22K-4187
SYED SAMROZE ALI

ASSIGNMENT #02

Ex: 4.1

— QUESTION # 03 —

Le V denote the set of all real no.

Axiom 1: $x+y$ is in V for all real x & y

Axiom 2: $x+y = y+x$ for all real x & y

Axiom 3: $x+(y+z) = (x+y)+z$ for all real x, y & z

Axiom 4: for each $v = x$, we have $0+x = x+0$ for all real x

Axiom 5: For each $v = x$ let $-v = -x$ the $x+(-x) = (-x)+x = 0$

Axiom 6: kx is in V for all real k, x ~~&y~~

Axiom 7: $k(x+y) = kx+ky$ for all real k, x & y

Axiom 8: $(k+m)x = kx+mx$ for all real k, m & x

Axiom 9: $k(mx) = (km)x$ for all real k, m & x

Axiom 10: $1x = x$ for all real x

It's a vector space.

— QUESTION #5 —

→ Axiom 5 fails whenever $x \neq 0$ since it is then impossible to find (x', y') satisfying $x' > 0$ for which $(x, y) + (x', y') = (0, 0)$.

→ Axiom 6 fails whenever $k < 0$ & $x \neq 0$

This is not a vector space.

Ex: 4.3

—QUESTION # 01—

(a)

$U = (0, -2, 2)$, $V = (1, 3, -1)$
$W = (2, 2, 2)$

$K_1 U + K_2 V = W$

$K_1 (0, -2, 2) + K_2 (1, 3, -1) = (2, 2, 2)$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$A = \begin{bmatrix} -2 & 3 & 2 \\ 0 & 1 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

$-\frac{1}{2} R_1 \rightarrow R_1$

$$A = \begin{bmatrix} 1 & -3/2 & -1 \\ 0 & 1 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

$R_3 - 2R_1 \rightarrow R_3$

$$A = \begin{bmatrix} 1 & -3/2 & -1 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$R_3 - 2R_2 \rightarrow R_3$

$$A = \begin{bmatrix} 1 & -3/2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 + \frac{3}{2} R_2 \rightarrow R_1$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$K_1 = 2$
$K_2 = 2$

The system is consistent & $(2, 2, 2)$ is linear combination of U & V.

(b)

$U = (0, -2, 2)$, $V = (1, 3, -1)$

$W = (0, 4, 5)$

$K_1 U + K_2 V = W$

$K_1 (0, -2, 2) + K_2 (1, 3, -1) = (0, 4, 5)$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 3 & 4 \\ 2 & -1 & 5 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$A = \begin{bmatrix} -2 & 3 & 4 \\ 0 & 1 & 0 \\ 2 & -1 & 5 \end{bmatrix}$$

$-\frac{1}{2} R_1 \rightarrow R_1$

$$A = \begin{bmatrix} 1 & -3/2 & -2 \\ 0 & 1 & 0 \\ 2 & -1 & 5 \end{bmatrix}$$

$R_3 - 2R_1 \rightarrow R_3$

$$A = \begin{bmatrix} 1 & -3/2 & -2 \\ 0 & 1 & 0 \\ 0 & 2 & 9 \end{bmatrix}$$

$R_3 - 2R_2 \rightarrow R_3$

$$A = \begin{bmatrix} 1 & -3/2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\frac{1}{9} R_3 \rightarrow R_3$

$$A = \begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 + 3/2 R_2 \to R_1$

$$A = \begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As the system is inconsistent so $(0,4,5)$ is not a linear combination of U & V.

(c)

$U = (0,-2,2)$
$V = (1,3,-1)$
$\omega = (0,0,0)$

$K_1 U + K_2 V = \omega$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 3 & 0 \\ 2 & -1 & 0 \end{bmatrix}$$

as we can see that $k_1 = 0$, $k_2 = 0$, $k_3 = 0$

so we can conclude that system is consistent & $(0,0,0)$ is a linear combination of U & V.

— QUESTION #12 —

(a)

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

~~the standard basis of R^2~~
the e_1 & e_2 for A is

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$U = (1,2)$

$K_1 e_1 + K_2 e_2 = U$

$K_1(1,0) + K_2(2,1) = (1,2)$

Augmented Matrix

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

The coefficent Matrix is

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right) = 1$$

Therefore system is consistent.

We can conclude that $U = (1,2)$ is span of $\{T_A(e_1), T_A(e_2)\}$

(b)

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The e_1 & e_2 for A is

$$e = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$K_1 e_1 + K_2 e_2 = U$

$K_1(1,1) + K_2(1,1) = (1,2)$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Coefficent matrix is

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right) = 0$$

~~Here~~ : $\det(B) = 0$ so system is inconsistant And therefore vector $U = (1,2)$ not a span of $\{T_A(e_1), T_A(e_2)\}$

Ex : 4.4

— QUESTION #02 —

(a)

$K_1 V_1 + K_2 V_2 + K_3 V_3 = 0$

$K_1(-3,0,4) + K_2(5,-1,2) +$

$K_3(1,1,3) = 0$

$$A = \left[\begin{array}{ccc|c} -3 & 5 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 4 & 2 & 3 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$R_3 + \frac{3}{4} R_1 \rightarrow R_3$

$$A = \left[\begin{array}{ccc|c} 4 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 13/2 & 13/4 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$R_3 + 2/13 R_2 \rightarrow R_3$

$$A = \left[\begin{array}{ccc|c} 4 & 2 & 3 & 0 \\ 0 & 13/2 & 13/4 & 0 \\ 0 & 0 & 3/2 & 0 \end{array}\right]$$

$2/3 R_3 \rightarrow R_3$

$R_2 - 13/4 R_3 \rightarrow R_2$

$$A = \left[\begin{array}{ccc|c} 4 & 2 & 3 & 0 \\ 0 & 13/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 - 3R_3 \rightarrow R_1$

$2/13 R_2 \rightarrow R_2$

$$A = \left[\begin{array}{ccc|c} 4 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 - 2R_2 \rightarrow R_1$

$1/4 R_1 \rightarrow R_1$

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\because K_1 = K_2 = K_3 = 0$

$\therefore$ The vectors are linearly independent.

(b)

$K_1(-2,0,1)+K_2(3,2,5)+K_3(6,-1,1)+K_4(7,0,-2)=0$

$$A=\left[\begin{array}{cccc|c} -2 & 3 & 6 & 7 & 0 \\ 0 & 2 & -1 & 0 & 0 \\ 1 & 5 & 1 & -2 & 0 \end{array}\right]$$

$R_3+\frac{1}{2}R_1 \to R_3$

$R_2 \leftrightarrow R_3$

$$A=\left[\begin{array}{cccc|c} -2 & 3 & 6 & 7 & 0 \\ 0 & 13/2 & 4 & 3/2 & 0 \\ 0 & 2 & -1 & 0 & 0 \end{array}\right]$$

$-\frac{4}{13}R_2+R_3 \to R_3$

$$A=\left[\begin{array}{cccc|c} -2 & 3 & 6 & 7 & 0 \\ 0 & 13/2 & 4 & 3/2 & 0 \\ 0 & 0 & -\frac{29}{13} & -\frac{6}{13} & 0 \end{array}\right]$$

$-\frac{13}{29}R_3 \to R_3$

$-4R_3+R_2 \to R_2$

$$A=\left[\begin{array}{cccc|c} -2 & 3 & 6 & 7 & 0 \\ 0 & 13/2 & 0 & 39/58 & 0 \\ 0 & 0 & 1 & 6/29 & 0 \end{array}\right]$$

$-6R_3+R_1 \to R_1$

$\frac{2}{13}R_2 \to R_2$

$$A=\left[\begin{array}{cccc|c} -2 & 3 & 0 & 167/29 & 0 \\ 0 & 1 & 0 & 3/29 & 0 \\ 0 & 0 & 1 & 6/29 & 0 \end{array}\right]$$

$R_1-3R_2 \to R_1$

$-\frac{1}{2}R_1 \to R_1$

$$A=\left[\begin{array}{cccc|c} 1 & 0 & 0 & -79/29 & 0 \\ 0 & 1 & 0 & 3/29 & 0 \\ 0 & 0 & 1 & 6/29 & 0 \end{array}\right]$$

$K_1 = \frac{79}{29}K_4$

$K_2 = -\frac{3}{29}K_4$

$K_3 = -\frac{6}{29}K_4$

$\therefore$ The vectors are linearly dependent.

— QUESTION # 12 —

The set with one vector is linearly dependent under condition that vector v is not zero vector

if $v = 0$, then

$k \cdot v = 0$

for all $k \in R$, which implies that it has a non-trival sol, hence set is dependent

if $v \neq 0$ then

$k \cdot v = 0$

has only trival sol $k = 0$, hence set is independent.

Ex: 4.5

— QUESTION # 01 —

$K_1 (2,1) + K_2 (3,0) = 0$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

$\det(A) = -3$

which proves that vector $(2,1)$ & $(3,0)$ form a basis for R^2

— QUESTION # 02 —

The given set is

$\{(3,1,-4), (2,5,6), (1,4,8)\}$

$K_1 (3,1,-4) + K_2 (2,5,6) + K_3 (1,4,8) = 0$

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 5 & 4 \\ 4 & 6 & 8 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \\ -4 & 6 & 8 \end{vmatrix}$$

$$= 3\begin{vmatrix} 5 & 4 \\ 6 & 8 \end{vmatrix} - 2\begin{vmatrix} 1 & 4 \\ -4 & 8 \end{vmatrix} + 1\begin{vmatrix} 1 & 5 \\ -4 & 6 \end{vmatrix}$$

$$= 3(40-24) - 2(8+16) + 8 + 16$$

$$= 26 \neq 0$$

Then $\det(A) = 26$, which proves that $(3,1,-4)$, $(2,5,6)$ & $(1,4,8)$ form a basis for R_3.

Ex: 4.6

— QUESTION #04 —

The homogenous linear system is,

$$x_1 - 4x_2 + 3x_3 - x_4 = 0$$
$$2x_1 - 8x_2 + 6x_3 - 2x_4 = 0$$

$$\left[\begin{array}{cccc|c} 1 & -4 & 3 & -1 & 0 \\ 2 & -8 & 6 & -2 & 0 \end{array}\right]$$

$-2R_1 + R_2$

$$\begin{bmatrix} 1 & -4 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let,

$x_4 = t$, $x_3 = s$, $x_2 = r$

$$x_1 - 4x_2 + 3x_3 - x_4 = 0$$
$$x_1 = 4r - 3s + t$$

$$(x_1, x_2, x_3, x_4) = (4r - 3s + t, r, s, t)$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4r - 3s + t \\ r \\ s \\ t \end{bmatrix}$$

$$= r\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Basis:

$U_1 = (4, 1, 0, 0)$

$U_2 = (-3, 0, -1, 0)$

$U_3 = (1, 0, 0, 1)$

— QUESTION # 10 —

We have the subspace of P_3 consisting all polynomials,

$a_0 + a_1 x + a_2 x^2 + a_3 x^3$

for which $a_0 = 0$

$\therefore$ $a_0 = 0$ then

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 = 0 + a_1 x + a_2 x^2 + a_3 x^3$

$= a_1 x + a_2 x^2 + a_3 x^3$

Hence we have 3 free variables, a_1, a_2 & a_3

we conclude that it is a dimension of the solution space is 3.

Ex: 4.8

— QUESTION # 01 —

(a)

$$\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 4 & 0 & -1 \\ 3 & 6 & 2 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}$$

$$= -2\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 6 \\ -1 \end{bmatrix} + 5\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

— QUESTION # 03 —

(a)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

Augmented Matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 2 \end{array}\right]$$

$-R_1 + R_2 \rightarrow R_2$

$-2R_1 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & -1 & -1 & 0 \end{array}\right]$$

$-R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

as system is inconsistent so b is not in the column space of A.

(b)

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 9 & 3 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{array}\right]$$

By reducing,

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x_1 = 1, \quad x_2 = 3, \quad x_3 = -1$

By linear Combination

$$\begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$$

$$= 1\begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- Vector b is in the col. space of A.

Ex: 4.9

QUESTION #01

(a)

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -3 & 3 \\ 4 & 8 & -4 & 4 \end{bmatrix}$$

$R_1 \leftrightarrow R_4$

$R_2 - \frac{1}{2}R_1 \rightarrow R_2$

$$A = \begin{bmatrix} 4 & 8 & -4 & 4 \\ 0 & 0 & 0 & 0 \\ 3 & 6 & -3 & 3 \\ 1 & 2 & -1 & 1 \end{bmatrix}$$

$R_3 - \frac{3}{4}R_1 \rightarrow R_3$

$R_4 - \frac{1}{4}R_1 \rightarrow R_4$

$\frac{1}{4}R_1 \rightarrow R_1$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank}(A) = 1}$

$1 + \text{Nullity}(A) = 4$

$\boxed{\text{Nullity}(A) = 3}$

(b)

$$A = \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & 4 & 5 & 8 & -4 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$R_2 + \frac{1}{3}R_1 \rightarrow R_2$

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 0 & 0 & 5/3 & 10/3 & -10/3 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

$R_3 + \frac{2}{3}R_1 \rightarrow R_3$

$\frac{3}{13}R_3 \rightarrow R_3$

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 0 & 0 & 5/3 & 10/3 & -10/3 \\ 0 & 0 & 1 & 2 & -2 \end{bmatrix}$$

$R_1 + R_3 \rightarrow R_1$

$R_2 \leftrightarrow R_3$

$-\frac{1}{3}R_1 \rightarrow R_1$

$$\begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\begin{array}{l} \text{Rank}(A) = 2 \\ \text{Nullity}(A) = 3 \end{array}}$$

— Assignment 03 —

(a)

Rank(A) = 3

Nullity (A) = 0

(b)

Rank(A) + Nullity (A) = n

∵ rank(A) = 3 & Null(A) = 0

3 + 0 = 3

(c)

∵ Rank(A) = 3 so

the no. of leading variable is 3.

∵ Nullity (A) = 0 so no. of parameters is 0

Ex: 5-1

— QUESTION#02 —

$$Ax = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$= 4x$$

Therefore x is an eigenvector of A corresponding ~~to~~ to the eigen value 4.

QUESTION # 15

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + 4y \\ 2x + 3y \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

St. Matrix,

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

The characteristic eqn of Matrix A is $\det(\lambda I - A) = 0$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det\left(\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{bmatrix}\right)$$

$$= (\lambda-1)(\lambda-3) - (-4)(-2)$$

$$= \lambda^2 - 4\lambda - 5$$

Now using characteristic eqn

$$\lambda^2 - 4\lambda - 5 = 0$$

$$\lambda_1 = -1 \quad \& \quad \lambda_2 = 5$$

When $\lambda_1 = -1$

$$\begin{bmatrix} -1-1 & -4 \\ -2 & -1-3 \end{bmatrix} x = 0$$

$$\begin{bmatrix} -2 & -4 \\ -2 & -4 \end{bmatrix} x = 0$$

By reduce Echelon form

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$x_2 = t$

$x_1 = -2t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2t \\ t \end{bmatrix}$$

$$= t\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ is eigen space

when $\lambda_1 = -1$

when $\lambda_2 = 5$

$$\begin{bmatrix} 5-1 & -4 \\ -2 & 5-3 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix} x = 0$$

Reduce Echelon form

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$x_2 = t$

$x_1 = t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix}$$

$$= t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is eigen vector

at $\lambda_2 = 5$

Ex: 5-2

— QUESTION # 02 —

If two sq. matrix are similar then they have the same determinants

$$\det(A) = \begin{vmatrix} 4 & -1 \\ 2 & 4 \end{vmatrix}$$

$$= 16 + 2 = \cancel{18}$$

$$\boxed{\det(A) = 18}$$

$$\det(B) = \begin{vmatrix} 4 & 1 \\ 2 & 4 \end{vmatrix}$$

$$= 16 - 2$$

$$\boxed{\det(B) = 14}$$

$\because \det(A) \neq \det(B)$

The matrix A & B are not similar.

— QUESTION # 05 —

By Characteristic Eqn

$$\det(\lambda I - A) = 0$$

$$\det\left(\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-1 & 0 \\ -6 & \lambda+1 \end{bmatrix}\right) = 0$$

$$(\lambda - 1)(\lambda + 1) + 0 = 0$$

$$\lambda_1 = 1 \quad , \quad \lambda_2 = -1$$

at $\lambda_1 = 1$

$$\begin{bmatrix} 1-1 & 0 \\ -6 & 1+1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} -6 & 2 \\ 0 & 0 \end{bmatrix}$$

$R_1 \div 6$

$$\begin{bmatrix} 1 & -1/3 \\ 0 & 0 \end{bmatrix}$$

$x_2 = t$

$x_1 = 1/3\, t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/3\, t \\ t \end{bmatrix}$$

$$= t \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$$

$$= t \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is eigenspace at $\lambda = 1$

at $\lambda = -1$

$$\begin{bmatrix} -1-1 & 0 \\ -6 & -1+1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 \\ -6 & 0 \end{bmatrix}$$

$-3R_1 + R_2 \rightarrow R_2$

$$\begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$$

$R_1 \div -2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$x_2 = t$

$x_1 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix}$$

$$= t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is eigen space at $\lambda_2 = -1$

$\rightarrow$

$$P = [P_1 \quad P_2]$$

$$P = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$P^{-1} \cdot A \cdot P = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P^{-1} A \cdot P = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$