Ex: 4.1

Numerical Differention

- Forward Difference:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} + O(h)$$

- Backward Difference:

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h} + O(h)$$

- Central Difference:

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} + O(h^2)$$

"3-Point Formulas"

- Endpoint Formula:

$$f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right] + \frac{h^2}{3}f'''(\xi_0)$$

- Midpoint Formula:

$$f'(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right] - \frac{h^2}{6}f'''(\xi_1)$$

more accurate!

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2nd Derivative Midpoint formula:

$$f''(x_0) = \frac{1}{h^2}\{f(x_0-h) - 2f(x_0) + f(x_0+h)\} - \frac{h^2}{12} f^{(4)}(\xi)$$

"5-POINT FORMULAS"

- Endpoint Formula:

$$f'(x_0) = \frac{1}{12h}[-25f(x_0) + 48f(x_0+h) - 36f(x_0+2h) + 16f(x_0+3h) - 3f(x_0+4h)] + \frac{h^4}{5} f^{(5)}(\xi)$$

- Midpoint Formula:

$$f'(x_0) = \frac{1}{12h}[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)] + \frac{h^4}{30} f^{(5)}(\xi)$$

Q: Use forward diff formula to appr. derivative of $f(x) = \ln x$ at $x_0 = 1.8$ using $h = 0.1, 0.05$ & 0.01. And determine error bounds.

$$f'(1.8) = \frac{f(1.8+h) - f(1.8)}{h}$$

at $h = 0.1$

$$= \frac{f(1.8+0.1) - f(1.8)}{0.1}$$

$$= \frac{\ln(1.9) - \ln(1.8)}{0.1}$$

$$= 0.5406722$$

For error bound:

$f''(x) = \frac{-1}{x^2}$ $\quad \frac{-1}{\xi^2}$ → minus sign mod ki waja se hat gaya

$$\frac{|h f''(\xi)|}{2} = \frac{|h|}{2\xi^2} < \frac{0.1}{2(1.8)^2}$$

$$= 0.0154321$$

h	$f'(1.8)$	$\lvert h\rvert / 2\xi^2$
0.05	0.54~~06722~~ 79795	0.0077160
0.01	0.5540180	0.0015432

Q1(b) Use forward & backward difference formulas to determine missing entry in the following tables.

x	$f(x)$	$f'(x)$
0.0	0.0000	
0.2	0.74140	
0.4	1.3718	

$f(0.0)$ & $f(0.2)$ ke liye forward & $f(0.4)$ ke liye backward.

$\boxed{h = 0.2}$

$$f'(0.0) = \frac{f(0.0000+0.2) - f(0.0000)}{0.2}$$

$$= \frac{0.74140 - 0.0000}{0.2}$$

$$\boxed{f'(0.0) = 3.707}$$

$$f'(0.2) = \frac{f(0.2+0.2) - f(0.2)}{0.2}$$

$$\boxed{f'(0.2) = 3.152}$$

$$f'(0.4) = \frac{f(0.4) - f(0.4-0.2)}{0.2}$$

$$\boxed{f'(0.4) = 3.152}$$

Q2(a):

x	$f(x)$	$f'(x)$
-0.3	1.9507	
-0.2	2.0421	
-0.1	2.0601	

$x_1 - x_0 = 0.1$ so $\boxed{h = 0.1}$

Using forward fo $f(0.3)$ & $f(0.2)$

$$f'(-0.3) = \frac{f(-0.3+0.1) - f(0.3)}{0.1}$$

$$= \frac{2.0421 - 1.9507}{0.1}$$

$$\boxed{f'(0.3) = 0.914}$$

$$f'(0.2) = \frac{f(-0.2+0.1) - f(-0.2)}{0.1}$$

$$= \frac{f(-0.1) - f(-0.2)}{0.1}$$

$$\boxed{f'(0.2) = 0.18}$$

$f'(-0.1) = \frac{f(-0.1) - f(-0.1-0.1)}{0.1}$

$= \frac{2.0601 - f2.0421}{0.1}$

$\boxed{f'(-0.1) = 0.18}$

Q5(b): Use most accurate 3-point formula to determine each missing entry

x	$f(x)$	$f'(x)$
8.1	16.94410	3.09205
8.3	17.56492	3.11615
8.5	18.19056	3.139975
8.7	18.82091	3.163525

$h = 0.2$

midpoint wala isliye use nhi kiya q k 7.9 ki value nhi di wi.

$f'(8.1) = \frac{1}{2(0.2)}[-3f(8.1) + 4f(8.3) - f(8.5)]$

$\boxed{f'(8.1) = 3.09205}$

endpt

$f'(8.3) = \frac{1}{2(0.2)}[-3f(8.3) + 4f(8.5) - f(8.7)]$

$f'(8.3) = 2.916425$ midpoint

$f'(8.3) = \frac{1}{0.4}[f(8.5) - f(8.1)]$

endpoint cannot be applied. for midpoint

$f'(8.5) = \frac{1}{0.4}[f(8.7) - f(8.3)]$

$\boxed{f'(8.5) = 3.139975}$

Taking $h = -0.2$

$f'(8.7) = \frac{1}{-0.4}[-3f(8.7) + 4f(8.5) - f(8.3)]$

$\boxed{f'(8.7) = 3.163525}$

Q6:

(a):

x	$f(x)$	$f'(x)$
-0.3	-0.27652	
-0.2	-0.25074	
-0.1	-0.16134	
0	0	

$h = 0.1$

$f'(-0.3) = \frac{1}{2(0.1)}[-3f(-0.3) + 4f(-0.2) - f(-0.1)]$

$\boxed{f'(-0.3) = -0.06030}$

$f'(-0.2) = \frac{1}{0.2}[f(-0.1) - f(-0.3)]$

$\boxed{f'(-0.2) = 0.5759}$

5/8

Ex: 4.2

"NUMERICAL INTEGRATION"

- CLOSED NEWTON-COTES FORMULA:

$n=1$: Trapezoidal Rule. [can be applied on any no. of intervals]

$\boxed{h=b-a}$

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0)+f(x_1)\right] - \frac{h^3}{12} f''(\xi)$$

$n=2$: Simpson Rule. One by 3 [applied only on even no. of intervals]
most accurate

$\boxed{h=\frac{b-a}{2}}$

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right] - \frac{h^5}{90} f^{(4)}(\xi)$$

$n=3$: Simpson's Three by Eight Rule [applied on intervals multiple of 3]
less accurate than 1/3 but more than trapezoid

$\boxed{h=\frac{b-a}{3}}$

$$\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\left[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right] - \frac{3h^5}{80} f^{(4)}(\xi)$$

Composite Trapezoidal (more accurate than trapezoidal.

Phele extreme values add honge (phir jitni bachi hai unko x2 by 2 krke add

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(0)+f(n) + 2\left[f(1)+f(2)+\dots+f(n-1)\right]\right]$$

Composite Simpson 1/3 :

$$\int_a^b f(x)\,dx = \frac{h}{3}\left\{f(0)+f(n) + 4\left(f(1)+f(3)+ \text{odd wale}\right) + 2\underbrace{\left(f(2)+f(4)+\dots\right)}_{\text{even wale}}\right\}$$

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Composite Simpson 3/8th Rule:-

$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(0)+f(n)+3(f(1)+f(2)+f(4)-)+2[f(3)\ldots\right]$

not multiple of 3 — multiple of 3

Q1(a):

$\int_{0.5}^{1} x^4\,dx$

Trapezoidal Rule:
$h = 0.5$

$= \frac{0.5}{2}[f(0.5)+f(1)]$

$= 0.265625$

Simpson 1/3 Rule:
$h = 0.25$

0.5+h

$= \frac{0.25}{3}[f(0.5)+4f(0.75)+f(1)]$

$= 0.19401$

Simpson 3/8 Rule:
$h = 0.1667$

$= \frac{3(0.1667)}{8}[f(0.5)+3f(0.6667)+3f(0.833)+f(1)]$

$= 0.19379$

Q1(e):

$\int_{1}^{1.6} \frac{2x}{x^2-4}\,dx$

Trapezoidal:
$h = 3/5$

$= \frac{(3/5)}{2}[f(1)+f(1.6)]$

$= -0.866667$

Simpson 1/3 Rule:
$h = 3/10$

$= \frac{(3/10)}{3}[f(1)+4f(1+3/10)+f(1.6)]$

$= -0.7391053$

Simpson 3/8 Rule:
$h = 1/5$

$= \frac{3(1/5)}{8}[f(1)+3f(1.2)+3f(1.4)+f(1.6)]$

$= -0.736428$

(g): $\int_0^{\pi/4} x \sin x \, dx$

Trapezoidal:

$h = \pi/4$

$= \frac{h}{2}[f(0) + f(\pi/4)]$

$= 0.2180895$

Simpson 1/3 Rule:

$h = \pi/8$

$f(0) = 0$

$= \frac{\frac{\pi}{8}}{3} \times [f(0) + 4f(\pi/8) + f(\pi/4)]$

$= 0.1513826$

Simpson 3/8 Rule:

$h = \pi/12$

$= \frac{3(\pi/12)}{8}[f(0) + 3f(\pi/12) + 3f(\pi/6) + f(\pi/4)]$

$= 0.1515852$

Q2(c): $\int_{0.75}^{1.3} ((\sin x)^2 - 2x\sin x + 1)\, dx$

Trapezoidal: $h = 11/20$

$= \frac{h}{2}[f(0.75) + f(1.3)]$

$= -0.037024$

Simpson 1/3 Rule: $h = 11/40$

$= \frac{h}{3}[f(0.75) + 4f(0.75 + 11/40) + f(1.3)]$

$= -0.0202715899$

Simpson 3/8 Rule:

~~$= -0.2099$~~

Q2(d): $\int_{e}^{e+1} \frac{1}{x \ln x} dx$

Trapezoidal: $h = 1$

$= \frac{1}{2}[f(e) + f(e+1)]$

$= 0.2863341726$

Simpson 1/3 rule: $h = 1/2$

$= \frac{1/2}{3}[f(e) + 4f(e+1/2) + f(e+1)]$

$= 0.2726704525$

Q3(a):

$f^{4}(x) = 12x^2$

Error bound $= \frac{h^3}{12} f^{4}(\xi)$

$= \frac{(0.5)^3}{12} \cdot 12(\xi)^2$ $\quad \xi = 1$

Error bnd $= 0.125$

Actual value $= 0.19375$

Abs soln $= |0.19375 - 0.265625|$

Abs $= 0.071875$

Q3(f):

$f^{4}(x) =$

• OPEN NEWTON – COTES FORMULA:

$n=0$: Midpoint Rule

$$\int_{x_{-1}}^{x_1} f(x)\,dx = 2h f(x_0) + \frac{h^3}{3} f''(\xi)$$

$n=1$:

$$\int_{x_{-1}}^{x_2} f(x)\,dx = \frac{3h}{2}\left[f(x_0)+f(x_1)\right] + \frac{3h^3}{4} f''(\xi)$$

Q1(c): $\int_1^{1.5} x^2 \ln x\,dx$

$h=0.25$

$1 \quad 1.25 \quad 1.5$

$x_{-1} \quad x_0 \quad x_1$

$= 2(0.25)\,f(1.25)$

$= 0.174330$

(d): $\int_0^{0.35} \frac{2}{x^2-4}\,dx$

0.175

$0 \quad 0.175 \quad 0.35$

$x_{-1} \quad x_0 \quad x_1$

$= 2(0.175)\,f(0.175)$

$= -0.1763501811$

$n=1$: $\frac{1.5-1}{3} = \frac{1}{6}$

$(1/6)$

$1 \quad 7/6 \quad 4/3 \quad 1.5$

$x_{-1} \quad x_0 \quad x_1 \quad x_2$

$= \frac{3(1/6)}{2}\left[f(7/6)+f(4/3)\right]$

$= 0.1803127$

$n=1$:

$7/60$

$0 \quad 7/60 \quad 7/30 \quad 0.35$

$x_0 \quad x_1$

$= \frac{3(7/60)}{2}\left[f(7/60)+f(7/30)\right]$

$=$

$x^2 e^{-x}$ $\quad 2xe^{-x} + -x^2 e^{-x} = 0$

Ex 4.4 $\quad xe^{-x}(2-x) = 0$

Q1 (h):

$xe^{-x} = 0 \quad 2-x=0$

$x = 2$

$$\int_0^{3\pi/8} \tan x \, dx \; ; n = 8$$

x_0	0	0
x_1	$3\pi/64$	0.1483360
x_2	$3\pi/32$	0.30334669
x_3	$9\pi/64$	0.47296478
x_4	$3\pi/16$	0.6681786
x_5	$15\pi/64$	0.9063472
x_6	$9/32\,\pi$	1.2185035
x_7	$21\pi/64$	1.6683992
x_8	$3\pi/8$	2.4142136

Composite Trapezoid.

$$I = \frac{\frac{3\pi}{64}}{2}\left[f(x_0) + f(x_8) + 2(f(1) + f(2) + f(3) + f(4) + f(5) + f(6) + f(7))\right]$$

=

Composite Simpson

$$f = \frac{\frac{3\pi}{64}}{3}\left[f(x_0) + f(x_8) + 4[f(x_1) + f(x_3) + f(x_5) + f(x_7)] + 2[f(2) + f(4) + f(6)]\right]$$

=