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AXIOMS OF CONSUMER PREFERENCES

"Assumption" is something that doesn't necessarily hold. "Axiom" is a well-established assumption.

(1) Completeness
(2) Transitivity
(3) Continuity
(4) Convexity
(5) Non-satiation

We assume that all 5 of these axioms hold

(1) **Completeness**: If d/t options are available to consumer. He has ability to rank the consumption bundle according to his preference. Whenever ranking is done, there are a few possibilities

x → bundles X → commodities

Let x′ & x″ are two consumption bundles

(i) x′ is strictly preferred over x″ $\boxed{x' > x''}$
(ii) x′ is strictly inferior to x″. $\boxed{x' < x''}$
(iii) x′ is atleast as good as x″
(iv) x′ is at most as good as x″
(v) x′ & x″ are equally preferred. $\boxed{x' \sim x''}$

→ x′ is better, on a lower side it is atleast as good as x″, but not worse

(iii) → x′ can be indifferent to x″ $\boxed{x' \geq x''}$
x′ is better than x″

(iv) → $\boxed{x' \leq x''}$

⇒ It is possible for consumption bundles as well as commodities.

(i), (ii) → Strict Preferences (v) → indifference

(iii), (iv) → Weak Preferences

⇒ Ranking is possible (completeness), it implies that there are no holes or gaps in consumer preference

⇒ No holes in preferences — Consumer has ability to construct indifference curve

(2) **Transitivity:** Let x', x'' and x''' are three consumption bundles. Then transitivity implies

(a) if $x' > x''$ and $x'' > x'''$ then it implies that $x' > x'''$

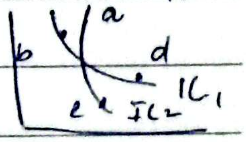
(b) if $x' \sim x''$ and $x'' \sim x'''$ then it implies that $x' \sim x'''$

⇒ Comparison of two enables us to compare third one as well.

e.g. to compare / or find out who is well-dressed among 40 people, then by comparing 2 using principle of transitivity one-by-one

⇒ It doesn't hold much (sentences game).

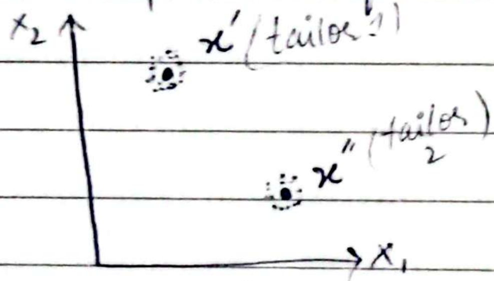
⇒ Consistency in preferences (it is not possible that $x' > x''$ & $x'' > x'''$ but a person is indifferent b/w x' & x'''.



⇒ IC never intersect each other.

a is preferred to b & it always should be this way. inconsistency such that composition of commodity remain the same

(3) **Continuity**: Minor changes in consumption bundles do not affect your preferences OR there are no sudden jumps in preferences
e.g Tea made on daily basis, the measurements are not the same, but we don't realize it small changes doesn't affect preferences/taste
"Minor" → which do not change the composition of commodity.
Stitch slightly big or small in a dress → Minor change
Yellow button attached to blue dress → Major (it changes the composition of commodity).



Two distinct consumption bundles. Let $x' > x''$
Then continuity implies that all bundles closer to x' will also be preferred over all bundles near x''

→ Changes made by tailor 1 are always preferred over changes by 2nd.

A footnote Concept of "Lexicographic ordering"

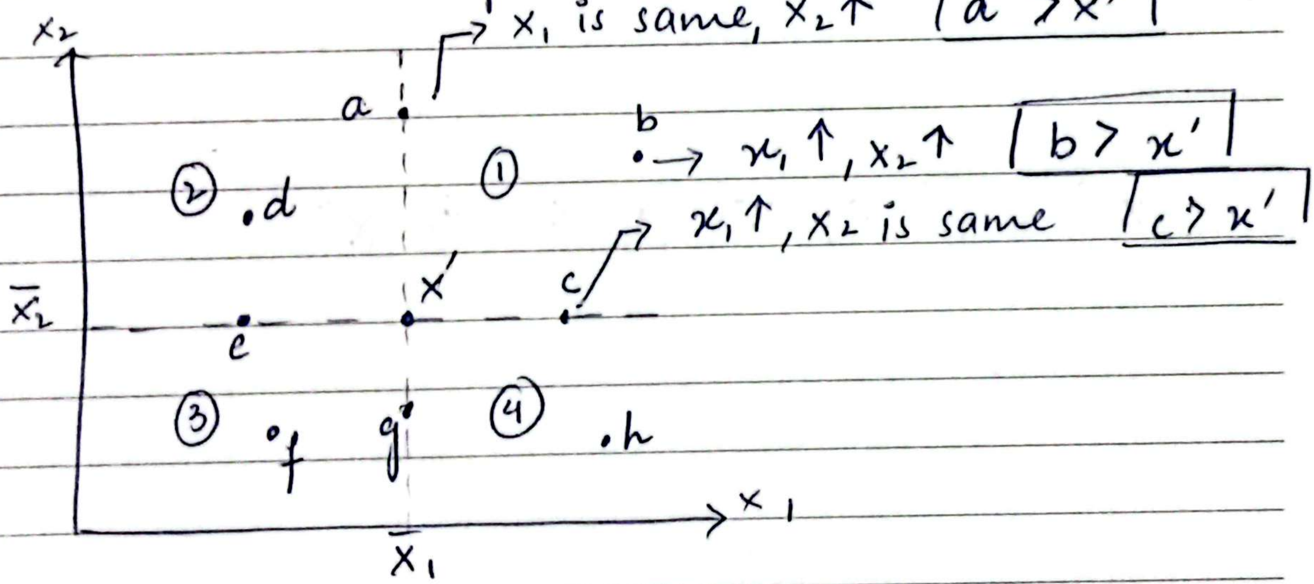
1. In utility bundle, commodities are expressed such that most preffered in listed first & so on
2. If you have 2 consumption bundles, where in 1st bundle, 1st commodity is preferred then 1st bundle will be picked no matter how desirable 2nd bundle is.

Example : (burger , fries , sauce , drink) lexicographic ordering

Let there are 2 bundles

① [burger , fries , sauce , pepsi] ② [burger, fries, sauce, pepsi]
② ④ ② ⑥ ① ⑧ ④ ④

① will always be preferred to ② because main preferrence is the burger.
If a third bundle is there, [burger③, 0, 0, 0]
now ③ will be preferred to the former ones.
If quantity of first commodity (burger) is same then choice is made on the basis of second commodity.



comparing all phases one-by-one
① (including boundaries) - 3 possibilities (a, b, c) bcz boundary is included ① > x'
② (excluding boundaries) ② < x'
③ (including boundaries) ③ < x'
④ excluding boundaries ④ > x'

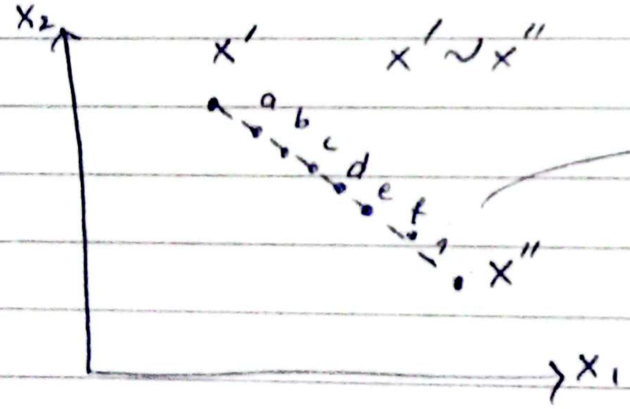
⇒ There is no consumption bundle that is indifferent to x', each has unique utility

distinct
bundles

→ In case of lexicographic ordering where continuity doesn't hold (discontinuous behaviour)
→ IC's cannot be constructed
→ Minor change in cons. bundle will affect your behaviour.

(4) **Convexity** Let x' and x'' are two consumption bundles such that $x' \sim x''$
The convexity implies that all linear combinations of x' & x'' will be strictly preferred over x' & x''
Linear Combination

$$\lambda(x') + (1-\lambda)(x'') > x' \text{ \& } x''$$



⇒ If convexity holds then all consumption bundles on the straight line will be strictly preferred over x' & x''

$\boxed{x' \sim x''}$ → Linear Combination are clearly better than these two

→ All combinations along the straight line belong to higher indifference curves

→ IC is never a straight line bcz a, b, c, d ... are better than x' & x"
→ It can be convex or concave

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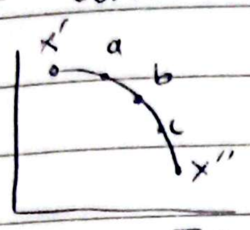
→ If concave



It means that x' & x" are better than a, b, c, d which is not the case

→ Hence, ICs are convex.
⇒ If strict convexity holds, IC will be convex
if weak convexity holds, IC can be a straight line as well.
⇒ It can never be concave
e.g 25 breads & 4 dips (extreme) - preference will be 6 breads & 4 dips.

(5) Non-Satiation

"Satiation point" is a point where beyond this point consumption will not give more utility - Utility is maximum here.
It is a point where last unit has some utility for you but not more than that
Any additional unit will not increase the total utility.
e.g Full at 3 breads, 4th was not needed - 3 was a satiation point.
⇒ As long as marginal utility is +ve, we're below the satiation point

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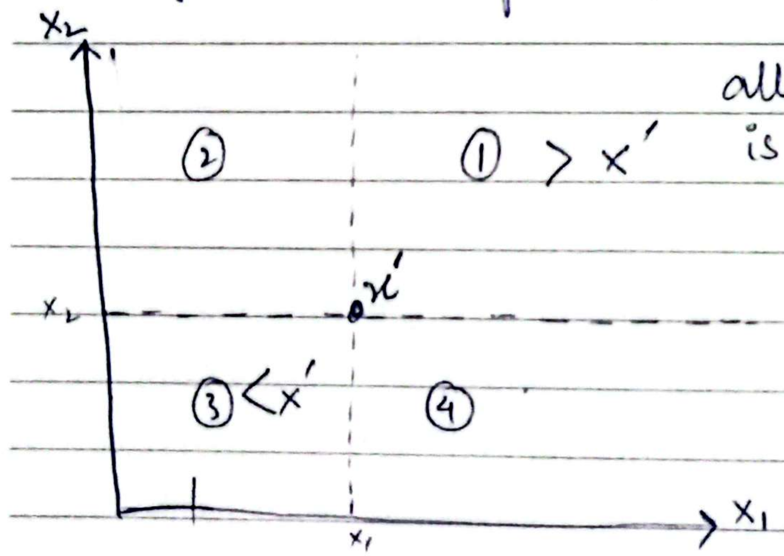
Non-Satiation $\boxed{MU > \text{zero}}$

All the commodities including in consumption bundle have positive marginal utility.

⇒ How much we consume will depend on our resources / budget constraint — but marginal utility (want) should be +ve.

⇒ More is preffered over less as long as marginal utility is positive.



all are ↑ or atleast one is

For ② & ④, one commodity is ↑ while other is ↓

Some Utility is gained, some is lost.

Gain = Lost

Indifferent point.

both are declining or atleast one is

⇒ If there is any indifference point, it will pass through ② & ④

⇒ A line passing through ② & ④ can be straight, concave or convex.

Convex IC is true/correct.

⇒ Non-satiation holds → ICs are negatively sloped.

UTILITY FUNCTION: $u(x_1, x_2 \ldots x_n)$

⇒ Sometimes utilities are additive (aggregation is possible - sometimes they are not additive.
⇒ Numbers are assigned but they are hypothetical and hold no significance.

$U(x_1) + U(x_2) = TU(x_1, x_2)$

$u(x_1) + u(x_2) \neq TU(x_1, x_2)$

Utility is additive, if aggregative give TU
If not - it is not additive.

Additive Case Example

$U(\text{markee}) + U(\text{wat}) = TU(M+C)$

Non-Additive Case

$u(B) + U(T) < U(T+B)$

Utility of both complement each. In case of complements - total utility is always greater

For Substitutes

$u(\text{pepsi}) + u(\text{coke}) < TU$

Utility are additive - if commodities are unrelated.

⇒ If all axioms hold, there is a well-established utility function.

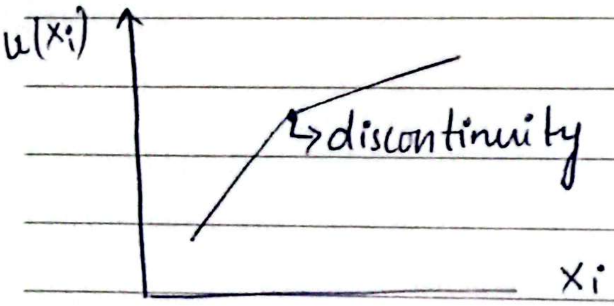
Fundamental Properties:

(1) **↑ing in X_i (commodity):** The more you consume, the more utility you get. $u(x_i)$ is ↑ing in X_i due to Axiom of Non-satiation.

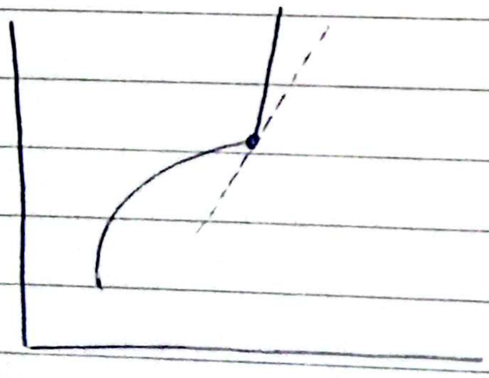
i.e. $MU > 0$

Mathematically, $\frac{d(u)}{dx_i} >$ zero. +vely sloped

(2) **Twice Differentiable:**



Kink showing change in consumer behaviour



Due to discontinuity unique slope is not possible

Axiom of Continuity

This rules out the possibility of discontinuity

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For any function, its level curve is convex — it is Quasi Concave
For UF, level curve is IC.

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(3) Quasi Concavity:

A function is Quasi-concave if its contour set is convex

Contour set is a set at which the value of function is fixed/same
It is also known level curve. IC is a level curve

IC is a contour set of utility maximization.

=> Utility Function will be Quasi Concave if the corresponding level curve (IC) is curve

Quasi-concavity of UF can be spelled in two ways
- corresponding curve is convex
- MRS is diminishing

x_1	x_2	$MRS = \Delta x_2 / \Delta x_1$
1	20	–
2	15	-5
3	11	-4
4	8	-3

Whenever behaviour of MRS is explained. Sign is ignored. We're interested in how many units are given up to obtain the second.

Mathematically, (can't ignore the sign)

second derivative is +ve.
diminishing MRS → in absolute terms
→ By taking second derivative of MRS, answer should be +ve. If it is property of Quasi-concavity hold.

$$\bar{u} = u(x_1, x_2)$$

$$du = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2$$

Since it is an IC - $dU = 0$

$$0 = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2$$

$$\frac{\partial u}{\partial x_1} = MU_1 = u_1 \qquad \frac{\partial u}{\partial x_2} = MU_2 = u_2$$

$$0 = u_1 dx_1 + U_2 dx_2$$

$$-u_2 dx_2 = u_1 dx_1$$

$$\boxed{\frac{dx_2}{dx_1} = -\frac{u_1}{U_2}} \rightarrow (MRS_{12})$$

For convexity of indifference curve

$$\frac{d^2 x_2}{dx_1^2} > \text{zero}$$

Amount of x_2 depends on x_1

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$$\frac{dx_2}{dx_1} = -\frac{u_1(x_1, x_2)}{u_2(x_1, x_2)}$$

Quotient Rule

$u_1 \; U_2$

$$\frac{u_2(u_1') - u_1(u_2')}{u_2^2}$$

$$\Rightarrow \frac{dx_2}{dx_1} = -\frac{u_1[x_1, x_2(x_1)]}{u_2[x_1, x_2(x_1)]}$$

$$\frac{d^2x_2}{dx_1^2} = -\frac{1}{u_2^2}\left[u_2\left\{\frac{dU_1}{dx_1} + \frac{dU_1}{dx_2}\left(\frac{dx_2}{dx_1}\right)\right\} - u_1\left\{\frac{dU_2}{dx_1} + \frac{dU_2}{dx_2}\left(\frac{dx_2}{dx_1}\right)\right\}\right]$$

$$\frac{d^2x_2}{dx_1^2} = -\frac{1}{u_2^2}\left[u_2\left\{u_{11} + u_{12}\left(-\frac{u_1}{u_2}\right)\right\} - u_1\left\{u_{21} + U_{22}\left(-\frac{u_1}{u_2}\right)\right\}\right]$$

$\therefore \frac{dx_2}{dx_1} = \frac{-u_1}{u_2}$

$$\frac{d^2x_2}{dx_1^2} = -\frac{1}{u_2^2}\left[u_2U_{11} - \frac{u_1U_{12}U_2}{u_2} - U_1U_{21} + \frac{U_1U_{22}U_1}{U_2}\right]$$

Taking u_2 common

$$\frac{d^2x_2}{dx_1^2} = -\frac{1}{u_2^3}\left[U_2^2U_{11} - u_1U_{12}U_2 - U_1U_2U_{21} + U_1^2U_{22}\right]$$

According to Young's Theorem $U_{12} = U_{21}$ can be added

$$= -\frac{1}{u_2^3}\left[U_2^2U_{11} - 2U_1U_2U_{12} + U_1^2U_{22}\right]$$

3rd Property

BlackGold Utility Function is Quasi Convex

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Condition $\boxed{\frac{d^2X_2}{dx_1^2} > 0}$

$$\frac{d^2x_2}{dx_1^2} = -\frac{1}{U_2^3}\left[U_2^2U_{11} - 2U_1U_2U_{12} + U_1^2U_{22}\right]$$

$\boxed{U_1 > 0}$ $\boxed{U_2 > 0}$

$U_i > 0$

U_{11} = derivative of MU

$\boxed{U_{11} \gtreqless 0}$ $\boxed{U_{22} \gtreqless 0}$

$U_{ii} > 0$ $MU_i \uparrow$

$U_{ii} < 0$ $MU_i \downarrow$

$U_{ii} = 0$ $\overline{MU_i}$

MU itself is always +ve, but the rate of change is always diminshing

$U_{12} \gtreqless 0$

$U_{ij} \gtreqless 0$

$U_{ij} \rightarrow$ Rate of Δ in MU of i^{th} commodity due to Δ in j^{th} commodity

$U_{ij} > 0$ for complements

$U_{ij} < 0$ for substitutes

$U_{ij} = 0$ unrelated

$$\frac{d^2x_2}{dx_1} = -\frac{1}{U_2^3}\left[U_2^2U_{11} - 2U_1U_2U_{12} + U_1^2U_{22}\right]$$

U_2^3: + ; U_2^2: + ; U_{11}: + / − / 0 ; 2: + ; U_1U_2: + ; U_{12}: + / − / 0 ; U_1^2: + ; U_{22}: + / 0 / −

Classify one-by-one according to diff categories

For Unrelated Commodities

$U_{12} = \text{zero}$

it should be -ve because we need overall sign to be +ve

$$\frac{d^2 x_2}{dx_1^2} = -\frac{1}{U_2^3}\left[U_2^2 U_{11} + U_1^2 U_{22}\right]$$

$U_2^3 \to (+)$; $U_2^2 \to (+)$; $U_{11} \to (+), (0), (-)$; $U_1^2 \to (+)$; $U_{22} \to (0), (-), (+)$

For Convexity

$$\frac{d^2 x_2}{dx_1^2} : \left[U_2^2 U_{11} + U_1^2 U_{22}\right] < \text{zero}$$

There are 9 possibilities in total.

$\Rightarrow U_{11} > 0 \quad U_{22} > 0$

$U_{11} > 0 \quad U_{22} = 0$

$U_{11} > 0 \quad U_{22} < 0$

$\Rightarrow U_{11} = 0 \quad U_{22} > 0$

$U_{11} = 0 \quad U_{22} = 0$

$U_{11} = 0 \quad U_{22} < 0$

$\Rightarrow U_{11} < 0 \quad U_{22} > 0$

$U_{11} < 0 \quad U_{22} = 0$

$U_{11} < 0 \quad U_{22} < 0$

$$\boxed{\begin{matrix} U_2^2 U_{11} + U_1^2 U_{22} \\ (+) U_{11} + (+) U_{22} \end{matrix}}$$

$U_{11} > 0$	$U_{22} > 0$	(+) (+) + (+) (+) = (+)
$U_{11} > 0$	$U_{22} = 0$	(+) (+) + (+) (0) = (+)
$U_{11} > 0$	$U_{22} < 0$	(+) (+) + (+) (-) = (?)
$U_{11} = 0$	$U_{22} > 0$	(+) (0) + (+) (+) = (+)
$U_{11} = 0$	$U_{22} = 0$	(+) (0) + (+) (0) = (0)
$U_{11} = 0$	$U_{22} < 0$	(+) (0) + (+) (-) = (-) ✗,

$U_2^2 U_{11} + U_1^2 U_{22}$

$U_{11} < 0$	$U_{22} > 0$	$(+)(-) + (+)(+) = ?$	
$U_{11} < 0$	$U_{22} = 0$	$(+)(-) + (+)(0) = (-)$	P_2
$U_{11} < 0$	$U_{22} < 0$	$(+)(-) + (+)(-) = (-)$	P_3

P_1 Convexity holds $\Rightarrow MU_2 \downarrow$ diminishing, $\overline{MU_2}$ - constant Exception

P_2 Convexity holds $\Rightarrow MU_1 \downarrow$, $\overline{MU_2}$ Exception

P_3 Convexity holds $\Rightarrow MU_1 \downarrow$, $MU_2 \downarrow$
Only this is the normal case

For Substitutes $\boxed{U_{12} < 0}$

$U_2^2 U_{11} - 2U_1U_2U_{12} + U_1^2 U_{22}$

$(+) U_{11} - (+)(+)(-) + (+) U_{22}$ → a +ve constant

$(+) U_{11} + (+) U_{22} +$ (a +ve constant)

$U_2^2 U_{11} + U_1^2 U_{22} + c$ where $c > 0$

$(+)\ \cancel{(+)}$

$U_{11} > 0$	$U_{22} > 0$	$(+)(+) + (+)(+) + (+) = (+)$
$U_{11} > 0$	$U_{22} = 0$	$(+)(+) + (+)(0) + (+) = (+)$
$U_{11} > 0$	$U_{22} < 0$	$(+)(+) + (+)(-) + (+) = ?$ ✓
$U_{11} = 0$	$U_{22} > 0$	$(+)(0) + (+)(+) + (+) = (+)$
$U_{11} = 0$	$U_{22} = 0$	$(+)(0) + (+)(0) + (+) = (+)$
$U_{11} = 0$	$U_{22} < 0$	$(+)(0) + (+)(-) + (+) = ?$ ✓
$U_{11} < 0$	$U_{22} > 0$	$(+)(-) + (+)(+) + (+) = ?$ ✓
$U_{11} < 0$	$U_{22} = 0$	$(+)(-) + (+)(0) + (+) = ?$ ✓
$U_{11} < 0$	$U_{22} < 0$	$(+)(-) + (+)(-) + (+) = ?$ ✓

✓ Convexity may hold - unsure about it
For Complement - DIY - one combination will be there where convexity will hold for sure

$U_2^2 U_{11} - 2 U_1 U_2 U_{12} + U_1^2 U_{22}$

$(+) U_{11} - (+)(+)(-) + (+) U_{22}$

CONSUMER CHOICE

Most of the time, consumer is utility maximizer but in some case he is expenditure minimizer as well.

1. Utility Maximization s.t Budget Constraint
2. Expenditure Minimization s.t fixed utility

Maximization of Utility subject to the Budget Constraint

$u(x_i) \rightarrow$ Objective Function

$M = \sum P_i X_i \rightarrow$ Constraint

$$\mathcal{L} = U(X_i) + \lambda\left[M - \sum_{i=1}^{n} P_i X_i\right]$$

F.O.C

$$\frac{\partial \mathcal{L}}{\partial X_1} = 0 \Rightarrow U_1 - \lambda P_1 = 0 \quad \text{--- (1)}$$

$$\frac{\partial \mathcal{L}}{\partial X_2} = 0 \Rightarrow U_2 - \lambda P_2 = 0 \quad \text{--- (11)}$$

$\vdots$

$$\frac{\partial \mathcal{L}}{\partial X_n} = 0 \Rightarrow U_n - \lambda P_n = 0 \quad \text{--- (n)}$$

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$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow M - \sum_{i=1}^{n} P_i X_i = 0 \quad \text{--- } (n+1)$$

Simultaneous solution of F.O.C will yield utility maximizing demand functions

$$\boxed{X_i = X_i (P_i , M)}$$ it has Many names

- Utility Maximizing demand function
- Money Income held constant demand function
- Marshallian Demand Function.

Example: $u = x_1^{\alpha} x_2^{\beta}$

$$M = P_1 X_1 + P_2 X_2$$

$$\mathcal{L} = x_1^{\alpha} x_2^{\beta} + \lambda [M - P_1 X_1 - P_2 X_2]$$

F.O.C → Assignment.

S.O.C

$$\begin{vmatrix} U_1 & U_{12} & -P_1 \\ U_{21} & U_{22} & -P_2 \\ -P_1 & -P_2 & 0 \end{vmatrix}$$ Border Hessian

Solve it to find an equation

Substitute FOC in the answer equation

$u_1 = \lambda P_1 \qquad u_2 = \lambda P_2$

that should be equal to answer of Quasi concavity.

Check homogenous of degree zero

apply elasticity equation to get there will be 2 commodities, there will be 2 elasticities - own & cross price.

Properties of Marshallian Demand Function

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$X_i = X_i(P_i, M)$

① Homogenous of degree zero in prices and income

Homogenous → Multiply equation by a constant & if the constant can be factored out (taken common), equation is said to be homogenous

The power of that constant is the degree of homogenity.

Example

$2P_1P_2 + M^2$

$2(cP_1)(cP_2) + (cM)^2$

$2c^2P_1P_2 + c^2M^2$

$c^2[2P_1P_2 + M^2]$

we multiplied by c but c^2 is factored out so degree of homogenity is 2

$2\frac{P_1}{P_2}$ → if both P_1 & P_2 are multiplied by 2, it will have no affect as it will be cancelled out. (homogenous of degree zero)

If all price & income change by same proportion - then the demand function will not change bcz it is not going to change the purchasing power.

→ Same proportionate change in Prices & Income

→ Euler's Theorem : is only applied when function is homogenous

$$Y = f(x_1, x_2, x_3 \ldots x_n)$$

let it be homogenous of degree "r"

$$\frac{\partial Y}{\partial x_1} x_1 + \frac{\partial Y}{\partial x_2} x_2 + \frac{\partial Y}{\partial x_3} x_3 + \cdots \frac{\partial Y}{\partial x_n} x_n = r(Y)$$

(1) MDF are H.O.D = zero in prices & income

Applying Euler's Theorem on it

$$x_i(P_i, M)$$

$$\frac{\partial x_i}{\partial P_1} P_1 + \frac{\partial x_i}{\partial P_2} P_2 + \frac{\partial x_i}{\partial P_3} P_3 + \cdots + \frac{\partial x_i}{\partial P_n} P_n + \frac{\partial x_i}{\partial M} M = (0) x_i$$

If we divide entire equation by x_i so we can get elasticity.

Sum of all own-price, cross-price & income elasticities is equal to zero

$$\frac{\partial x_i}{\partial P_1} \cdot \frac{P_1}{x_i} + \frac{\partial x_i}{\partial P_2} \cdot \frac{P_2}{x_i} + \frac{\partial x_i}{\partial P_3} \cdot \frac{P_3}{x_i} \cdots \frac{\partial x_i}{\partial P_n} \cdot \frac{P_n}{x_i} + \frac{\partial x_i}{\partial M} \cdot \frac{M}{x_i} = \frac{(0) x_i}{x_i}$$

$$e_{i1} + e_{i2} + e_{i3} + \cdots e_{in} + \mu_{iM} = 0$$

own price effect $<$ Normal
$>$ Giffen
$=0$ perfectly inelastic commodity / income

(2) MDF satisfy the Budget Constraint

$$M = P_1X_1 + P_2X_2 + P_3X_3 + \cdots + P_nX_n$$

$$M = P_1[X_1(P_i, M)] + P_2[X_2(P_i, M)] + P_3[X_3(P_i, M)] + \cdots + P_n[X_n(P_i, M)]$$

Differentiate w.r.t M

$$1 = P_1\frac{\partial X_1}{\partial M} + P_2\frac{\partial X_2}{\partial M} + P_3\frac{\partial X_3}{\partial M} + \cdots + P_n\frac{\partial X}{\partial M}$$

Converting into income elasticities

Div by multiplying by M

Expenditure on commodity 1 → income elasticity of d for commodity 1

$$1 = \frac{P_1X_1}{M}\left(\frac{\partial X_1}{\partial M}\cdot\frac{M}{X_1}\right) + \frac{P_2X_2}{M}\left(\frac{\partial X_2}{\partial M}\cdot\frac{M}{X_2}\right) + \frac{P_3X_3}{M}\left(\frac{\partial X_3}{\partial M}\cdot\frac{M}{X_3}\right) + \cdots + \frac{P_nX_n}{M}\left(\frac{\partial X_n}{\partial M}\cdot\frac{M}{X_n}\right)$$

$\frac{P_1X_1}{M}$ → Share of Expenditure on commodity 1.

$$1 = S_1\eta_{1M} + S_2\eta_{2M} + S_3\eta_{3M} + \cdots + S_n\eta_{nM}$$

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$$\boxed{1 = \sum_{i=1}^{n} S_i \eta_{iM}}$$

Engel Aggregation

→ Weighted sum of all income elasticities is 1 where the weights are S/shares

$$S_1 + S_2 + S_3 + \dots S_n = 1$$

implications of Engel Aggregation ?

Differentiate w.r.t P_1 ∴ Using Product Rule

derivative of 1st Term

$$\frac{dM}{dP_1} = \underbrace{P_1 \frac{\partial X_1}{\partial P_1}}_{} + X_1(1) + P_2 \frac{\partial X_2}{\partial P_1} + P_3 \frac{\partial X_3}{\partial P_1} + \dots + P_n \frac{\partial X_n}{\partial P_1}$$

Income is not the function of price, so $\frac{dM}{dP_1} = 0$

$$0 = X_1 + P_1 \frac{\partial X_1}{\partial P_1} + P_2 \frac{\partial X_2}{\partial P_1} + P_3 \frac{\partial X_3}{\partial P_1} + \dots P_n \frac{\partial X_n}{\partial P_1}$$

Converting into elasticities

$$0 = X_1 + \frac{P_1 X_1}{P_1}\left[\left(\frac{\partial X_1}{\partial P_1}\right)\frac{P_1}{X_1}\right] + \frac{P_2 X_2}{P_1}\left(\frac{\partial X_2}{\partial P_1} \frac{P_1}{X_2}\right)$$

$$+ \frac{P_3 X_3}{P_1}\left(\frac{\partial X_3}{\partial P_1} \frac{P_1}{X_3}\right) + \dots + \frac{P_n X_n}{P_1}\left(\frac{\partial X_n}{\partial P_1} \cdot \frac{P_1}{X_n}\right)$$

Multiply entire term by P_1 & div by M

P_1's will be eliminated

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$$0 = \frac{P_1 x_1}{M} + \frac{P_1 X_1}{M}\left[\left(\frac{\partial X_1}{\partial P_1}\right)\frac{P_1}{X_1}\right] + \frac{P_2 X_2}{M}\left[\frac{\partial X_2}{\partial P_1}\frac{P_1}{X_2}\right]$$

$$+ \frac{P_3 X_3}{M}\left[\frac{\partial X_3}{\partial P_1}\cdot\frac{P_1}{X_3}\right] + \cdots + \frac{P_n X_n}{M}\left[\frac{\partial X_n}{\partial P_1}\frac{P_1}{X_n}\right]$$

$$\boxed{S_1 \mathcal{E}_{11} + S_2 \mathcal{E}_{21} + S_3 \mathcal{E}_{31} + \cdots + S_n \mathcal{E}_{n1} = -S_1}$$

Cournot Aggregation

weighted sum of all own-price & cross-price elasticities is equal to the negative share of commodity 1.

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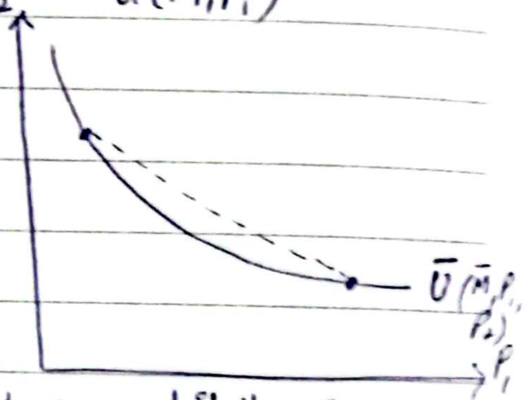
5th Property - INDIRECT UTILITY FUNCTION
IUF is Quasi Convex

Level curve is which along which value of function is constant

Taking two points on level - if linear combination of that two points is not preferred over original IC than it is Quasi Convex



⇒ u along IC is greater than u along the points showing linear combinations of two points on IC curve.

⇒ Higher the curve, lower the utility func

Indirect indifference curve

$$\boxed{u = x_1^2 x_2^2}$$

$$x_1 = \frac{M}{2P_1}, \quad x_2 = \frac{M}{2P_2}$$

$$\boxed{IUF = \frac{M^4}{16P_1^2 P_2^2}}$$

① IUF is H.O.D = zero in prices and income

$$\frac{(kM)^4}{16(kP_1)^2(kP_2)^2} = \frac{k^4}{k^4}\left[\frac{M^4}{16P_1^2P_2^2}\right] = k^0\left[\frac{M^4}{16P_1^2P_2^2}\right]$$

② IUF is decreasing in price

if differentiated w.r.t P - ans is -ve

if differentiated w.r.t M - ans is +ve

$$V = \frac{M^4}{16 P_1^2 P_2^2} = \frac{M^4 (P_1)^{-2}}{16 P_2^2}$$

$$\frac{\partial V}{\partial P_1} = \frac{-2 M^4 (P_1)^{-2-1}}{16 P_2^2} = -\frac{2 M^4 P_1^{-3}}{16 P_2^2}$$

$$= -\frac{M^4}{8 P_1^3 P_2^2}$$

$$\Rightarrow \frac{\partial V}{\partial P_1} < \text{zero}$$

(3) $$V = \frac{M^4}{16 P_1^2 P_2^2}$$

$$\frac{\partial V}{\partial M} = \frac{4M^3}{16 P_1^2 P_2^2} = \frac{1}{4} \cdot \frac{M^3}{P_1^2 P_2^2}$$

$$\frac{\partial V}{\partial M} = \frac{M^3}{4 P_1^2 P_2^2} > \text{zero}$$

(4) <u>Roy's Identity</u>

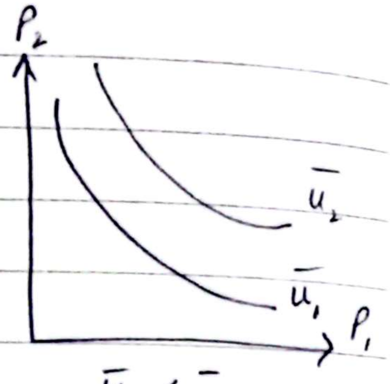
$$\frac{\partial V / \partial P_1}{\partial V / \partial M} = \frac{-M^4 / 8P_1^3 P_2^2}{M^3 / 4 P_1^2 P_2^2} = -\frac{M^4}{8 P_1^3 P_2^2} \cdot \frac{4 P_1^2 P_2^2}{M^3}$$

$$= \frac{-M}{2 P_1}$$

$\Rightarrow$ MDF (with a -ve sign)

(5) Higher IC yields less utility.

$$IUF = \frac{M^4}{16P_1^2P_2^2} \rightarrow \text{Quasi convex}$$



$\bar{u}_2 < \bar{u}_1$

Let $V = 20$

$M = 10$

$$\Rightarrow 20 = \frac{10,0000}{16P_1^2P_2^2}$$

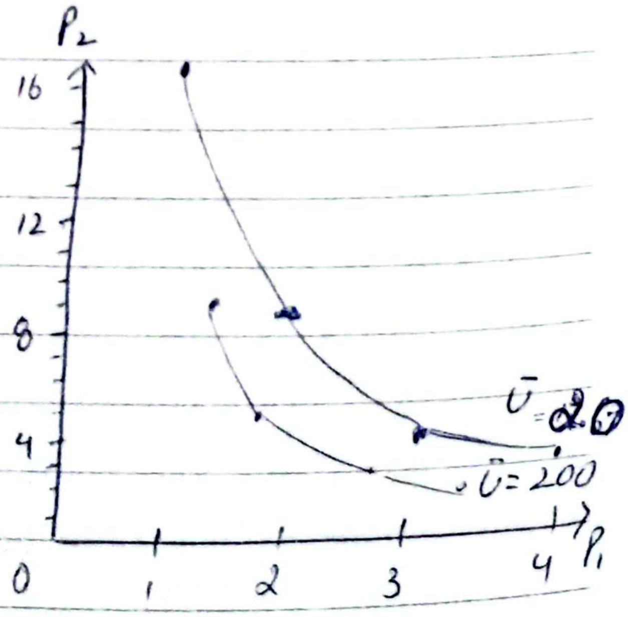
$$\Rightarrow \sqrt{P_1^2P_2^2} = \sqrt{312.5}$$

$$\Rightarrow \boxed{P_1P_2 = 17.67}$$

$$P_2 = \frac{17.67}{P_1}$$

P_1	P_2	$u = 17.67$
1	17.67	
2	8.83	
3	5.89	
4	4.41	

UN



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Now $Y = 200$ $M = 10$

$\Rightarrow 200 = \frac{10,0000}{16 P_1^2 P_2^2}$

$\Rightarrow P_1^2 P_2^2 = 31.25$

$\Rightarrow \boxed{P_1 P_2 = 5.59}$

P_1	P_2
1	5.59
2	2.79
3	1.86
4	1.39

$U = 5.59$

EXPENDITURE MINIMIZATION

Objective Function : $M = \sum_{i=1}^{n} P_i X_i$

Constraint : $\bar{u} = U(X_i)$

$\mathcal{L} = \sum P_i X_i + \lambda[\bar{U} - u(X_i)]$

$\frac{\partial \mathcal{L}}{\partial X_1} = 0$ $\quad P_1 - \lambda^* \partial u / \partial X_1 = 0$ — (I)

$\frac{\partial \mathcal{L}}{\partial X_2} = 0$ $\quad P_2 - \lambda^* \partial u / \partial X_2 = 0$ — (II)

⋮

$\frac{\partial \mathcal{L}}{\partial X_n} = 0$ $\quad P_n - \lambda^* \partial u / \partial X_n = 0$ — (III)

$\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ $\quad u - U(X_i) = 0$ — (IV)

$P_1 = \lambda^* u_1 \Rightarrow \lambda^* = P_1 / u_1$

$P_2 = \lambda^* u_2 \Rightarrow \lambda^* = P_2 / U_2$

⋮

$P_n = \lambda^* U_N$

$u = u(X_i)$

$\boxed{\lambda^* = \frac{1}{\lambda}}$

In UM
$\lambda = u/P$
Here
$\lambda = P/u$
reciprocal of each other

F.O.C $\frac{P_1}{U_1} = \frac{P_2}{U_2}$

$\frac{u_2}{u_1} = \frac{P_2}{P_1}$

(Slope of IC) MRS = slope of BC

⇒ On one side there are prices, which are not our self evaluation (individuals have no influence) – determined in market

⇒ Other (u_2/u_1) is subject – individual evaluation

⇒ Eqm → Mkt Evaluation = self Evaluation

Assignment

S.O.C → same

Border Hessian will be positive.

⇒ MDF are UM demand functions (where M is held constant)

⇒ Hicksian DF are EM demand functions (where utility is held constant.

Example: $u = x_1^2 x_2^2$

$$\mathcal{L} = P_1 x_1 + P_2 x_2 + \lambda^* [\bar{u} - x_1^2 x_2^2]$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \mathcal{L}_1 = P_1 - 2\lambda^* x_1 x_2 \quad \text{— (i)}$$

$$\mathcal{L}_2 = P_2 - 2\lambda^* x_1^2 x_2 \quad \text{— (ii)}$$

$$\mathcal{L}_\lambda = u - x_1^2 x_2^2 \quad \text{— (iii)}$$

Taking Ratio of (i) & (ii)

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(M) (T) (W) (T) (F)

$$\frac{P_1}{P_2} = \frac{X_2}{X_2}$$

$$\rightarrow X_1 = \frac{P_2 X_2}{P_1}$$

Expenditure on both commodities is the same when utility func is symmetric (weight of both is same in utility function. - equally preferred

$$P_1 X_1 = P_2 X_2$$

Substituting in (iii)

$$\bar{u} = \frac{P_2^2 X_2^2}{P_1^2} X_2^2 = 0$$

$$\bar{u} = X_2^4 \frac{P_2^2}{P_1^2}$$

Taking square root (2 times)

$$(\bar{u})^{1/4} = X_2 \left[\frac{P_2}{P_1}\right]^{1/2}$$

$$\boxed{X_2 = \frac{(\bar{u})^{1/4}}{(P_2/P_1)^{1/2}}}$$

Expenditure Minimization

$$\mathcal{L} = \sum_{i=1}^{n} P_i X_i + \lambda^* [\bar{u} - u(X_i)]$$

F.O.C

$$\frac{\partial \mathcal{L}}{\partial X_i} = P_i - \lambda^* u' = 0 - \wedge \text{eq}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda^*} = \bar{u} - u(X_i) = 0$$

Hicksian Demand Function

Properties of H.D.F

(1) H.O.D = zero in prices

$$X_i = X_i(P_i, \bar{u})$$
$$= X_i(kP_i - \bar{u})$$
$$= k^0 [X_i(P_i, u)$$

(2) HDF satisfy the utility constraint

$$\bar{u} = u(X_i)$$
$$\bar{u} = u[x_1(P_i, \bar{u}), x_2(P_i, \bar{u}) \ldots X_n(P_i, \bar{u})]$$

Expenditure Function

If we substitute Hicksian DF in our objective function we'll get Expenditure function like we got IUF by putting marshallian in utility maximization

Expenditure Function is obtained by substituting held constant / HDF in the Budget equation

$$M = \Sigma P_i X_i$$
$$\boxed{M = \Sigma P_i [X_i(P_i, \bar{u})]}$$ expenditure function

IUF → maximum achieveable utility at the given prices & income

* Minimum expenditure at given $\bar{u}$

EF tells us the minimum expenditure required to achieve the given level of utility.

$$e = e(P_i, \bar{u})$$

If we have budget equation

$$M = P_1 X_1 + P_2 X_2 + \dots P_n X_n$$

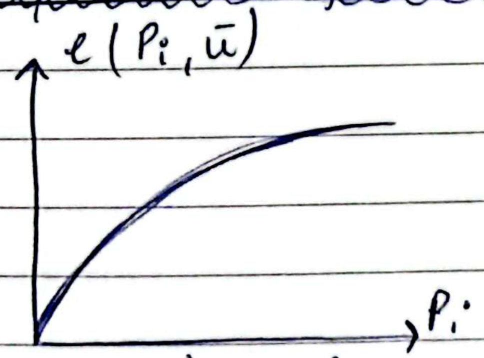
converting into expenditure function

$$e = P_1 [X_1(P_i, \bar{u})] + P_2 [X_2(P_i, \bar{u})] + \dots + P_n [X_n(P_i, \bar{u})]$$

Expenditure function

① Expenditure function is H.O.D = one in prices
if all prices change by same propotion, to keep given utility constant, expenditure will also rise by the same propotion

② Expenditure function is concave



⇒ Prop Δ in expenditure will be less than prop Δ in price of a commodity when all other prices are held constant.

increasing at decreasing rate.

Property (1) states that if prices e.g ↑es by 5% of all commodities then exp will also ↑ by 5%.

Property (2) states that if price of only one commodity changes among all then the change in expenditure will be less than the change in prices

$\phi\ \lambda\left[e\{p',\bar{u}\}\right]+(1-\lambda)\left[e\{p'',\bar{u}\}\right]$

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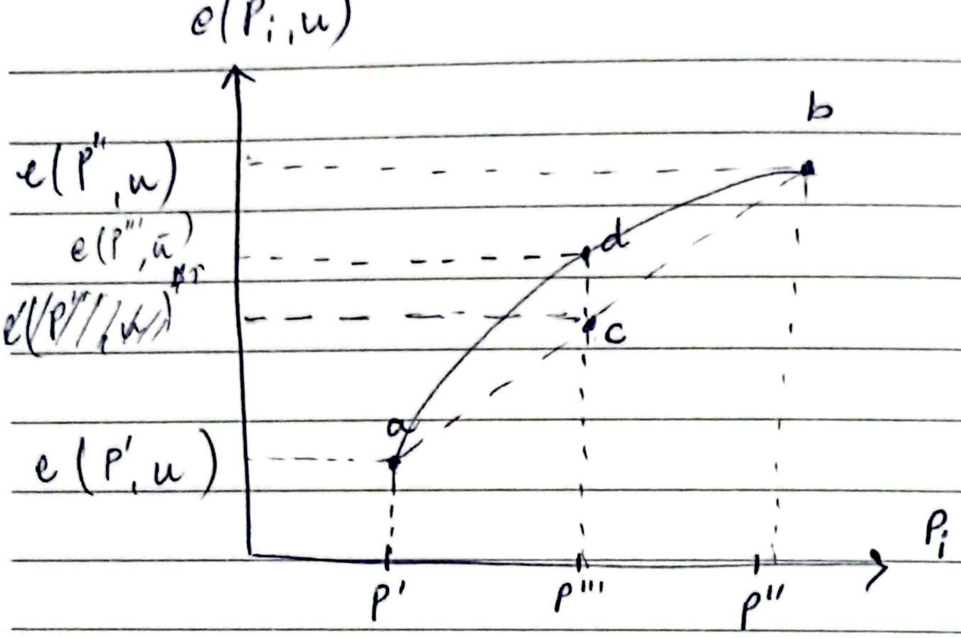

on c - straight
above c - concave
below c - convex

P''' is a price which is somewhere between the two other prices.

$P''' \to$ linear combo of P' & P''

$P''' = \lambda P' + (1-\lambda)(P'')$

Let P' and P'' be two price vectors such that x' & x'' are corresponding quantity where expenditure is minimum

$P'x' \to e(P',\bar{u})$

$P''x'' \to e(P'',\bar{u})$

Lets define P''' as a linear combination of P' and P''

$P''' = \lambda P' + (1-\lambda)P''$

Corresponding to P''', the expenditure minimizing quantity is x'''

$P'''x''' \to e(P''',\bar{u})$

$\boxed{P'x' < P'x'''}$ — (1)

$$\boxed{p''x'' < p''x'''}$$ ① Now we're disturbing the expenditure minimize

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combinations

$$\lambda(p'x') + (1-\lambda)p''x'' < \lambda p'x''' + (1-\lambda)p''x'''$$

$$\lambda[p'x'] + (1-\lambda)[p''x''] < [\lambda p' + (1-\lambda)p'']x'''$$

$$\lambda[p'x'] + (1-\lambda)[p''x''] < p'''x'''$$

$$\boxed{\lambda[e(p',\bar{u})] + (1-\lambda)[e(p'',\bar{u})] < e(p''',\bar{u})}$$

③ Expenditure function is increasing in prices

$$\left.\frac{e(P_i,\bar{u})}{\partial P_i}\right|_{\bar{u}} > \text{zero}$$

$$e = P_1[x_1(P_i,\bar{u}) + P_2[x_2(P_i,\bar{u})] + \dots + P_n[x_n(P_i,\bar{u})]$$

Differentiate w.r.t P_1 {Any derivative w.r.t P will be +ve}

$$\frac{\partial e}{\partial P_1} = x_1(P_i,\bar{u}) + \frac{\partial x_1}{\partial P_1}\underset{\lambda^* u_1}{\underset{\downarrow}{P}} + \frac{\partial x_2}{\partial P_1}\underset{\lambda^* u_2}{\underset{\downarrow}{P_2}} + \dots \frac{\partial x_n}{\partial P_1}\underset{\lambda^* u_n}{\underset{\downarrow}{P_n}}$$

According to F.O.C

$$P_1 = \lambda u'$$

$$\Rightarrow \frac{\partial e}{\partial P_1} = x_1(P_i,\bar{u}) + \lambda^*\left[\frac{\partial x_1}{\partial P_1}u_1 + \frac{\partial x_2}{\partial P_1}u_2 + \dots + \frac{\partial x_n}{\partial P_1}u_n\right] \quad \text{—— ⓐ}$$

consider $u = u(x_i)$ → This is the constraint here
$u = u(x_1, x_2 \dots x_n)$

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$$\frac{du}{dP_1} = \frac{\partial u}{\partial x_1} \cdot \frac{\partial x_1}{\partial P_1} + \frac{\partial u}{\partial x_2} \cdot \frac{\partial x_2}{\partial P_1} + \dots + \frac{\partial u}{\partial x_n} \cdot \frac{\partial x_n}{\partial P_1}$$

Comparing with equation (†), it is the same term in the bracket.

$$0 = u_1 \frac{\partial x_1}{\partial P_1} + u_2 \frac{\partial x_2}{\partial P_1} + \dots + u_n \frac{\partial x_n}{\partial P_1}$$

$$\boxed{\frac{\partial e}{\partial P_1} = x_i(P_i, \bar{u})} \to \text{Shephard Lemma}$$

⇒ If expenditure function is differentiated w.r.t P_1 it, will give Hicksian Demand Function of i^{th} commodity

$$\frac{\partial e}{\partial u} > \text{zero}$$

e = obj function, u = constant

Langragian (def): It is Δ in the optimal value of obj function due to given change in the constant.

$$\frac{\partial e}{\partial u} = \lambda^*$$

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$\boxed{\begin{matrix} M = 100 \\ x_i = 15 \end{matrix}}$ Put it here

$x_i(P_i, M)$ - Utility: max $u = 50$

$\downarrow$

$\boxed{c(P_i, u)}$

$\downarrow$

exp. min $\cong$ HDF

$u = 50$

$x_i(P_i, u)$ — H.D.F

$\downarrow$

constraint

$\downarrow$

IUF

$u^*(P_i; M)$

Hicksian DF $\rightarrow$ substitution Effect Slutsky Equation

x_i

19/12/2024

~~Especially for durable commodities & insurance~~ is

Income t - Budget constraint

To be consumed

To be saved

$C_t = f(Y_t \pm i)$

Overtime Consumption Model

Two period Model (Current & Future)

we have the option of borrowing & lending

Two Assumptions

(a) two period model

(b) Perfectly competitive market with given price of lending & borrowing

x_0 = Current Consumption $\rightarrow x_0 > 0$

x_1 = Future Consumption $\rightarrow x_1 > 0$

w_0 = current Wealth $w_0 > 0$

w_1 = Future Wealth. $w_1 > 0$

A = Amount lended or borrowed   A > 0 (borrowing)
A < 0 (lending)
A = 0 (neither borrowing nor lending)

The Model

$X_0 = W_0 + A$ — (i)   if borrowed (+ve)
if lended (-ve)
if nothing (0)

$x_1 = w_1 - A$

r = Rate of borrowing / lending

$$x_1 = w_1 - A(1+r) \quad \text{— (ii)}$$

(i) - current consumption
(ii) - future consumption

$$\text{(i)} \Rightarrow X_0 - W_0 = A$$

Substitute in eq (ii)

$$X_1 = w_1 - [X_0 - w_0](1+r)$$

$$x_1 = w_1 - x_0(1+r) + w_0(1+r)$$

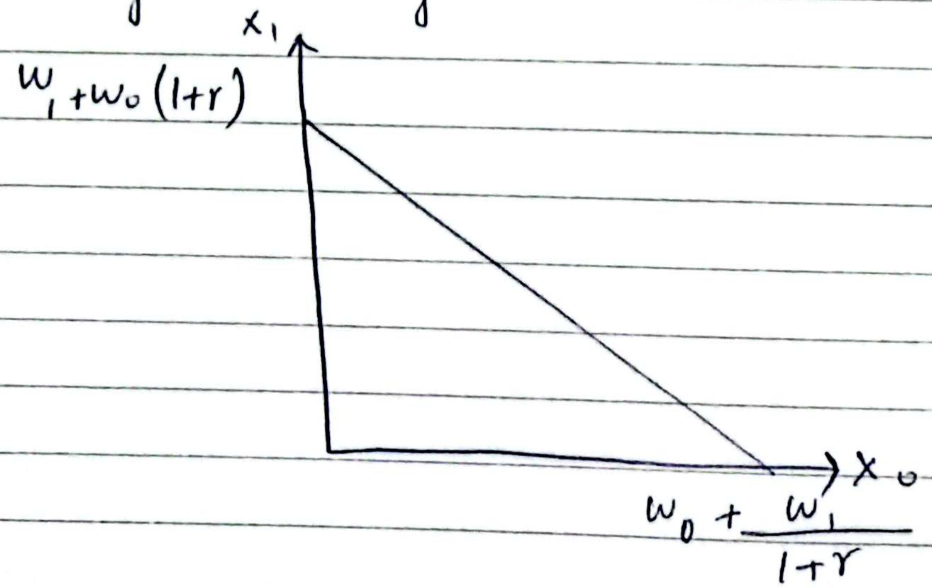
Taking consumptions on one side

$$x_1 + x_0(1+r) = w_1 + w_0(1+r)$$

x_1 is for future and r is more associated with the future

$$\boxed{\frac{x_1}{1+r} + x_0 = \frac{w_1}{1+r} + w_0}$$ Budget constraint

Drawing the budget line



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Working Intercepts

$$\boxed{X_0 = 0 \text{ (for vertical intercepts)} \quad X_1 = w_1 + w_0(1+r)}$$

$$\boxed{X_1 = 0 \text{ (for horizontal intercept)} \quad X_0 = \frac{w_1}{1+r} + w_0}$$

Working out slope of the budget line

$$\text{Slope} = -\frac{\text{Perpendicular}}{\text{Base}}$$

$$= -\frac{w_1 + w_0(1+r)}{w_0 + \frac{w_1}{1+r}}$$

$$= -\frac{w_1 + w_0(1+r)}{\frac{w_0(1+r) + w_1}{1+r}} = -(1+r)$$

$$\boxed{\text{Slope} = -(1+r)}$$

To keeps things simple, lending & borrowing rates are assumed to be same.

Utility Function $U(x_0, x_1) = \underbrace{u(x_0) + u(x_1)}$

Based on two assumptions

a) Utility Function is additive

Total utility depends on what is consumed today plus what is consumed in future (not interdependent)

CamScanner

(b) functional forms are identical

Same utility functions for same consumption bundles

Optimization

$$\mathcal{L} = u(x_0) + u(x_1) + \lambda\left[\frac{x_1}{(1+r)} + x_0 - \frac{w_1}{1+r} - w_0\right]$$

F.O.C

marginal utility

$$\frac{\partial \mathcal{L}}{\partial x_0} = 0 \Rightarrow u'(x_0) + \lambda = 0 \quad \text{— (i)}$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0 \Rightarrow u'(x_1) + \frac{\lambda}{1+r} = 0 \quad \text{— (ii)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \frac{x_1}{1+r} + x_0 - \frac{w_1}{1+r} - w_0 = 0 \quad \text{— (iii)}$$

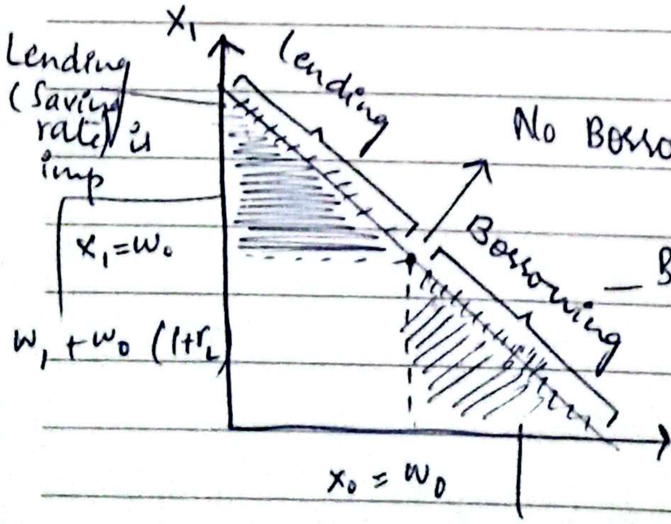
Ratio of (i) & (ii)

$$\frac{u'(x_0)}{u'(x_1)} = \frac{-\lambda}{-\lambda/1+r}$$

$$\Rightarrow \frac{u'(x_0)}{u'(x_1)} = \underbrace{-(1+r)}$$

$$MRS_{x_0,x_1} = \text{Slope of Budget line}$$

Slope of BL is tangent to IC, their slopes are equal



No Borrowing, No Lending

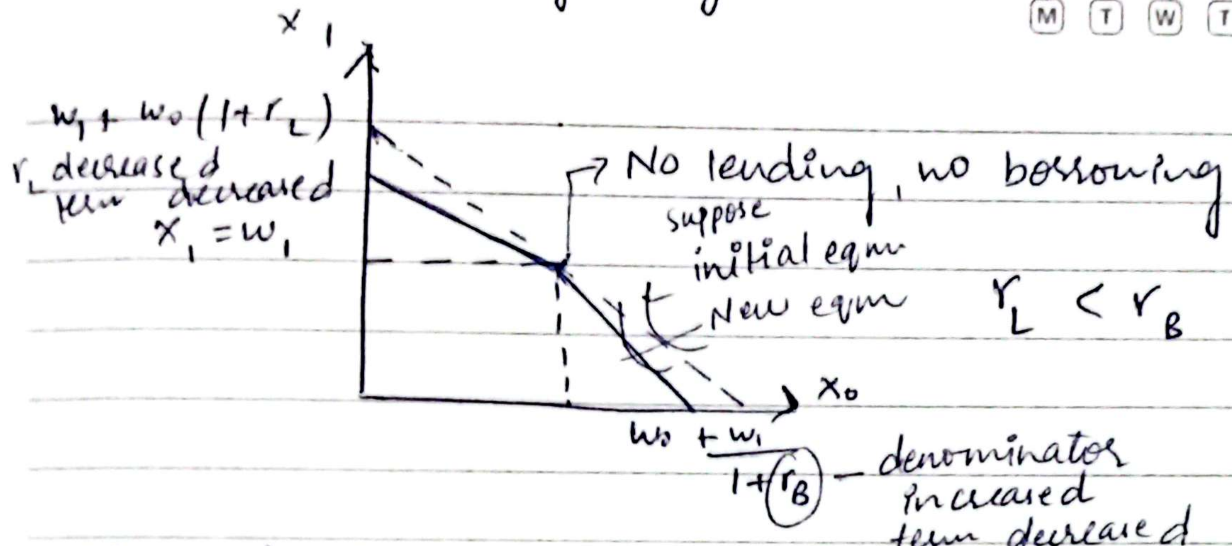
— Borrowing Rate is imp

$$r_L = r_B$$

Zone of borrower

$$w_0 + \frac{w_1}{1+r_B}$$

But normally borrowing rate is always higher



Normally the BL is kinked — outward

Assumption (a) is quite realistic — Utility function is additively separable
Assumption (b) is not valid in some cases

Time preferences are important — it may be possible that sometime future consumptions is more realistic than current consumption

→ This utility function is not incorporating time preference

Incorporating time preferences : (Irving Fisher Intertemporal choice)

$$u(x_0, x_1) = u(x_0) + \frac{u(x_1)}{u(1+\rho)}$$

$\rho = 0$ (No time preferences)
$\rho > 0$ (Same amount of good will give more utility in current period rather than the future)
$\rho < 0$ (Same amount of good will give more utility in future rather than current)

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Idea:

A Special Case: fixed amount of wealth in current year & has to spend it over the course of his lifetime.

"Fixed wealth in current period which is to be spend overtime".

W_0 = Wealth in current period

X_0 = Consumption in current period

X_1 = Future consumption (investment)

$$X_1 = \underbrace{W_0 - X_0}_{\text{plan for future}}$$

Now it is not kept in a locker consumer invests it.

v. close to model of inheritance

→ We keep small amount with us a a precautionary measure (it is not invested)

→ Now for investment there are always two option — risky & non-risky

Returns are always related with uncertainity

→ A rational consumer should always form a diverse portfolio - some risky & some relatively safe

Return low but certainity OR Return High but uncertainity.

Three parts of $\rightarrow$ $\boxed{W_0 - X_0}$ — Risky Investments

Precautionary Measure

Safe Projects

It can be more than 3 as well

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$$X_1 = W_0 - X_0$$ — High Risk w/ high return (R_H) — Low Risk w/ low return (R_L)

Total unconsumed wealth $= W_0 - X_0$ — y, $1-y$ $\quad 0 \leq y \leq 1$

y = Proportion of $(W_0 - X_0)$ spend on risky projects

We haven't incorporated the precautionary amount, but it is possible to include it in model

— No return (y_1)
— low return (y_2)
— High return ($1-y_1-y_2$)

$$X_1 = [(W_0 - X_0)y]R_H + [(W_0 - X_0)(1-y)]R_L$$

$$u(X_0, X_1) = u(X_0) + \frac{u(X_1)}{\rho}$$

$\therefore$ it is additively separable

$\therefore$ Functional form is identical.

If $\rho = 1$

Functional form is same for both, same amount of good is giving more utility in both periods

$\rho = \phi$

$\rho > 1 \quad \rho < 1$

If $\rho \neq 0$

current or future, one has more utility in a certain period.

Varian
Ch 11 consumption over time

Future utility is actually expected utility
$E[u(X_1)]$

Varian has taken ρ in numerator so interpretation will be diff

$$u(X_0, X_1) = u(X_0) + \frac{E[u(X_1)]}{\rho}$$

$$u(X_0, X_1) = u(X_0) + \frac{1}{\rho} E\left[u\left(\{W_0 - X_0\}y\right\}R_H + \{(W_0 - X_0)(1-y)\}R_L\right]$$

$$u(X_0, X_1) = u(X_0) + \frac{1}{\rho} E\left[u\left[\{(W_0 - X_0)y\}R_H + \{(W_0 - X_0)(1-y)R_L\}\right]\right]$$

$\partial u / \partial X_0 = 0$

$$u(X_0, X_1) = u(X_0) + \frac{1}{\rho} E\left[u'\left[\text{''}\right]\right] = 0$$

$(W_0 - X_0)^{\dagger}(W_0 - X_0)(-y)$

$\frac{\partial u}{\partial y} = 0$

Model given in Varian chapter
In Exercise - model is modified
DIY

For 3 choice variables (imp for exam)

$\frac{\partial u}{\partial X_0}, \frac{\partial u}{\partial y_1}, \frac{\partial u}{\partial y_2}$

↳ if some is kept as precaution then something is kept as a constant

$\partial u / \partial y_3$

CHOICE UNDER UNCERTAINITY :

uncertain situation very closely resemble a lottery or a gamble
choice under uncertainity is discussed under the framework of lottery / gamble
whenever participating in a gamble, first "possibility of winning" is considered but along with it also the "amount which you'll win or lose"
That is how expected value of a lottery is calculated.

$$E(L) = p_1(x_1) + p_2(x_2) \qquad \text{coin}$$

$$= \frac{1}{2}(10) + \frac{1}{2}(-10) = 0$$

Gamble
- Fair Gamble → $E(V) = 0$
- Favorable Gamble → $E(V) = +ve$
- Unfavorable Gamble → $E(V) = -ve$

$$E(L) = p_1(x_1) + p_2(x_2)$$

$$= \frac{1}{2}(200) + \frac{1}{2}(-10) = 95$$

probability is always b/w 0 & 1, can't be 0 or 1 else it'll be a certain situation

Certain Outcome (CO) compare Expected (EO) outcome

10   95

There are 3 possibilities now

$\boxed{CO = EO}$ → Risk averse will avoid gamble, He will prefer CO over EO of same value.

$CO < EO$

$CO > EO$

Let CO = 10 – it will have some utility
u(CO)

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For Risk averse, utility of CO is greater than EO even if EV of EO = CO

For Risk averse,

$$u(CO) > u(EO)$$
$$\text{if} \quad EV(EO) = CO$$

For Risk Neutral,

$$u(CO) = u(EO)$$
$$\text{if} \quad EV(EO) = CO$$

For Risk Lover,

$$u(CO) < u(EO)$$
$$\text{if} \quad EV(EO) = CO$$

Example

$p_1 = 0.50$ $\quad$ $p_2 = 0.50$

$x_1 = 100$ $\quad$ $x_2 = 33.33$

$$EV = (0.50)(100) + (0.50)(-33.334)$$
$$= 50 - 16.667$$
$$\boxed{EV = 33.33}$$

$EV = (0.50)(100) + (0.5)$

$X = 50 + 0.50(-x)$

$X + 0.50X = 50$

$1.5X = 5$

$X = \frac{50}{1.5}$

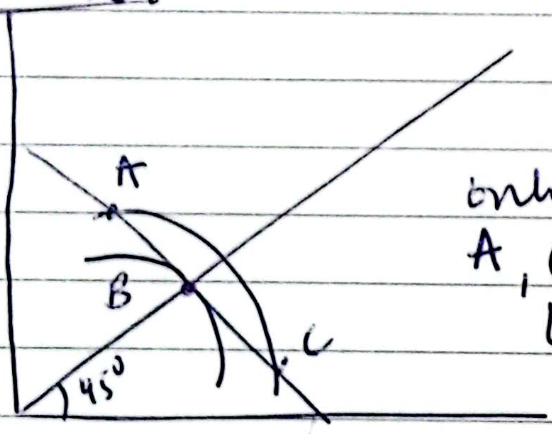
$X = 33.334$

Risk lover

A>B

C>B

B is certain



Budget Line

$x_1 = x_2$ (Line of Certainty)

A, B, C are all on BL

only B is certain

A, C behaviour of Risk Lovers

Higher IC higher satisfaction

Two same outcome
Risk averse – choose certain one
Risk neutral – indifferent
Risk lover – uncertain

BlackGold
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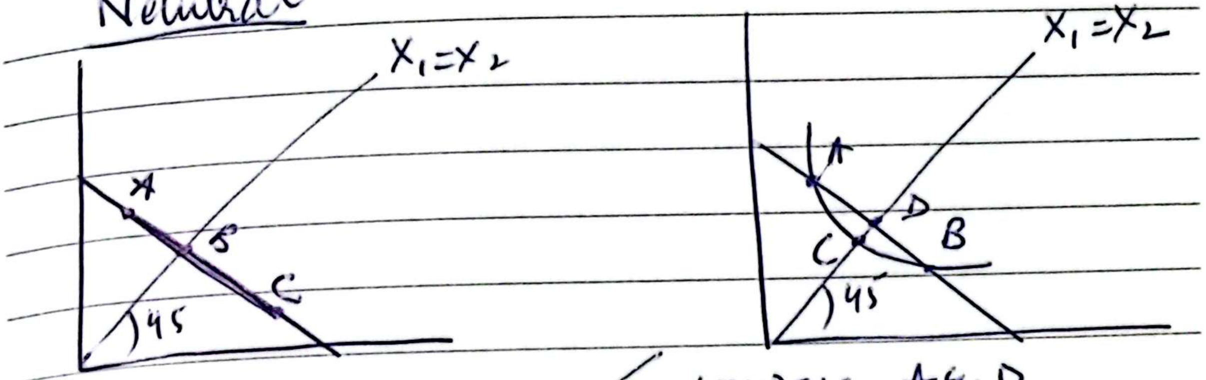
Neutral



compare A & D
A < D (higher IC)
B < D & there is no uncertainity in D

A < C
B < C (in C there is certainity)

In case of this IC, consumer is indifferent b/w A, C, B. But consumer always prefer some sort of certain outcome over uncertain one
→ This is not an accurate representation of situation.

⇒ The IC of a consumer under uncertainity is not only "convex" but it will also make a "tangent with the line of uncertainity".

For Risk lover → MRS ↑es
Risk averse → MRS diminishing (IC should be tangent to the BL) where MRS is diminishing

$u(x_0, x_1)$

$$\boxed{\bar{u} = p_0(x_0) + p_1(x_1)}$$

Diff b/w old & new utility

1. Here probabilities are included
2. Here utility function is additive

Expected Value

$u(\bar{u}) = u(EV)$

if $\bar{u} = EV$

$$\bar{u} = [p_0 x_0, p_1 x_1]$$

where $x_1 = x_1(x_0)$

$$d\bar{u} = \underbrace{\left(\frac{\partial u}{\partial x_0}\right)}_{(u_0)} p_0 + \underbrace{\left(\frac{\partial u}{\partial x_1}\right)}_{(u_1)} \frac{dx_1}{dx_0} p_1$$

$$0 = u_0 p_0 + u_1 p_1 \frac{dx_1}{dx_0}$$

before $u_{11} \to +ve$

$\boxed{\text{strictly } u_{12} \to 0}$ can't be +ve or -ve.

$$\boxed{\frac{dx_1}{dx_0} = -\frac{u_0 p_0}{u_1 p_1}}$$

Take derivative (DIY)

$\frac{d^2 x_1}{dx_0^2} > 0$ $MRS_{0,1}$ diminishing (Risk averse)

$\frac{d^2 x_1}{dx_2^2} < 0$ $MRS_{0,1}$ increasing (Risk lover)

$\frac{d^2 x_1}{dx_1^2} = 0$ $MRS_{0,1}$ constant (Risk Neutral)

$u_{12} = 0$ - we've diff time periods

solve DIY

Three cases

MU — u_1 $\oplus$ u_2 $\oplus$ $u_{12} = 0$

u_{11}, u_{22} — Behaviour of MU can be +ve, −ve, 0

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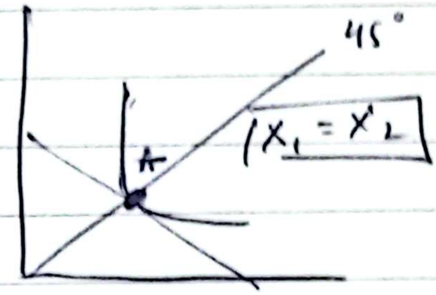
DIY

At the end, the simplified equation will represent the point



$\leftrightarrow$

$$\bar{u} = [P_0 X_0 + P_2 X_2] \quad \text{where } x_1 = x_1(x_0)$$

$$\partial \bar{u} = \frac{\partial u}{\partial X_0} P_0 + \left(\frac{\partial u}{\partial X_1}\right) P_1 \left(\frac{\partial X_1}{\partial X_0}\right)$$

$$0 = u_0 P_0 + u_1 P_1 \frac{\partial x_1}{\partial x_0}$$

$$u_1 P_1 \frac{\partial x_1}{\partial x_0} = -u_0 P_0$$

$$\frac{\partial x_1}{\partial x_0} = -\frac{u_0}{u_1}\frac{P_0}{P_1}$$

$$\frac{\partial^2 x_1}{\partial x_0^2} = -\frac{u_0 P_0 (x_0, x_1(x_0))}{u_1 P_1 (x_0, x_1(x_0))}$$

$$\frac{\partial^2 x_1}{\partial x_0^2} = \frac{u_1 P_1 \left\{\left(\frac{\partial u_0 P_0}{\partial x_0} + \frac{\partial u_0 P_0}{\partial x_1}\left(\frac{\partial x_1}{\partial x_0}\right)\right)\right\} - u_0 P_0 \left\{\frac{\partial u_1 P_1}{\partial x_0} + \frac{\partial u_1 P_1}{\partial x_1}\left(\frac{\partial x_1}{\partial x_0}\right)\right\}}{(u_1 P_1)^2}$$

Same thing is available for consumption in different states — Concept of contingent commodities
"Same physical good is available for consumption in different states of world."
If analyzed empirically, it is something very close to a lottery.

Whenever we going to a consume or purchase a contingent commodity, its just like a lottery, & it will have multiple possibilities.
⇒ where there is a probability of bad outcome but cost is low so risk will be taken by the consumer.

* Define Lottery
* Axioms of Lottery

If all axioms are fulfilled, there is a well behaved utility function

DEFINING LOTTERY:- $L_j = [(Y_i, P_i)\ i = 1,2,3,\ldots n]$

Y = Outcome
P_i = corresponding probability

$$\sum_{i=1}^{n} P_i = 1$$

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AXIOMS OF OUTCOMES:

(1) **Completeness** → Ranking of outcomes is possible

$Y_i > Y_j$ $\qquad$ $Y_j > Y_i$ $\qquad$ $Y_i \sim Y_j$

if $Y_i > Y_j \Rightarrow Y_i > Y_j$

if $Y_i < Y_j \Rightarrow Y_j > Y_i$

if $Y_i = Y_j \Rightarrow Y_i \sim Y_j$

(2) **Transitivity** if $Y_1 > Y_2$ and $Y_2 > Y_3$
then transitivity implies that $Y_1 > Y_3$

(3) **Non-Satiation** More is preferred over less
MU of $Y_i > 0$

AXIOMS OF LOTTERY:

(1) Complete Ordering
(2) Transitive Ordering
(3) Strong Independence
(4) Independent Ordering
(5) Standard Lottery
(6) Compound Lottery.

(1) **Complete Ordering:**

L_i & L_j

$L_i > L_j$ OR $L_j > L_i$ OR $L_i \sim L_j$

If asked between multiple options to participate in a lottery, one should be able to answer with clarity & that is only possible if lotteries are ranked.

(2) **Transitive Ordering:** whatever the ranking is ~~done~~ we're consistent in it.

If $L_i > L_j$
and $L_j > L_w$
$\Rightarrow L_i > L_w$

order doesn't matter

(3) **Strong Independence:** let there be two lotteries, identical in all aspects except one outcome

$L_1 = [(Y_1, p_1), (Y_2, p_2)]$

$L_2 = [(Y_1, p_1)\ (Y_3, p_3)]$

If given two lotteries are identical except one outcome & you're asked to rank the lotteries, the ranking will depend solely on your preference of the different outcome.

if $Y_2 > Y_3$
$\Rightarrow L_1 > L_2$ OR
if $Y_3 > Y_2$ $\qquad Y_2 \sim Y_3$
$\Rightarrow L_2 > L_1$ $\qquad \Rightarrow L_2 \sim L_3$

Our outcomes are not complements or substitutes, since it is not so we'll not get them together.

(4) Indepedent Ordering

No concept of substitute or complement

Our consumer is rational - he won't be affected by the convincing tactics) - He will only and focus on the outcomes & probabilities

$$L_{12} = [(Y_1, P_1), (Y_2, P_2)]$$

$$L_{21} = [(Y_2, P_2), (Y_1, P_1)]$$

Outcomes & Probabilities are important not the order in which they are expressed.

Axiom of

(5) Standard Lottery (Imp)

For Example,

$$L[(Y_1, P_1), (Y_2, P_2)]$$

$P_1 = P_2 = 0.50$

$Y_1 = 10{,}000$

$Y_2 = 100$

If Price = 500

$\boxed{Y_2 < 500 < Y_1 \text{ (play)}}$

$\boxed{Y_2 < 1000 < Y_1 \text{ (not play)}}$

Given this lottery a consumer is indifferent b/w participating or keeping an amount of 725

700 ~ 100

at 700 Yes

at 750 No

Above 725 Not play

Below 725 Play

At 725 Indifferent.

⇒ SL rules out discontinuity - shows continuity in consumer behaviour.

$L\left[(\overset{Y_1}{10,000}, \overset{P_1}{0.50})\ (\overset{Y_2}{100}, \overset{P_2}{0.50})\right]$

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Outcomes & Probabilities given - at a certain outcome you becomes indifferent
- At a certain probability you become indifferent.

Let the outcomes are

$$Y_1 > Y_2 > Y_3 > Y_4 \ldots\ldots > Y_n$$

$\downarrow$ Best $\qquad\qquad \downarrow$ Worst

Define a lottery with Best (Y_1) and Worst (Y_n) outcome.

$$L_{1n} = \left[(Y_1, p_1), (Y_n, p_n)\right]$$

$$p_1 + p_n = 1$$

$$Y_1 > Y_n$$

Outcome will be somewhere b/w best & the worst necessarily

At that outcome consumer will be indifferent b/w outcome & participating

Q At what conditions will it be equal to the best or worse?

$L_{1n} = \left[(Y_1, p_1), (Y_n, p_n)\right] \sim Y_1$

P_1 is v high (close to one) $\quad p_1 = 1$

$L_{1n} = \left[(Y_1, p_1), (Y_n, p_n)\right] \sim Y_n$

$p_n = 1$

Since probabilities cannot be 0 or 1 so it has to be b/w best & worse.

// Listen to last point in rec

(6) Axiom of Compound Lottery:

Such a lottery where outcome are itself a lottery e.g if you win, you'll get a prize bond

$$L_{12} = [(L_1, q_1)(L_2, q_2)]$$

$$L_1 = [(Y_1, p_1), (Y_2, p_2)]$$

$$L_2 = [(Y_1', P_1'), (Y_2', P_2')]$$

Now consumer will be interested in learning details of the lottery.

Decision maker while making a decision whether to participate or not, will decide considering the outcome & probabilities of lotteries.

=> If all axioms are fulfilled, there is a well behaved utility function

Expected Utility Hypothesis:

$$p_1\{U(Y_1)\} + p_2\{U(Y_2)\} + p_3\{u(Y_3)\} + \cdots$$

$$+ P_n\{u(Y_n)\}.$$

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Expected Utility Hypothesis

$$u(Y_i) = p_1 u(Y_1) + p_2 u(Y_2) + \dots + p_n u(Y_n)$$
$$i = 1, 2, \dots n$$

Properties

(1) Increasing in Y_i
(2) Continuous in Y_i
(3) It can be either linear convex or concave

(Explanations)

① if $Y_i > Y_j$

$\Rightarrow Y_i > Y_j \Rightarrow u(Y_i) > u(Y_j)$

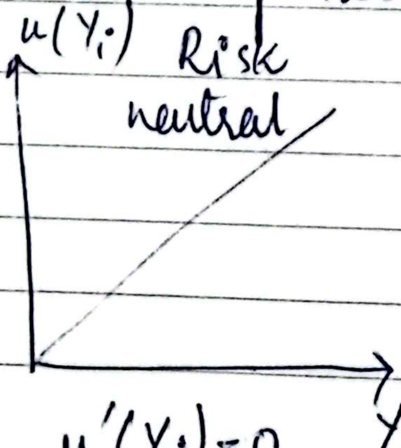
② $\Rightarrow u(Y_i) > 0$
$u'(Y_i) > 0$
$u''(Y_i) \gtrless 0$

③ Shape of expected utility function
$u(Y_i) > 0$ (first Property)

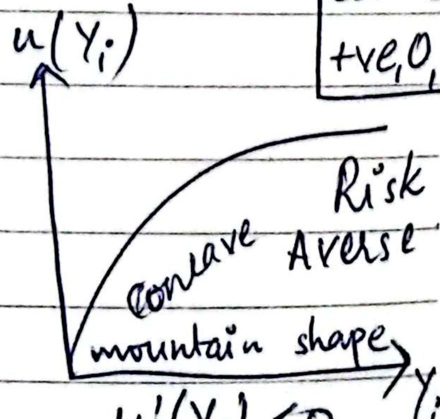
$\Rightarrow$ $u''(Y_i) > 0$ (+ve) $\Rightarrow \uparrow$es at $\uparrow$esing rate
$u''(Y_i) = 0$ (zero) $\Rightarrow \uparrow$es at constant rate
$u''(Y_i) < 0$ (-ve)
$\Downarrow$
$\uparrow$esing at decreasing rate.

2nd derivative is rate of change can be +ve, 0, -ve

$u'(Y_i) > 0$
$u''(Y_i) > 0$



$u'(Y_i) = 0$
$u''(Y_i) = 0$



$u'(Y_i) < 0$
$u''(Y_i) < 0$

→ normal behaviour of everyone is the "Risk Averse" one.

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Childhood
Pocket money = 10
↳ 5 lost, too much loss

Now
Pocket Money = 50
↳ 5 lost, not even felt

With time, income increase so for same amount the utility doesn't remain the same.

For Risk-lover → UF is convex
For Risk-Neutral → UF is linear
For Risk-Averse → UF is concave.

Let there be a lottery with only two outcomes (Y_i & Y_j) such that p_i is the probability of Y_i and P_j is the probability of Y_j

$$L_{ij} = [(P_i, Y_i)(P_j, Y_j)]$$

lets calculate expected value of Y_i & Y_j

$$Y_e = P_i(Y_i) + P_j(Y_j)$$

Lets construct a standard lottery corresponding to L_{ij} i.e,

$$Y_c \cong [(P_i, Y_i), (P_j, Y_j)]$$

such that

$$Y_i < Y_c < Y_j$$

$Y_i = 100$, $Y_j = 1000$, $P_i = 0.3 =$ $P_j = 0.7$

cost of = 500 lottery

500 – Participate
600 – Participate
700 – Do not participate

b/w 600 & 700, there is a number at which player is indifferent b/w participating & not participating
That number is Y_c
Since consumer is indifferent Y_c and L_{ij}

$$u(Y_c) = u(L_{ij})$$

utility of a certain outcome

$$P_i\{u(Y_i) + P_j u(Y_j)\}$$

expected utility of a lottery involving Y_i & Y_j with probabilities P_i & P_j

Comparing $u(Y_e)$ and $u(Y_c)$
expected utility & utility of certain outcome

⇒ Comparing utility of the expected value of lottery with the expected utility of (involving Y_i & Y_j) lottery involving Y_i & Y_j

$$u(Y_e) \rightarrow u[P_iY_i + P_jY_j]$$

$$u(Y_c) \rightarrow P_iu(Y_i) + P_j u(Y_j)$$

There are three possibilities
* Expected utility of lottery ≷ utility of the expected value of lottery

$$\underbrace{P_i u(Y_i) + P_j u(Y_j)}_{u(Y_c)} \qquad \underbrace{u[P_iY_i + P_jY_j]}_{u(Y_e)}$$

① $u[P_i Y_i + P_j Y_j] = P_i u(Y_i) + P_j u(Y_j)$

utility of the expected value of lottery with Y_i & Y_j

Expected utility of lottery with Y_i & Y_j

$u(Y_e) = u(Y_c)$

Decision maker takes lottery at its expected value

② $u[P_i Y_i + P_j Y_j] < P_i u(Y_i) + P_j u(Y_j)$

$u(Y_e) < u(Y_c)$

Decision maker takes lottery below its expected value

③ Decision maker takes lottery above its expected value

$u[P_i Y_i + P_j Y_j] > P_i u(Y_i) + P_j u(Y_j)$

$u(Y_e) > u(Y_c)$

Example: Expected value of lottery = 100 cost = 100

Indifferent → possibility ① Riske neutral

considers cost too much → possibility ② Averse

happily paritiicipate → possibility ③ Lover.

Shape of utility function depends upon individual attitude towards risk

$$L_{i,j} = [(Y_i, p_i), (Y_j, P_j)]$$

Standard lottery $Y_c \cong L_{i,j}[(Y_i, P_i)(Y_j, P_j)]$

$u(Y_c) \cong P_i\, u(u_i) + P_j\, u(u_j) \rightarrow$ Expected utility of lottery

Expected Value of Lottery

$$Y_e = P_i(Y_i) + P_j(Y_j)$$

$u(Y_e) = u[P_i Y_i + P_j Y_j] \rightarrow$ utility of the expected value of lottery

Comparing expected utility of lottery with the utility of the expected value

three possibilities

1. Expected utility of lottery $>$ Utility of the expected value of lottery

$$P_i[u(Y_i)] + P_j[u(Y_j)] > u(Y_e)$$

2. Expected utility of lottery $<$ Utility of the expected value of lottery

$$P_i[u(Y_i)] + P_j[u(Y_j)] < u(Y_e)$$

3. Expected utility of lottery $=$ Utility of the expected value of lottery

$$P_i[u(Y_i)] + P_j[u(Y_j)] = u(Y_e)$$

$\therefore\ Y_e = P_i Y_i + P_j Y_j$

In Previous lecture, "above" & "below" words are used, but concept is same. Date: ...../...../..........

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① Lottery is given more weight as compared to its expected value

└ RISK LOVER

② Lottery is given less weight as compared to its expected value

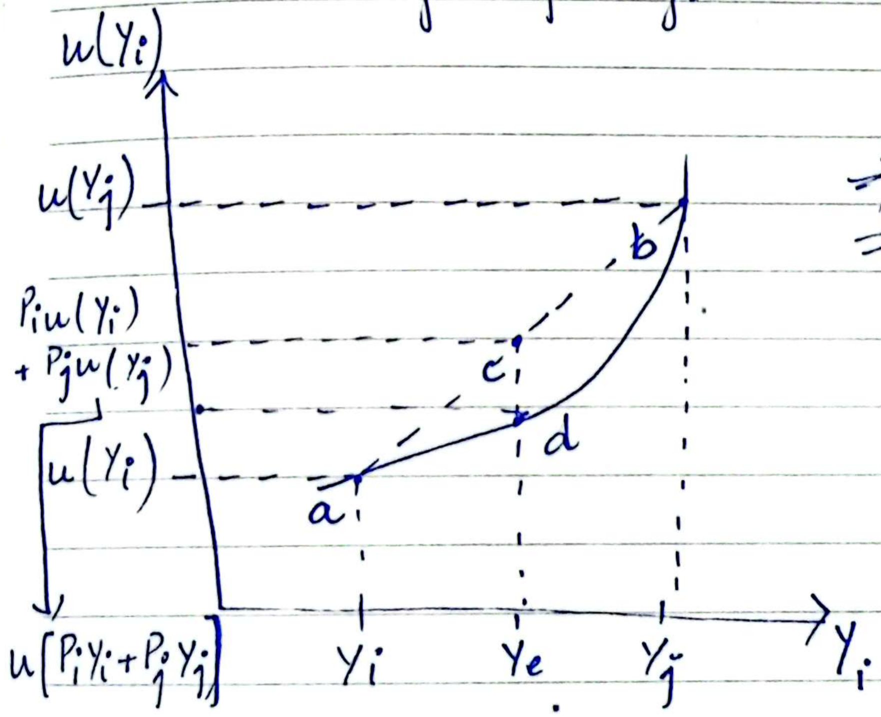
└ RISK AVERSE

③ Lottery is given same weight as of its expected value

└ RISK NEUTRAL

Graphically,

Assuming Y_j is greater than Y_i



→ Convex

⇒ Individual is Risk lover

$$Y_e = P_i Y_i + P_j Y_j$$

→ First Derivative is always +ve

→ The sign of second derivative will help determine the behaviour of individual

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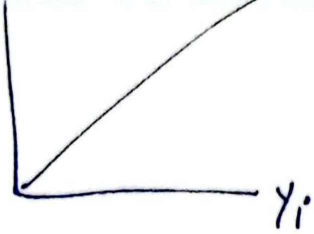
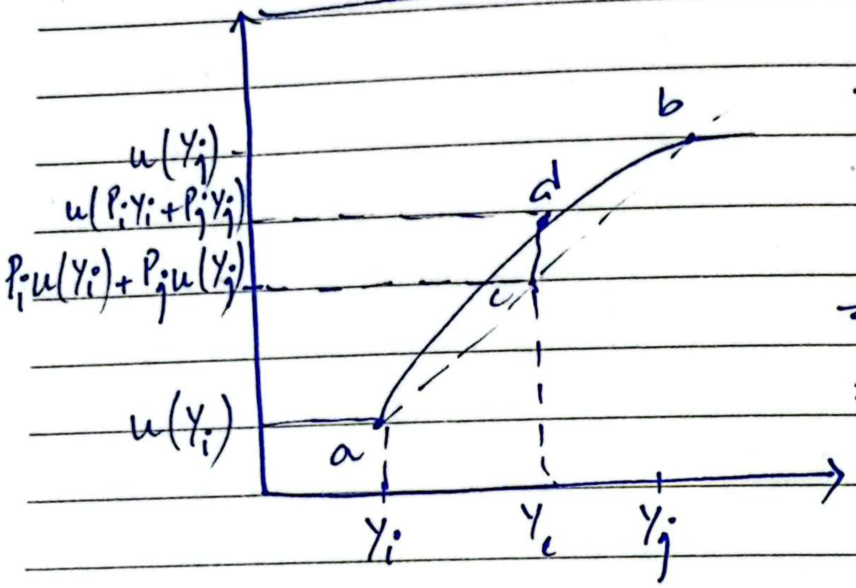
$u'(Y_i) > 0$
$u''(Y_i) = 0$





⇒ Concave
⇒ Indivinal is Risk Averse

⇒ Linear
⇒ Indivinal is Risk Neutral.

Numerical Example : {1st Method}

$u'(Y) > 0$ such that $Y_i > 0$
$u''(Y) < 0$

$$u = Y^{1/2}$$

$$\frac{du}{dY} = u'(Y) = \frac{1}{2}(Y)^{-1/2} = \frac{1}{2}\left(\frac{1}{Y^{1/2}}\right) = \frac{1}{2Y^{1/2}}$$

$$= \frac{1}{2}(Y)^{-1/2} > \text{zero}$$

⇒ By being positive, it means that whatever value of Y is there, answer will always be +ve.

$$\frac{d^2u}{dY^2} = u''(Y) = -\frac{1}{2}\left[\frac{1}{2}(Y)^{-1/2^{-1}}\right] = \frac{-1}{4}(Y)^{-3/2}$$

$$= \frac{-1}{4(Y)^{3/2}} < \text{zero}$$

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⇒ Y can never be -ve, it is outcome
so it'll always be +ve.

Utility Function: $u(Y) = [Y]^{1/2}$

sometimes utility functions are complicated merely taking derivatives doesn't increase the sign.

our attitude towards risk is not constant. For some income levels an individual might be risk lover & from others, he might be risk averse.

$u = Y^{1/2}$ → This utility function demonstrates a consumer who is constantly risk averse. However, if by taking second derivative, utility func. becomes quadrate - it will be hard to determine the sign.

Lets check individual's behaviour for two different values of Y

$Y_1 = 25 \qquad Y_2 = 100$

$P_1 = 0.50 \qquad P_2 = 0.50$

Expected value (Y_e)

$Y_e = P_1 Y_1 + P_2 Y_2$

$Y_e = 0.50(25) + 0.50(100)$

$= 12.5 + 50$

$\boxed{Y_e = 62.5}$

expected value

Utility of Expected Value

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$u = Y^{1/2}$ $\boxed{\text{expected value} = 62.5}$ Y_e

$u = [62.5]^{1/2}$

$\boxed{u(Y_e) = 7.90}$

Expected Utility of Lottery

$= p_1 u(Y_1) + p_2 u(Y_2)$

we need $u(Y_1)$ & $u(Y_2)$

$u(25) = [25]^{1/2} \Rightarrow \boxed{u(Y_1 = 25) = 5}$

$u(100) = [100]^{1/2} \Rightarrow \boxed{u(Y_2 = 100) = 10}$

Expected utility of lottery with outcomes 25 and 100

$= 0.50(5) + 0.50(10)$

$= 2.5 + 5$

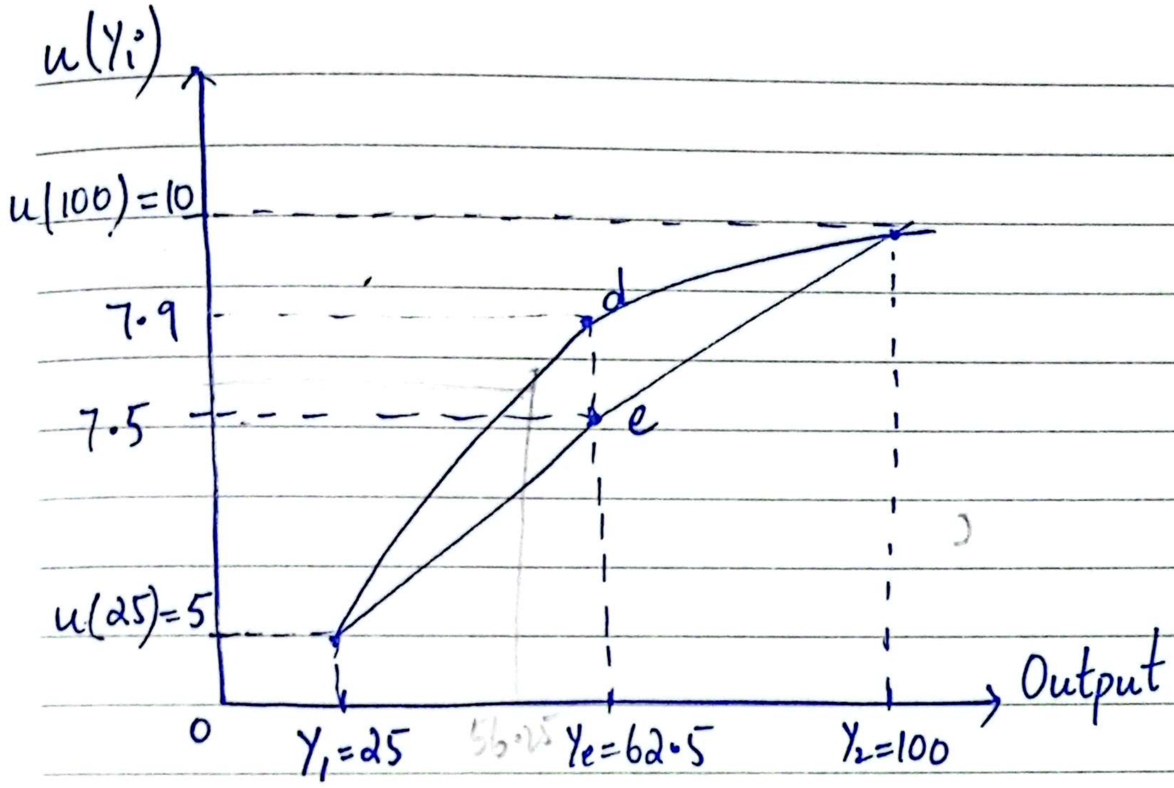
$= 7.5$

$\Rightarrow$ Expected utility of lottery with $Y_1 = 25$ and $Y_2 = 100$ is 7.5

$\Rightarrow$ Utility of the expected value of $Y_1 = 25$ & $Y_2 = 100$ is 7.9

Utility of expected value of 25 & 100 > Expected utility of lottery

$7.9 > 7.5$



$$u = Y^{3/2}$$

Q What can be the value so that EU is 7.5?

$$u = (Y)^{1/2}$$

$$7.5 = (Y)^{1/2}$$

Taking square on both sides

$$56.25 = Y$$