Chapter NO 4

(Partial equilibrium)

Previously we studied individual consumer and firms, assuming fixed market prices. Now we examine how these agents interact within the markets, where prices and quantities are collectively determined. First, we will explore prices and quantity setting in a single or related markets, then evaluate these markets from a social perspective. The key focus is on how a markets ~~equili-~~ ~~-brium~~ competitive structure affects its social performance and overall societal welfare.

4.1 Perfect market Overview :

In perfectly competitive markets, numerous buyers and sellers exists such that no single participant can influence market prices. Buyers and sellers are "price takers" meaning they accept the market prices as given. Each buyer maximizes utility, and each seller maximizes profit based on this price.

Demand and supply:

↳ Market demand for a good is the sum of individual demands, depending on its price, the prices of other goods, and the buyers income distribution.

↳ Short-Run supply is determined by existing firms, while long-run supply allows entry and exit of the firms based on profitability.

Short Run Equilibrium:

Occurs where market demand equals market supply, so each buyer and seller operates at an optimal level without incentive to change behavior.

As individual demand is $q_i(P, P', y_i)$.
so market demand is the sum of the individual demand

$$q^d(P) = \sum_{i \in I} q_i(P, P', y_i)$$

And similarly Individual supply of firm is

$$q_j(P, w)$$

So market supply is the sum of all individual firm supplies

$$q^s(P) = \sum_{j \in J} q_j(P, \omega)$$

So the equilibrium in the market occurs when

$$q^d(P) = q^s(P)$$

for the competitive market, individual firm's short run supply function using the cobb-douglas function is

$$q = x^\alpha K^{1-\alpha}$$

where x is a variable input like labor, and K is a fixed input like plant size

↦ Short-Run-Profit functions

↳ So the short run profit function is

Revenue ⇒ $R = Pq$

price ↙ ↘ quantity produced

Costs: The costs include variable plus fixed costs

Variable cost: $C_V = w_x x$, where w_x is the wage rate for input x

fixed cost: $C_f = W_K K$, where W_K is the cost of the fixed input K.

Putting it all together.

$$\text{Profit} = TR - TC$$

$$\pi = Pq - (w_x X + W_K K)$$

Using the cobb-douglas PF:
Given the production function $q = X^{\alpha} K^{1-\alpha}$, we can solve it for X in terms of q.

$$x = \left(\frac{q}{K^{1-\alpha}}\right)^{1/2}$$

Now putting X value in profit function

$$\pi = Pq - \left[w_x \left(\frac{q}{K^{1-\alpha}}\right)^{1/2} + w_K K\right]$$

lets simplify
$\alpha = 1/2$ so

$$x = \left(\frac{q}{K^{1-1/2}}\right)^{\frac{1}{1/2}} = \left(\frac{q^2}{K}\right)$$

So profit equation becomes.

$$\pi = Pq - \left(w_x \frac{q^2}{K} + w_K K \right)$$

Given $w_x = 4$, $w_K = 1$, and $K = 1$

$$\pi = Pq - \left(4 \frac{q^2}{1} + (1)(1) \right)$$

$$\pi = Pq - 4q^2 - 1$$

1) Short run equilibrium:

*) Individual firm Supply:

↳ Given $\alpha = 1/2$, $w_x = 4$, $w_K = 1$ and $K = 1$

↳ firm output supply $= q_j = P/8$

↳ Market supply with $j = 48$ firms

so

$$q^s = 48 \; P/8 = 6P$$

$$q^s = 6P$$

*) Market demand:

Market demand $= q^d = \frac{294}{P}$

*) Equilibrium price and quantity:

Set demand equal to supply

$$q^s = q^d$$

$$6P = \frac{294}{P}$$

$$\frac{6P^2}{6} = \frac{294}{6} \Rightarrow P^2 = 49$$

$$P = 7$$

So the equilibrium price = 7

and the equilibrium quantity becomes

$$q^* = q_d(P^*) = \frac{294}{7} = 6$$

$$q_d = \frac{q^*}{P^*} = \frac{294}{7} \Rightarrow 42$$

(4) firms output and profit :

↳ output per form ⇒

as $q^d = 42$

and the total number of firms are 48 so

output per form becomes

$$q_{kj} = \frac{42}{48} = \frac{7}{8}$$

$$q_{kj} = 7/8$$

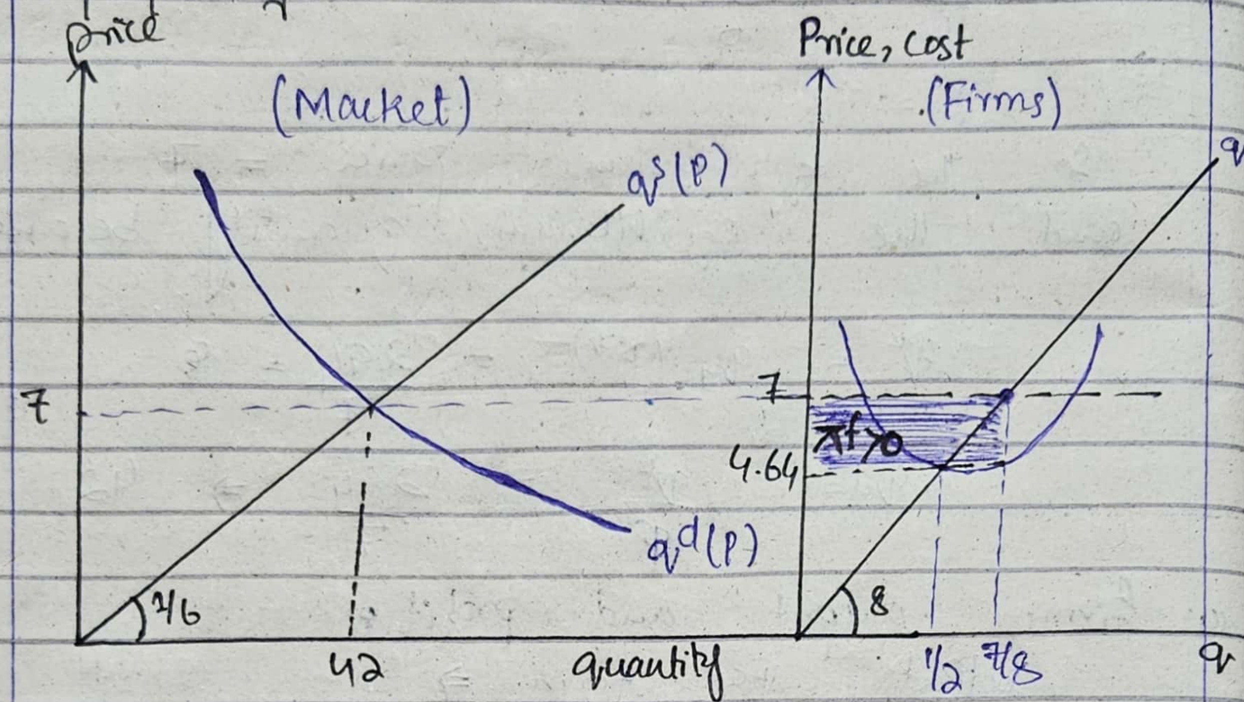
↳ And the profit becomes.

$$\pi = P \cdot q - (w_x X + w_K K)$$

and the π becomes

$$\pi_j = \frac{7^2 \cdot 1}{16} - 1 = 2.0625$$

Graphically :



Short-Run equilibrium in a Single market.

⇒ Long-Run Equilibrium :

In the long-Run

(i) All the firms freely adjust all inputs, including plant size.

(ii) Positive profits attract new firms, driving profits to zero and establishing an equilibrium price.

*) Conditions for long run Equilibrium :

i) Market clearing condition :

$$q^d = q^s$$

↳ Given demand : $q_d(P) = \frac{294}{P}$

↳ Market supply (with j firms each producing q_j) :

$$q_s = \sum_{j=1}^{J} q_j$$

2) Zero profit :
long-run profit must be zero :

$$\pi = Pq - (w_x X + w_K K) = 0$$

(3) applying the Conditions :
from the previous steps

$$P^* = 7$$
$$q^* = 42$$
$$q_j = P/8 \quad \text{(individual firm output)}$$

⇒ Long-Run Adjustment : (Example 4.3)

(i) Zero-profit condition

$$\pi_j = \frac{P^2 K}{16} - K$$

(ii) set $\pi_j = 0$

→ Market demand and supply at $P = 4$

$$q_d = \frac{294}{4} = 73.4$$

→ Market supply : $q_s = J \cdot \frac{4K}{8}$.

→ Simplified : $q_s = J \cdot K/2$

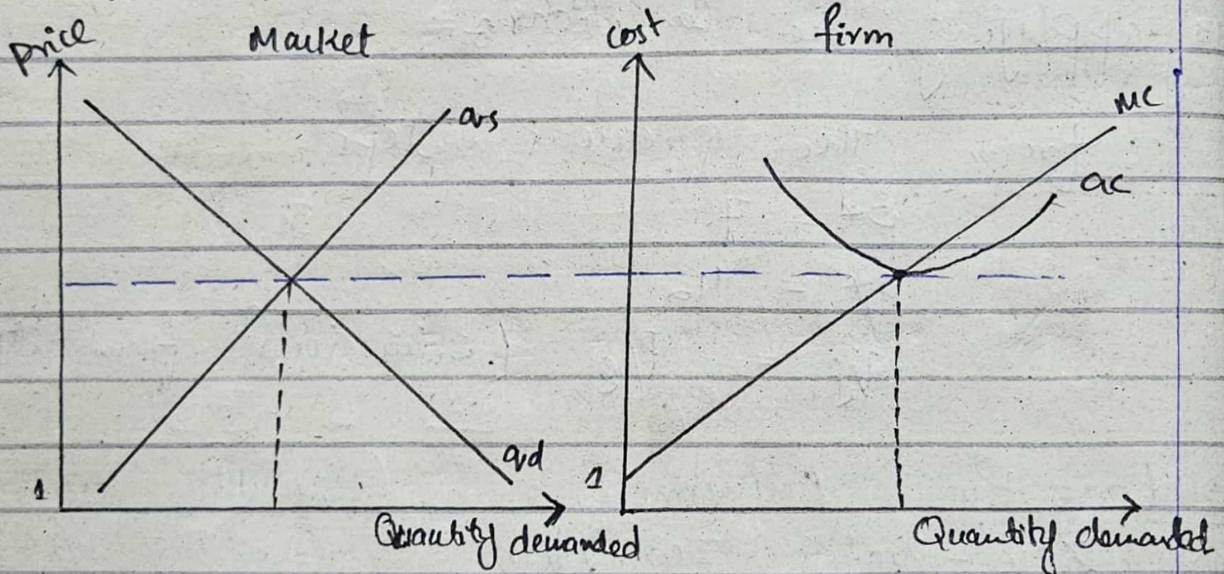
Market clearing.

$q_s = q_d$

$\frac{294}{4} = \frac{4}{8} JK$

$147 = JK$

Graphically : (not linked with above example)



Long-Run equilibrium in a competitive firm :

4.2 Imperfect Competition :

This world lies between the extreams of perfect competition and pure monopoly, unlike perfect

Perfect competition with numerous small firms or pure monopoly with just one firm, imperfect competition sees a few firms dominating the market, creating interdependence and strategic interactions.

↳ In a pure monopoly.
Single seller : No close substitutes, high barriers to entry.
Profit maximization: The monopolists sets quantity q to maximize profit.

→ Revenue and profits:
↳ Revenue (R) depends on price and quantity:

$$R(q) = P(q) \cdot q$$

↳ Cost (C) depends on the quantity produced : $C(q)$

↳ Profit is the difference between Revenue and cost.

$$\pi(q) = R(q) - C(q)$$

↳ Profit Maximizing:
To maximize profit, the monopolist will set output at the point

$$R(q) = P(q) \cdot q$$

$$MR = P(q) \cdot (1) + \frac{dP}{dq} q$$

So $MR = P(q) + q \cdot \frac{dP}{dq}$

where $MR = MC$.

↳ **Marginal Revenue:**

↳ Marginal Revenue is the change when increasing production by a small amount. if the Revenue is

$$R(q) = P(q) \cdot q \quad \text{Then}$$

Then by increasing the quantity we should take derivative of Revenue w.r.t q.

$$dR(q) = P(q) + q \cdot dP/dq$$

↓

by product rule (noted above).

So

$$MR = \frac{dR}{dq} = P(q) + q \cdot dP/dq$$

where dP/dq reflects how prices changes with output.

↳ **Marginal Costs:**

Marginal cost is the change in cost when production increases

$$MC = \frac{dc}{dq}$$

At the profit-maximizing point, we set

$$MR = MC$$

Elasticity of Demand &

The elasticity of demand helps us understand how sensitive quantity demanded is to price changes.

$$\varepsilon = \frac{dq}{dp} \cdot P/q$$

In monopoly pricing, the monopolists sets :

$$MC = P(q)\left(1 + \frac{1}{\varepsilon}\right)$$

This equation shows that the monopolists price will depend on both the marginal ~~revenue~~ cost and the price elasticity of demand.

↳ if $\varepsilon > 1 \Rightarrow$ quantity demanded is more sensitive to price changes, so $\frac{1}{\varepsilon}$ is small, so the monopolists sets prices close to Marginal cost.

↳ But if $\varepsilon < 1$ Qd is less sensitive to price changes allowing the monopolists to charge a price significantly above marginal cost. This is because lowering the price wouldn't increase sales much, but it would reduce revenue.

2) Imperfect Competition and Nash equilibrium:

In imperfect competition, multiple firms interact, and each firm considers the action of others in deciding its output. This setting often leads to a Nash equilibrium:

Nash equilibrium concept:

In a Nash equilibrium

i) Each firm chooses its best strategy given what others are doing.

ii) No firm can improve its profit by changing its output alone if others keep their outputs unchanged.

Suppose there are J firms (eg Firm 1, Firm 2, Firm 3, ... Firm J), and each produces q_j units. The profit for each firm j depends on both its output and the output of other

$$\pi_j = R_j(q_1, q_2 \cdots, q_j, \cdots q_J) - C_j(q_j)$$

→ Conditions for Nash equilibrium

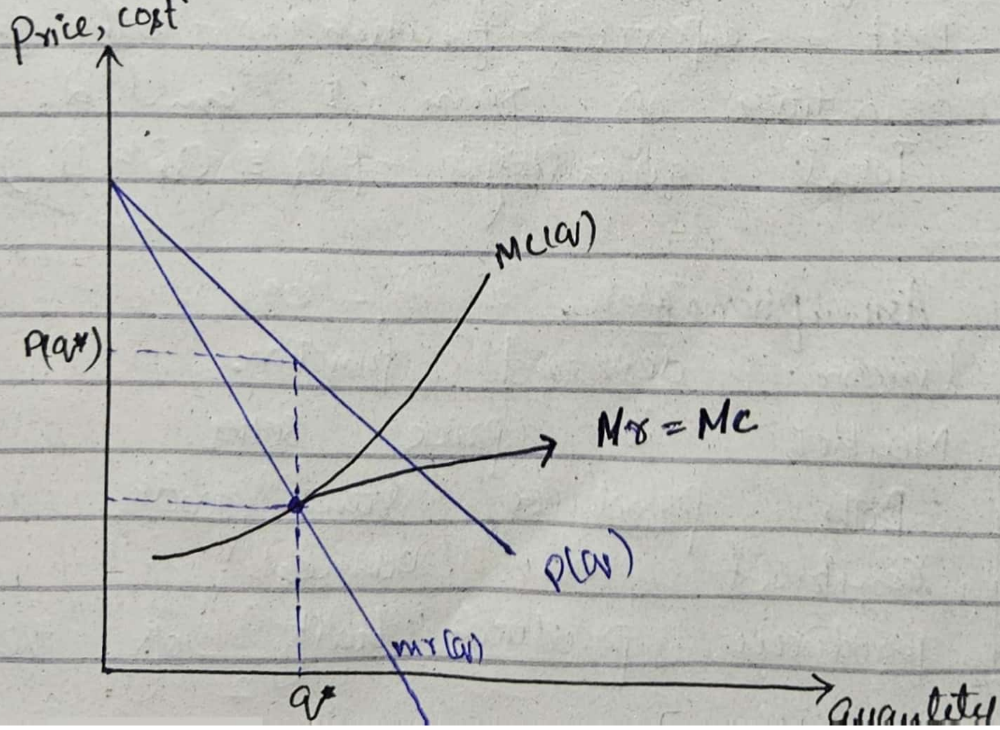
For each firm j in equilibrium

$$\frac{\partial \pi_j}{\partial q_j} = 0.$$

This means that each firm maximizes its profit assuming others don't change their production.

↳ If all the firms are profit-maximizing given the other's choices, no firm has an incentive to change. This stable condition is what defines the Nash equilibrium.

Graphically equilibrium in a pure monopolys



⇒ Cournot Oligopoly:

Developed by Auguste Cournot in 1838, This model describes how firm in an oligopoly (a market with a few firms) compete on quantity rather than price. Each firm's profit depends on the total market output, as this determines the market price, Here's a breakdown of how the model works, including the mathematics behind equilibrium output and price.

↳ Cournot model Steps:

To find.

1) Price
2) Profit function for firm 1 and 2
3) Best response function of both firms.
4) Quantities of firm 1 and 2
5) Total quantity ($Q = Q_1 + Q_2$)

⇒ Assumptions:

(i) Linear demand function.
(ii) Market is price setter.
(iii) Both produces same goods.
(iv) Constant cost curves
(v) producers produce half of the

total demand.

⇒ Market demand :

The market price depends on the total output sold by all firms

inverse demand function is

$$q = \alpha - \beta p$$

$$\beta p = \alpha - q$$

$$p = \frac{\alpha - q}{\beta}$$

$$p = \alpha/\beta - \frac{q}{\beta}$$

$$p = \alpha/\beta - \frac{1}{\beta} q$$

lets $\alpha/\beta = a$, $1/\beta = b$

So $\boxed{p = a - bq}$

Profit function for firm A :

$$\pi_A = TR - TC$$

$$\pi_A = p \cdot q_A - Cq_A$$

as $p = a - bq$

$$\pi_A = (a - bq) q_A - Cq_A$$

$$\pi_A = aq_A - bqq_A - cq_A \quad \& \; q = q_A + q_B$$

$$\pi_A = aq_A - b(q_A + q_B)q_A - cq_A$$

$$\boxed{\pi_A = aq_A - bq_A^2 - bq_A q_B - cq_A}$$

profit equation for firm A

Similarly, for firm B.

$$\boxed{\pi_B = aq_B - bq_B^2 - bq_A q_B - cq_B}$$

Now for firm A

$$\pi_A = aq_A - bq_A^2 - bq_A q_B - cq_B$$

differentiate w.r.t q_A

$$\frac{\partial \pi_A}{\partial q_A} = \frac{\partial}{\partial q_A}\left(aq_A - bq_A^2 - bq_A q_B - cq_A\right)$$

$$\frac{\partial \pi_A}{\partial q_A} = a - 2bq_A - bq_B - c = 0$$

$$a - bq_B - c = 2bq_A$$

$$\frac{\cancel{2b}q_A}{\cancel{2b}} = \frac{a - bq_B - c}{2b}$$

$$q_A = \frac{a - bq_B - c}{2b}$$

$$\boxed{q_A = \frac{a-c}{2b} - \frac{q_B}{2}} \rightarrow \text{Firm 1 Best response}$$

Similarly for firm B.

$$\boxed{q_B = \frac{a-c}{2b} - \frac{Q_A}{2}}$$

↳ Firm 2 best response.

Now solve this equation for Q_A / q_A

$$Q_A / q_A = \frac{a-c}{2b} - \frac{Q_B / Q_{VB}}{2}$$

Put value of q_B in above equ-

$$q_A = \frac{a-c}{2b} - \frac{1}{2}\left[\frac{a-c}{2b} - \frac{q_A}{2}\right]$$

$$q_A = \frac{a-c}{2b} - \frac{1}{2}\left[\frac{a-c-bq_A}{2b}\right]$$

$$q_A = \frac{a-c}{2b} - \frac{a+c+bq_A}{4b}$$

$$q_A = \frac{2a-2c-a+c+bq_A}{4b}$$

$$q_A = \frac{a-c+bq_A}{4b}$$

$$q_A = \frac{a-c}{4b} + \frac{\cancel{b}q_A}{4\cancel{b}}$$

$$q_A = \frac{a-c}{4b} + \frac{q_A}{4}$$

$$q_A - \frac{q_A}{4} = \frac{a-c}{4b}$$

$$\frac{4q_A - q_A}{4} = \frac{a-c}{4b}$$

$$\frac{3q_A}{4} = \frac{a-c}{4b}$$

$$q_A = \frac{4(a-c)}{(4b)3} \Rightarrow q_A = \frac{\cancel{4}(a-c)}{\cancel{12}_3 b}$$

$$\boxed{q_A^* = \frac{a-c}{3b}}$$ optimal value for firm A

Similarly for firm B.

$$\boxed{q_B^* = \frac{a-c}{3b}}$$ optimal value for firm B.

Now the total quantity becomes:

$$q = q_A + q_B$$

$$q = \frac{a-c}{3b} + \frac{a-c}{3b}$$

$$\boxed{q = \frac{2(a-c)}{3b}}$$

Total product produced by both the firms

→ Bertrand Oligopoly:

The Bertrand model, introduced by Joseph Bertrand in 1883.

Shifts the focus from quantity competition (cournot) to price competition. Lets simplify and break down the key points.

Assumptions :

(i) Each firm simultaneously chose price
(ii) They produced homogeneous goods.
(iii) Same marginal cost of production.
(iv) Both the firms objective is profit maximization.
(v) Consumer is indifferent b/w the firms.
(vi) Both prefer lower prices to higher prices.

Example of Bertrand Oligopoly :

For firm A and B.

$$Q = \alpha - \beta(P),$$

$$C_A = C_A Q_A$$

$$C_B = C_B Q_B$$

profit function for firm A

$$\pi_A = \text{Revenue} - \text{Cost}.$$

$$\pi_A = P_A Q_A - C_A Q_A$$

Put Q value.

$$\pi_A = P_A(\alpha - \beta P_A) - C_A(\alpha - \beta P_A)$$

$$\boxed{\pi_A = (P_A - C_A)(\alpha - \beta P_A)}$$

$$\pi_A \begin{cases} (P_A - C_A)(\alpha - \beta P_A) & \text{if } P_A < P_B \\ \frac{1}{2}(P_A - C_A)(\alpha - \beta P_A) & \text{if } P_A = P_B \\ \text{zero} & \text{if } P_A > P_B \end{cases}$$

Similarly for the π_B.

$$\pi_B = (P_B - C_B)(\alpha - \beta P_B)$$

$$\pi_B \begin{cases} (P_B - C_B)(\alpha - \beta P_B) & \text{if } P_B < P_A \\ \frac{1}{2}(P_A - C_A)(\alpha - \beta P_A) & \text{if } P_A = P_B \\ \text{zero} & P_B > P_A \end{cases}$$

↳ Optimal Strategies :

1) $P_A = P_B = MC \rightarrow$ Sustainable

2) $P_A = P_B > MC$

3) $P_A = P_B < MC$ → not sustainable strategies

↳ Strategy profile :

(A)	(B)	
P_L	P_L	(low prices)
P_H	P_H	(high prices)
P_N	P_N	(Normal prices)

$(P_L, \overset{\times}{P_L})$, $(P_L, \overset{\times}{P_H})$, $P(L, \overset{\times}{P_N})$
$(P_H, \overset{\times}{P_L})$, $(P_H, \overset{\times}{P_H})$, $P(P_H, \overset{\times}{P_N})$
$(P_N, \overset{\times}{P_L})$, $(P_N, \overset{\times}{P_H})$, $\boxed{(P_N, P_N)}$

Nash equilibrium

In all above 9 strategies, only (P_N, P_N) is Nash and can be proved by logic and reasons.

⇒ Monopolistic Competition

In monopolistic Competition several firms offer similar ~~but~~ yet differentiated products, granting each some control over their pricing. Unlike pure monopoly or perfect competition, firms here face a downward - sloping demand curve because each product variant has close, though not perfect substitutes.

Key Aspects of Monopolistic Competition

1) S.R equilibrium.

↳ Firms select prices to maximize profits, given the prices of rivals

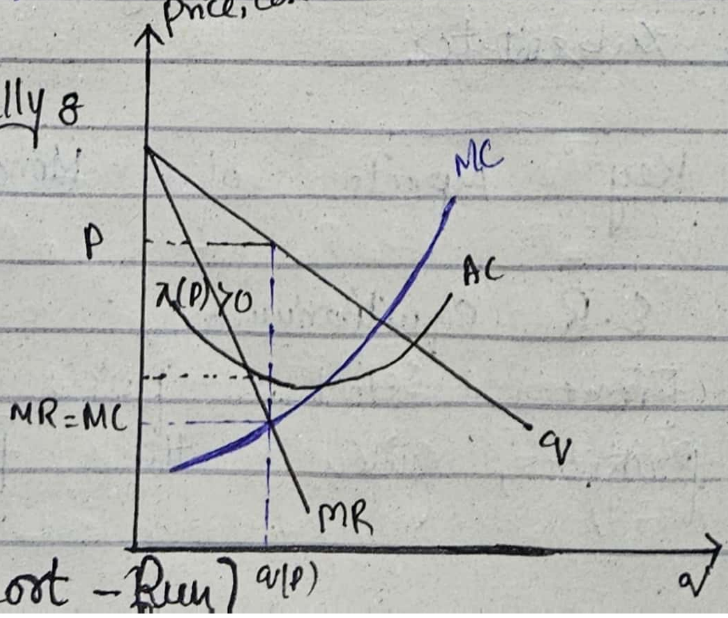
↳ Profit maximization leads to firms setting price and output where marginal Revenue (MR) equals Marginal cost (MC). In the short-run firms can have a positive, zero, or negative profits.

2) Long-Run Equilibrium:

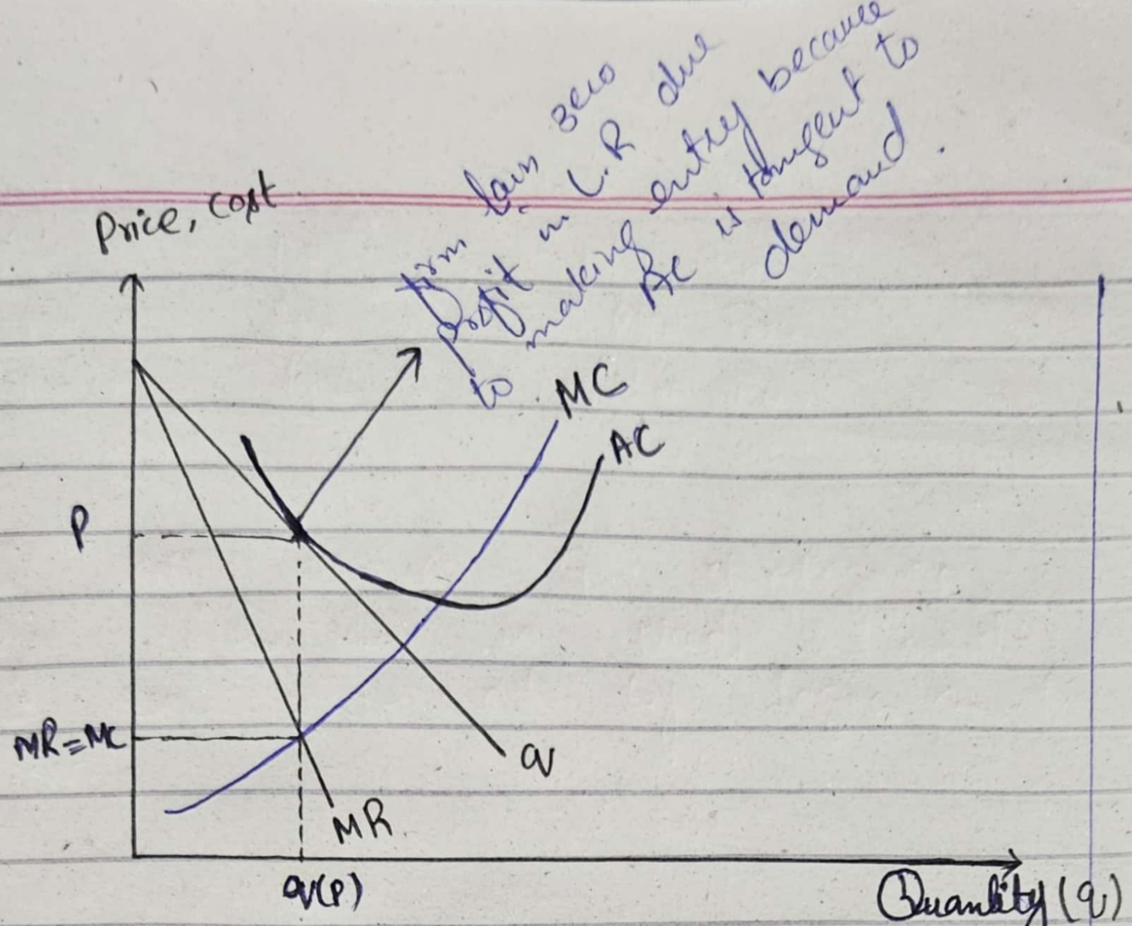
↳ If firms are making a profit, new firms will enter, adding similar products. This increases competition, driving into prices and profits.

↳ Eventually, each firm earns zero economic profit in the long-run due to making entry, resulting in a stable situation where demand is tangent to average cost (AC).

Graphically:



(a) (Short-Run)



(b) Long-run equilibrium in a Monopolistic Competition :