Analysis of Difference Equation of a System (Filter) in Z-Domain

Syed Nazmus Sakib
ID : AE-172-009

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Objective

This experiment's objective is to use MATLAB to examine a Discrete-Time (D-T) system specified by its difference equation. Plotting the pole-zero diagram, identifying poles and zeros, calculating the transfer function $H(z)$, and examining the frequency response to ascertain the filter type, center frequency, and bandwidth are all included in the analysis. Additionally, a thorough sensitivity analysis is carried out to show how pole placement influences filter properties.

Theoretical Background

The given difference equation for the D-T system is:

$$y[n] = 1.2y[n-1] - 0.75y[n-2] + x[n] + 2x[n-1] + x[n-2] \quad (1)$$

Rearranging terms to group outputs $y[n]$ and inputs $x[n]$:

$$y[n] - 1.2y[n-1] + 0.75y[n-2] = x[n] + 2x[n-1] + x[n-2] \quad (2)$$

Taking the Z-transform of both sides (assuming zero initial conditions):

$$Y(z)(1 - 1.2z^{-1} + 0.75z^{-2}) = X(z)(1 + 2z^{-1} + z^{-2}) \quad (3)$$

The Transfer Function $H(z)$ is defined as the ratio of output to input:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.2z^{-1} + 0.75z^{-2}} \quad (4)$$

Where the sampling frequency is given as $F_s = 10$ kHz.

1 Part 1: System Analysis (Poles and Zeros)

Transfer Function Definition

Using MATLAB, the coefficients vectors were defined as:

- Numerator (a): [1 2 1]
- Denominator (b): [1 -1.2 0.75]

Pole and Zero Locations

Using the `tf2zp` function, the roots of the numerator (zeros) and denominator (poles) were calculated:
Zeros (z):

$$z_{1,2} = -1 \quad (5)$$

Analysis: The system has a double zero at $z = -1$, which corresponds to the Nyquist frequency ($F_s/2$). This indicates the filter completely suppresses the highest possible frequency.

Poles (p)**:**

$$p_1 = 0.6000 + 0.6245i \tag{6}$$
$$p_2 = 0.6000 - 0.6245i \tag{7}$$

Analysis: The poles are complex conjugates located inside the unit circle, ensuring the system is stable.

Pole-Zero Diagram

The pole-zero map visualizes these locations relative to the Unit Circle.

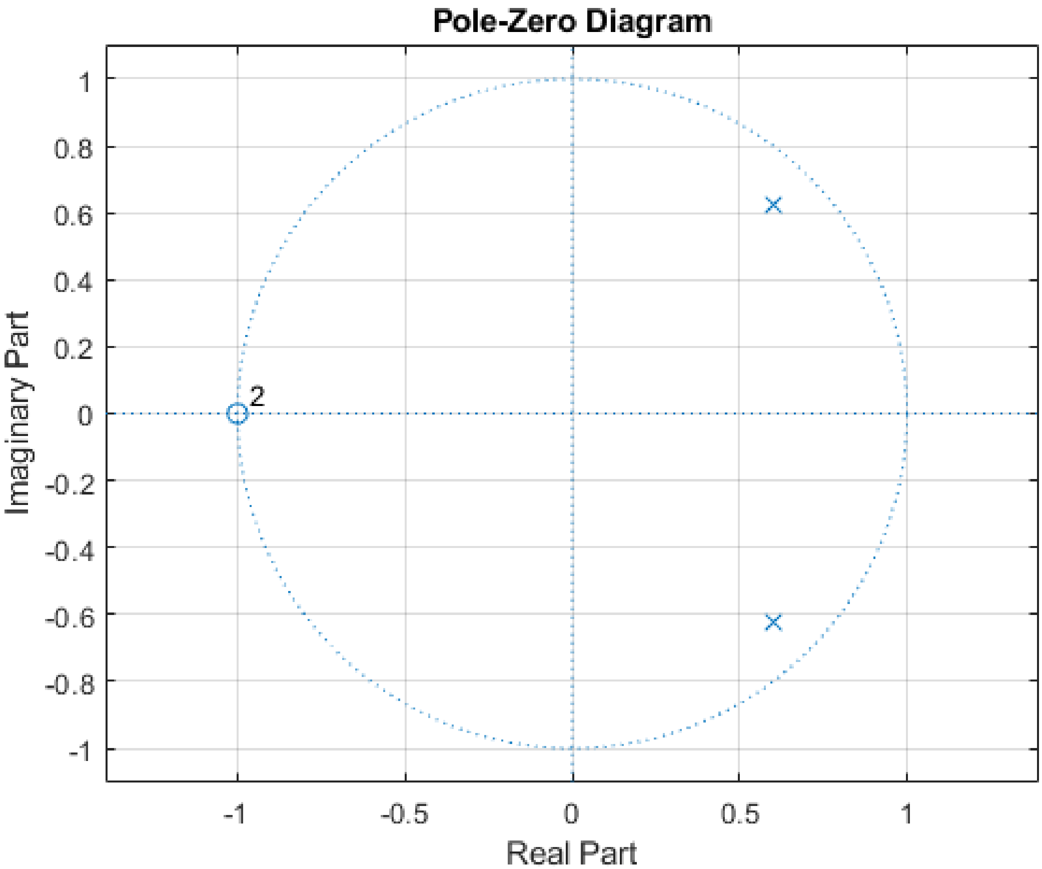

Figure 1: Pole-Zero Diagram

Observation: The proximity of the poles to the unit circle (indicated by the dotted line) suggests a peak in magnitude response at the angle corresponding to the poles.

2 Part 2: Frequency Response and Bandwidth

Magnitude and Phase Response

The frequency response was calculated using `freqz` with $N = 1024$ points over the range 0 to F_s.

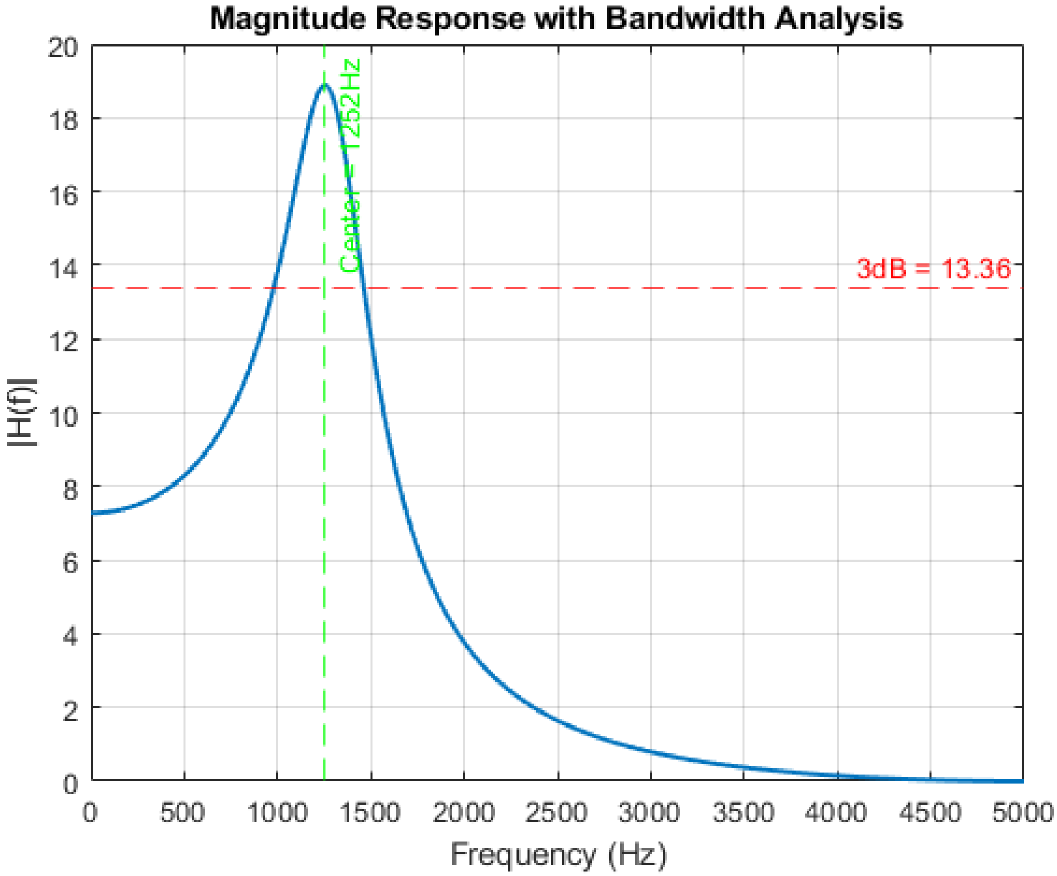

Figure 2: Magnitude and Phase Response (Note: Place the figure showing the two subplots: Magnitude on top, Phase on bottom)

Bandwidth and Peak Analysis

A detailed analysis was performed on the magnitude response in the range 0 to $F_s/2$.

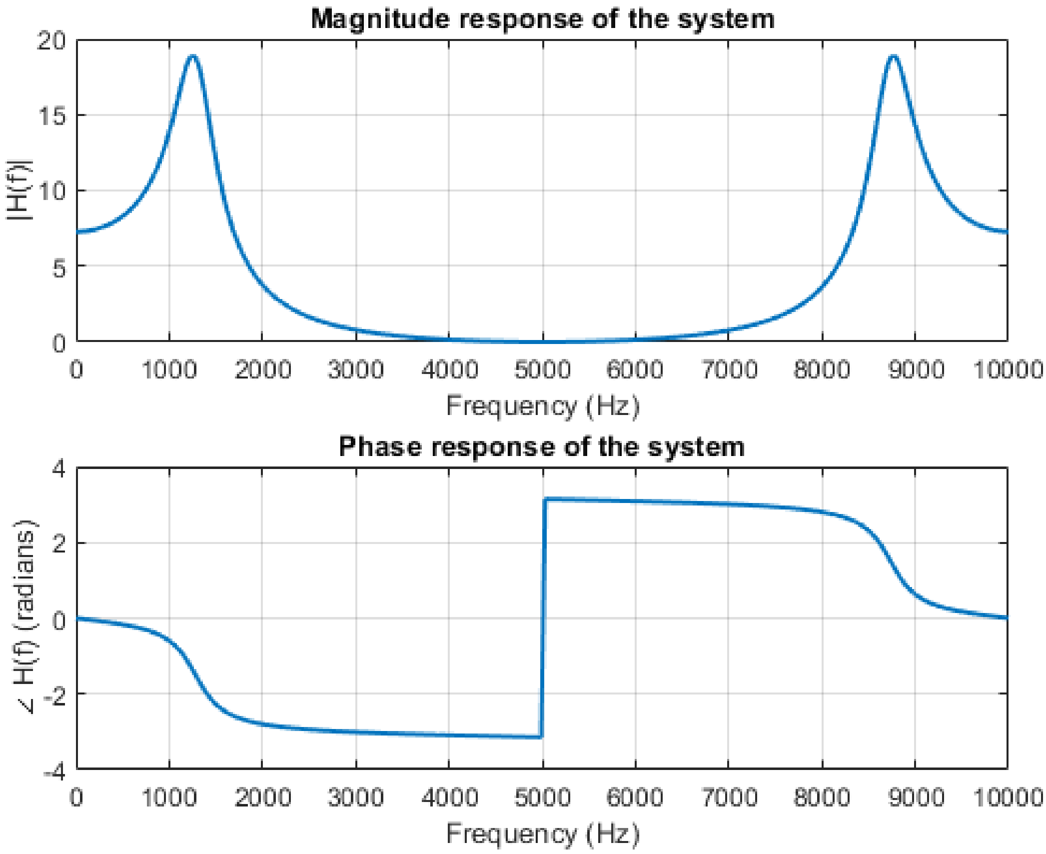

Figure 3: Magnitude Response with Bandwidth Analysis (Note: Place the figure showing the peak with the green dashed line and the red 3dB threshold line)

Experimental Results:

- Maximum Magnitude: 18.89
- Frequency at Maximum (f_c): 1252.45 Hz
- 3dB Cutoff Magnitude: $18.89/\sqrt{2} = 13.36$
- Bandwidth: 469.67 Hz

Filter Classification

Based on the magnitude response:

- **Low Frequency (DC):** At 0 Hz, magnitude is finite but low (≈ 7.2).
- **High Frequency ($F_s/2$):** At 5000 Hz, magnitude drops to 0 (due to zeros at $z = -1$).
- **Mid Frequency:** There is a distinct resonant peak at ≈ 1252 Hz.

Conclusion: The system acts as a Band Pass Filter (BP).

3 Part 3: In-Depth Parameter Sensitivity Analysis

To better understand the behavior of the filter, parameters determining the pole locations (Radius r and Angle θ) were varied.

Experiment 1: Effect of Pole Radius (r)

The radius of the poles determines the distance from the origin to the pole. In this experiment, the angle was held constant while the radius was varied from 0.50 to 0.98.

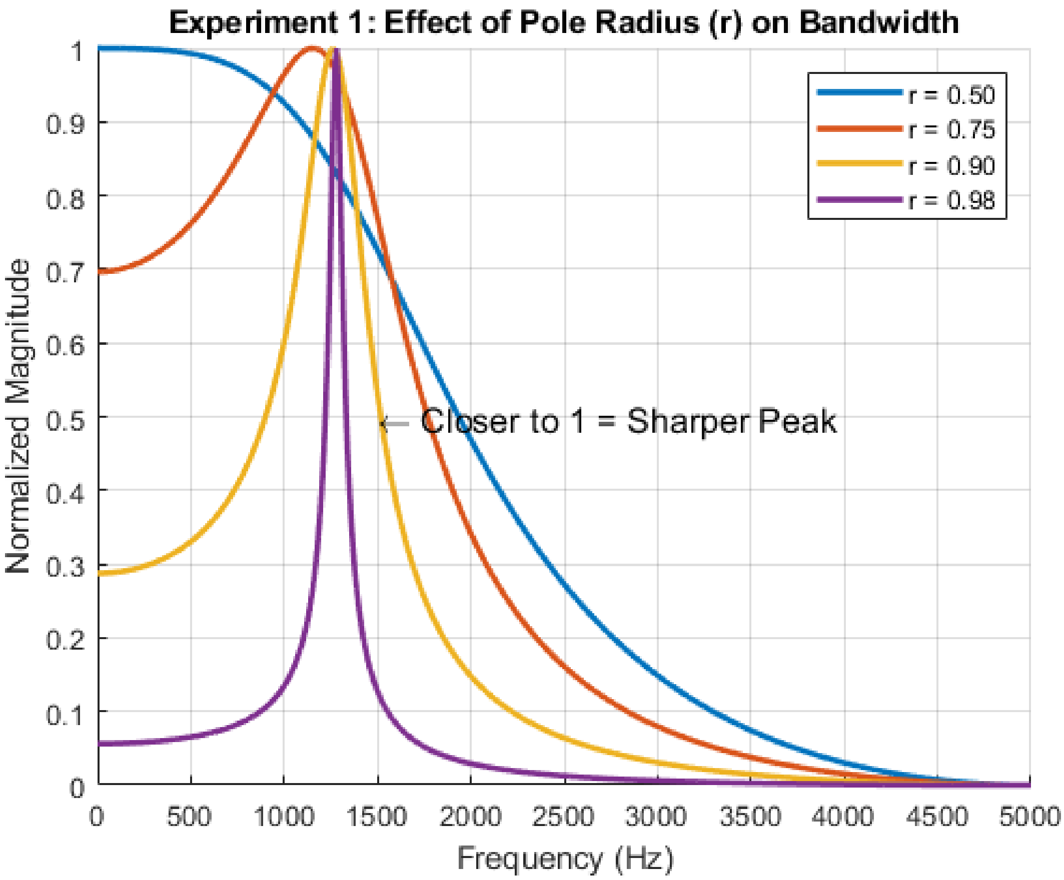

Figure 4: Effect of Pole Radius (r) on Bandwidth

Analysis:

- As $r \to 1$ (Purple line, $r = 0.98$), the poles move closer to the unit circle. This results in a much sharper peak (higher Q-factor) and narrower bandwidth.
- As $r \to 0$ (Blue line, $r = 0.50$), the poles move toward the origin, resulting in a flatter, wider response.

Experiment 2: Effect of Pole Angle (θ)

The angle of the poles determines the resonant frequency. The radius was held constant while the angle was swept from 20° to 90°.

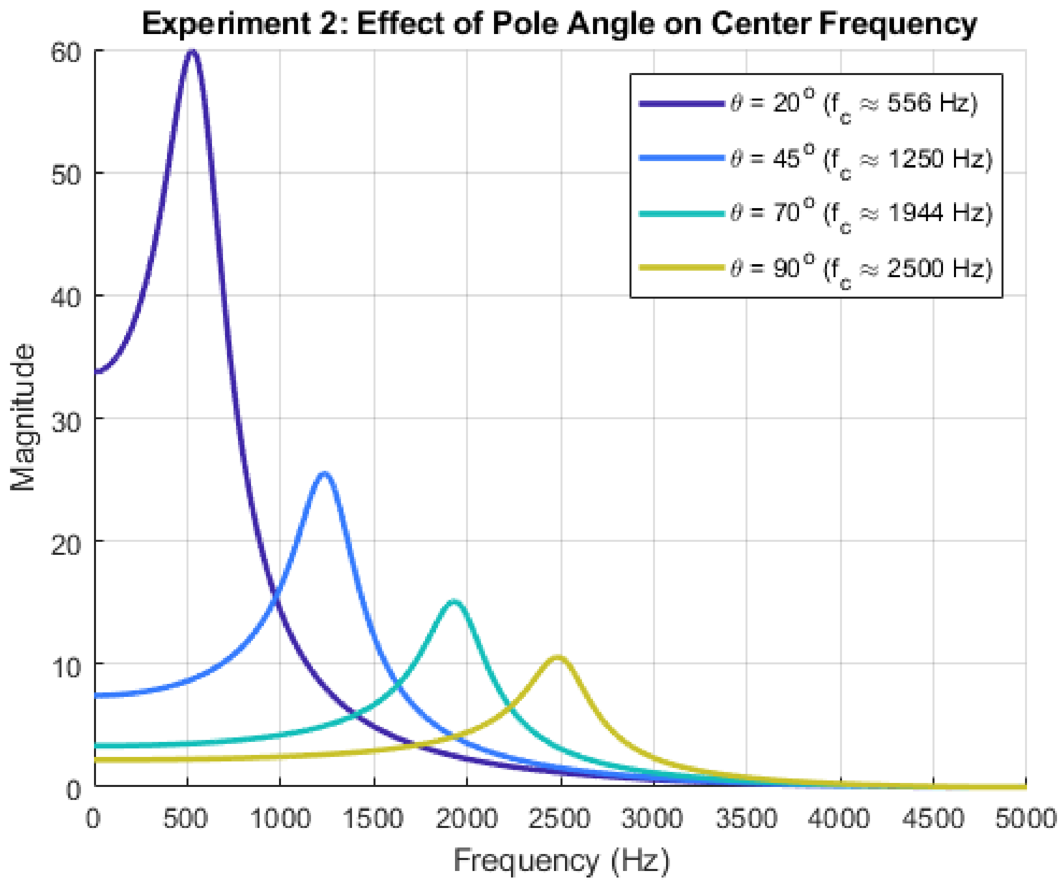

Figure 5: Effect of Pole Angle on Center Frequency

Analysis:

- The center frequency follows the relationship $f_c = \frac{\theta}{360^\circ} \times F_s$.
- For $\theta = 45^\circ$, the calculated frequency is $\frac{45}{360} \times 10000 = 1250$ Hz, which matches our system's behavior perfectly.

Experiment 3: 3D Z-Domain Visualization

To visualize the "Rubber Sheet" analogy of the Z-transform, a 3D surface plot of magnitude $|H(z)|$ was generated.

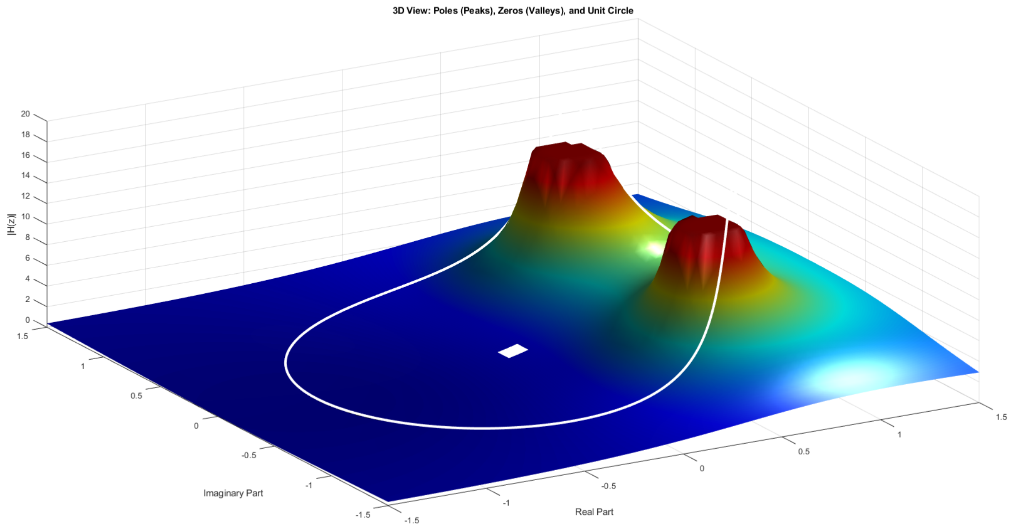

Figure 6: 3D View: Poles (Peaks), Zeros (Valleys), and Unit Circle (Note: Place the 3D colored surface plot here)

Interpretation:

- **Peaks (Red):** Represent the Poles. These push the magnitude up toward infinity.
- **Valleys (Dark Blue):** Represent the Zeros (at $z = -1$). These pull the magnitude down to zero.
- **White Line:** Represents the Unit Circle. The height of the surface along this white line corresponds exactly to the 2D Frequency Response graph derived in Part 2.

Conclusion

In this lab, a difference equation was successfully analyzed in the Z-domain.

- The system was identified as a Band Pass Filter with a center frequency of 1252 Hz and a bandwidth of 470 Hz.
- The poles were found to be complex conjugates at $0.6 \pm 0.6245i$.
- The sensitivity analysis demonstrated that the Pole Angle controls the center frequency, while the Pole Radius controls the filter selectivity (sharpness/bandwidth).
- The 3D visualization confirmed that the frequency response is simply the magnitude of the transfer function evaluated along the unit circle.