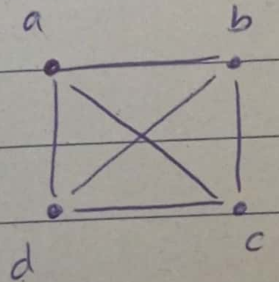
Date Day

"CHAPTER # 07"

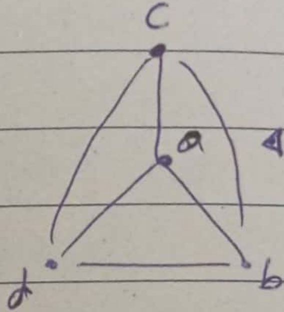
— PLANARITY —

→ A graph G is planar iff the vertices can be arranged on the page so that edges donot cross (or touch) at any point other than at a vertex.

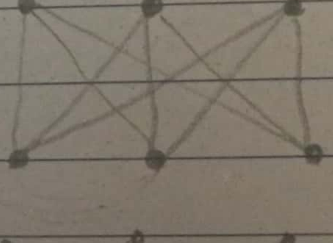
→ for a graph to be planar at least one drawing of G should exist whithout edge crossing.



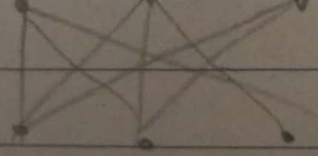
K_4



planar drawing

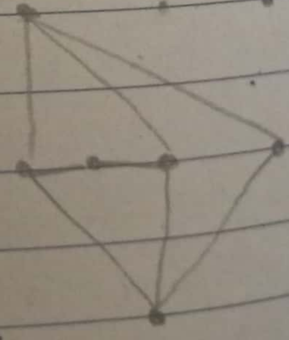


} they cannot be planar!



removed one edge

⇒



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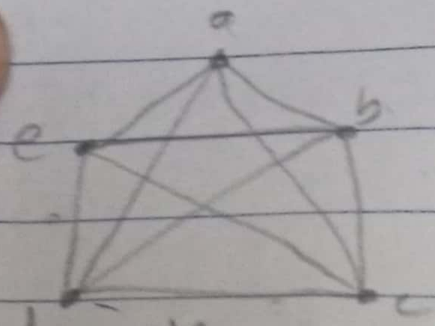
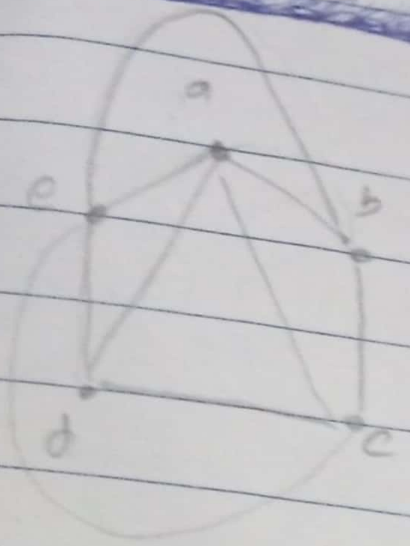
lists, namely the one exhibited by the fact that $\chi(G) = k$. However, if we remove the same one element from each of these lists, then G cannot be colored since otherwise $\chi(G) < k$.

→ for any simple graph G, $ch(G) \leq \Delta(G) + 1$.



K_5

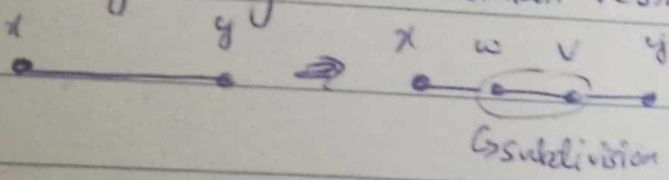


Finally the pic

So K_5 is no planar!

→ Having either K_5 or $K_{3,3}$ as a subgraph will gurantee that a graph is non-planar
→ If a subgraph is non planar than whole graph is non-planar.

Subdivision: Ek edge "xy" k darmian vertices insert krna.



↳ Subdivision of a graph can be obtained by dividing one, two or even all of the edges.

Kuratowski's Theorem:

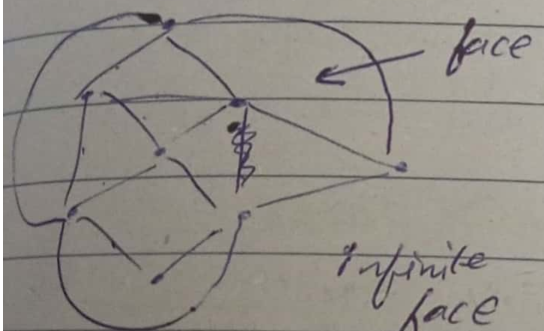
A graph G is planar iff it doesnot contain a subdivision of $K_{3,3}$ or K_5.

↳ In order to contain $K_{3,3}$ a graph must have at least 6 vertices ~~at least~~ of degree 3 or greater.

↳ For K_5 subdivision graph must have at least 5 vertices of deg 4 or greater.

Euler's Formula:

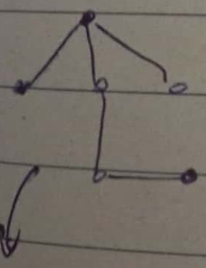
↳ Region: Given a planar drawing of G, a region is a portion of the plane completely bounded by the edges of the graph.



Size = 15 (no. of ~~vertices~~ edges)

Order = 8 (no. of vertices)

region/faces = 9 (includining infinite one)



order = 6

size = 5

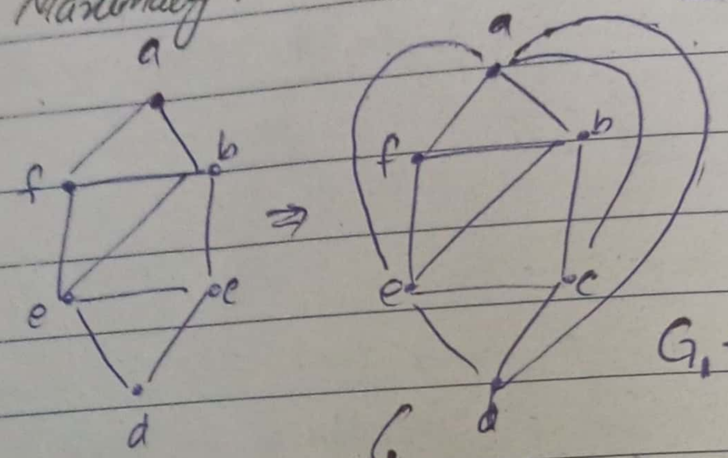
face = 1 (only infinite one)

tree only has 1 face.

"If G is a connected planar graph with n vertices, m edges, & r regions then

$$\boxed{n - m + r = 2}$$

- **Maximally Planar:**



$G_1 + \{ac, ae, ad\}$

G_1

a border for infinite region is created b/w edges of vertice a, d, e.

Now we cannot add any more edges on right hand side graph, as it will make edge-crossing.

So this is called maximally planar

"If G is a maximally planar simple graph with $n \geq 3$ vertices and m edges, then

$$\boxed{m = 3n - 6}$$

If G is a simple planar graph with m edges and $n \geq 3$ vertices then,

$$\boxed{m \leq 3n-6}$$

- If G is a simple planar graph with m edges and $n \geq 3$ and no cycles of length 3, then

$$\boxed{m \leq 2n-4}$$

"If graph doesnot satisfy the inequality, then it must be nonplanar."

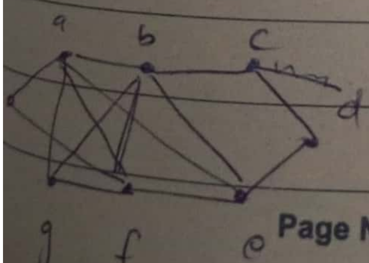
But if inequality satisfied nothing canbe defined.

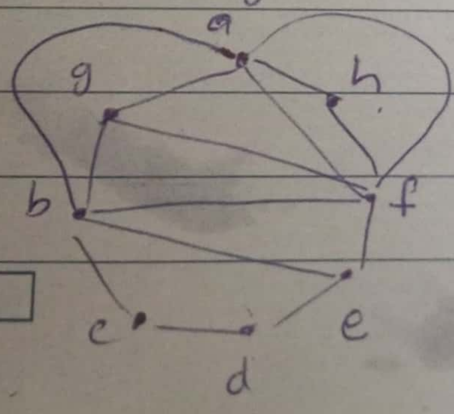
$n = 7$ $15 \leq$

$m = 15$

Cycle Chord Method:

- Technique for making planar graph
- arrange all vertices in a circle but with some care in arrangements
- Uske baad edges lgayenge circle k andar
- Or phir Circle k bahir agr andar jaga na ho.





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- **Edge Crossing:**

→ For any simple graph G the crossing number of G → cr(G), is the minimum no. of edge crossing in any drawing of G satisfying the conditions below;

i) No edge crosses another more than once

ii) at most two edges cross at a given point.

"Let G be a simple graph with m edges and n vertices. then

$$cr(G) \geq m - 3n + 6$$

$15 - 3(6) + 6$

" If G is bipartite then, $cr(G) \geq m - 2n + 4$

→ Edge crossing upper bounds:

Complete Graphs:

$$cr(K_n) \leq \frac{1}{4}\left\lfloor\frac{n}{2}\right\rfloor\left\lfloor\frac{n-1}{2}\right\rfloor\left\lfloor\frac{n-2}{2}\right\rfloor\left\lfloor\frac{n-3}{2}\right\rfloor$$

For Bipartite Graphs:

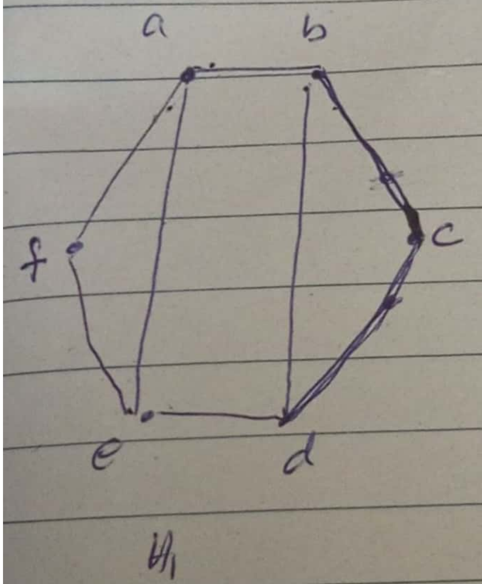
$$cr(K_{m,n}) \leq \left\lfloor\frac{m}{2}\right\rfloor\left\lfloor\frac{m-1}{2}\right\rfloor\left\lfloor\frac{n}{2}\right\rfloor\left\lfloor\frac{n-1}{2}\right\rfloor$$

Thickness :

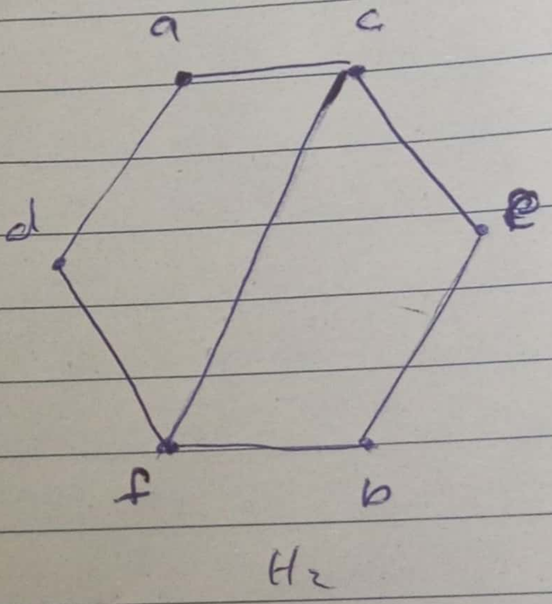
→ Let $T = \{H_1, H_2, \ldots, H_t\}$ be a set of spanning sub-graphs of G so that each H_i is planar and every edge of G appears in exactly one graph from T.

The thickness of a graph G → $\theta(G)$, is the minimum size of T among all possible such collections.

If a graph is planar than $\theta(G) = 1$.



H_1



H_2

For K_6 → $\theta(K_6) = 2$.

Simple Graphs → $\theta(G) \geq \left\lfloor \frac{m}{3n-6} \right\rfloor$

Bipartite → $\theta(G) \geq \left\lfloor \frac{m}{2n-4} \right\rfloor$

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For complete → $$\Theta(K_n) = \begin{cases} \left\lfloor \dfrac{n+7}{6} \right\rfloor & n \neq 9, 10 \\ 3 & n = 9, 10 \end{cases}$$