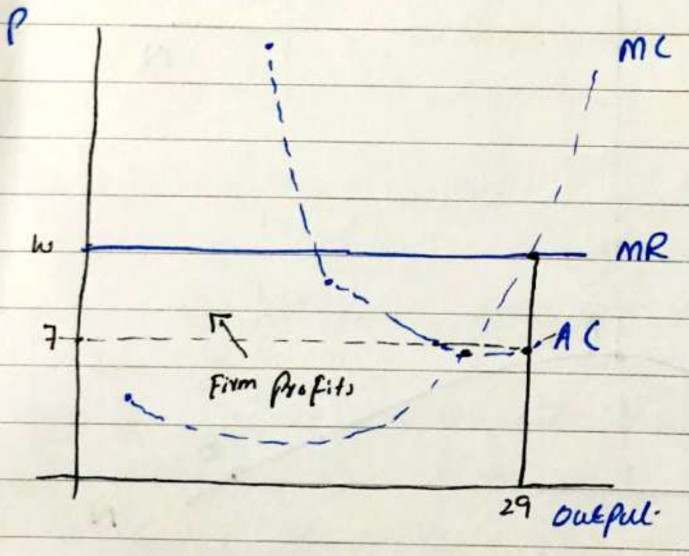
Total Product (Q)	Total Revenue	MR	Total Cost	ATC	MC	Firms Profit
0	\$0	–	\$100	–	–	-\$100
5	50	\$10	120	\$24	\$4.0	-70
15	150	10	140	9.33	2.0	10
23	230	10	160	6.96	2.5	70
27	270	10	180	6.66	5.0	90
29	290	10	200	6.90	10	90
30	300	10	220	7.33	20	80

→ Graphical illustration of short-run profit maximization:-



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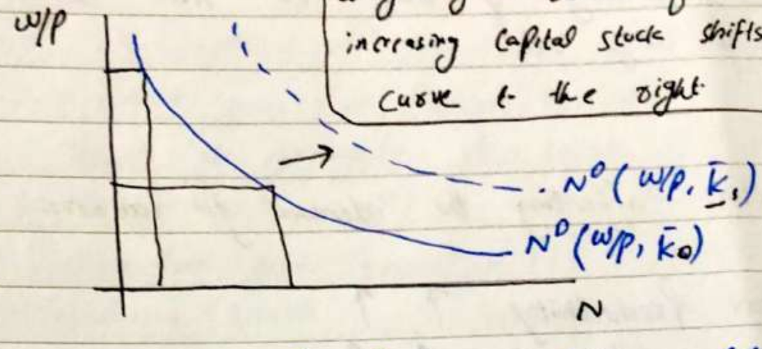
→ The demand for labour:-



varying the real wage rate implies a movement along a given demand for labour curve, whilst increasing capital stock shifts the demand curve to the right.

→ Since $F_{NN} < 0$, the marginal product of labour falls as more units of labour are employed. As a result

$$dN^D = \frac{-F_{NK}}{F_{NN}} d\bar{K} + \frac{1}{F_{NN}} d(w/P)$$

→ A higher real wage $(d(w/P) > 0)$ diminishes the demand for labour $(dN^D < 0)$ ceteris paribus. $(\bar{K})$.

In summary $dN^D = \frac{-F_{NK}}{F_{NN}} d\bar{K} + \frac{1}{F_{NN}} d(w/P)$

$$N^D = N^D(w/P, \bar{K}) \qquad N^D_{w/P} = \frac{1}{F_{NN(-)}} < 0$$

$$N^D_{\bar{K}} = \frac{-F_{NK}}{\underset{(-)}{F_{NN}}} > 0.$$

→ Since labour & capital are cooperative factors of production, increasing the capital stock raises the marginal product of labour.

→ A higher cost of labour or a lower capital stock necessitates a higher marginal product of labour & thus a lower demand for labour

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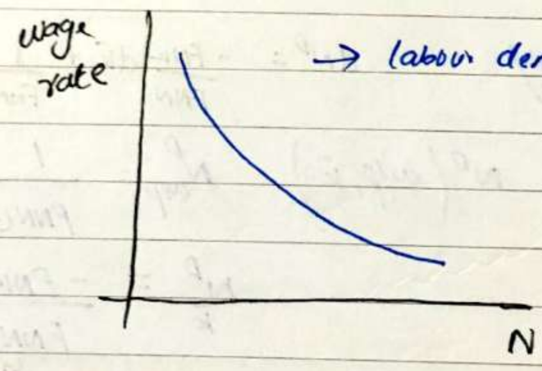
∴ The demand for labour shows how many workers the firms are willing & able to hire at a given time & wage rate.

· Factors affecting the demand for labour:-

→ Labour Productivity ↑, ↑
→ Changes in technology ↑, ↓↑
→ Changes in the number of Firms
→ Changes in demand for a Product that labour Produces

· Theory of Marginal Productivity of labour & diminishing marginal Returns.



→ labour demand graph

household derives utility from goods consumption (C) & leisure $(1-N^s)$. household 'owns' one unit of time of which N^s units are spent working, so that time available for leisure is equal to $1-N^s$.

$U(C, 1-N^s)$

$U_C > 0$, $U_{1-N} > 0$ | $U_{CC} < 0$, $U_{1-N,1-N} < 0$

1.1.2) The Supply of labour:-

- Households → Suppliers of labours
- Household → cannot guess their consumption level → so they forms a guess about the Aggregate Price level P^e.

utility function → $U(C, 1-N^s)$ $\quad U > 0,\ U_{1-N} > 0$, Positive but diminishing.

- Derives utility from goods consumption (C) & leisure $(1-N^s)$

$U_{CC} < 0$, $U_{1-N,1-N} < 0$

Some extra consumption of goods & leisure is fun, but less so if you already consume a lot or have plenty of spare time

. Household chooses combination of C & $1-N^s$.

$$P^e C = W N^s$$

$$C = \left(\frac{W}{P^e}\right) N^s$$

$$\max_{(N^s)} U = U\left((W/P^e)N^s,\ 1-N^s\right) \rightarrow \text{labour supply choice household}$$

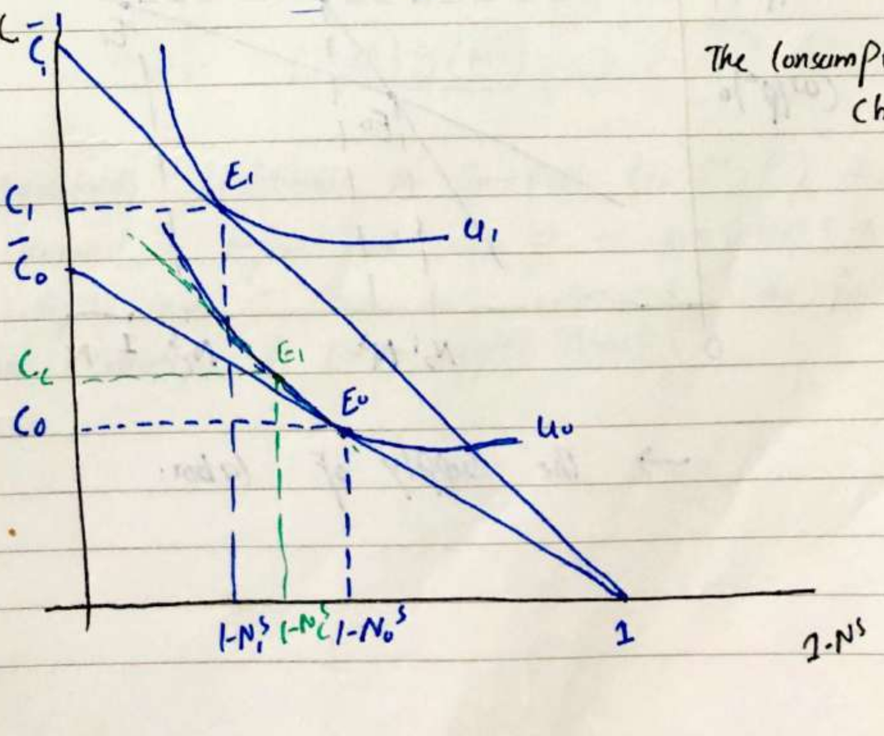
Decision Rule for household ←

$$\frac{dU}{dN^s} = 0, \quad (W/P^e)U_C - U_{1-N} = 0. \rightarrow (1.1)$$

→ $\left(\frac{W}{P^e}\right)U_C$ measures the marginal benefit of supplying one extra unit of labor to labor mkt.

→ U_{1-N} measures the marginal cost of that extra unit.

optimum $MC = MB$



The Consumption-leisure Choice

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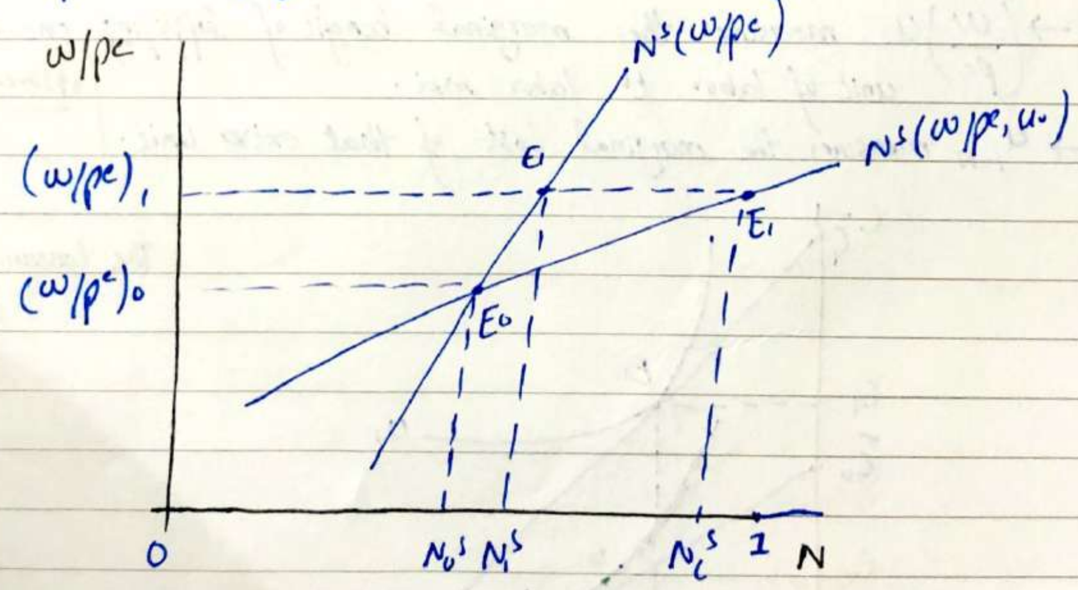
→ labour supply curve is not necessarily positive, like the labour demand curve, which always slopes downward. Reason is that there are two, potentially offsetting effects.

1) Pure Substitution effect

↳ Move from E^0 to the hypothetical compensated point E' constitutes the pure substitution effect.

↳ Intuitively the pure substitution effect says that a household will buy less of anything for which the price has risen. A rise in the expected real wage rate means that the price of leisure has gone up. Consequently the household buys less of it.

↳ the compensated labor supply curve is always upward sloping.



→ The supply of labor.

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2) The Second effect is Called the income effect.

- N_0^s, a higher expected real wage implies a higher expected real income, or, $(w/p^e)_1 N_0^s > (w/p^e)_0 N_0^s$,
- Provided leisure is a normal good, the household would react to this higher income by purchasing more leisure, not less. (P
- Hence Income effect (IE) E' to E_1 works in opposite direction to the pure Substitution effect.

Mathematically,

$$w/p^e = g(N^s) \qquad g_N \gtrless 0 \iff SE \gtrless |IE|$$

So, A higher real wage has two effects on labour supply,

1) It makes leisure more expensive which induces households to have less leisure & works more hours (SE).

2) A higher RW raises the income of households s. they become lazier & work less hours.

$$w/p = (P^e/P)\, g(N^s) \rightarrow (1.12)$$

- If households overestimate the price level (ie $P^e > P$), they will demand a higher real wage for a given level of labour supply than if they had estimated the price level correctly. (Lucas supply curve).

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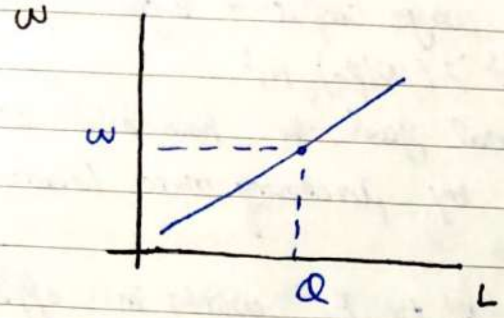
· Supply of labour:-

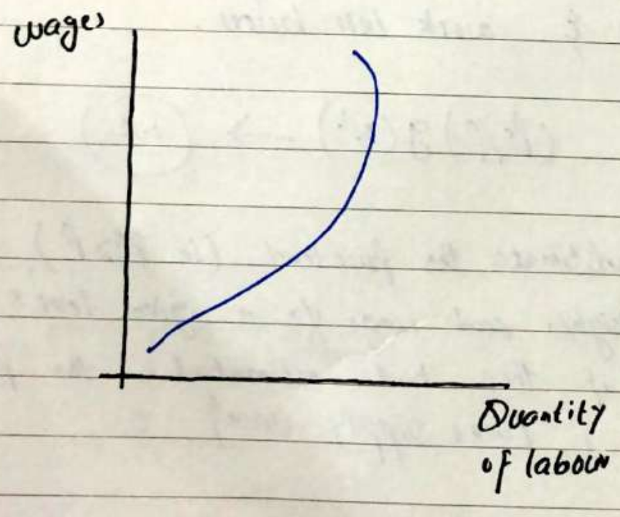
The number of workers willing & able to work,



· Factors other than wages that will shift the supply curve.

→ The size of the working population
→ Migration
→ Peoples preferences for work
→ Net advantages of work
→ Work & leisure.

- Individual labour supply



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1.1.3) Aggregate supply in goods market

i): Adaptive expectations (AEH)

ii): PERFECT foresight hypothesis (PFH)

i)

→ under AEH the expected price level is given in short run, but moves slowly to correct for past expectational errors.

$$P^e_{t+1} = P_t + (1-\alpha)\left[P^e_t - P_t\right], \quad 0<\alpha<1 \rightarrow (1.13)$$

$\alpha \rightarrow$ speed with which households update their price expectations.

$$\Delta P^e_{t+1} = \alpha\left(P_t - P^e_t\right], \quad 0<\alpha<1 \quad (AEH) \rightarrow (1.14)$$

where $\Delta P^{e}_{t+1} = P^e_{t+1} - P^e_t$.

→ ii) PFH → states that households expect the price level that actually holds:

$$P^e_t = P_t. \quad (PFH) \rightarrow (1.15)$$

AEH: This equation is households expect that price in future period t+1 to be equal to the actual price in current period t if their expectations proved correct in current period if $(P^e_t \neq P_t)$, they incorporate part of the expectational error in the revision of their expectation in current period, when α represent the speed with which households update their price expectations.

w = wages N = Employment
Y = output P = Price level

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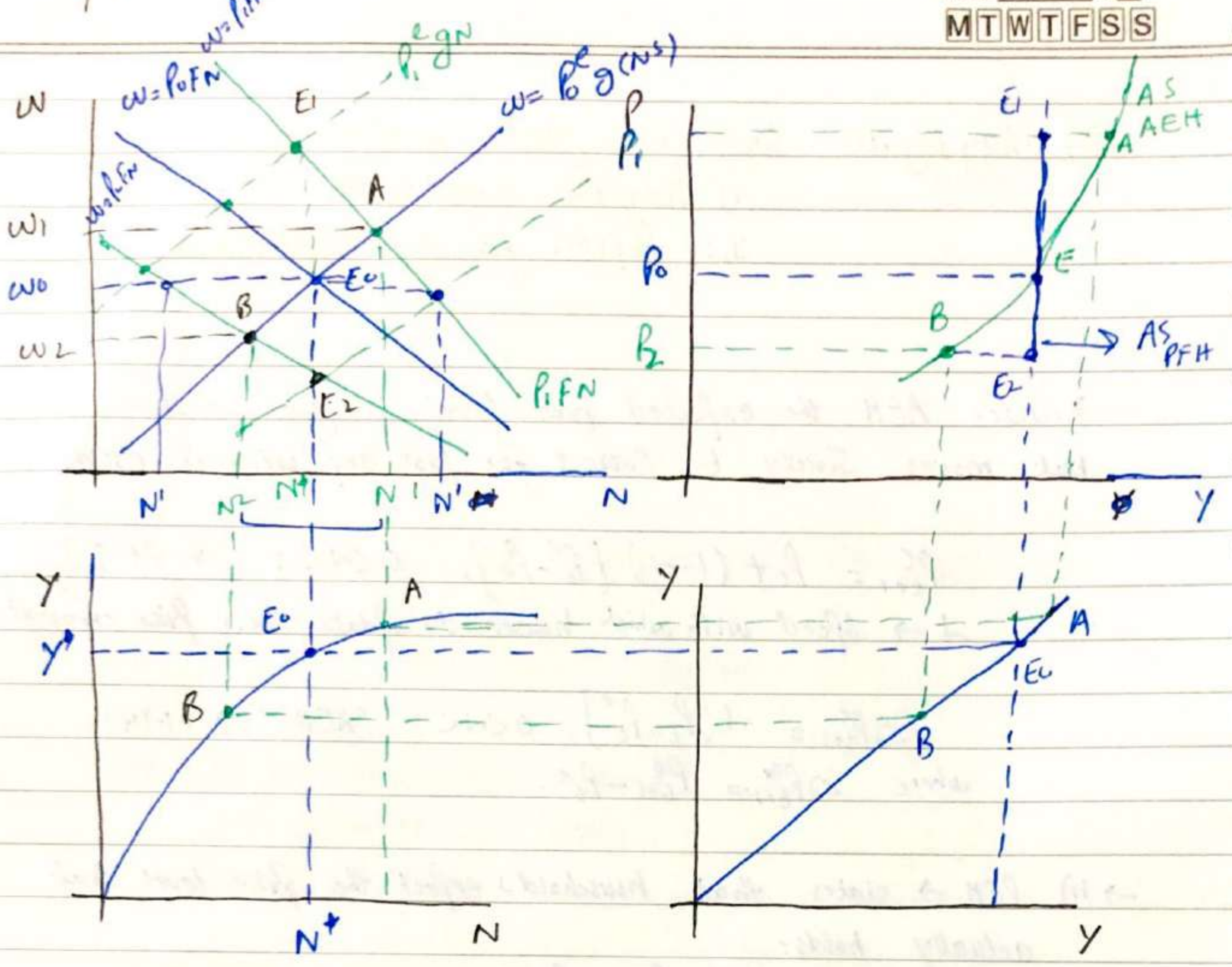

1) $P_1 > P_0$ → PFH → $P_e = P_0$ → Expected & actual prices always coincide.
2) $P_2 < P_0$

→ Short Run AEH → Positively sloped
→ long Run AEH → Vertically sloped

→ Short Run PFH → Vertically sloped
→ long Run PFH → ~~P~~ Vertically sloped

Shift in labor Supply
→ Population, technology, Taste/Preference
→ Expected price level

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- The labour demand & labor Supply curves may be written in terms of elasticities

$$\frac{dN^D}{N^D} = \frac{d\bar{k}}{\bar{k}} - \varepsilon_D\left[\frac{dw}{w} - \frac{dP}{P}\right] \rightarrow (1.16)$$

$$\frac{dN^s}{N^s} = \varepsilon_s\left[\frac{dw}{w} - \frac{dP^e}{P^e}\right] \rightarrow (1.17)$$

→ where $\varepsilon_D = -F_N/(NF_{NN})$ $\varepsilon_s = g(N)/(Ng_N)$ → wage elasticies of labour demand & Supply.

→ $N^d = N^s$

$$\frac{dw}{w} - \frac{dP}{P} = \frac{1}{\varepsilon_D+\varepsilon_s}\left(\frac{d\bar{k}}{\bar{k}} - \varepsilon_s\left(\frac{dP}{P} - \frac{dP^e}{P^e}\right)\right)$$

$$\frac{dY}{Y} = \frac{F_N}{Y}dN + \frac{F_K}{Y}d\bar{k} = \omega_N\frac{dN}{N} + (1-\omega_N)\frac{d\bar{k}}{\bar{k}}$$

where $\omega_N = WN/PY$

$$\frac{dY}{Y} = \frac{\omega_N\varepsilon_D\varepsilon_s}{\varepsilon_D+\varepsilon_s}\left(\frac{dP}{P} - \frac{dP^e}{P^e}\right) + \frac{(1-\omega_N)\varepsilon_D + \varepsilon_s}{\varepsilon_D+\varepsilon_s}\frac{d\bar{k}}{\bar{k}} \quad (AS) \quad (1.20)$$

- a bigger capital stock boosts the MP_K & thus real wage. This attenuates the rise in the AS of goods. $dP^e/P^e = dP/P$ do not affect real wages, employment, or AS of goods. Unanticipated price do effect these variables. for example $P > P^e$, the real wage falls & thus employment & AS of output rise.

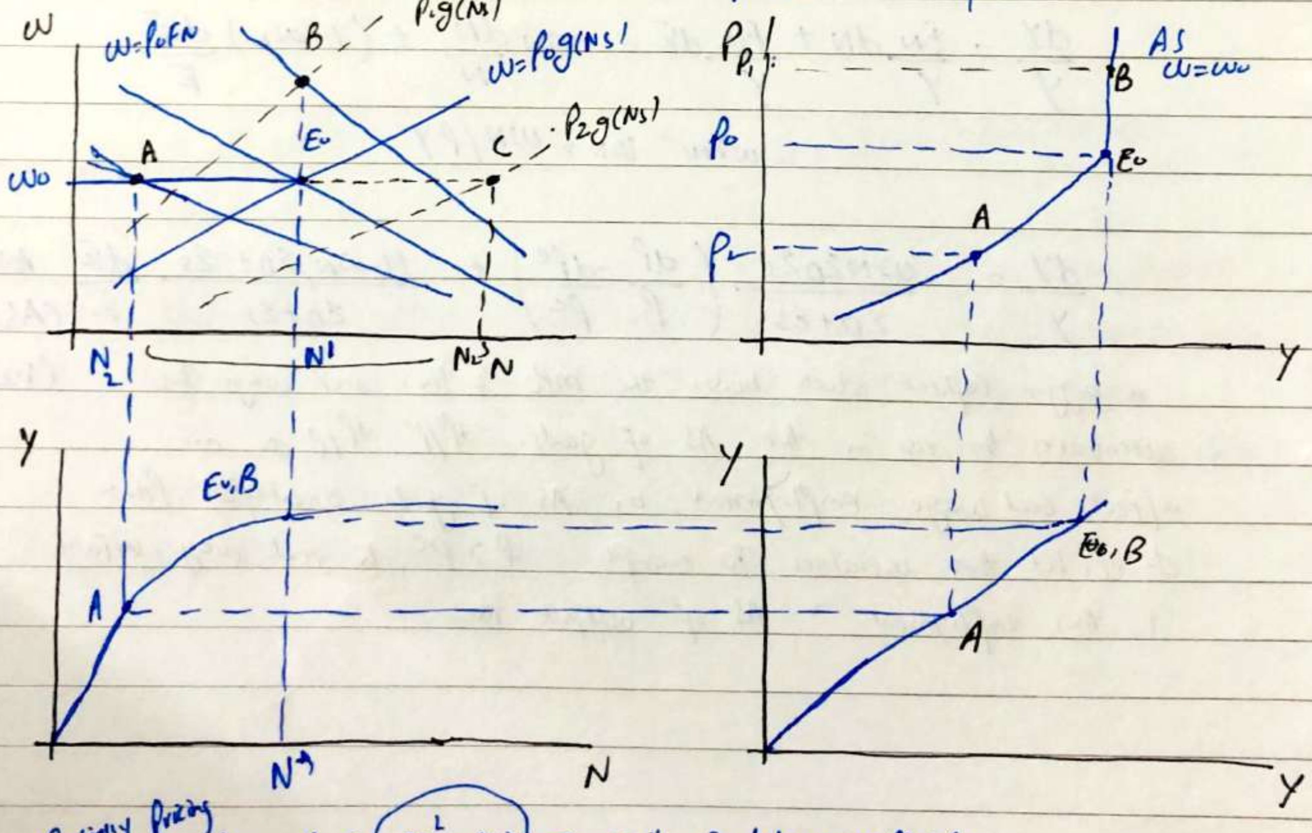
1.14) Nominal Wage Rigidities:-

- John Maynard Keynes, "nominal rigidity" of wages. Wages are often said to be sticky-down, meaning that they can move up easily but move down only with difficulty.
- Sticky wage theory argues that employee pay is resistant to decline even under deteriorating economic conditions.

→ Modigliani (1994)

- Nominal wages are inflexible downwards, but perfectly flexible in the upward direction.
- workers hate wage cuts but love a rise, wages are rigid downward.
- under PFH A segment AS curve is +ve. | Situation is different if $P = P_2$ (lower price.)

$N_S - N_L^S \rightarrow$ unemployed.



Perfectly pricing themselves, out of labour market ← (→ $N_S^L - N^L$ → units of labour unemployed

→ Nominal wage rate is not allowed to fall, Employment equal $N_L (< N)$

~

1.2) Aggregate demand : Review of IS-LM Model:-

- Demand side of the economy can be described by means of the IS-LM Model → (For closed economy)

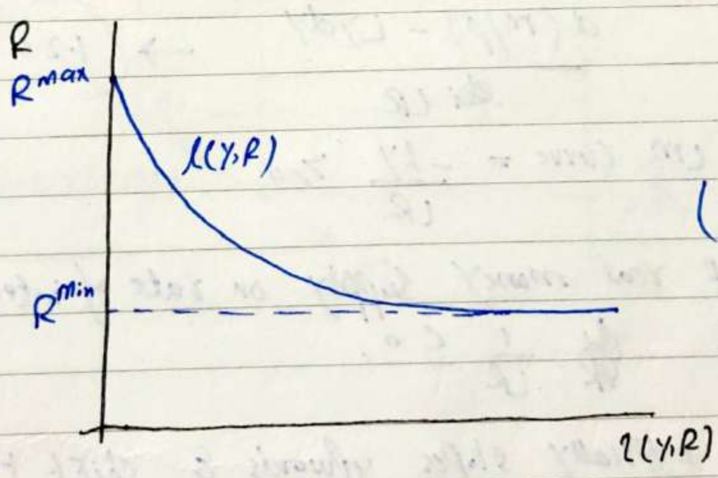
$$
\left.\begin{array}{lll}
Y = C + I + G, & & (1.21) \\
C = C(Y-T) & 0 < C_{Y-T} < 1 & (1.22) \\
I = I(R) & I_R < 0 & (1.23) \\
T = T(Y) & 0 < T_Y < 1 & (1.24)
\end{array}\right\} IS
$$

$$
\left.\begin{array}{lll}
M/P = L(Y,R) & L_Y > 0, \ L_R \leq 0. & (1.25)
\end{array}\right] LM
$$

- The demand for Money:-

  - Two motives

  1) Transaction motives/demand   2) Speculative motive/demand.



(The liquidity Preference function)

$C_Y, I_Y > 0$	$M/P = L(Y,R)$
$I_Y < 0$	$L_Y > 0$
$\frac{\partial I(Y,V)}{\partial V} = I_V < 0$	$L_i < 0$
MPC $0 < C_{Y-T} < 1$	

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1.2.2) The IS-LM Model:-

LM Curve = $M/P = L(Y,R)$

$$d\left(\frac{M}{P}\right) = d(L(Y,R))$$

$$\frac{PdM - MdP}{P^2} = L_Y dY + L_R dR$$

$$\frac{PdM}{P^2} - \frac{MdP}{P^2} = L_Y dY + L_R dR$$

$$\frac{1}{P}dM - \frac{M}{P^2}dP = L_Y dY + L_R dR \rightarrow (9)$$

OR

$$d\left(\frac{M}{P}\right) = L_Y dY + L_R dR$$

$$L_R dR = d(M/P) - L_Y dY$$

$$dR = \frac{d(M/P) - L_Y dY}{L_R} \rightarrow (1.26)$$

Slope of LM Curve = $\frac{-L_Y}{L_R} \geq 0$

effect of real money supply on rate of interest is $\frac{1}{L_R} \leq 0$.

- LM Curve typically slopes upwards & shift to right if real money balances expand.
- A higher money supply or a lower price level pushes up bond prices & thus lowers the interest rate.
- A higher interest rate lowers money demand.

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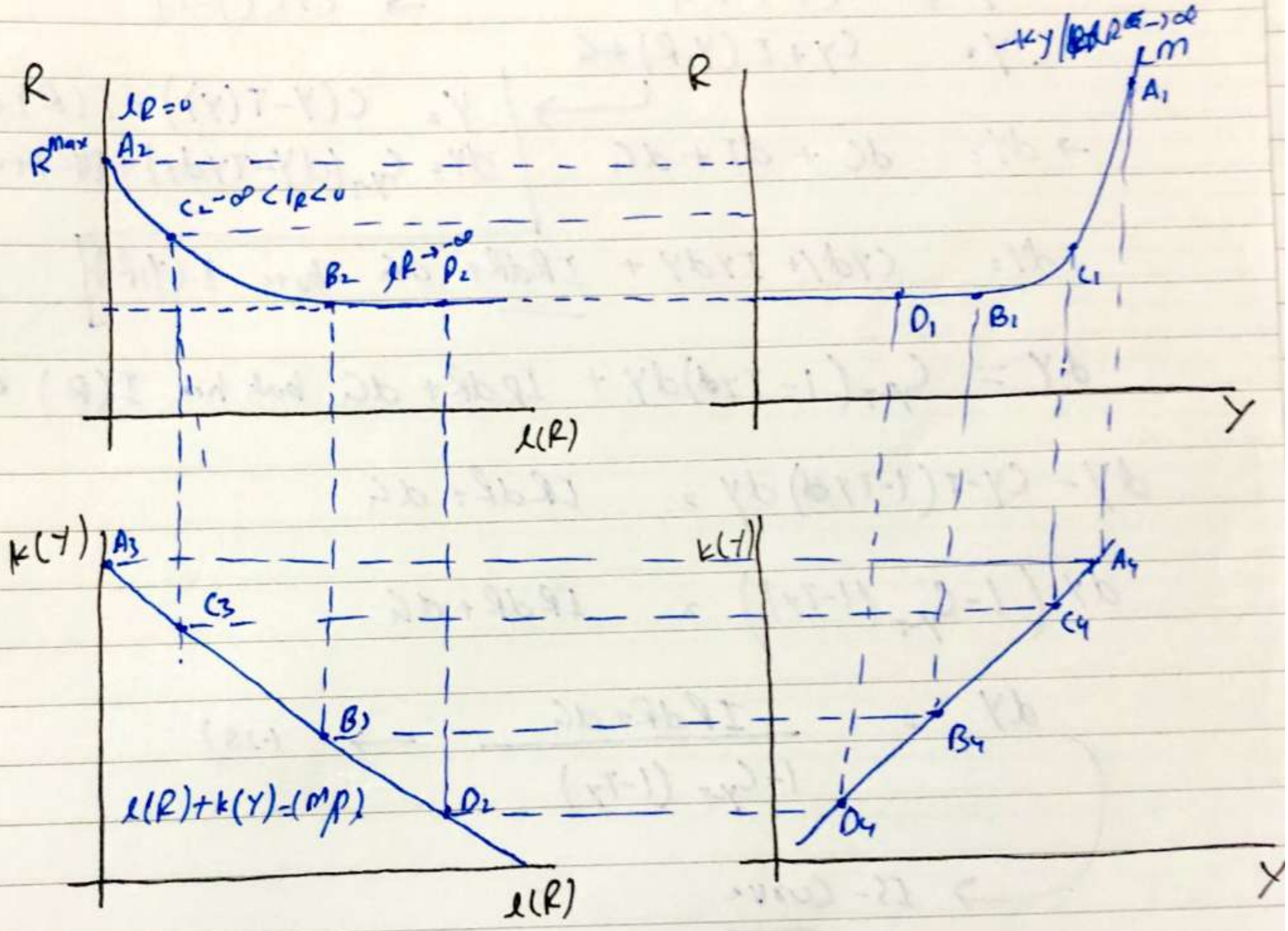
· Derivation of the LM Curve:-

→ LM Curve is vertical for high rates of interest & horizontal for low rates of interest. (Phase diagram)



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→ The IS - Curve:- represents combinations of output y & rate of interest R, for which there exist aggregate spending balance.

$$Y = C + I + G \qquad \rightarrow C = C(Y-T)$$

$$\therefore Y = CY + I(Y,R) + G$$

$$Y = C(Y - T(Y)) + I(R) + G$$

$$dY = C_{Y-T}(dY - T_Y dY) + I(R) + G$$

$$\rightarrow dY = dC + dI + dG$$

$$dY = C_Y dY + I_Y dY + \underline{I_R dR} + dG \quad \text{here } I(Y,R)$$

$$dY = C_{Y-T}(1 - T_Y)dY + I_R dR + dG \quad \text{but here } I(R)$$

$$dY - C_{Y-T}(1 - T_Y)dY = I_R dR + dG$$

$$dY \cdot (1 - C_{Y-T}(1 - T_Y)) = I_R dR + dG$$

$$dY = \frac{I_R dR + dG}{1 - C_{Y-T}(1 - T_Y)} \rightarrow (1.25)$$

→ IS - Curve

$$\rightarrow dY = \frac{1}{1 - C_{Y-T}(1 - T_Y)} I_R dR + dG$$

$$\frac{I_R dR}{dR}$$

$$\rightarrow \frac{dY}{dG} = \frac{d}{dG}\left(\frac{1}{1 - C_{Y-T}(1 - T_Y)}\right) I_R dR + dG \rightarrow$$

$$\frac{dY}{dG} = \frac{1}{1 - C_{Y-T}(1 - T_Y)} \qquad \frac{dY}{dR} = \frac{1}{1 - C_{Y-T}(1 - T_Y)} I_R$$

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Chapter:- 02

Dynamics in aggregate demand & Supply

- stability will play a fundamental role. A stable system may be defined as one in which the unique equilibrium (also called stationary state) is eventually restored following a shock to one or more of exogenous variables.
- When the system has multiple equilibria (or stationary points) then may be stable & unstable equilibria.
- Unstable system are not very useful for understanding the economy. An unstable system has no stable equilibria.
- A very useful piece of methodological advice is contained in the so called correspondence principle.

2.1) Adaptive expectations & stability:

- Under flexible wages & prices by assuming that price expectations are formed according to the adaptive expectations. (AEH).

$$Y = AD(G, M/P) \qquad 2.1) \rightarrow \text{AD Curve}$$
$$Y = Y^* + \phi[P - P^e] \qquad \phi > 0 \qquad 2.2)$$
$$\dot{P}^e = \lambda(P - P^e). \qquad \lambda > 0 \qquad 2.3)$$

Equation 2.1) is the AD curve, which summarizes the simultaneous occurrence of money market equilibrium & spending equilibrium. The AD curve depends on two exogenous variables, namely government consumption, G (via IS) & nominal money supply (via the LM curve).

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$$AD_1 = \frac{+}{1 - C_{Y-T}(1-T_Y) + L_Y I_R / L_R} \rightarrow 2.4)$$

$$AD_{m/P} = \frac{I_R / L_R}{1 - C_{Y-T}(1-T_Y) + L_Y I_R / L_R} \rightarrow 2.5)$$

- C_{Y-T} marginal propensity to consume, T_Y marginal tax rate, I_R interest sensitivity of investment, L_Y & L_R denote income & interest sensitivity of money demand.

→ $0 < C_{Y-T} < 1$, $0 < T_Y < 1$, $I_R < 0$, $L_Y > 0$, $L_R < 0$.

→ Aggregate demand rises if government spending or real money balances are increased.

→ Flatter the LM curve more effective monetary policy & steeper the LM curve less effective monetary policy.

→ Effectiveness of monetary policy depends upon the slope of LM curve.

Equation 2.2) $Y = Y^* + \phi \left[P - P^e\right]$ specification of aggregate supply in the goods market. Potential output, also called the full employment level of output, Y^*, depends on supply side variables.

Due to the fact that expected and the actual price level do not always coincide under the assumption of adaptive expectations, labour supply & consequently output can differ (in the short run) from their respective full-employment levels.

$P > P^e$

ϕ is the reciprocal of the slope of AS.

ϕ is high → AS curve will be flatter

$\frac{dP}{dY} \rightarrow \frac{dY}{dP}$

$dY/dP \rightarrow$ slope of AS $= \frac{dP}{dY}$

AS

→ the Parameter ϕ determines the slope of the Short Run AS Curve - the higher a value of ϕ, the flatter the short Run AS - Curve, and the larger the output fluctuations that occur as a result of a given shift in aggregate demand.

$$AS \quad Y = \dot{Y} + \phi(P - P^e)$$

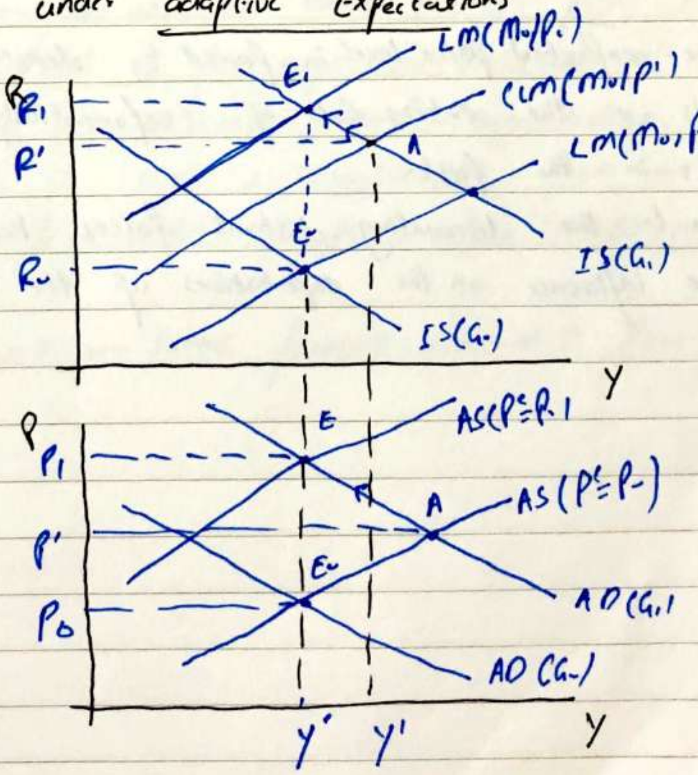
$$Y = \dot{Y} + \phi P - \phi P^e$$

$$\phi P = \phi P^e + Y - \dot{Y}$$

$$P = P^e + \frac{1}{\phi}(Y - \dot{Y}) \rightarrow (2.7)$$

$$\frac{dP}{dY} = \frac{1}{\phi} \rightarrow \phi = \frac{dY}{dP} \rightarrow \text{reciprocal of (AS-Slope) supply curve}$$

· Fiscal Policy under adaptive Expectations



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→ Note that the difference b/w the full employment level of output and the actual level of output $Y^e - Y$, is sometimes called okuns gap.

· Finally equation 2.3

$$\dot{P}^e = \lambda(P - P^e) \qquad \lambda > 0$$

is the continuous time version of the AEH expressed in equation (1.114) → $\Delta P^e_{t+1} = \lambda(P_t - P^e_t)$

→ The expected price level $P^e(t)$ depends on actual price levels from period t into the indefinite past.

→ under AEH the expected price level, P^e is treated just like the capital stock, namely as something that is determined in the past

$$\boxed{P^e(t) = \int_{-\infty}^{t} \lambda P(\tau) e^{-\lambda(t-\tau)} d\tau} \qquad (2.8)$$

The expected price level in period t, denoted by $P^e(t)$, depends on the entire path of (exponentially weighted) price levels in the past.

· Due to the discounting, distant prices have relatively little influence on the expectations of the current price level

· In the Figure of Fiscal Policy under adaptive expectations, does the economy automatically return to an equilibrium after a Shock, Say in increase in govt Spending?

→ Following the increase in govt Spending ($dG > 0$), the IS Curve & hence AD Curve both Shift to the right.

→ At Point A the Price level has increased from P_0 to P' & output has also increased (to Y'). Is A stationary Point. A?

→ Clearly there is an equilibrium in the Sense that AD Curve & Short Run AS Curve intersect.

→ There is however a disequilibrium regarding expectations, at Point A households base their plans on the expectation that the Price is P_0 but the actual Price level is higher ($P' > P_0$).

↳ Hence A is not a stationary Point.

As the expected Price level is increased, the short term AS curve will start to Shift up and to the left and the economy will move along the new AD Curve towards Point E_1.

→ Point E_1 is a Point of full equilibrium, because all markets clear & there is an expectational equilibrium.

→ Hence Point E_1 is both an equilibrium from an economic point of view & a stationary Point. (IS-LM-AS model is Stable)

→ Now we will use some formal method to Prove this

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As

$$Y = AD(G, m/P) \rightarrow (2.1)$$

$$dY = AD\,dG + AD\,d\left(\frac{m}{P}\right)$$

$$dY = AD\,dG + \frac{P\,dm - m\,dP}{P^2}$$

$$dY = AD\,dG + \frac{1}{P}dm - \frac{m}{P^2}dP \rightarrow \alpha = m/P^2$$

$$\boxed{dY = AD_G\,dG + \frac{1}{P}AD_{m/P}\,dm - \alpha\,dP} \rightarrow (2.9)$$

→ From AD

Now as 2.21

$$Y = \bar{Y} + \phi(P - P^e)$$

Supply

$$\boxed{dY = d\bar{Y} + \phi(dP - dP^e)} \rightarrow (2.10)$$

These (2.9) & (2.10) can be solved for the change in price level, dP, & in output, dY:

Equating Equation (2.9) & (2.10)

$$AD_G\,dG + \frac{1}{P}AD_{m/P}\,dm - \alpha\,dP = d\bar{Y} + \phi(dP - dP^e)$$

$$AD_G\,dG + \frac{1}{P}AD_{m/P}\,dm - \alpha\,dP = d\bar{Y} + \phi\,dP - \phi\,dP^e$$

$$AD_G\,dG + \frac{1}{P}AD_{m/P}\,dm - d\bar{Y} + \phi\,dP^e = \phi\,dP + \alpha\,dP$$

$$AD_G\,dG + \frac{1}{P}AD_{m/P}\,dm - d\bar{Y} + \phi\,dP^e = (\phi + \alpha)\,dP$$

$$\boxed{dP = \frac{AD_G\,dG + 1/P\,AD_{m/P}\,dm - d\bar{Y} + \phi\,dP^e}{\phi + \alpha}} \rightarrow (2.11)$$

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Now

$$dY = AD_G\,dG + \frac{1}{P} AD_{m/p}\,dm - \alpha\,dP$$

Put value of dP

$$dY = AD_G\,dG + \frac{1}{P} AD_{m/p}\,dm - \alpha\left(\frac{AD_G\,dG + 1/P\,AD_{m/p}\,dm - d\dot{Y} + \phi\,dP^e}{\phi+\alpha}\right)$$

$$dY = AD_G\,dG + \frac{1}{P} AD_{m/p}\,dM - \frac{\alpha AD_G\,dG - \alpha\,1/P\,AD_{m/p}\,dm + \alpha\,d\dot{Y} - \alpha\phi\,dP^e}{\phi+\alpha}$$

$$dY = \frac{(\phi+\alpha)(AD_G\,dG + 1/P\,AD_{m/p}\,dm - \text{" "}}{\phi+\alpha}$$

$$dY = \frac{\phi AD_G\,dG + \phi\,1/P\,AD_{m/p}\,dm + \alpha \cancel{AD_G}\,dG + \alpha/P\,\cancel{AD_{m/p}}\,dm - \alpha\cancel{AD_G}\,dG - \alpha\,1/P\,\cancel{AD_{m/p}}\,dm + \alpha\,d\dot{Y} - \alpha\phi\,dP^e}{\phi+\alpha}$$

$$\boxed{dY = \frac{\phi AD_G\,dG + \phi/P\,AD_{m/p}\,dm + \alpha\,d\dot{Y} - \alpha\phi\,dP^e}{\phi+\alpha}} \rightarrow (2.12)$$

Since both ϕ & α are positives so the denominator of (2.11) & (2.12) is guaranteed to be positive.

→ Hence Equ 2.11 says, P is an increasing function of P^e, G, & M but a decreasing function of $\dot{Y}$.

→ Expression (2.12) shows that the keynesian multiplier which is relevant when prices are sticky i.e AD_G is weakened on account of the rise in the price level & the associated contraction in real money balances. $\phi/\phi+\alpha$ → which is positive but less than unity

- The flatter the AS curve i.e. the smaller the change in the price level caused by a change in aggregate demand (the higher is ϕ), the smaller is the rise in the price level and the dampening of the short-run Keynesian multiplier.

→ A very steep AS curve (a low value of ϕ) implies that a rise in govt spending yields a relatively large boost to the price level & a small rise in employment & output.

$\phi\uparrow$ flatter, $\phi\downarrow$ steeper] SS curve

- Equ (2.11) is very useful for our stability analysis.

SS slope $\frac{dP}{dY}$

$\phi = \frac{dY}{dP}$

As

$$\dot{P}^e = \alpha(P - P^e) \rightarrow \text{continuous time version of AEH}$$

$$d\dot{P}^e = \alpha(dP - dP^e) \quad (2.3)$$

$$\text{As } dP = \frac{\phi dP^e + AD_G dG + 1/P\, AD_{m/P}\, dm - d\bar{Y}}{\phi + \alpha} \quad \text{put in } d\dot{P}^e$$

$$d\dot{P}^e = \alpha\left(\frac{\phi dP^e + AD_G dG + 1/P\, AD_{m/P}\, dm - d\bar{Y}}{\phi + \alpha}\right) - \alpha\, dP^e$$

$$d\dot{P}^e = \frac{\alpha\phi dP^e + \alpha AD_G dG + \alpha/P\, AD_{m/P}\, dm - \alpha d\bar{Y} - (\phi + \alpha)\alpha dP^e}{\phi + \alpha}$$

$$d\dot{P}^e = \frac{\cancel{\alpha\phi dP^e} + \alpha AD_G dG + \alpha/P\, AD_{m/P}\, dm - \alpha d\bar{Y} - \cancel{\phi\alpha dP^e} - \alpha\alpha dP^e}{\phi + \alpha}$$

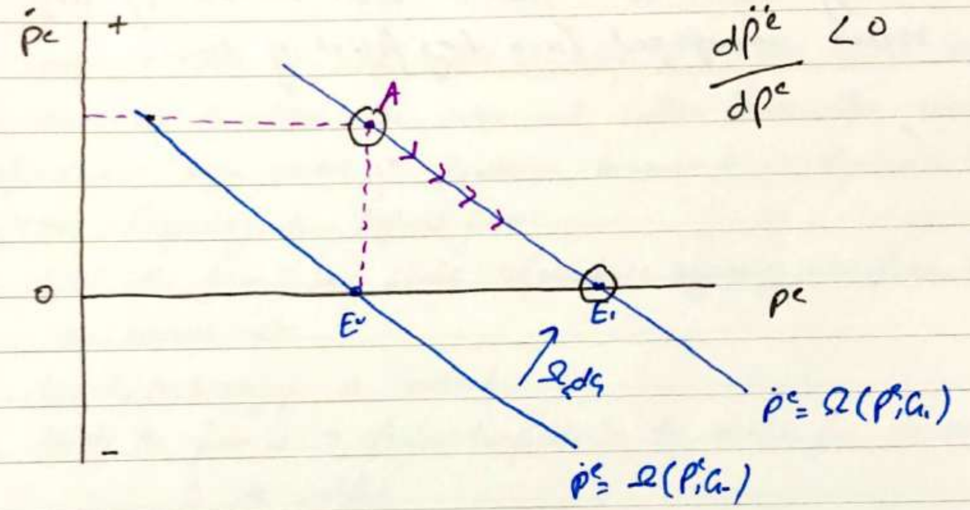
$$d\dot{P}^e = \frac{\alpha AD_G dG + \alpha/P\, AD_{m/P}\, dm - \alpha d\bar{Y} - \alpha\alpha dP^e}{\phi + \alpha}$$

$$\boxed{d\dot{P}^e = \frac{-\alpha\alpha}{\phi + \alpha} dP^e + \frac{\alpha AD_G dG + \alpha/P\, AD_{m/P}\, dm - \alpha d\bar{Y}}{\phi + \alpha}} \quad (2.13)$$

Interpretation (2.13): implies an implicit function, $\dot{p}^e = \Omega(p^e; G, m, y^*)$, relating the time rate of change in the expected price level to that price level and to the exogenous variables.

$\Omega_{p^e} < 0$, $\Omega_G > 0$, $\Omega_m > 0$, $\Omega_{y^*} < 0$ → partial derivative is given in equation (2.13) left-side

· Stability & Adaptive expectations (Phase diagram)



· The initial equilibrium or steady state is given by E0. If govt spending increased the $\dot{p}^e$ line shifts up & to the right (Since $\Omega_G > 0$). Even though p^e is fixed in the short run, $\dot{p}^e$ jumps to a positive value (point A). The expected price level start to rise, which is represented by the arrows along the new $\dot{p}^e$ line.

· Eventually the economy reaches reaches point E_1, which is new equilibrium & steady state. Stability property → changes in expected price level should taper off.

$$\boxed{\frac{\partial \dot{p}^e}{\partial p^e} < 0 \quad \frac{d\dot{p}^e \downarrow}{dp \uparrow} < 0}$$

→ stability implies, if this stability condition holds, the model is y (also) stable in face of shocks to other kinds of exogenous variable as well.

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2.2) A first look at hysteresis:

The hysteresis property, we mean a System whose steady state is not given, but can wander about and depends on the past path of the economy.

→ System with hysteresis can be viewed as being in the twilight zone b/w stable & unstable systems.

→ The best example, of hysteresis is due to people becoming allienated from the labour market if they remain unemployed for a long period of time.

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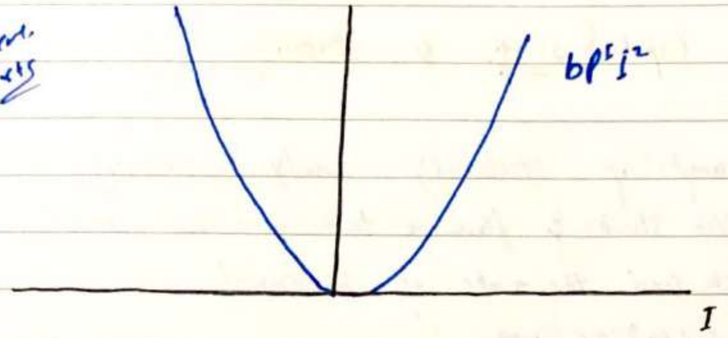
2.3) Investment, the Capital stock, & stability

- We saw an example of stability analysis involving expectations, example of interaction b/w stocks & flows is the one b/w the level of the Capital stock and the rate of investment; we will introduce a theory of Capital as well.

2.3.1) Adjustment Costs & investment:

- Firms invest in order to add units of Capital to the stock they already have & to replace the worn out Capital stock. The objective is to expand our basic model of producer behavior to a dynamic setting.
- Assumption Regarding the typical firm.
  - → First, the firm has static expectations regarding all prices & the interest rate.
  - → Second, technology is Constant
  - → Third, the firm is a Perfect Competitor in the mkts for its inputs & its output.
  - → Fourth, the investment process is Subject to adjustment Costs.
- For low levels of investment these adjustment costs are low, but these costs rise more than proportionally with level of investment.
- The adjustment Cost function is assumed to be quadratic: $b P^I I^2$, b is positive Constant, P^I is price of new machines & I is the level of gross investment by the firm
  - The Production function is still given by $Y = F(N, K)$ & has usual properties.

→ $P_F(N_t, K_t)$ = Sale Revenue (value of output, $P_t Y$)
→ WN^t = wage bill (Payment to labor)
→ P^I = Price of investment (machine) goods
→ $P^I I_t$ = current outlay / expenditure on invest goods
→ $bP^I I_t^2$ = Adjustment costs



Adjustment costs of investment

Finally, we assume that the typical firm maximizes the present value of the net payments it can make to the owners of its capital stock (ie shareholders) subject to the restrictions of the production function & the capital accumulation identity.
· The market rate of interest on bonds, R, is used as the discount factor.

· Since the problem of the firm is essentially dynamic, all variables must be given a time index. In order to obtain the simplest possible action, the derivation proceeds in discrete time, Nominal cash flows at the begining of period t, Λ_t is defined -

$$\Lambda_t = PF(N_t, K_t) - WN_t - P^I I_t - bP^I I^2 t \longrightarrow (2.36)$$

· N_t employment, K_t is capital stock at begining of period t, I_t is the level of investment in period t, The prices of goods & labour (P, P^I, W) have no time index because we assume that firms expect them to be constant over time.

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The first two terms on Right hand Side ($PF(N_t, K_t) - WN_t$) represent Sales revenue minus the wage bills. The third term ($P^I I_t$) represents the current outlays on new investment goods & the fourth term represents the adjustment Costs. ($bP^I I_t^2$)

- The identity linking rates of investment & the Capital Stock is given in discrete time

$$K_{t+1} - K_t = I_t - \delta K_t, \quad 0 < \delta < 1 \rightarrow \text{shows link b/w invst \& capital}$$

$$\Delta K_t = I_t - \delta K_t \quad (2.37)$$

& δ represents the Constant rate of Physical deterioration of the Capital stock due to wear & tear (Depreciation rate)

- In period 0 ('today') the objective function of the firm, i.e the present value of present & future Cash flows Streams Can be written as

PV of Cash flow.

$$\bar{V}_0 = \sum_{t=0}^{\infty} \left(1/1+R \right)^t \bar{\lambda}_t$$

$$= \sum_{t=0}^{\infty} \left(\frac{1}{1+R} \right)^t \left(PF(N_t, K_t) - WN_t - P^I I_t - bP^I I_t^2 \right) \rightarrow 2.38$$

- Due to the dynamic nature of the Problem, the firm must formulate plans regarding production now & in the indefinite future (i.e for $t = 0, 1, \dots \infty$) It does so by Chosing paths (for time periods $t = 0, 1, 2, \dots \infty$) for employment (N_t) investment (I_t) & the Capital stock (K_{t+1}) Such that (2.38) is maximized Subject to (2.37).

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· Two things are noteworthy about the firms optimization problem.

1) First, the choices regarding investment & the capital stock are not independent because the capital accumulation identity (2.37 $K_{t+1} - K_t = I_t - \delta K_t$ implies a path of the capital stock once a path for investment is chosen.

2). Second, the planning period, $t=0$, the firm has an installed capital stock already, so that K_0 is not a choice variable to the firm. formally the maximization problem can be solved by means of the lagrange multiplier method

$$\mathcal{L}_0 = \sum_{t=0}^{\infty}\left(\frac{1}{1+R}\right)^t\left(PF(N_t,K_t) - WN_t - P^t I_t - bP^t I_t^2\right) - \sum_{t=0}^{\infty}\left(\frac{1}{1+R}\right)^t \cdot \lambda_t \cdot \left[K_{t+1} - (1-\delta)K_t - I_t\right) \rightarrow (2.39)$$

As $K_{t+1} - K_t = I_t - \delta K_t$

$K_{t+1} - K_t + \delta K_t - I_t = 0$

$K_{t+1} - (1-\delta)K_t - I_t = 0$

where $\lambda_t \rightarrow$ ~~long~~ lagrange multiplier for capital accumulation.

F.O.C:

$$\frac{\partial \mathcal{L}_0}{\partial N_t} = \left(\frac{1}{1+R}\right)\left(PF_N(N_t,K_t) - W\right) = 0 \rightarrow (2.40)$$

$$= \left(\frac{1}{1+R}\right)^t\left(PF_N - W\right) = 0$$

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2) $\frac{\partial \mathcal{L}_0}{\partial K_{t+1}} = \left(\frac{1}{1+R}\right)^t \left(\frac{P f_K (N_{t+1}, K_{t+1}) + (1-\delta)\lambda_{t+1}}{1+R} - \lambda_t\right) = 0 \rightarrow 2.41$

$\hookrightarrow$ 9t is for period $(t+1)$ so discounted by $(1+R)$

3) $\frac{\partial \mathcal{L}_0}{\partial I_t} = \left(\frac{1}{1+R}\right)^t \left(-P^I - 2bP^I I_t + \lambda_t\right) = 0 \rightarrow (2.42)$

Note that $(1+R)^{-t} > 0$ so that the terms in square brackets on the right-hand side of (2.40-2.42) must be zero to satisfy the first-order conditions.

. Hence, equation (2.40) amounts to the marginal productivity condition for the labor input.

$\hookrightarrow$ Labour is fully flexible factor of production & the choice of how much labour to use is not a dynamic one.

Equations (2.41. $\frac{\partial \mathcal{L}_0}{\partial K_{t+1}}$, 2.42 $\frac{\partial \mathcal{L}_0}{\partial I_t}$ can be combined to yield an expression for the optimal path of investment.

From 2.42 $\rightarrow$ $-P^I - 2bP^I I_t + \lambda_t = 0$

$-(P^I + 2bP^I I_t) = -\lambda_t$

$\lambda_t = P^I(1 + 2bI_t)$

$$\left.\begin{aligned} \text{If } \lambda_t &= P^I(1+2bI_t) \\ \lambda_{t+1} &= P^I(1+2bI_{t+1}) \end{aligned}\right] \rightarrow (2.43)$$

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By substituting these expressions (2.41) we obtain the F.O.C for investment:

$$\frac{PF_K(N_{t+1}, K_{t+1}) + \lambda_{t+1}(1-\delta)}{1+R} - \lambda_t = 0$$

$$PF_K(N_{t+1}, K_{t+1}) + \lambda_{t+1}(1-\delta) - \lambda_t(1+R) = 0$$

Put $(\lambda_{t+1}, \lambda_t)$

$$PF_K(N_{t+1}, K_{t+1}) + \underline{(1-\delta)\,P^I}(1+2bI_{t+1}) - \underline{(1+R)}(P^I(1+2bI_t)) = 0$$

$$PF_K(N_{t+1}, K_{t+1}) + (1-\delta)\underline{P^I} + (1-\delta)(2bP^I I_{t+1}) - (1+R)\underline{P^I} - (1+R)(P^I)2bI_t = 0$$

$$PF_K(N_{t+1}, K_{t+1}) + (\cancel{1}-\delta-\cancel{1}-R)P^I + (1-\delta)(2bP^I I_{t+1}) - (1+R)2bI_tP^I = 0$$

$$(1-\delta)(2bP^I I_{t+1}) - (1+R)(2bP^I I_t) + PF_K(\ ) - P^I(R+\delta) = 0$$

divide the equation by $(1-\delta)(2bP^I$ we get

$$I_{t+1} - \frac{(1+R)}{(1-\delta)}I_t + \frac{PF_K}{2bP^I(1-\delta)} - \frac{P^I(R+\delta)}{2bP^I(1-\delta)} = 0 \rightarrow (2.42)$$

This equation is an unstable difference equation, for investment because the coefficients for I_t is greater than unity.

→ The steady state solution for investment is found by setting $\Delta I_{t+1} = 0$, $I_{t+1} = I_t = I$:

S. above equation

$$I - \left(\frac{1+R}{1-\delta}\right)I + \frac{PF_K}{2bP^I(1-\delta)} - \frac{R+\delta}{(1-\delta)2b} = 0$$

$$\left(1 - \frac{1+R}{1-\delta}\right)I + \cdots - \qquad = 0$$

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$$\left(\frac{1-\delta-\gamma-R}{1-\delta}\right) I + \frac{PF_K}{2bP^I(1-\delta)} - \frac{R+\delta}{(2b)(1-\delta)} = 0$$

$$-\left[\frac{(R+\delta)}{(1-\delta)}\right] I + \ldots = 0$$

multiply / divide equation by $\left(\frac{1-\delta}{R+\delta}\right)$

$$-\left(\frac{\cancel{1-\delta}}{\cancel{R+\delta}}\right)\left(\frac{\cancel{R+\delta}}{\cancel{1-\delta}}\right) I + \frac{PF_K}{2bP^I(\cancel{1-\delta})} \times \frac{\cancel{1-\delta}}{R+\delta} - \frac{\cancel{R+\delta}}{(2b)(\cancel{1-\delta})} \times \frac{(\cancel{1-\delta})}{\cancel{R+\delta}} = 0$$

$$-I + \frac{PF_K(N,K)}{2bP^I(R+\delta)} - \frac{1}{2b} = 0$$

$$\boxed{I = \frac{1}{2b}\left[\frac{PF_K(N,K)}{P^I(R+\delta)} - 1\right]} \rightarrow 2.45$$

- If the value of the marginal product of capital (PF_K) is greater than the rental price of capital (ie the opportunity cost of capital plus the depreciation charge, $(R+\delta)P^I$) the firm should invest. VMPK (PF_K) > Rental price of capital
- In absence of adjustment cost ($b=0$) the firm has no well defined optimal investment policy. In absence of adjustment costs, the firm has an infinite speed of investment and immediately adjust its capital stock to the optimal level

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· One final remark, $I = \frac{1}{\partial b}\left(\frac{PF_K(N,K)}{P^I(R+S)} - 1\right)$ concerns the price of investment goods, P^I.

· The IS-LM model is essentially a one-good model, so one would expect that the investment good is actually the same as the consumption good & thus $P = P^I$. There is however, a reason why the two prices can diverge, even in a one-good setting.

· Suppose that the govt wishes to stimulate investment, it could do so by subsidizing investment goods. In that case the price of investment goods faced by firms is equal to $P_I = (1 - S_I)P$, where S_I is the subsidy.

So Equat (2.45)

$$\boxed{I = \frac{1}{2b}\left[\frac{F_K(N,K)}{(1-S_I)(R+S)} - 1\right] \quad (2.46)}$$

$P = P^I$

$$I = \frac{1}{\partial b}\left(\frac{PF_K(N,K)}{P(R+S)} - 1\right)$$

· It is clear, investment subsidy is successful in stimulating investment, i.e. $\partial I / \partial S_I > 0$.

$P^I = (1 - S_I)P$

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→ Stability:- The investment theory summarised

$$I = I(R, K, Y) \quad I_R < 0, \; I_K < 0, \; I_Y > 0$$

· where we assume that there is no investment subsidy ($P^F = P$).

$F_K = (1-\alpha) Y/K$, $\partial F_K/\partial Y > 0$, and $\partial F_K / \partial K < 0$.

↳ ~~Accelerat~~ Accelerator theory of investment

$K^D(Y, R)$ → Desired level of capital stock. $K_Y^D > 0$, $K_R^D < 0$.

& assuming that investment takes place in order to close the gap b/w the desired & the actual level of capital stock,

$I = b(K^D - K)$, with b being the speed of adjustment.

→ Stability in IS-LM model (Price level is constant) AS curve is perfectly elastic ($P = 1$). Money supply constant,

Model of AD with dynamics in capital stock can be written as:

$$Y = C(Y - T(Y)) + I(R, K, Y) + G \quad (2.48) \rightarrow \text{IS Curve}$$

$$M = L(Y, R) \quad (2.49) \rightarrow \text{LM curve}$$

$$\dot{K} = I(R, K, Y) - \delta K \quad (2.50) \rightarrow \text{Capital accumulation identity}$$

$K_{t+1} - K_t$

We assume that the IS curve is downward sloping

$$0 < C_{Y-T}(1 - T_Y) + I_Y < 1.$$

· The capital stock is pre-determined in short-run, so that IS-LM equation (2.48, 2.49) jointly determines short run equilibrium value for output Y & rate of interest R, in terms of K & G:

IS - Equation

$Y = AD(\bar{K}, \overset{+}{G})$

$R = H(\underset{-}{K}, \underset{+}{G})$

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$$Y = C(Y - T(Y)) + I(R, K, Y) + G$$

$$dY = C_{Y-T}(dY - T_Y dY) + I_R dR + I_K dK + I_Y dY + dG$$

$$dY(1 - C_{Y-T}(1 - T_Y) - I_Y) = I_R dR + I_K dK + dG$$

$$\frac{dR}{dY} = \frac{dY(1 - C_{Y-T}(1-T_Y) - I_Y) - I_K dK - dG}{I_R} = dR$$

$$\hookrightarrow \frac{1}{I_R}\frac{d}{dY}\left(1 - C'(1-T') - I_Y\right)$$

$$\frac{dR}{dY} = \frac{1 - C_{Y-T}(1-T_Y) - I_Y}{I_R}$$

$$\rightarrow \quad 1 - C_{Y-T}(1-T_Y) - \overset{+}{I_Y} < 1 - C_{Y-T}(1-T_Y)$$

$$1 - C_{Y-T}(1-T_Y) < 1 - C_{Y-T}(1-T_Y) + I_Y$$

. IS curve is steeper than dR/dY when $\underline{I_R}$.

LM Equation: $M = L(Y, R)$

$$dM = L_Y dY + L_R dR$$

$$L_R dR = dM - L_Y dY$$

$$dR = \frac{dM - L_Y dY}{L_R}$$

$$\boxed{dR = \frac{1}{L_R} dM - \frac{L_Y}{L_R} dY} \rightarrow (A)$$

Also $L_Y dY = dM - L_R dR$

$$\boxed{dY = \frac{1}{L_Y} dM - \frac{L_R}{L_Y} dR} \rightarrow (B)$$

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Now as we find

$$dY(1-C_{Y-T}(1-T_Y)-I_Y) = dG + I_R dR + I_K dK$$
$$dY(1-C_{Y-T}(1-T_Y)-I_Y) = dG + I_K dK + I_R\left(\frac{1}{L_R}dM - \frac{L_Y}{L_R}dY\right)$$

$$\boxed{dY\left(1-C_{Y-T}(1-T_Y)-I_Y+I_R\frac{L_Y}{L_R}\right) = dG + I_K dK + \frac{I_R}{L_R}dM}$$

(1)

$$\boxed{\frac{dY}{dG} = \frac{1}{1-C_{Y-T}(1-T_Y)-I_Y+I_R L_Y/L_R}} \rightarrow AD_G$$

$\rightarrow AD_G > 0$

$+ \quad +$

↳ higher → income increase in Y more than $> \frac{1}{1-C_{Y-T}(1-T_Y)+I_R L_Y/L_R}$

$$\frac{dY}{dK} = I_K AD_G < 0$$

- The positive output effect in investment $(L_Y > 0)$ ensures that the multiplier is larger than its counterpart in the standard IS-LM model (24). In terms of Fig (24) an increase in govt consumption shifts the IS curve to the right, & moves the equilibrium from point E_0 to point A

Now same (2)

$$\boxed{\frac{dY}{dM} = \frac{I_R/L_R}{1-C_{Y-T}(1-T_Y)-I_Y+I_R L_Y/L_R}} \rightarrow \frac{I_R}{L_R}AD_G > 0 \quad M\uparrow Y\uparrow$$

Now Next from (B) Same Procedure

(3)

$$\boxed{\frac{dY}{dK} = \frac{I_K}{1-C_{Y-T}(1-T_Y)-I_Y+I_R L_Y/L_R}} \rightarrow I_K AD_G < 0 \rightarrow AD_K < 0$$

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$$dY\left(1-C_{Y-T}(1-T_Y)-I_Y\right) = dG + I_R dk + I_R dR$$

$d\cancel{R}/d\cancel{G}$ As from (B) $dY = \frac{1}{L_Y} dm - \frac{L_R}{L_Y} dR$ Put in (dY)

$$\left(\frac{1}{L_Y} dm - \frac{L_R}{L_Y} dR\right)\left(1-C_{Y-T}(1-T_Y)-I_Y\right) = dG + I_k dk + I_R dR$$

$$\cancel{\frac{dR}{dG}} =$$

$$\frac{1}{L_Y} dm \left(1-C_{Y-T}(1-T_Y)-I_Y\right) = dG + I_k dk + I_R dR + \frac{L_R}{L_Y} dR\left(1-C_{Y-}\cdots\right)$$

" " " " $+ dR\left(I_R + \frac{L_R}{L_Y}\right)$

(1 - d-)

$$\cancel{\frac{dR}{dG}} =$$

$$dR\left(I_R + \frac{L_R}{L_Y}\left(1-\right.\right.$$

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From (B)

$$\frac{dR}{dG} = \frac{-L_Y}{L_R} AD_G \rightarrow H_G > 0$$

$$\& \quad \frac{dR}{dk} = \frac{-L_Y I_K}{L_R} AD_G < 0$$

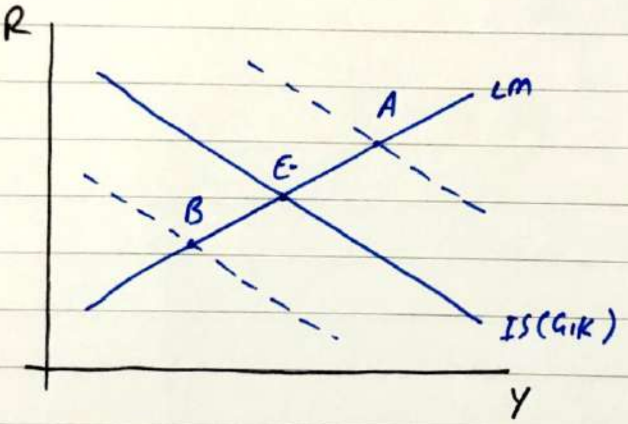
$$Y = AD(\overset{+}{G}, \overset{-}{K}) \rightarrow AD_G, AD_K$$

$$R = H(\underset{+}{G}, \underset{-}{K}) \rightarrow H_G, H_K$$

$\rightarrow$ $AD_G > 0$, $\quad AD_K = I_K AD_G < 0$,

$$H_G = \frac{-L_Y}{L_R} AD_G > 0, \quad H_K = \frac{-L_Y I_K}{L_R} AD_G < 0 \quad (2.53)$$

Fig (2.4)



The interested reader should verify that move from E. to A explains the sign of AD_G & H_G, whilst the move from E. to B explains the signs of AD_K, H_K.

$\rightarrow$ Clearly a fiscal contraction or a higher capital stock lowers the interest rate & depresses aggregate demand & hence output.

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$\dot{k} = I(R, K, Y) - \delta K$ (2.54)

$\dot{k}$ depends on k directly & indirectly via induced effect on Y & R.

$\dot{k} = I - \delta K$

$\dot{k} = I(R, K, Y) - \delta K$

$\dot{k} = I\left(\underbrace{H(G,K)}_{R}, K, \underbrace{AD(G,K)}_{Y}\right) - \delta K$

$d\dot{k} = I_k dk$

Acceleration principal

$\dot{k} = \psi(G, K)$

$I_k < 0$

$I_Y > 0$

$AD_k < 0$

where the partial deriv of $\psi(K, G)$

$$\frac{\partial \dot{k}}{\partial K} = \overset{(+)}{\overset{(-)(-)}{I_R H_K}} + \overset{(-)}{I_K} + \overset{(+)}{I_Y}\overset{(-)}{AD_K} - \delta \quad (2.55)$$

For stability

$$\boxed{\psi_K = I_R H_K + I_K + I_Y AD_K - \delta}$$

→ stabilizing effect

Also $\dfrac{\partial \dot{k}}{\partial G} = I_R H_G + I_Y AD_G$

$\psi_G = I_R H_G + I_Y AD_G \rightarrow$ (2.56)

→ distabilizing effect

Recall, stability requirement is that changes in the capital stock must taper off. $\dfrac{\partial \dot{k}}{\partial K} = \psi_K < 0$ hold, But is ψ_K negative?

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Looking at (2.55)

$$\frac{\partial \dot{k}}{\partial k} = \Psi_k = \overset{(+)}{\overset{(-)(-)}{I_R H_k}} + \overset{(-)}{I_k} + \overset{(-)}{I_Y AD_k} - \overset{(-)}{\delta}$$

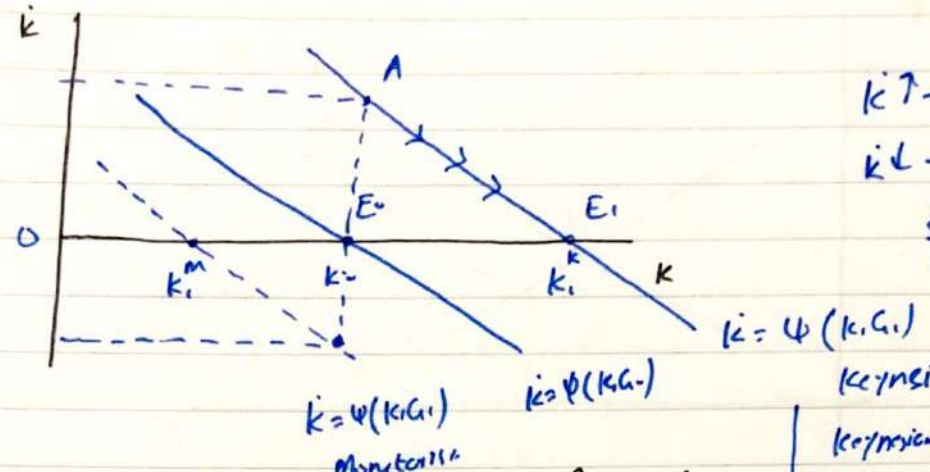
$$I_k < 0, \quad I_Y AD_k < 0, \quad -\delta < 0$$

$$\frac{\partial \dot{K}}{\partial k} < 0 \rightarrow \quad I_R H_K < I_K + I_Y AD_K - \delta \quad \Big| \text{ Samuelson Correspondance Principal}$$

$$\Psi K < 0$$

- A high Capital stock & (thus) a low level of Aggregate Demand both imply a low level of gross investment
- In addition, a high Capital stock implies a high level of depreciation. Hence, net investment is at a low level & the Capital stock will fall in future periods back to its equilibrium value.
- However, a "destabilizing" influence is clearly the term $I_R H_K > 0$. Intuitively the destabilizing effect is due to the fact that a higher Capital stock induces a lower interest rate ($H_K < 0$) & stimulates investment as $I_R < 0$.

→ Well-trained Economists? one would appeal to Samuelsons Correspondance principal & simply assume stability, i.e postulate that the destabilizing effect of $I_R H_K > 0$ is dominated by sum of ~~Sab~~ stabilizing effects $(I_k + I_Y AD_k - \delta) < 0$, so that Ψ_k is negative & the $\dot{k}$ lines in fig 2.5 are downward sloping. This is approach all take here.



$\dot{k}\uparrow \rightarrow k\downarrow$

$\dot{k}\downarrow \rightarrow k\uparrow$

$\frac{\partial \dot{k}}{\partial k} < 0$

→ The effect on capital on rise in public spending.

Keynesian $G\uparrow$
Monetarist $|IR|$ large
large effect on R
$G\uparrow \rightarrow R\uparrow \rightarrow I\downarrow$
Keynesian
$G\uparrow \rightarrow R\uparrow \rightarrow I$ small increase

· Given that stability has been assumed, what happens if the government increases its expenditure $dG > 0$?

$\psi(k, G) \rightarrow$ may shift down or up on sign of $\partial \dot{k}/\partial G$?

Recall → $I_R H_G$ is negative & $I_Y AD_G$ is positive

1) A typical monetarist would suggest a strong interest rate effect on investment ($|I_R|$ large), & a large effect on interest rate but a small effect on output of a rise in govt spending (H_G is large & AD_G is small). Consequently a monetary might suggest that ψ_G is negative.

→ According to monetarist the $\dot{k}$ line shifts down & in the long run the capital stock is crowded out by govt. spending.

2) A typical keynesian might argue reverse ($|I_R|$ small, H_G small & AD_G large, so that $\psi_G > 0$. This implies that $\dot{k}$ line shifts up & to the right, so that capital stock is stimulated in the long run by a rise in govt spending.

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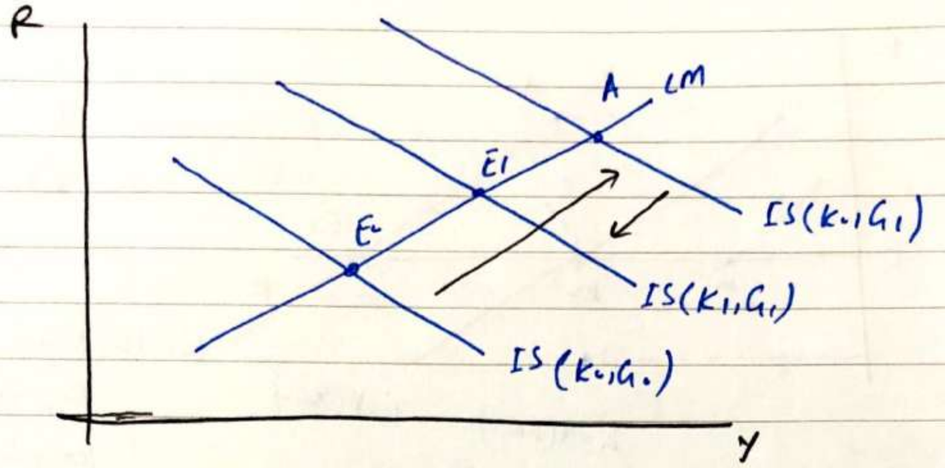

· Capital accumulation & the keynesian effects of fiscal policy

· In the short Run the Capital stock is fixed (at K_0) & the IS Curve shifts to the right (From $IS(K_0,G_0)$ to $IS(K_0,G_1)$) as a result of the increase in government Consumption. (E_0 to A)

· Over time the Capital stock increases, & the IS Curve gradually shift to the left. In the new steady state $\dot{k}=0$. the Capital stock is equal to K_1, $IS(K_1,G_1)$, is the relevant IS Curve. & Equilibrium point E_1.

→ The long Run effect on output is guaranteed to be positive.

$$\dot{k}^{LR} = \psi(K^{LR}, G) = 0 \qquad LR = \text{long Run}$$

$$\left(\frac{dK}{dG}\right)^{LR} = \frac{\psi_G}{-\psi_K}$$

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$$\dot{k}^{LR} = \psi(k^{LR}, G)$$

$$dk^{LR} = \psi_k dk^{LR} + \psi dG -$$

$$= \psi_k \left(\frac{dk}{dG}\right)^{LR} \cdot dG + \psi dG = 0$$

$$dG\left(\psi\left(\frac{dk}{dG}\right)^{LR} + \psi_G\right) = 0$$

$$\psi_k \left(\frac{dk}{dG}\right)^{LR} + \psi_G = 0$$

$$\psi_G = -\psi_k \left(\frac{dk}{dG}\right)^{LR}$$

$$\boxed{\left(\frac{dk}{dG}\right)^{LR} = \frac{\psi_G}{-\psi_{k(-)}}} \qquad (2.57)$$

where stability ensures that denominator is positive

. in a keynesian the additional govt spending "crowds in" the Capital Stock & the numerator is positive & reverse holds for a monetarist.

Now, long Run Capital Stock $= \left(\frac{dk}{dG}\right)^{LR} = \frac{\psi_G}{-\psi_k}$

$$\text{LR AD Curve} = Y^{LR} = AD(k^{LR}, G)$$

$$dY^{LR} = AD_k dk^{LR} + AD_G dG$$

$$= AD_k \left(\frac{dk}{dG}\right)^{LR} \cdot dG + AD_G dG$$

$$dY^{LR} = dG\left(AD_k\left(\frac{dk}{dG}\right)^{LR} + AD_G\right)$$

$$\left(\frac{dY}{dG}\right)^{LR} = AD_k\left(\frac{dk}{dG}\right)^{LR} + AD_G \qquad \Big|\ AD_k = I_k AD_G$$

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$$\left(\frac{dY}{dG}\right)^{LR} = I_k AD_k \left(\frac{dk}{dG}\right)^{LR} + AD_G$$

$$\left(\frac{dY}{dG}\right)^{LR} = AD_G \left(I_k \left(\frac{dk}{dG}\right)^{LR} + 1 \right) \qquad \left(\frac{dk}{dG}\right)^{LR} = \frac{\psi_G}{-\psi_k}$$

$$\left(\frac{dY}{dG}\right)^{LR} = AD_G \left(I_k \left(\frac{\psi_G}{-\psi_k}\right) + 1 \right)$$

As $\psi_k = I_R H_k + I_k + I_Y AD_k - \delta$

$\psi_G = I_R H_G + I_Y AD_G$

$$\left(\frac{dY}{dG}\right)^{LR} = AD_G \left(I_k \left(\frac{I_R H_G + I_Y AD_G}{I_R H_k + I_k + I_Y AD_k - \delta}\right) + 1 \right) \qquad H_G = \frac{-L_Y AD_G}{L_R}, \quad H_k = \frac{-L_Y AD_k}{L_R}$$

to manipulate

$$\boxed{\left(\frac{dY}{dG}\right)^{LR} = \frac{AD_G \overbrace{(\delta - I_k)}^{(-)}}{\underbrace{-\psi_k}_{(-)}}} > 0 \qquad (2.58)$$

→ long-Run Govt expenditure multiplier (Public spending)

→ Increase in $G \uparrow \to k \uparrow$ & Y in long Run

→ under prediction of keynesian

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2.4) Wealth effects And the government budget constraint:

→ Another example of stock-flow interaction are the intrinsic dynamics in the IS-LM model, that arise once we allow for the wealth effects in consumption & money demand if the government issues extra bonds or prints more money to finance its deficit.

→ Blinder & Solow (1973) suggest that this issue can be fruitfully studied with the aid of the IS-LM model, with fixed price level

→ The government can issue consuls (bonds of infinite term to maturity) that promise the owner a fixed periodic payment of 1 euro from now to infinity.

$$P_B = \int_0^{\infty} 1 \cdot e^{-Rt} \, dt = -\frac{1}{R} \cdot e^{-Rt} \Big|_0^{\infty} = 1/R$$

P_B = The price of bond

$$P_B = \frac{1}{1+R} + \frac{1}{(1+R)^2} + \cdots$$

$$= \frac{1}{1+R}\left(1 + \frac{1}{1+R} + \frac{1}{(1+R)^2} \cdots\right)$$

$$P_B = \left(\frac{1}{1+R}\right)\left(1 + x + x^2 + x^3 \cdots\right)$$

$$P_B = \left(\frac{1}{1+R}\right)\left(\frac{1}{1-x}\right)$$

$$P_B = \frac{1}{1+R}\left(\frac{1}{1 - 1/1+R}\right)$$

$$\left(P_B = \frac{1}{R}\right)$$

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→ If the government has issued B of such bonds in the past, then the payments it must make each period are equal to B times 1 euro.

→ B represents both the number of consols in the hands of the public & the interest payments of the govt to the public.

· if the govt issues new consols ($\dot{B}>0$), it recieves $P_B\dot{B}$ in revenue from this bond sale.

· The govt can meet its obligations by simply printing money ($\dot{M}>0$). With goods prices fixed at unity, the govt budget restrictions can be written as:

$$G+B = T+\dot{M}+(1/R)\dot{B}$$

· The left hand-side represents the nominal spending level of the govt inclusive of interest (i.e coupon) payments to private agents.

· The right hand-side of the govt budget restrictions shows the three financing methods open to the govt, namely taxation, money finance, & bond finance.

$$T = T(Y+B) \qquad 0 < T_{Y+B} < 1$$

↳ depends on all income recieved by households.

· The total amount of real private financial wealth in the economy, A, is the sum of the fixed capital stock, K, the real money supply, & the real value of bond holdings by the public.

$$A = K + M + B/R.$$

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As

$$\left.\begin{aligned} Y &= C + I(R) + G \\ C &= C(Y+B-T(Y+B), A) \end{aligned}\right\} IS$$

$$LM.\quad m = L(Y, R, A)$$

$C_A > 0$
mPC out of wealth
mPC → mPC out of income
→ Sensitivity of Consumption

$$Y = C(Y+B-T(Y+B), A) + I(R) + G$$

$$dY = C_{Y+B-T}(dY + dB - T_{Y+B}(dY + dB) + dA + I_R dR + dG$$

$0 < C_{Y+B-T} < 1$, $C_A > 0$
$L_Y > 0$, $I_R < 0$,
$0 < L_A < 1$.

As $A = \bar{k} + M + \frac{1}{R} B$

$$dA = \underbrace{C_A d\bar{k}}_{\text{constant}=0} + C_A dm + C_A \frac{1}{R} dB - C_A \frac{B}{R^2} dR$$

$$dY = C_{Y+B-T}(dY + dB - T_{Y+B}(dY + dB) + C_A dm + C_A\left(\frac{1}{R} dB - \frac{B}{R^2} dR\right)$$

$$+ I_R dR + dG$$

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2.4.1) Short-run macroeconomic equilibrium:-

- In the short Run, the money supply & the level of government debt are Predetermined variables.

$$dY = C_{Y+B-T}\left(dY + dB - T_{Y+B}(dY + dB)\right) + C_A dm + C_A\left(\frac{1}{R}dB - \frac{B}{R^2}dR\right) + I_R dR + dG$$

- The IS curve is obtained by combining (2.61-2.63) with standard investment function $I = I(R)$ & closed economy $Y = C + I + G$

$$dY = \frac{1}{1 - C_{Y+B-T}(1 - T_{Y+B})}\left[dG + \left[C_{Y+B-T}(1 - T_{Y+B}) + C_A/R\right]dB + \left[I_R - C_A\frac{B}{R^2}\right]dR + C_A dm\right] \rightarrow (2.66)$$

$$Y = AD(\overset{+}{G}, \overset{+}{M}, \overset{?}{B})$$

$$R = I(\overset{+}{G}, \overset{?}{M}, \overset{+}{B})$$

Also.
$2.64 \rightarrow \quad M = L(Y, R, A). \quad L_Y > 0,\ L_R < 0,\ 0 < L_A < 1.$

$2.65 \rightarrow \quad Y = C(Y + B - T(Y+B),\ \bar{E} + M + B/R) + I(R) + G$

$$dR = \frac{-(L_A/R)dB - L_Y dY + (1 - L_A)dm}{L_R - L_A B/R^2}$$

The IS curve is downward sloping & the LM curve slopes up, just as in the basic IS-LM model

$$\left.\begin{aligned} Y &= AD(\overset{+}{G}, \overset{?}{B}, \overset{+}{M}) \\ R &= H(\underset{+}{G}, \underset{+}{B}, \underset{?}{M}) \end{aligned}\right\} \rightarrow (2.68)$$

By using 2.66, 2.67 for partial derivatives can be obtained.
For AD curve.

$$\frac{dY}{dG} = AD_G = \frac{1}{\underbrace{1 - C_{Y+B-T}(1 - T_{Y+B})}_{+} + \underbrace{\xi L_Y}_{+}} > 0$$

where $\xi = C_A \frac{B}{R^2} + I_R \Big/ L_A B/R^2 + I_R$

$$\frac{dY}{dB} = AD_B = \frac{C_{Y+B-T}(1 - T_{Y+B}) + C_A/R - \xi L_A/R}{1 - C_{Y+B-T}(1 - T_{Y+B}) + \xi L_Y} \gtrless 0$$

$$\frac{dY}{dM} = AD_M = \frac{C_A + \xi(1 - L_A)}{1 - C_{Y+B-T}(1 - T_{Y+B}) + \xi L_Y} > 0$$

C_A: Responsive of sensitivity of consumption of wealthy asset

L_A: Response of sensitivity of demand for money to change in wealth

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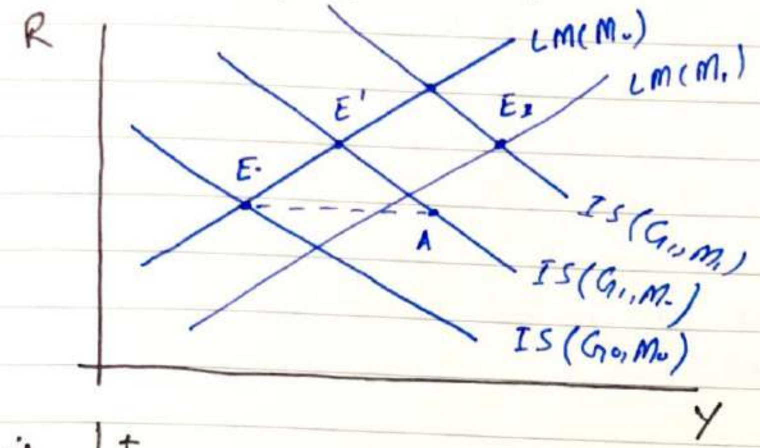
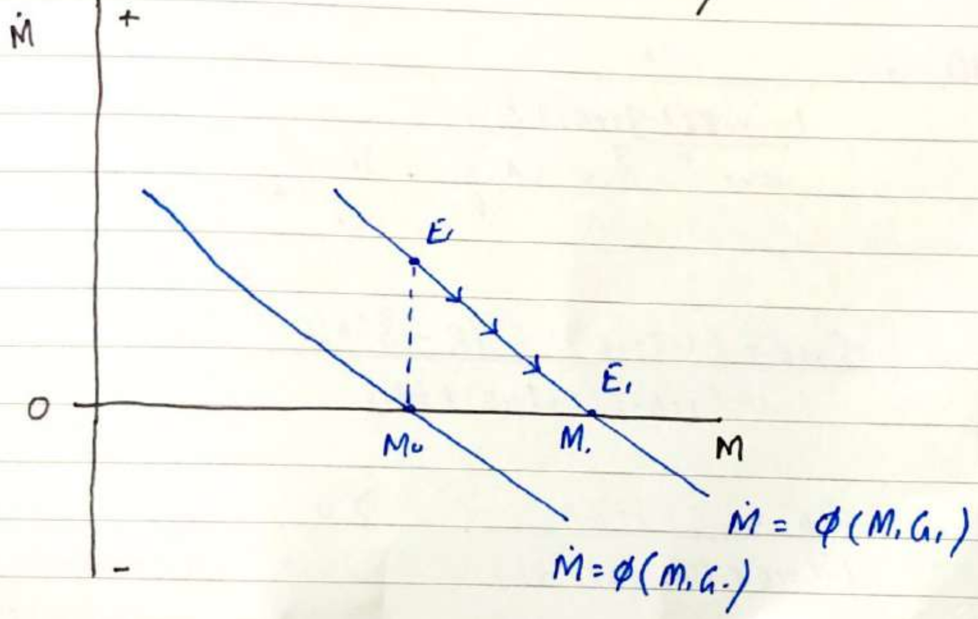
→ The effects of fiscal policy under money finance:- (2.7)





- In t-p panel of fig 2.7, the initial equilibrium is at point E_0, An increase in govt spending shifts the IS curve from IS (G_0, M_0) to IS (G_1, M_0).
- At point A income is higher than before & there is an excess demand for money (an excess supply of ~~neon~~ bonds).
- This causes a fall in bond prices i.e a rise in interest rate which moves the economy at point E'. In terms of 2.7 both output & the rate of interest are higher, hence $AD_a > 0$.

- We have shown an increase in govt spending causes a short-run increase in output, Y, & fall of interest, R.
- We have not taken the govt ~~spending causes~~ budget restrictions into account.

→ Blinder & Solow (1973) consider two extreme cases.

1). The government prints new money to finance the additional govt spending. $\dot{M} \neq 0, \dot{B} = 0$

2) The govt balances its books by issuing additional bonds, i.e $\dot{M} = 0$ & $\dot{B} \neq 0$

Question i) is the model stable under both financing methods

ii) what is relationship b/w diff. output multipliers for govt spending with respect to diff modes of govt. finance

At first blush,

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2.4.2) Money finance:-

under money finance the government budget restriction. reduces to $\dot{M} = G + B - T(Y+B)$, where B is fixed.

$$G + B = T(Y+B) + \dot{m} + \frac{1}{R}\underbrace{\dot{B}}_{\text{Fixed}} \quad (2.60)$$

$$\dot{m} = \underbrace{G+B}_{\text{Spending}} - \underbrace{T(Y+B)}_{\text{Revenue}} \quad \left(\text{Budget deficit}\right)$$

$\dot{m}$ depends upon G & Y, Y itself depends on G, M.

$$\dot{m} = G + B - T\left(AD(G, m, B) + B\right) = \phi(M, G)$$

$$\frac{d\dot{m}}{dm} = \phi_m = 0 - T_{Y+B}\, AD_m$$

$$\frac{d\dot{m}}{dm} = -\underset{+ve}{\text{Tax rate}}\; AD_m^{+ve} < 0 \quad (2.76)$$

$$\frac{d\dot{m}}{dm} < 0$$

Also $\dot{m} = G + B - T\left(AD(G, m, B) + B\right)$

$$d\dot{m} = dG - T_{Y+B}\, AD_G\, dG$$

$$\frac{d\dot{m}}{dG} = \left(1 - T_{Y+B}\, AD_G\right) > 0$$

$$\frac{d\dot{m}}{dG} = \frac{(1 - C_{Y+B-T})(1 - T_{Y+B}) + \xi L_Y}{1 - C_{Y+B-T}(1 - T_{Y+B}) + \xi L_Y} > 0$$

- $G\uparrow$ creates budget deficit
- to Finance the budget deficit, growth in money supply > 0..

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· long Run - Govt. Expenditure Multiplier:-

$$G + B = T(Y + B)$$

$$dG + \underset{0}{dB} = T_{Y+B}\, dY^{LR} + T_{Y+B}\, \underset{0}{dB}$$

$$dG = T_{Y+B}\, dY^{LR}$$

$$\left(\frac{dY}{dG}\right)^{LR}_{mF} = \frac{1}{T_{Y+B}} > \left(\frac{dY}{dG}\right)^{SR} = AD_g$$

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2.4.3) Bond finance:-

under Bond finance the government budget restriction, reduces to $\left(\frac{1}{R}\right)\dot{B} = G + B - T(Y+B)$ & m is fixed

$$G + B = T(Y+B) + \underset{0}{\dot{m}} + \frac{1}{R}\dot{B}$$

$$\frac{1}{R}\dot{B} = (G+B) - T(Y+B)$$

$$\dot{B} = R\left[(G+B) - T(AD(G,m,B)+B)\right]$$

$$\dot{B} = H(G,B,m) \cdot \left[G+B - T(AD(G,B,m)+B)\right]$$

$$\dot{B} = \Delta(B,G).$$

$$\frac{d\dot{B}}{dB} = \Delta_B = R\left[dB - T_{Y+B}\,\overset{AD_B\,dB}{\cancel{(1+AD_B)B}} - T_{Y+B}\,dB\right]$$

$$= R\left(1 - T_{Y+B}(1+AD_B)\,dB\right)$$

$$\frac{d\dot{B}}{dB} = R\left[1 - T_{Y+B}(1+AD_B)\,dB\right] \gtrless 0 \qquad AD_B > 0,\ AD_B < 0$$

Now

$$\frac{d\dot{B}}{dG} = R\left(dG - T_{Y+B}\,AD_G\,dG\right) \qquad (2.8)$$

$$\frac{d\dot{B}}{dG} = R\left(1 - T_{Y+B}\,AD_G\right) > 0 \qquad (2.81)$$

→ increase in G creates the budget deficit.

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· Model is stable if (and only if) changes in debt eventually dampen over time $\frac{\partial \dot{B}}{\partial B} \equiv \Delta_B < 0$ is negative.

$$\Delta_B = \frac{\partial \dot{B}}{\partial B} < 0 \qquad 1 - T_{Y+B}(1 + AD_B) < 0$$

$$AD_B > \frac{1 - T_{Y+B}}{T_{Y+B}} > 0 \quad (2.82)$$

→

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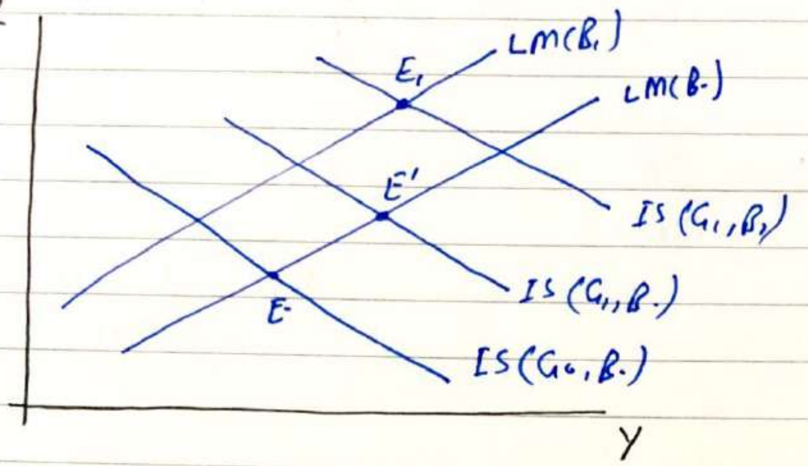
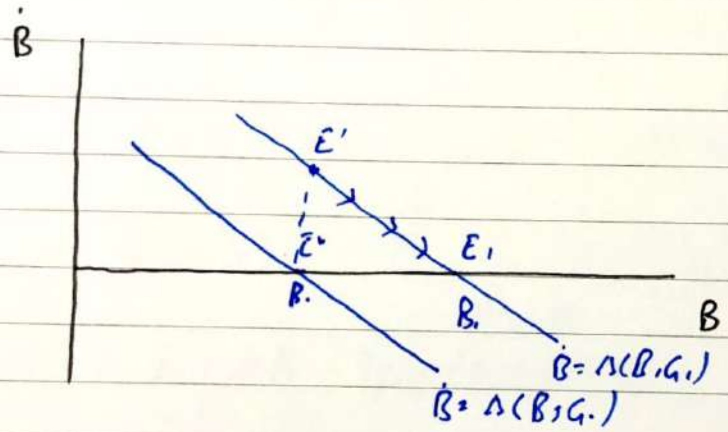
2.8) Fiscal Policy under (stable) bond financing





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long Run:

$$Y^{LR} = AD(G, B, m)$$

$$dY^{LR} = AD_G\, dG + AD_B\, dB + AD_m\, \overset{\times}{dm}$$

$$\left(\frac{dY}{dG}\right)^{LR}_{BF} = AD_G + AD_B\left(\frac{(1-T_Y+B)\, AD_G}{T_Y+B(1+AD_B)-1}\right)$$

$$\hookrightarrow \frac{dB}{dG} = \frac{\Delta G}{-\Delta B} = \frac{1-T_Y+B\, AD_G}{T_Y+B(1+AD_B)-1}$$

$$\left(\frac{dY}{dG}\right)^{LR}_{BF} > \left(\frac{dY}{dG}\right)^{SR}_{BF}$$

$\rightarrow$ Also $\left(\frac{dY}{dG}\right)^{LR}_{BF} > \left(\frac{dY}{dG}\right)^{LR}_{mF}$ # chH02