

# Ex: 4.1

## NUMERICAL DIFFERENTIATION

- Forward Difference:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} + O(h)$$

- Backward Difference:

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h} + O(h)$$

- Central Difference:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} + O(h^2)$$

### "3-Point Formulas"

- Endpoint Formula:

$$f'(x_0) = \frac{1}{2h} [-3f(x_0) + 4f(x_0+h) - f(x_0+2h)] + \frac{h^2}{3} f'''(\xi)$$

- Midpoint Formula:

$$f'(x_0) = \frac{1}{2h} [f(x_0+h) - f(x_0-h)] - \frac{h^2}{6} f'''(\xi)$$

→ more accurate!

$dx = 0.1$

$dt = 0.05$

2nd Derivative Midpoint formula:  

$$f''(x_0) = \frac{1}{h^2} [f(x_0-h) - 2f(x_0) + f(x_0+h)] - \frac{h^2}{12} f^{(4)}(\xi)$$

### "5- POINT FORMULAS"

• Endpoint Formula:

$$f'(x_0) = \frac{1}{12h} [-25f(x_0) + 48f(x_0+h) - 36f(x_0+2h) + 16f(x_0+3h) - 3f(x_0+4h)] + \frac{h^4}{5} f^{(5)}(\xi)$$

• Midpoint Formula:

$$f'(x_0) = \frac{1}{12h} [f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)] + \frac{h^3}{30} f^{(5)}(\xi)$$

Use forward diff formula to approx. derivative of  $f(x) = \ln x$  at  $x_0 = 1.8$  w/  $h = 0.1, 0.05$  &  $0.01$ . And determine error bounds.

$$f'(1.8) = \frac{f(1.8+h) - f(1.8)}{h}$$

at  $h = 0.1$

$$= \frac{f(1.8+0.1) - f(1.8)}{0.1}$$

$$= \frac{\ln(1.9) - \ln(1.8)}{0.1}$$

$$= 0.5406722$$

For error band:

$$f''(x) = \frac{-1}{x^2} \rightarrow \frac{-1}{\xi^2} \rightarrow \text{minus sign mod ki waja se hat gaye}$$

$$\frac{|h f''(\xi)|}{2} = \frac{|h|}{2 \xi^2} < 0.1$$

$$= 0.0154321$$

$h$	$f'(1.8)$	$ h /2\xi^2$
0.05	0.5406722	0.0077160
0.01	0.5540180	0.0015432



Q1(b) Use forward & backward difference formula to determine missing entry in the following tables.

$x$	$f(x)$	$f'(x)$
0.0	0.0000	
0.2	0.74140	
0.4	1.3718	

$f(0.0)$  &  $f(0.2)$  using forward &  $f(0.4)$  using backward.

$$h = 0.2$$

$$f'(0.0) = \frac{f(0.0000 + 0.2) - f(0.0000)}{0.2}$$

$$= \frac{0.74140 - 0.0000}{0.2}$$

$$f'(0.0) = 3.707$$

$$f'(0.2) = \frac{f(0.2 + 0.2) - f(0.2)}{0.2}$$

$$f'(0.2) = 3.152$$

$$f'(0.4) = \frac{f(0.4) - f(0.4 - 0.2)}{0.2}$$

$$f'(0.4) = 3.152$$

Q2(a):

$x$	$f(x)$	$f'(x)$
-0.3	1.9507	
-0.2	2.0421	
-0.1	2.0601	

$$x_1 - x_0 = 0.1 \text{ so } h = 0.1$$

Using Forward for  $f'(0.3)$  &  $f'(0.2)$

$$f'(-0.3) = \frac{f(-0.3 + 0.1) - f(-0.3)}{0.1}$$

$$= \frac{2.0421 - 1.9507}{0.1}$$

$$f'(-0.3) = 0.914$$

$$f'(-0.2) = \frac{f(-0.2 + 0.1) - f(-0.2)}{0.1}$$

$$= \frac{f(-0.1) - f(-0.2)}{0.1}$$

$$f'(-0.2) = 0.18$$

→ endpoint cannot be applied.  
midpoint

$$f'(-0.1) = \frac{f(-0.1) - f(-0.1 - 0.1)}{0.1}$$

$$= \frac{2.0601 - f(2.0421)}{0.1}$$

$$f'(-0.1) = 0.18$$

$$f'(8.5) = \frac{1}{0.4} [f(8.7) - f(8.3)]$$

$$f'(8.5) = 3.139975$$

Taking  $h = -0.2$

$$f'(8.7) = \frac{1}{-0.4} [-3f(8.7) + 4f(8.5) - f(8.3)]$$

$$f'(8.7) = 3.163525$$

Q5(h): Use most accurate 3-point formula to determine each missing entry

$x$	$f(x)$	$f'(x)$
2.1	16.94410	3.09205
2.3	17.86492	3.11615
2.5	18.19056	3.139975
2.7	18.82091	3.163525

$$h = 0.2$$

midpoint wala formula use krni karna  
7.9 ki value mu di deti.

$$f'(8.1) = \frac{1}{2(0.2)} [-3f(8.1) + 4f(8.3) - f(8.5)]$$

$$f'(8.1) = 3.04205$$

→ endpoint

$$f'(8.3) = \frac{1}{2(0.2)} [-3f(8.3) + 4f(8.5) - f(8.7)]$$

$$f'(8.3) = 2.916425$$

$$f'(8.5) = \frac{1}{2(0.2)} [f(8.5) - f(8.1)]$$

Q6:

$x$	$f(x)$	$f'(x)$
-0.3		
-0.2	-0.87652	
-0.1	-0.25074	
0	-0.16134	
0.1	0	

$$h = 0.1$$

$$f'(-0.3) = \frac{1}{2(0.1)} [-3f(-0.3) + 4f(-0.2) - f(-0.1)]$$

$$f'(-0.3) = -0.66030$$

$$f'(-0.2) = \frac{1}{0.2} [f(-0.2) - f(-0.3)]$$

$$f'(-0.2) = 0.5759$$



# Ex 4.2

## "NUMERICAL INTEGRATION"

### ⑥ CLOSED NEWTON-COTES FORMULA:

$n=1$  : Trapezoidal Rule. [can be applied on any no. of intervals]  
 $\int_a^b f(x) dx$

$$\int_{x_0}^{x_1} f(x) dx = \frac{h}{2} [f(x_0) + f(x_1)] - \frac{h^3}{12} f''(\xi)$$

$n=2$  : Simpson Rule. One by 3 [applied only on even no. of intervals]  
 $\int_a^b f(x) dx$  most accurate

$$\int_{x_0}^{x_2} f(x) dx = \frac{h}{3} [f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi)$$

$n=3$  : Simpson's Three by Eight Rule [applied on intervals multiple of 3]  
 $\int_a^b f(x) dx$  less accurate than  $\frac{4}{3}$  but more than trapezoid

$$\int_{x_0}^{x_3} f(x) dx = \frac{3h}{8} [f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)] - \frac{3h^5}{80} f^{(4)}(\xi)$$

Composite Trapezoidal (more accurate than Trapezoidal.)

prelo extreme values odd range (plus jitni bachni hai unko x2 by 2 krke add)

$$\int_a^b f(x) dx = \frac{h}{2} [f(0) + f(n) + 2[f(1) + f(2) + \dots + f(n-1)]]$$

Composite Simpson  $\frac{4}{3}$  :

$$\int_a^b f(x) dx = \frac{h}{3} [f(0) + f(n) + 4(f(1) + f(3) + \dots + \text{odd range}) + 2(f(2) + f(4) + \dots + \text{even range})]$$

Composite Simpson  $\frac{3}{8}^{\text{th}}$  Rule:

$$\int_a^b f(x) dx = \frac{3h}{8} [f(x_0) + f(x_n) + 3(f(x_1) + f(x_2) + f(x_4) + \dots) + 2(f(x_3) + \dots)]$$

not multiple of 3                      multiple of 3

Q1(a):

$$\int_{0.5}^1 x^2 dx$$

Trapezoidal Rule:

$$h = 0.5$$

$$= \frac{0.5}{2} [f(0.5) + f(1)]$$

$$= 0.265625$$

Simpson  $\frac{1}{3}$  Rule:

$$h = 0.25$$

$$= \frac{0.25}{3} [f(0.5) + 4f(0.75) + f(1)]$$

$$= 0.19401$$

Simpson  $\frac{3}{8}$  Rule:

$$h = 0.1667$$

$$= \frac{3(0.1667)}{8} [f(0.5) + 3f(0.6667) + 3f(0.8333) + f(1)]$$

$$= 0.19379$$

Q1(e):

$$\int_1^{1.6} \frac{dx}{x^2 - 4}$$

Trapezoidal Rule:

$$h = \frac{3}{5}$$

$$= \frac{(3/5)}{2} [f(1) + f(1.6)]$$

$$= -0.846667$$

Simpson  $\frac{1}{3}$  Rule:

$$h = \frac{3}{10}$$

$$= \frac{(3/10)}{3} [f(1) + 4f(1.3) + f(1.6)]$$

$$= -0.7391053$$

Simpson  $\frac{3}{8}$  Rule:

$$h = \frac{1}{5}$$

$$= \frac{3(1/5)}{8} [f(1) + 3f(1.2) + 3f(1.4) + f(1.6)]$$

$$= -0.736428$$



$$(g): \int_0^{\pi/4} x \sin x \, dx$$

Trapezoidal:

$$h = \pi/4$$

$$= \frac{h}{2} [f(0) + f(\pi/4)]$$

$$= 0.2180895$$

Simpson  $1/3$  Rule:

$$h = \pi/8$$

$$f(0) = 0$$

$$f(\pi/8)$$

$$= \frac{h}{3} [f(0) + 4f(\pi/8) + f(\pi/4)]$$

$$= 0.1513826$$

Simpson  $3/8$  Rule:

$$h = \pi/12$$

$$= \frac{3h}{8} [f(0) + 3f(\pi/12) + 3f(\pi/6) + f(\pi/4)]$$

$$= 0.1515852$$

$$Q2(c): \int_{0.75}^{1.3} (\sin x^2 - 2x \sin x + 1) \, dx$$

Trapezoidal,  $h = 1/20$

$$= \frac{h}{2} [f(0.75) + f(1.3)]$$

$$= -0.037024$$

Simpson  $1/3$  Rule:  $h = 1/40$

$$= \frac{h}{3} [f(0.75) + 4f(0.75 + 1/40) + f(1.3)]$$

$$= -0.02027158991$$

Simpson  $3/8$  Rule:

$$= -0.0499$$

$$Q2(a): \int_e^{e+1} \frac{1}{x \ln x} dx$$

Trapezoidal:  $h=1$

$$= \frac{1}{2} [f(e) + f(e+1)]$$

$$= 0.2863341726$$

Simpson  $1/3$  rule:  $h=1/2$

$$= \frac{1}{6} [f(e) + 4f(e+1/2) + f(e+1)]$$

$$= 0.2726704525$$

Q3(a):

$$f''(x) = 12x^2$$

$$\text{Error bound} = \frac{h^3 f'''(\xi)}{12}$$

$$= \frac{(0.5)^3 \cdot 12(\xi)^2}{12}$$

$$\xi = 1$$

$$|\text{Error bound}| = 0.125$$

$$\text{Actual value} = 0.19375$$

$$\text{Absch} = |0.19375 - 0.265625|$$

$$|Abs| = 0.071875$$

Q3(b):

$$f''(x) =$$



# • OPEN NEWTON-COTES FORMULA:

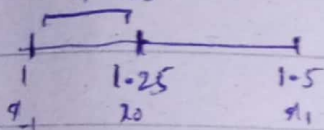
$n=0$ : Midpoint Rule

$$\int_{x_1}^{x_2} f(x) dx = 2h f(x_0) + \frac{h^3}{3} f'''(\xi)$$

$n=1$ :

$$\int_{x_1}^{x_2} f(x) dx = \frac{8h}{2} [f(x_0) + f(x_1)] + \frac{8h^3}{4} f'''(\xi)$$

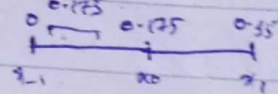
Q1(c):  $\int_1^{1.5} x^2 \ln x dx$   
 $h=0.25$



$$\approx 2(0.25) f(1.25)$$

$$\approx 0.174330$$

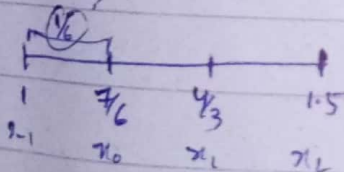
Q1(d):  $\int_0^{0.35} \frac{2}{x^2-4} dx$



$$\approx 2(0.175) f(0.175)$$

$$\approx -0.1763501811$$

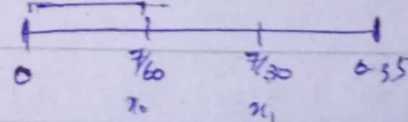
$n=2$ :  $\frac{1.5-1}{3} = \frac{1}{6}$



$$\approx \frac{3(1/6)}{2} [f(7/6) + f(4/3)]$$

$$\approx 0.1803127$$

$n=2$ :  $\frac{0.35-0}{3} = \frac{7}{60}$



$$\approx \frac{3(7/60)}{2} [f(7/60) + f(7/30)]$$

$$\approx$$

$$x^2 e^{-x}$$

$$2xe^{-x} - x^2 e^{-x} = 0$$

Ex 4.4

$$xe^{-x}(2-x) = 0$$

Q1 (h):

$$xe^{-x} = 0 \quad 2-x=0$$

$$x=2 \quad \checkmark$$

$$\int_0^{3\pi/8} \tan x \, dx, \quad n=8$$

$x_0$	0	0
$x_1$	$3\pi/64$	0.1483360
$x_2$	$3\pi/32$	0.30334669
$x_3$	$9\pi/64$	0.47296478
$x_4$	$9\pi/16$	0.6681786
$x_5$	$15\pi/64$	0.9063472
$x_6$	$9\pi/32$	0.2185035
$x_7$	$21\pi/64$	1.6683992
$x_8$	$3\pi/8$	2.4142136

Composite Trapezoid:

$$P = \frac{\frac{3\pi}{64}}{2} \left[ f(x_0) + f(x_8) + 2[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7)] \right]$$

Composite Simpson:

$$P = \frac{\frac{3\pi}{64}}{3} \left[ f(x_0) + f(x_8) + 4[f(x_1) + f(x_3) + f(x_5) + f(x_7)] + 2[f(x_2) + f(x_4) + f(x_6)] \right]$$