22K-4187
SYED SAMROZE ALI

ASSIGNMENT #01
Ex: 1-8

QUESTION #29

(a)

The standard Matrix for reflection about x-axis is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Therefor $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

we have that reflection about x-axis of (-1,-2) is (-1, -2)

(b)

The standard Matrix for reflection about y-axis is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. therefore $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

we have reflection about y-axis of (-1;2) is (1;2)

(c)

The standard Matrix for reflection about $y=x$ line is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Therefore $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

QUESTION # 30

(a)

St. Matrix of reflection on x-axis : $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

∴ reflection of (a, b) = $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ -b \end{bmatrix}$

(b)

St. Matrix of reflection on y-axis : $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

∴ reflection of (a, b) = $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -a \\ b \end{bmatrix}$

(c)

St. Matrix of reflection on x=y line : $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

∴ reflection of (a, b) = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$

Question # 31

(a)

The eqn of reflection about the xy-plane is

$w_1 = x + 0y + 0z$, $w_2 = 0x + y + 0z$, $w_3 = 0x + 0y - z$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The reflection of $(2, -5, 3)$ is

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$$

ie $\boxed{(2, -5, -3)}$

(b)

The eqn of reflection about xz-plane is,

$w_1 = x$, $w_2 = -y$, $w_3 = z$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The reflection of $(2, -5, 3)$ is

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

ie $\boxed{(2, 5, 3)}$

(c)

Equation of xz are, $w_1 = -x$, $w_2 = y$, $w_3 = z$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

reflection of $(2, -5, 3)$ is

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -5 \\ 3 \end{bmatrix}$$ i.e $\boxed{(-2, -5, 3)}$

QUESTION # 32

(a)

Eqn of xy plane are; $w_1 = x$, $w_2 = y$, $w_3 = -z$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

reflection of (a, b, c) is

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ -c \end{bmatrix}$$ i.e $(a, b, -c)$

(b)

Eqn of xz-plane is : $\omega_1 = x$, $\omega_2 = -y$, $\omega_3 = z$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The reflection point of (a, b, c) is

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ -b \\ c \end{bmatrix} \text{ i.e } \boxed{(a, -b, c)}$$

(c)

Eqn of yz-plane is : $\omega_1 = -x$, $\omega_2 = y$, $\omega_3 = z$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The reflection point of (a, b, c) is

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ i.e } \boxed{(-a, b, c)}$$

QUESTION # 33

(a)

Orthogonal projection of x-axis is

$$T(x,y) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$T(2,-5) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$ i.e $\boxed{(2,0)}$

(b)

Orthogonal projection of y-axis is

$$T(x,y) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$T(2,-5) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \end{bmatrix}$$ i.e $\boxed{(0,-5)}$

QUESTION # 34

(a)

Orthogonal Projection of x-axis is

$$T(x,y) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$T(a,b) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix}$$ i.e $\boxed{(a,0)}$

(b)

The orthogonal projection on y-axis is

$$T(x,y) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$T(a,b) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$ i.e $\boxed{(0,b)}$

QUESTION #35

(a)

The orthogonal projection on xy-plane is

$$T(x,y,z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$T(-2,1,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$ i.e $\boxed{(-2,1,0)}$

(b)

The orthogonal projection on xz-plane is

$$T(x,y,z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore T(-2,1,3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} \text{ i.e } \boxed{(-2,0,3)}$$

(c)

The orthogonal projection on yz-plane is

$$T(x,y,z) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore$

$$T(-2,1,3) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \text{ i.e } \boxed{(0,1,3)}$$

QUESTION # 36

(a)

The ~~points~~ orthogonal projection on xy-plane is given by

$$T(a,b,c) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

i.e $\boxed{(a,b,0)}$

(b)

The ~~points~~ orthogonal projection on xz-plane is given by

$$T(a,b,c) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ c \end{bmatrix}$$

i.e $\boxed{(a,0,c)}$

(c)

The ~~points~~ orthogonal projection on yz-plane is,

$$T(a,b,c) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ c \end{bmatrix}$$

i.e $\boxed{(0,b,c)}$

Question # 37

The standard matrix of rotation through an angle "θ" is,

$$T(x,y) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a)

$\theta = 30$ & vector $(3,-4)$.

$$T(3,-4) = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$T(3,-4) = \begin{bmatrix} \frac{3\sqrt{3}+4}{2} \\ \frac{3-4\sqrt{3}}{2} \end{bmatrix}$$

(b)

$\theta = -60^\circ$ & vector $(3,-4)$

$$T(3,-4) = \begin{bmatrix} \cos(-60) & -\sin(-60) \\ \sin(-60) & \cos(-60) \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\boxed{T(3,-4) = \begin{bmatrix} \frac{3-4\sqrt{3}}{2} \\ -\frac{3\sqrt{3}+4}{2} \end{bmatrix}}$$

(c)

$\theta = 45$ & vector $(3,-4)$

$$T(3,-4) = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\boxed{T(3,-4) = \begin{bmatrix} \frac{7\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}}$$

(b)

$\theta = -60^\circ$ & vector $(3,-4)$

$$T(3,-4) = \begin{bmatrix} \cos(-60) & -\sin(-60) \\ \sin(-60) & \cos(-60) \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\boxed{T(3,-4) = \begin{bmatrix} \frac{3-4\sqrt{3}}{2} \\ -\frac{3\sqrt{3}+4}{2} \end{bmatrix}}$$

(c)

$\theta = 45$ & vector $(3,-4)$

$$T(3,-4) = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\boxed{T(3,-4) = \begin{bmatrix} \frac{7\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}}$$

(d)

$\theta = 90$, vectors $(3, -4)$

$$T(3,-4) = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

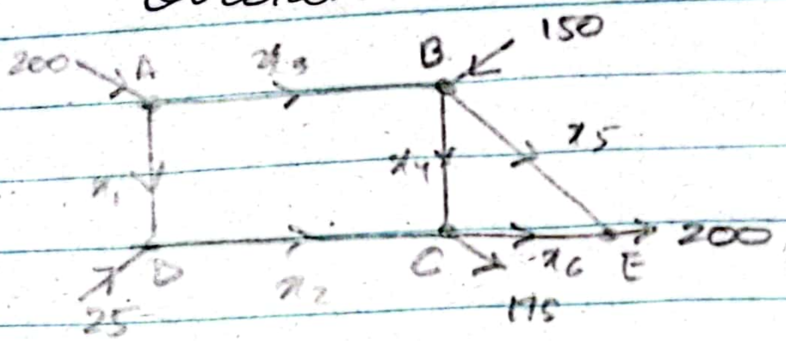
$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\boxed{T(3,-4) = \begin{bmatrix} 4 \\ 3 \end{bmatrix}}$$

CamScanner

Ex : 1-10

QUESTION #02



Node	Flow In	Flow Out
A	200	$x_1 + x_3$
B	$150 + x_3$	$x_5 + x_4$
C	$x_2 + x_4$	$x_6 + 175$
D	$x_1 + 25$	x_2
E	$x_6 + x_5$	200

The system can be rearranged as:

$$x_1 + x_3 = 200$$
$$-x_3 + x_5 + x_4 = 150$$
$$x_2 + x_4 - x_6 = 175$$
$$x_1 - x_2 = -25$$
$$x_6 + x_5 = 200$$

(b)
$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & -1 & 1 & 1 & 0 & 150 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$R_3 \times -1$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array}\right]$$

$R_1 - R_4 \rightarrow R_4$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 1 & 1 & 0 & 0 & 0 & 225 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array}\right]$$

$R_2 - R_4 = R_4$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & -1 & 1 & 0 & -1 & -50 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array}\right]$$

$R_3 + R_4 \to R_3$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & -1 & -1 & -200 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array}\right]$$

$R_4 + R_5 \to R_5$

$R_4 \times -1$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_3 = 200$

$x_2 + x_4 - x_6 = 175$

$x_3 - x_4 - x_5 = -150$

$x_5 + x_6 = 200$

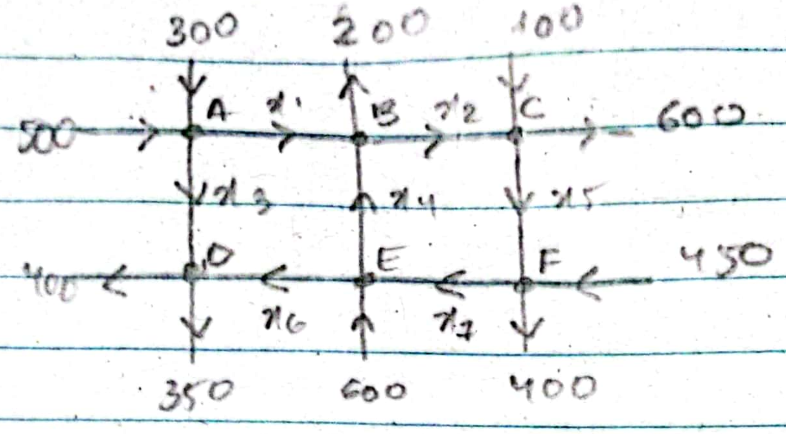
let, $x_4 = s$, $x_6 = t$

then,

$$\boxed{\begin{aligned} x_1 &= 150 + t - s \\ x_2 &= 175 - s + t \\ x_3 &= 50 + s - t \\ x_4 &= s \\ x_5 &= 200 - t \\ x_6 &= t \end{aligned}}$$

(c) When $x_4 = 50$ and $x_6 = 0$ the remaining flow rates become $x_1 = 100$, $x_2 = 125$, $x_3 = 100$, $x_5 = 200$. The direction of flow agree with the arrow in the diagram.

Question # 04



(a)

Node	Flow In	Flow Out
A	$300 + 500$	$x_1 + x_3$
B	$x_4 + x_1$	$200 + x_2$
C	$100 + x_2$	$600 + x_5$
D	$x_6 + x_3$	$400 + 350$
E	$x_7 + 600$	$x_4 + x_6$
F	$450 + x_5$	$400 + x_7$

1	2	3	4	5	6	7	=	
x_1		$+x_3$					=	800
x_1	$-x_2$		$+x_4$				=	200
	x_2			$-x_5$			=	500
		x_3			$+x_6$		=	750
			x_4		$+x_6$	$-x_7$	=	600
				x_5		$-x_7$	=	-50

1 2 3 4 5 6 7

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \end{array}\right]$$

$R_1 - R_2 \to R_2$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 600 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \end{array}\right]$$

$R_2 - R_3 \to R_3$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 600 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 100 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 750 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \end{array}\right]$$

$R_3 - R_4 \to R_4$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 600 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 100 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & -650 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 600 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \end{array}\right]$$

$R_4 + R_5 \to R_5$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 600 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 100 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & -650 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \end{array}\right]$$

$R_5 - R_6 \to R_6$ & $R_4 \times -1$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 800 \\ 0 & 1 & 1 & -1 & 0 & 0 & 0 & 600 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 100 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 650 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_3 = 800$

$x_2 + x_3 - x_4 = 600$

$x_3 - x_4 + x_5 = 100$

$x_4 - x_5 + x_6 = 650$

$x_5 - x_7 = -50$

Let, $x_6 = s$, $x_7 = t$

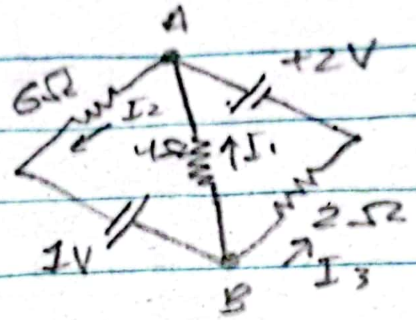
Then,

$$\boxed{\begin{aligned} x_1 &= 50 + s \\ x_2 &= 450 + t \\ x_3 &= 750 - s \\ x_4 &= 600 - s + t \\ x_5 &= -50 + t \\ x_6 &= s \\ x_7 &= t \end{aligned}}$$

For all +ve values s must be ≥ 0 & t must be ≥ 50

(c) For $x_1 = 0$, $s = -5$ which is not allowed and traffic will flow in wrong direction along path x_6.

QUESTION # 06



Node	Voltage Rise		Voltage Drops.
A	$4I_2 + 6I_2$	=	1
B	$2I_3$	=	$2+4I_1$

By Krichoff's current law at each node, we have

$I_1 - I_2 + I_3 = 0$

$$I_1 - I_2 + I_3 = 0$$
$$4I_1 + 6I_2 = 1$$
$$4I_1 - 2I_3 = -2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & 6 & 0 & 1 \\ 4 & 0 & -2 & -2 \end{array}\right]$$

$-4R_1 + R_2 \rightarrow R_2$

$-4R_1 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & -4 & 1 \\ 0 & 4 & -6 & -2 \end{array}\right]$$

$R_2 \div 10$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -2/5 & 1/10 \\ 0 & 4 & -6 & -2 \end{array}\right]$$

$-4R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -2/5 & 1/10 \\ 0 & 0 & -22/5 & -12/5 \end{array}\right]$$

$-5/22 \times R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -2/5 & 1/10 \\ 0 & 0 & 1 & 6/11 \end{array}\right]$$

~~[illegible]~~

$2/5 R_3 + R_2 \rightarrow R_2$

$-R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & -6/11 \\ 0 & 1 & 0 & 7/22 \\ 0 & 0 & 1 & 6/11 \end{array}\right]$$

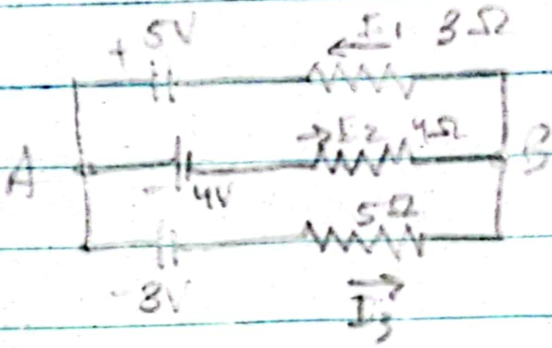
$R_2 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5/22 \\ 0 & 1 & 0 & 7/22 \\ 0 & 0 & 1 & 6/11 \end{array}\right]$$

$$\boxed{\begin{array}{l} I_1 = -5/22\ A \\ I_2 = 7/22\ A \\ I_3 = 6/11\ A \end{array}}$$

$\because I_1$ is -ve, the current is opposite to the direction shown in the diagram.

QUESTION # 08



Node	Voltage Rises	Voltage Drops
A	$3I_1 + 4I_2$	$5+4$
B	$4+5I_3$	$3+4I_2$

From Krichoff's current law at each node we have

$I_1 - I_2 - I_3 = 0$.

$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ 3I_1 + 4I_2 &= 9 \\ -4I_2 + 5I_3 &= -1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 3 & 4 & 0 & 9 \\ 0 & -4 & 5 & -1 \end{array}\right]$$

$-3R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 7 & 3 & 9 \\ 0 & -4 & 5 & -1 \end{array}\right]$$

$\frac{1}{7} \times R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & 3/7 & 9/7 \\ 0 & -4 & 5 & -1 \end{array}\right]$$

$R_2 + R_1 , \quad 4R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4/7 & 9/7 \\ 0 & 1 & 3/7 & 9/7 \\ 0 & 0 & 47/7 & 29/7 \end{array}\right]$$

$7/47 \times R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4/7 & 9/7 \\ 0 & 1 & 3/7 & 9/7 \\ 0 & 0 & 1 & 29/47 \end{array}\right]$$

$4/7 R_3 + R_1 \rightarrow R_1$

$-3/7 R_3 + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 77/47 \\ 0 & 1 & 0 & 48/47 \\ 0 & 0 & 1 & 29/47 \end{array}\right]$$

$$\boxed{\begin{aligned} I_1 &= 77/47 \text{ A} \\ I_2 &= 48/47 \text{ A} \\ I_3 &= 29/47 \text{ A} \end{aligned}}$$

Ex: 2.1

Question # 03

(a)

$$M_{13} = 4\begin{bmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= 4\left(0\begin{bmatrix} 1 & 14 \\ 1 & 2 \end{bmatrix} - 0\begin{bmatrix} 4 & 14 \\ 4 & 2 \end{bmatrix} + 3\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix} \right)$$

$$= 4(0 - 0 + 3(4-4))$$

$$\boxed{M_{13} = 0}$$

$$C_{13} = (-1)^{1+3}(0) = \boxed{0}$$

(b)

$$M_{23} = -3\begin{bmatrix} 4 & -1 & 6 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 & 14 \\ 1 & 2 \end{bmatrix} + 1\begin{bmatrix} 4 & 14 \\ 4 & 2 \end{bmatrix} + 6\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}$$

$$= 4(-12) + 1(-48) + 0)$$

$$\boxed{M_{23} = -96}$$

$C_{23} = (-1)^{2+3}(-96)$

$\boxed{C_{23} = 96}$

(c)

$$M_{22} = \begin{bmatrix} 4 & 1 & 6 \\ 4 & 0 & 14 \\ 4 & 3 & 2 \end{bmatrix}$$

$$= 4\begin{bmatrix} 0 & 14 \\ 3 & 2 \end{bmatrix} - 1\begin{bmatrix} 4 & 14 \\ 4 & 2 \end{bmatrix} + 6\begin{bmatrix} 4 & 0 \\ 4 & 3 \end{bmatrix}$$

$$= 4(-42) - 1(-48) + 6(12)$$

$\boxed{M_{22} = -48}$

$C_{22} = (-1)^{2+2}(-48)$

$\boxed{C_{22} = -48}$

(d)

$$M_{21} = \begin{bmatrix} -1 & 1 & 6 \\ 1 & 0 & 14 \\ 1 & 3 & 2 \end{bmatrix}$$

$C_{21} = (-1)^{2+1}(72)$

$\boxed{C_{21} = -72}$

$$= -1\begin{bmatrix} 0 & 14 \\ 3 & 2 \end{bmatrix} - 1\begin{bmatrix} 1 & 14 \\ 1 & 2 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$$

$$= -1(-42) - 1(-12) + 6(3)$$

$\boxed{M_{21} = 72}$

Question # 08

$$\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix} = \sqrt{2}(\sqrt{3}) - 4(\sqrt{6})$$

$\boxed{Det(A) = -3\sqrt{6}}$ $\quad -3\sqrt{6} \neq 0$ so inverse possible

Inverse : $\begin{bmatrix} -\sqrt{3}/3\sqrt{6} & \sqrt{6}/3\sqrt{6} \\ 4/3\sqrt{6} & -\sqrt{2}/3\sqrt{6} \end{bmatrix}$

$$\boxed{A^{-1} = \begin{bmatrix} -\sqrt{2}/6 & 1/3 \\ 4/3\sqrt{6} & -\sqrt{3}/9 \end{bmatrix}}$$

Question # 16

$$A = \begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix}$$

$Det(A) = 0$

$$0 = (\lambda-4)\begin{vmatrix} \lambda & 2 \\ 3 & \lambda-1 \end{vmatrix} - 0\begin{vmatrix} 0 & 2 \\ 0 & \lambda-1 \end{vmatrix} + 0\begin{vmatrix} 0 & \lambda \\ 0 & 3 \end{vmatrix}$$

$$= (\lambda-4)(\lambda^2-\lambda-6)$$

$$0 = (\lambda-4)(\lambda-3)(\lambda+2)$$

$\therefore \lambda - 4 = 0 \quad \lambda - 3 = 0 \quad \lambda + 2 = 0$

$\boxed{\lambda = 4}$, $\boxed{\lambda = 3}$, $\boxed{\lambda = -2}$

Ex : 2.2

QUESTION # 04

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 2 & -3 \\ -1 & 1 & 5 \end{bmatrix} \qquad A^t = \begin{bmatrix} 4 & 0 & -1 \\ 2 & 2 & 1 \\ -1 & -3 & 5 \end{bmatrix}$$

$$Det(A) = 4\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} - 2\begin{bmatrix} 0 & -3 \\ -1 & 5 \end{bmatrix} - 1\begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix}$$

$$= 4(10+3) - 2(0-3) - 1(0+2)$$

$$\boxed{Det(A) = 56}$$

$$Det(A^t) = 4\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} - 0 - 1\begin{bmatrix} 2 & 2 \\ -1 & -3 \end{bmatrix}$$

$$= 4(13) - 1(-4)$$

$$\boxed{Det(A^t) = 56}$$

$$\therefore \boxed{Det(A) = Det(A^t)}$$

QUESTION # 08

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = -1/3 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= -1/3 \left[1 \times 1 \times 1 \times 1 \right] = -1/3$$

$\det(A) = -1/3$

QUESTION # 12

$$= \begin{bmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{bmatrix}$$

$2R_1 + R_2 \to R_2$

$-5R_1 + R_3 \to R_3$

$$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & -2 & 1 \\ 0 & 13 & 2 \end{bmatrix}$$

taking -2 common from R_2

$$(-2) \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 13 & 2 \end{bmatrix}$$

$-13R_2 + R_3 \to R_3$

$$(-2) \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 17/2 \end{bmatrix}$$

Taking $17/2$ as common

$$(-2)(17/2)\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (1\times1\times1)(-2)(17/2)$$

$$\boxed{det(A) = -17}$$

QUESTION # 18

$$A = \begin{bmatrix} a+d & b+e & c+j \\ -d & -e & -j \\ g & h & i \end{bmatrix}$$

Adding $R_1 + R_2$ then,
Taking -1 from R_2

$$A = (-1)\begin{bmatrix} a & b & c \\ d & e & j \\ g & h & i \end{bmatrix}$$

$$= (-1)(-6) = 6$$

$$\boxed{det(A) = -6}$$

QUESTION # 20

$$A = \begin{bmatrix} a & b & c \\ 2d & 2e & 2j \\ g+3a & h+3b & i+3c \end{bmatrix}$$

Taking 2 common from R_2

$$A = (2)\begin{bmatrix} a & b & c \\ d & e & j \\ g+3a & h+3b & i+3c \end{bmatrix}$$

$$-3R_1 + R_3 \rightarrow R_3$$

$$A = (2)\begin{bmatrix} a & b & c \\ d & e & j \\ g & h & i \end{bmatrix}$$

$\det(A) = 2(-6)$

$\boxed{\det(A) = -12}$

QUESTION #23

$$\begin{bmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{bmatrix} = (b-a)(c-a)(c-b)$$

Proof:

$-aR_1 + R_2 \to R_2$

$-a^2R_1 + R_3 \to R_3$

$$\begin{bmatrix}1 & 1 & 1\\ 0 & b-a & c-a\\ 0 & b^2-a^2 & c^2-a^2\end{bmatrix}$$

$R_2 \times -(b+a) + R_3 \to R_3$

$$\begin{bmatrix}1 & 1 & 1\\ 0 & b^2-a^2 & c-a\\ 0 & 0 & \underset{c^2-a^2-(c-a)(b+a)}{\cancel{\quad}}\end{bmatrix}$$

~~$(1)(b-a)[(c-a)(b+a)-(a-a)(c+a)]$~~

~~$(1)(b-a)(c-a)[b+a-c-a]$~~

~~$(b-a)(c-a)$~~

$(1)(b-a)[(c-a)(c+a)-(c-a)(b+a)]$

$(b-a)(c-a)[c+\cancel{a}-b+\cancel{a}]$

$\boxed{(b-a)(c-a)(c-b)}$ Hence proved!