Motion : 2/3 II

Date

Position vector:-

Position vector is a vector that extends from reference point (usually origin).

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$r = (-3m)\hat{i} + (2m)j + (5m)\hat{k}$$

Displacement vector:-

It represents a particle's position during a certain time interval

$$\Delta r = r_2 - r_1 = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$$

Average Velocity:

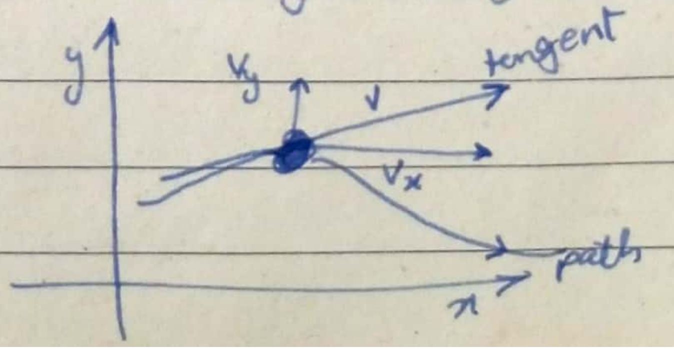
$$= \frac{\text{displacement}}{\text{time interval}}$$

$$\vec{V}_{avg} = \frac{\Delta r}{\Delta t} = \frac{\Delta x\hat{i}}{\Delta t} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

Instantaneous Velocity:-

$$\vec{V} = v_x\hat{i} + v_y\hat{j} + V_z\hat{k}$$

It's direction is always tangent to the path of particle.



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- AVERAGE ACC :

$$\vec{a}_{avg} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

- INSTANTANEOUS ACC :-

$$a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

(i) $\frac{dx}{dt} = -6t + 4$ $\frac{dy}{dt} = 12t - 4$

$\frac{d^2x}{dt^2} = -6$ $\frac{d^2y}{dt^2} = 12$

(iii) $4t\hat{i} - 4t\hat{j}$

$4\hat{i} - 4\hat{j}$

(ii) $-9t^2 - 4$, $-10t + 6$

Date

Example : (ppt)

For r :

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

At $t = 15\ sec$,

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66m$$

$$y = (0.22)(15)^2 - (9.1)(15) + 30 = -5$$

so,

$$\vec{r} = (66m)\hat{i} - (57m)\hat{j}$$

$$r = \sqrt{(66)^2 + (-57)^2}$$

$$r = 87m$$

$$\theta = \tan^{-1} y/x = \tan^{-1}\left(\frac{-57}{66}\right) = -41^\circ$$

For V:-

$$V_x = dx/dt = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$

$$V_x = -0.62t + 7.2$$

At $t = 15\ sec$

$$V_x = -2.1 m/s$$

Date

$$V_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$

$$= 0.44t - 9.1$$

At $t = 15$ sec

$$V_y = -2.5\ m/s$$

$$V = (-2.1 m/s)i + (-2.5 m/s)j$$

$$V = \sqrt{(-2.1)^2 + (-2.5)^2}$$

$$V = 3.3 m/s$$

$$\theta = \tan^{-1}\frac{V_y}{V_x} = -130°$$

For a:

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62 m/s^2$$

$$a_y = \frac{dv_y}{dt} = (0.44t - 9.1) = 0.44 m/s^2$$

$$a = (-0.62 m/s^2)i + (0.44 m/s^2)j$$

$$a = 0.76 m/s^2$$

$$\theta = -35°$$

PROJECTILE MOTION

EQUATION OF MOTIONS :

$S = Vt \longrightarrow \Delta x = V_{0x} \cdot t$

$x - x_0 = V_{0x} \cdot t$

$x - x_0 = (V_0 \cos\theta_0) \cdot t \quad \because V_{0x} = V_0 \cos$

$S = V_i t + \frac{1}{2} a t^2 \longrightarrow \Delta Y = V_{0y} t - \frac{1}{2} g t^2$

$y - y_0 = (V_0 \sin\theta_0) t - \frac{1}{2} g t^2$

$V_f = V_i + at \longrightarrow V_y = V_{0y} - gt$

$V_y = (V_0 \sin\theta) - gt$

$V_f^2 - V_i^2 = 2as \longrightarrow V_y^2 - V_{0y}^2 = -2g \Delta y$

$V_y^2 = (V_0 \sin\theta)^2 - 2g(y - y_0)$

- In projectile $a = -g$

Date

RANGE OF PROJECTILE:-

$$R = \frac{V_0^2 \sin 2\theta}{g}$$ for max $\Rightarrow \theta = 45°$

HEIGHT:-

$$H = \frac{V_0^2 \sin^2\theta}{2g}$$

TRAGECTORY PATH OF PARTICLE:-

$$Y = (\tan\theta_0)x - \frac{gx^2}{2(V_0\cos\theta)^2}$$

Initial Velocity of projectile:-

$$\vec{V_0} = \vec{V}_{0x}\hat{i} + \vec{V}_{0y}\hat{j}$$

& $V_{0x} = V_0 \cos\theta$, $V_{0y} = V_0 \sin\theta$

TYPES OF PROJECTILE MOTION

Date

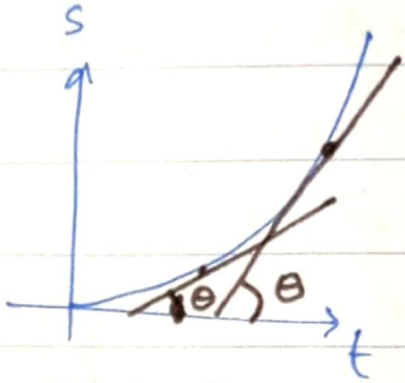
- HORIZONTAL :- Horizontal velocity always remains constant
- VERTICAL :- Vertical ~~ve~~ component changes with time. It is minimum at highest point &
- PARABOLIC :- Path traced by an object accelerating only in the vertical direction while moving at constant ~~velocity~~ horizontal velocity.

FINAL HORIZONTAL & VERTICAL COMPONENT OF VELOCITY :-

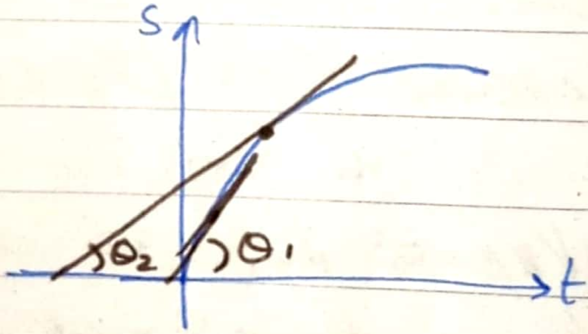
$$V_{xf} = V_0 \cos\theta \rightarrow \text{Horizontal}$$
$$V_{yf} = V_0 \sin\theta - gt \rightarrow \text{vertical}$$

Uniform Circular Motion

- $S = r\theta$
- $V = r\omega$
- $a = r\alpha$
- $T = \frac{2\pi r}{V}$
- $a_c = \frac{V^2}{r}$
- $\omega = \frac{\Delta\theta}{\Delta t}$
- $\alpha = \frac{\Delta\omega}{\Delta t}$
- $T = \frac{2\pi}{\omega}$



- $\theta < 90°$ so slope +ve
- if slope is +ve velocity is also positive (increasing)



- $\theta < 90°$ so slope +ve
- slope +ve $\therefore$ a velocity is also positive.
- lekn aage jate hoe ~~slope~~ θ kam hota jaarha hai to iska matlab velocity kam horhi hai. (decreasing)
- Velocity ki direction +ve hai leken decrease horhi hai.

$\therefore$ acceleration will be −ve. (opp to velocity's direction

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Q: A particle moves along axis. Determine direction of acceleration when:-

(a) +ve direction with increasing speed.
direction positive to velocity +ve or velocity +ve to acc +ve q k incrse. speed hai.

(b) in +ve direction with decr speed.
direction +ve to velocity +ve lekin speed decrease horhi hai to acc leration will be opp to velocity i.e in -ve.

(c) -ve direction with incr speed.
direction -ve to velocity -ve or velocity -ve to acc bhi -ve q k increasing speed main velocity or acc ki direction same hoti hai.

(d) in -ve direction with decreasing speed.
direction -ve to velocity -ve or acc is +ve bc3 speed is decreasing.

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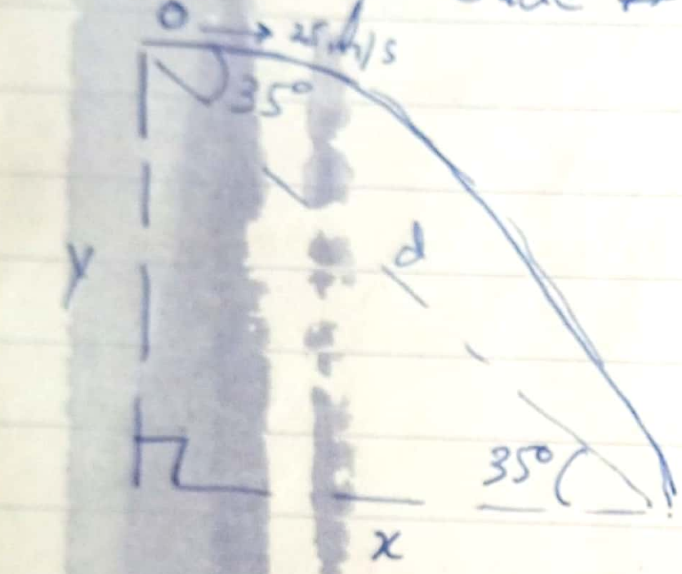
Question of slide # 37 (Motion 2d)



$S = vt$

$x = 25t$ — (1)

$S = V_i t + \frac{1}{2}at^2$ → bcz no movement vertically

$y = V_i t^{\to 0} + 0.5 \times (-9.8)\, t^2$

$y = -4.9t^2$ — (2)

From right angle Δ,

$\cos\theta = \frac{\text{base}}{\text{hy}} = \frac{x}{d}$

$x = d\cos 35$ — (3)

$\sin 35 = y/d$

$y = d\sin 35$ — (4)

Equating (1) & (3)

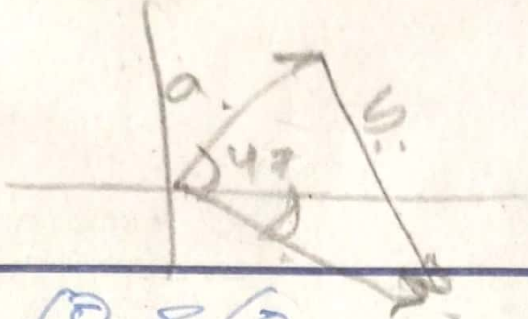
$25t = d\cos 35$

$\frac{25t}{\cos 35} = d$ — (5)

comparing 2 & (4)

$-4.9t^2 = d\sin 35$

$\frac{-4.9t^2}{\sin 35} = d$ — (6)



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comparing, (5) & (6)

$$\frac{25t}{\cos 35} = \frac{-4.9t^2}{\sin 35}$$

$$\frac{\sin 35}{\cos 35} = \frac{-4.9t}{25}$$

$$\tan 35 = 0.196$$

$$t = -3.57$$

$$X = 25(-3.57)$$

$$x = -89.3$$

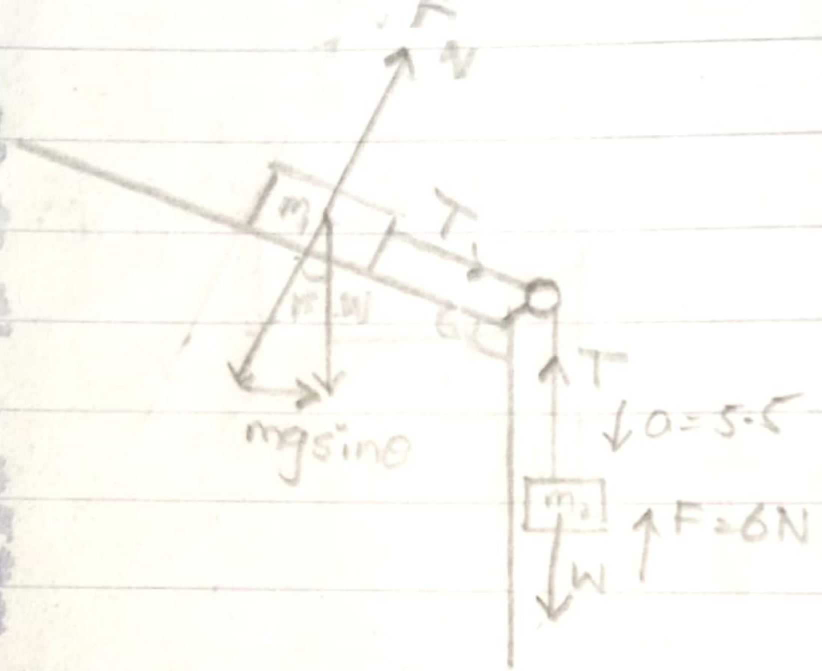
$$Y = -4.9(-3.57)^2$$

$$y = -62.5\,m$$

$$d = \frac{-89.3}{\cos 35} = -109\,m = 109\,m$$

Past Paper

Date



For m_1:

$F_{net_x} = T + mg\sin\theta$

$m_1 a = T + mg\sin\beta \rightarrow (1)$

$F_{net_y} = F_N - mg\cos\theta$

$0 = F_N - mg\cos\theta$

For m_2:

$F_{net_x} = W - (T+6)$

$m_2 a = m_2 g - (T+6)$

$m_2 a = m_2 g - T - 6$

$(2)(5.5) = 2(9.8) - T - 6$

$\boxed{T = 2.6} \rightarrow$ put in eq (1)

eq (1) $mg\sin\beta = m_1 a - T$

$\sin\beta = \frac{m_1 a - T}{mg} \Rightarrow \beta = 17.21°$

PAST PAPER Q1: Date

Q: (a)

$$2A - 6B + 3C = 2j$$

$$A = i - 2k \quad \& \quad B = -j + k/2$$

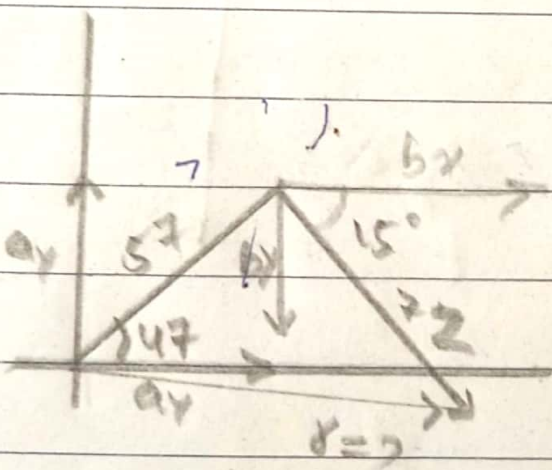
$$2(i - 2k) - 6(-j + k/2) - 2j = -3C$$

$$2i - 4k + 6j - 3k - 2j = -3C$$

$$2i + 2j - 7k = -3C$$

$$C = \frac{-2i}{3} - \frac{2j}{3} + \frac{7k}{3}$$

Q(b):



$$r = \sqrt{108.34^2 + 23.8^2}$$

$$r = 110.4$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = 12.02^\circ$$

$$r_x = a_x + b_x$$

$$= a\cos\theta + b\cos\theta$$

$$= 57\cos 47 + 72\cos 15$$

$$r_x = 108.34$$

$$r_y = a_x - b_x$$

$$= a\sin\theta - b\sin\theta$$

$$r_y = 23.08$$

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2 (b):

$h = 50m \qquad t = ?$

$S = \cancel{V_i t} + \frac{1}{2}at^2$

$50 = +\frac{1}{2}(9.8)t^2$

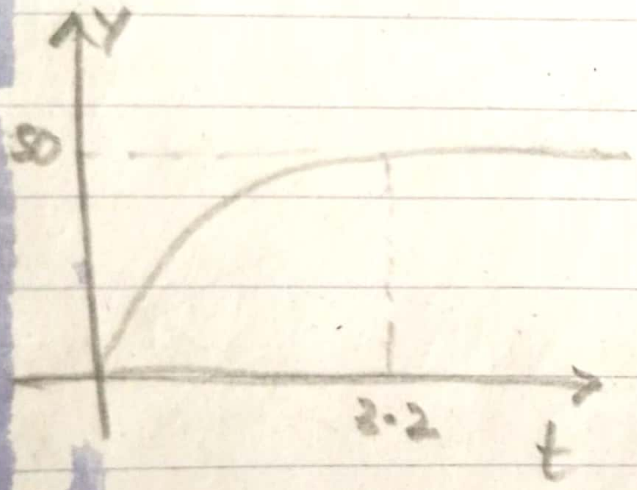
$t = 3.19$

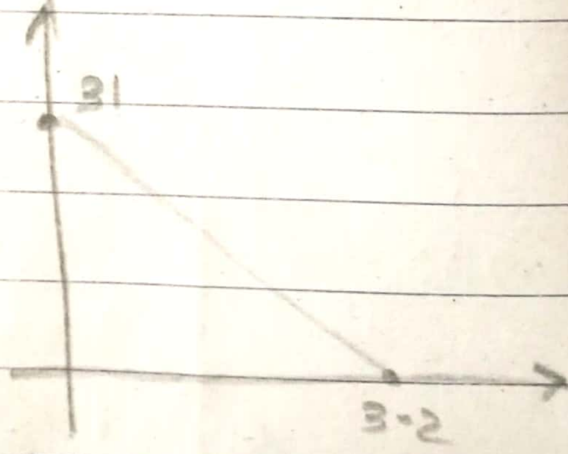
For V_i:

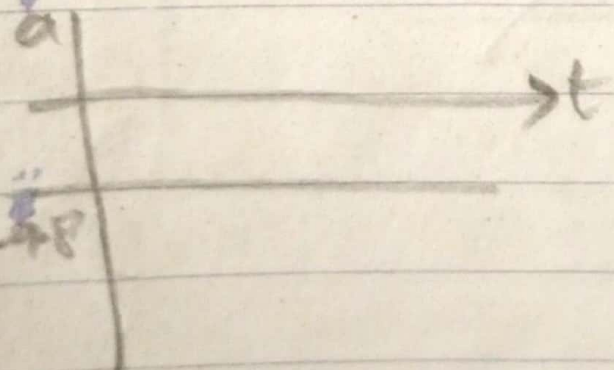
$V_f = V_i + at$

$0 = V_i - 9.8(3.19$

$V_i =$







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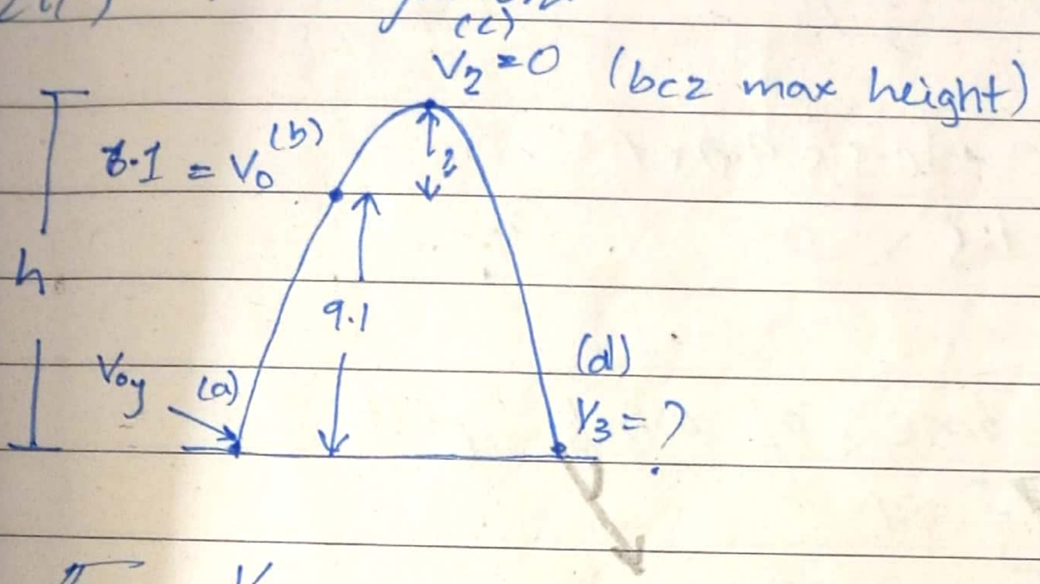
Q11) → Assignment



For V_{oy}:

$\cancel{V_0 = V_{oy} + at}$

$\cancel{6.1}$ $2as = V_0^2 - V_{oy}^2$

$2(-9.8)(9.1) = (6.1)^2 - (V_{oy})^2$

$V_{oy} = \sqrt{6.1^2 + 178.36}$

$\boxed{V_{oy} = 14.7}$

For H:

$2ah = V_2^2 - V_{oy}^2$

$2(-9.8)(h) = 0^2 - (14.7)^2$

$h = \dfrac{-(14.7)^2}{-(2 \times 9.8)}$

$\boxed{h = 11.02}$

or

$H_{max} = \dfrac{V_0^2}{2g}$

$H_{max} = 11.02$

bcz at max heigh initial will be zero.
we are calculating from c to d

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$S = V_i t + \frac{1}{2} g t^2$

$11 = 0 + 0.5 \times 9.8 \times t^2$

$t = 1.5$

For time of whole path

$T = 2t$

$T = 3sec$

$V_{3y} = V_{oy} + gt$

$= V_{oy} + (-9.8)(3se)$

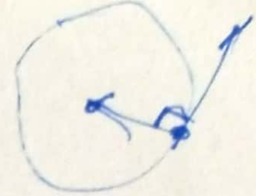
$V_{3y} = -14.7$

$V_{3x} = 7.6$, $V_{3y} = -14.7$

$V_3 = \sqrt{7.6^2 + (-14.7)^2}$

$V_3 = 16.5$

$\theta = \tan^{-1}\left(\frac{V_{3y}}{V_{3x}}\right) = 62.66°$



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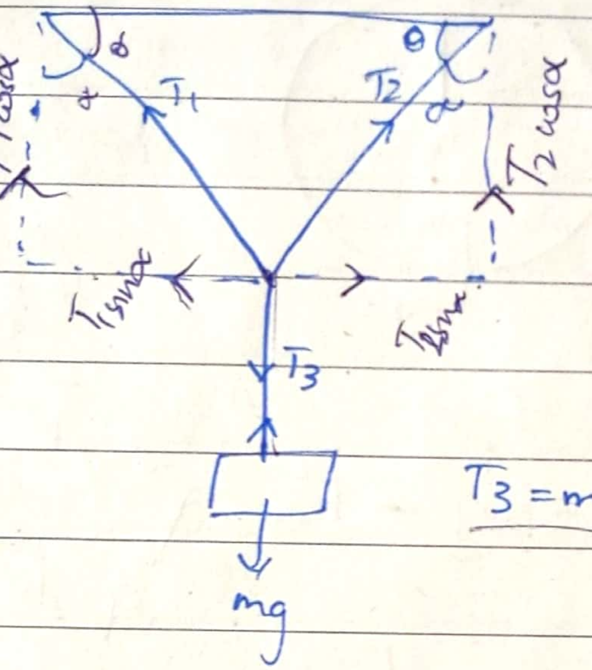


$T_3 = mg$

x direction:

$T_1\sin\alpha - T_2\sin\alpha = 0$

$\boxed{T_1\sin\alpha = T_2\sin\alpha}$ $\qquad T_1 = 1.66\,T_2$

y direction:

$T_3 - (T_1\cos\alpha + T_2\cos\alpha) = 0$

$T_3 = T_1\cos\alpha + T_2\cos\alpha$

$mg = (1.66\,T_2\cos\alpha) + T_2\cos\alpha$

" FRICTION "

Date

STATIC FRICTION

When tendancy of relative Motion is there. (Rest)
[motion ho nhi rha, ho ne wala hai]

KINETIC FRICTION

When relative motion is there. (Motion). [jab body move krrhi ho.

" KINETI FRICTION "

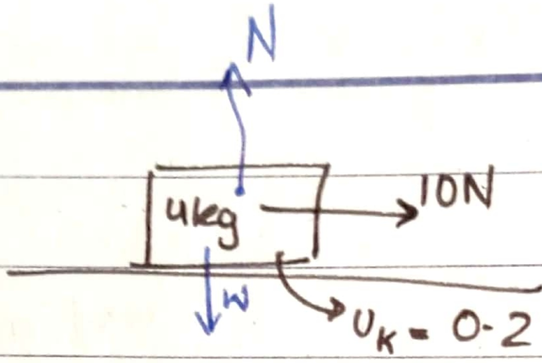
- Relative motio
- it is constant
  does not depend how fast or slow a body is moving
- $f_K \propto$ Normal Force
  $f_K = \mu_K N$
  ↳ constant coefficent of kinetic friction

- It's direction will be opp to relative motion.

Assume that all questions are of moving body.

Date

Q:



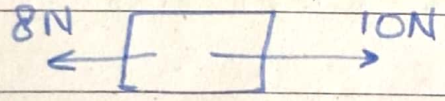
(1) Find frictional force (2) $\vec{a}$ (acceleration)

$f_k = \mu_k N$

$N = mg$

$N = 4g$

$f_k = u_k \cdot 4g = 8\,N$

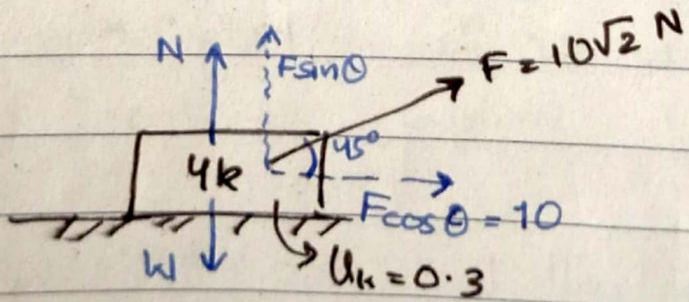


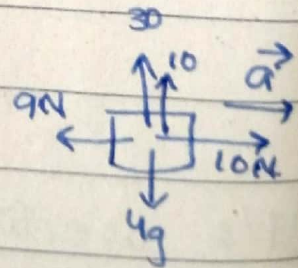
$F - f_k = ma$

$10 - 8 = 4 \times a$

$\boxed{a = 0.5\,m/s^2}$

Q:

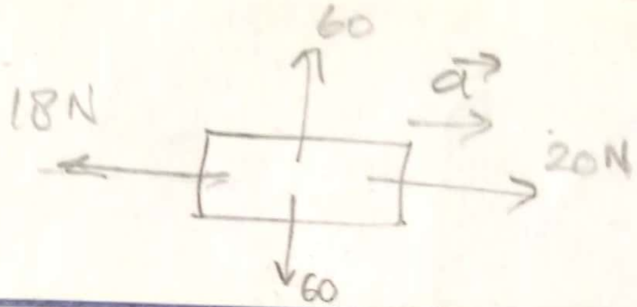




$N + F\sin\theta = mg$

$N = mg - F\sin\theta$

$N = 4 \times 10 - 10\sqrt{2}\sin 45$



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$N = 30$ Newton

$f_K = \mu_K N$

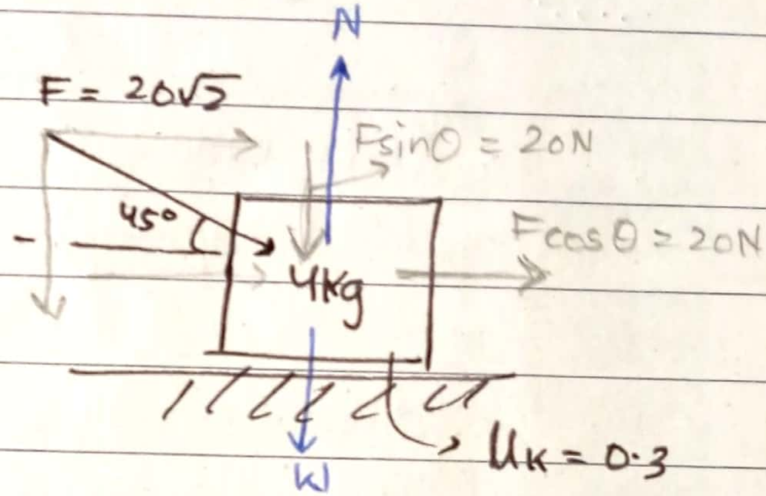
$= 0.3 \times 30$

$\boxed{f_K = 9 N}$

$ma = F - f_K$

$4a = 10 - 9$

$\boxed{a = 0.25 \, m/s^2}$



• Find Normal force, friction and acceleration.

$N - F\sin\theta = mg$

$N = mg + F\sin\theta$

$N = 60$ Newton

$f_K = \mu_K N$

$f_K = 18$

$ma = F\cos\theta - f_K$

$a = 0.5 \, m/s^2$

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"STATIC FRICTION"

- it is variable (self adjusting)

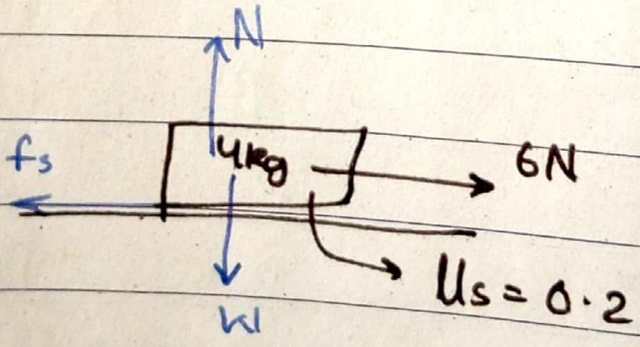
$$0 \leq f_s \leq F_{limiting}$$

$$F_{limiting} = \mu_s N$$

- direction will be opposite to motion.
- its minimum value is ZERO and max. is limiting friction (F_{limitg})

* Body only moves when Applied Force > Max static friction ($F_{limiting}$)

Q:



- First checking wheter body is at rest or motion. Therefore evaluate F_{limit} if it is greater than 6 N it means body is in motion.

Date

$F_{limit} = \mu_s N$

$= \cancel{4\times10\times}\ 0.2 \times 4g$

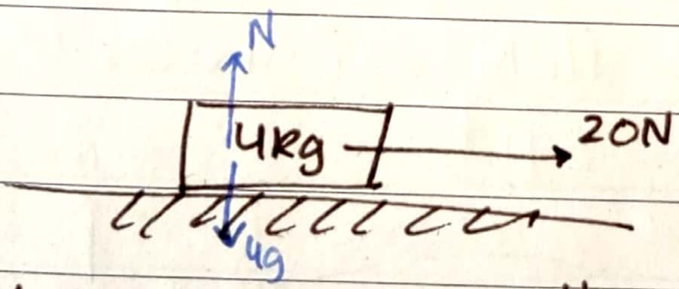
$F_{limit} = 8N$

$\because F_{limit} > F \quad \therefore$ it is static friction.
if it was moving than it would be kinetic.

As body is at Rest therefore we must have

f_s = Applied Force

$\therefore \boxed{f_s = 6N}$.



Find frictional force and a.

$\mu_s = 0.6$

$\mu_k = 0.4$

$F_L = \mu_s N$

$= 0.6 \times 4g$

$F_L = 24N$

$\because F_L > F$

$\therefore$ body is at rest.

since body is at rest

$f_s = F$

$\boxed{f_s = 20N}$

static friction khud ko adjust karleti hai.

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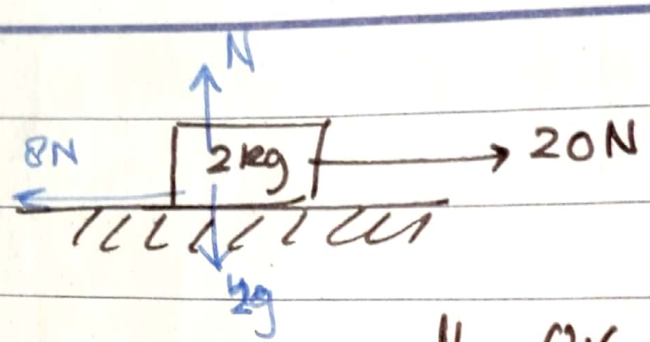


$\mu_s = 0.6 , \mu_k = 0.4$

$F_L = \mu_s N$

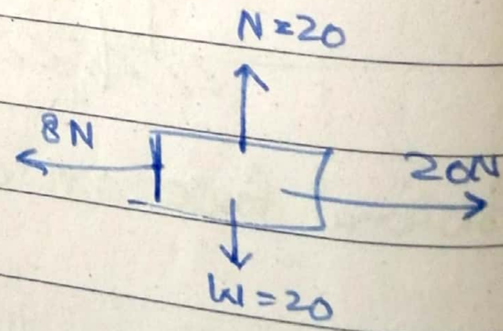
$= 0.6 \times 2 \times 10$

$\boxed{F_L = 12 N}$

$\therefore F_L < F \Rightarrow$ body will be moving and there will be kinetic friction.

$f_k = \mu_k N = 0.4 \times 2 \times 10$

$\boxed{f_k = 8 N}$



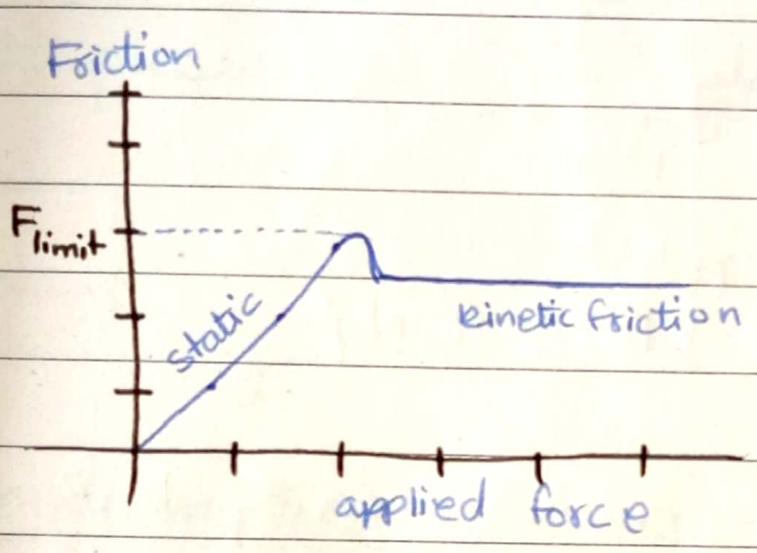
$ma = 20 - 8$

$\therefore a = \frac{12}{2} = 6 m/s^2$

$F_{limit} > F_{kinetic}$

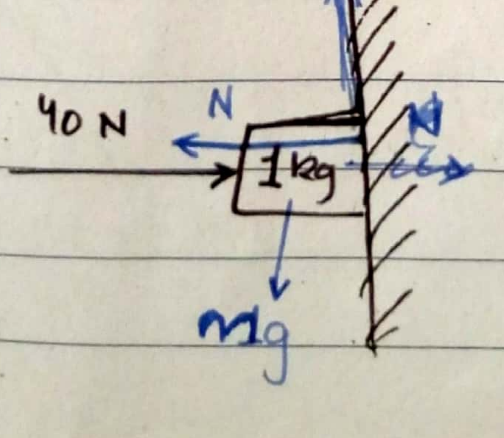
Date

(GRAPH B/W FRICTION AND APPLIED FORCE)



Shroo mein static friction or applied force brabr hote hai to constant graph jese hi applied force limiting friction ko cross ki to hamara friction ab kinetic hogai or kinetic hamesha limiting se kam hoti hai is waja se graph neeche aya or choonke kinetic is constant to is waja se straight line ban gaye.

Q:



$\mu_s = 0.3$

$\mu_k = 0.2$

- Find frictional force and "acc"

Date

- Applied force will be "mg" because 40 N force is not causing the body move only "mg" is causing the body to move ∴

$$F_{applied} = mg$$

$$\boxed{F = 10\,N}$$

∵ Body is not moving horizontally

∴ $N = 40\,N$

Now, $F_{limit} = \mu_s N$

$F_{limit} = 0.3 \times 40$

$F_{limit} = 12\,N$

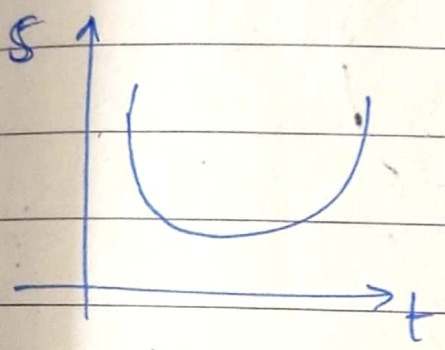
∵ $F_{applied} < F_{limit}$ ∴ body is at rest.

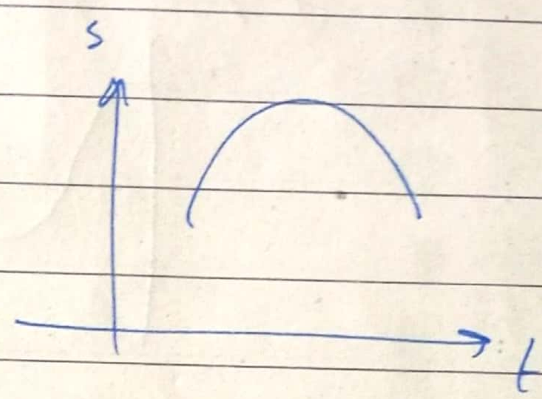
∴ As it is at rest $f_s = 10\,N$

S-V-a → GRAPHS.

Date

SHORT TRICK:

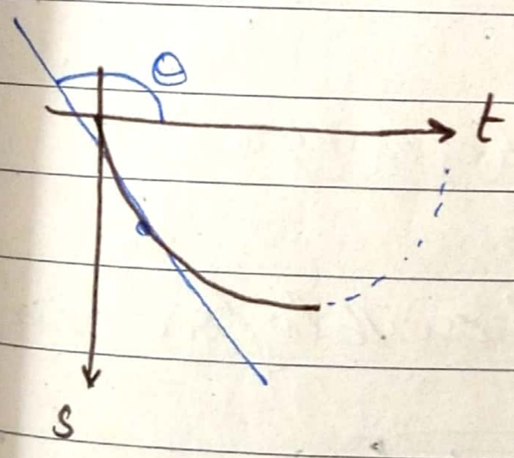




concave upward to "acc" +ve

Concave downward to "acc" negative.

Q:



$\because \theta > 90^\circ \therefore$

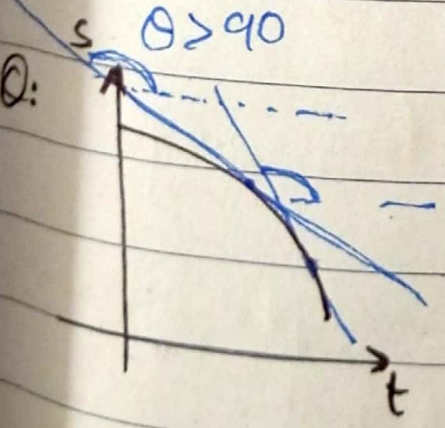
V → V is −ve

S → −ve

a → +ve (because concave upper)

Speed → decrease (bcz V & a have opp signs)

Q: $\theta > 90$



V → $\theta > 90 \therefore$ v → −ve

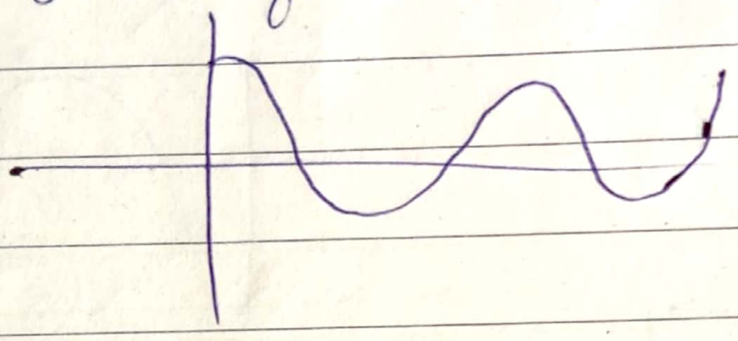
a → $\because \theta$ is decreasing wrt. time $\therefore$ a → −ve

Speed → increase (bcz 'V' & 'a' has same sign.

(SHM)

Date

Simple harmonic motion is ~~cosine~~ fun has a graph of cosine function



The time for one full cycle is the period 'T' of the oscillation.

$$T = \frac{1}{f} \rightarrow \text{frequency}$$

$1 Hz = 1 \text{ Hertz} = 1 \text{ oscillatio/sec} = 1 sec^{-1}$

$$\boxed{x(t) = x_m \cos(\underbrace{\omega t + \phi}_{\text{phase}})} \quad [\text{displacement}]$$

displacement (t) time

ω = angular ~~velocity~~ frequency

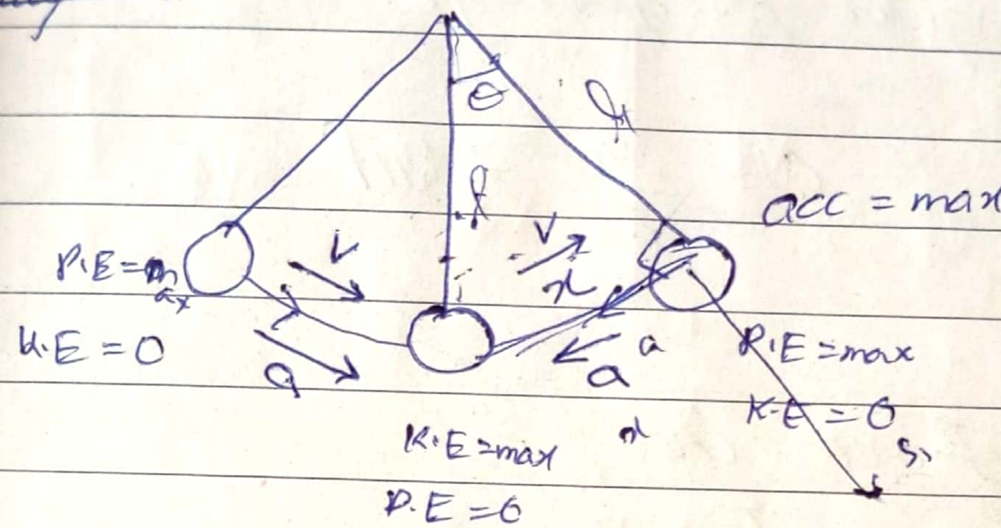
t = Time

ϕ = ~~angular displacement~~ phase constant

x_m = max- displa (Amplitude)

Date

• ~~Velocity~~



Velocity :

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$$

$$\boxed{v(t) = -\omega x_m \sin(\omega t + \phi)}$$

Acceleration:

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}[-\omega x_m \sin(\omega t + \phi)]$$

$$\boxed{a(t) = -\omega^2 x_m \cos(\omega t + \phi)}$$

$\because x_m \cos(\omega t + \phi) = x$

$\therefore \boxed{a(t) = -\omega^2 x}$

Date

Angular Frequency:

$$\omega = \frac{2\pi}{t} = 2\pi f$$

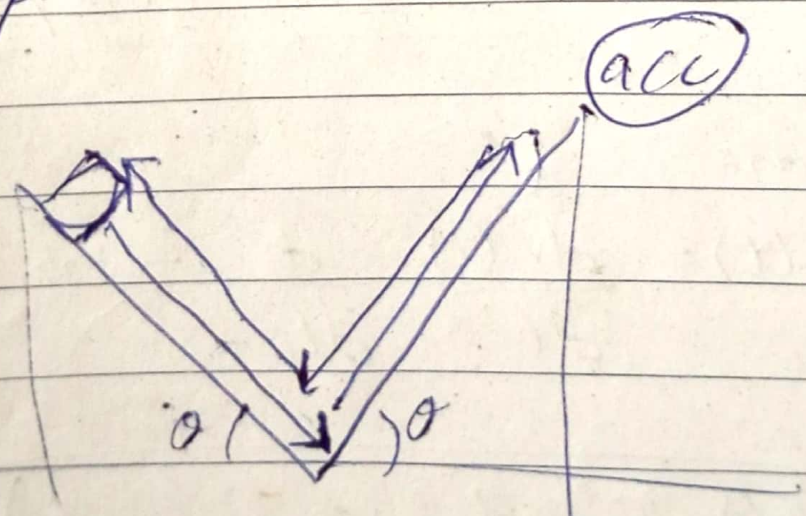
Force Law:

$$F = ma$$

$$F = m(-\omega^2 x)$$

$$\boxed{F = -\omega^2 m x}$$

Shadow of an obj. tells that the motion of a body in circular motion is a Simple harmonic motion.



$m = 1.8$ kg

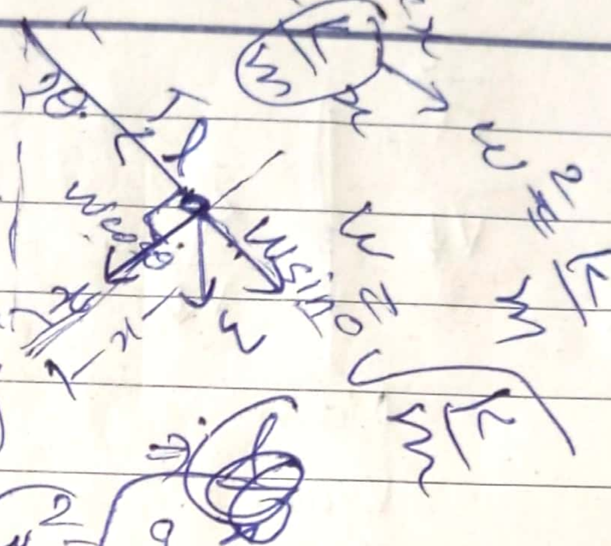
$M = 10$ kg

$x_m = ?$

$a = -\frac{k}{m}x$

$a = -\omega^2 x$

$\omega = \sqrt{\frac{k}{m}}$



$F = mg\cos\theta =$

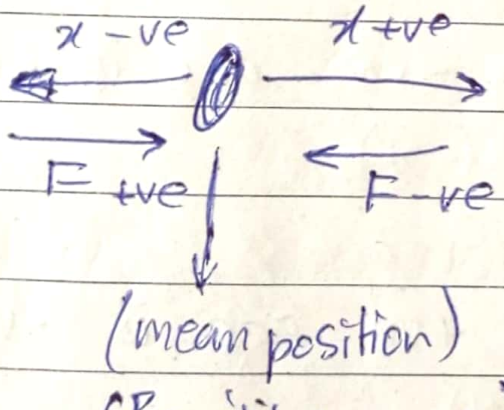
$\cancel{m}a = \cancel{m}g\cos\theta$

$\boxed{a = g\cos\theta}$

$a = \frac{g}{l}x$

$\omega^2 = \frac{g}{l}$

Oscillation:-



Force and displa are opposite when displa is taken from mean position

Force is always directed to mean position

<u>Stable</u> → $x=0$, $F=0$

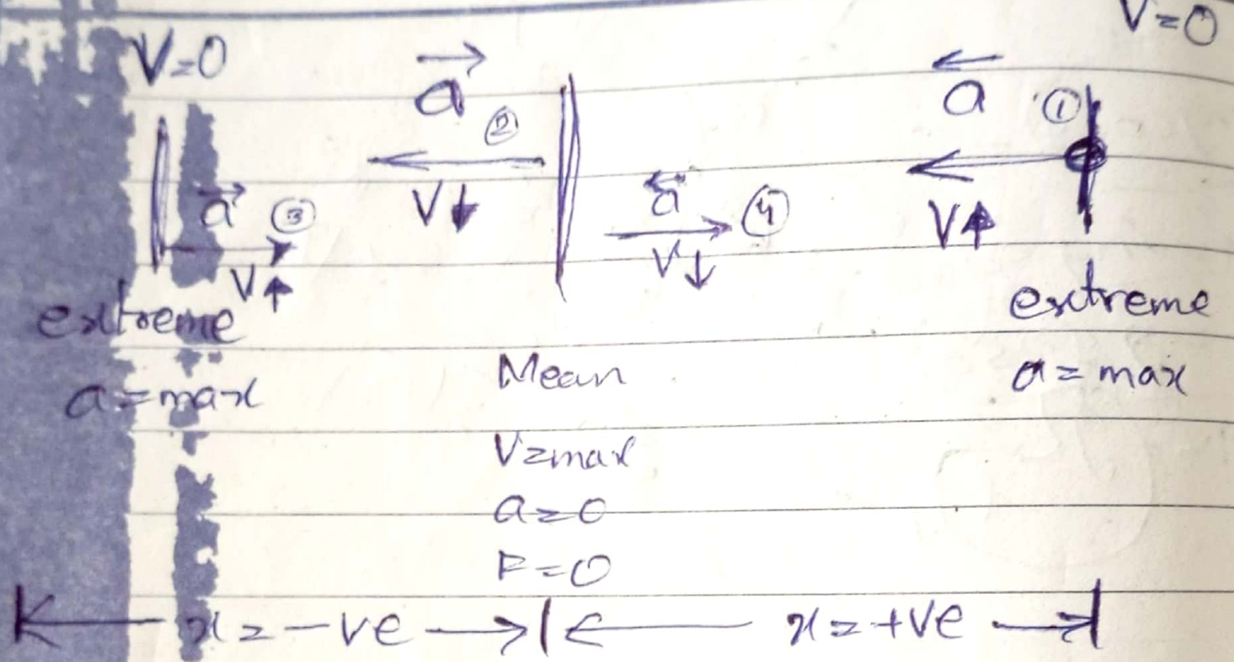
$F = -kx^n$

where n = odd integer

- Yes all oscillatory are periodic except in which energy is lost.
- All oscillatory are not SHM but All SHM are oscillatory. The oscillatory in which $F = -kx$ are called Simple Harmonic Motion.

'x' is always measured from mean position.

Date



$F = -kx \Rightarrow a = F/m \Rightarrow a = -\frac{k}{m}x$

When x is −ve acc is +ve

$a = -(-x)$

$a = +ve$

When x is +ve acc is −ve

$a = -(x)$

$a = -ve$

$a = -\frac{k}{m}x$

$a = -(\text{constant})\, x$ ∵ k & m remain constant

$\boxed{a = -\omega^2 x}$

$\boxed{a_{max} = -\omega^2 A}$

A = amplitude

$\boxed{\omega = \sqrt{\frac{k}{m}}}$

Longitudinal
Transverse
Magnetic / Electromagnetic
matter waves.

Date

$$\boxed{V = \pm \omega \sqrt{A^2 - x^2}}$$

where,

A = Amplitude

when $V = max$, $x = 0$

$$V = \pm \omega \sqrt{A^2 - 0}$$

$$= \pm \omega \sqrt{A^2}$$

$$\boxed{V_{max} = \pm \omega A}$$

- in time domain we have to study variation in amplitude w.r.t time.
- Amplitude of sine & cosine waves remains same
- $K = \frac{2\pi}{\lambda}$ (wave number).

CHECKPOINT # 01:

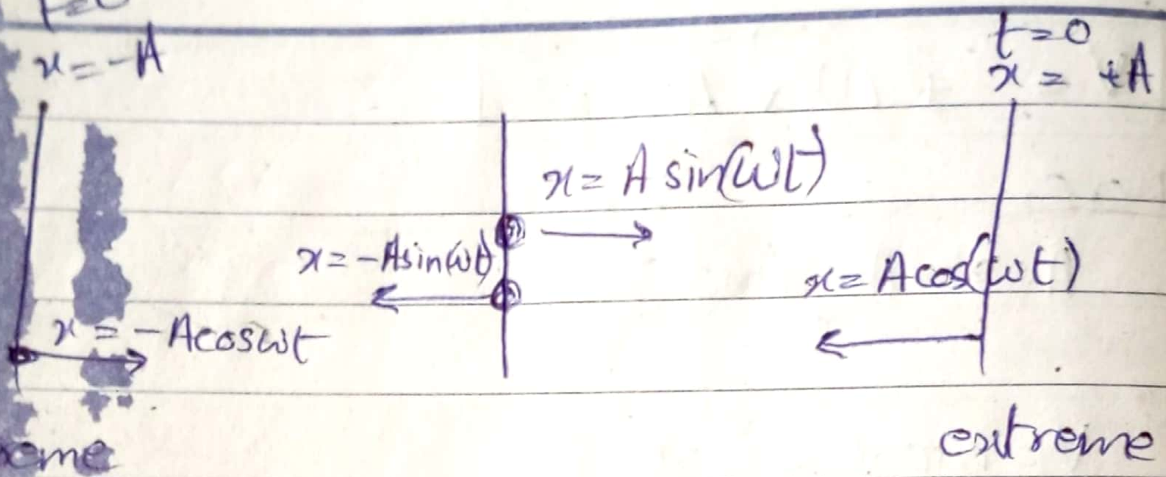
humein unho ne $(kx - \omega t)$ diya wa hai to $\omega = 2\pi f$ to jiska Omega zyad hoga uski frequency bhi zyada hogi.

(c) $8x - 16t \Rightarrow 16t = \omega t = 2\pi f$ to f ki value sabse zyada hai.

sabse zyada frequency 1st wave ki hai.

6, 15, 16.1, 16.5

Date



sin waali eq/s jab lagengi jab t=0 ho or particle mean position par ho.
cos waali jab lagegi jab t=0 hor or particle extreme position par ho

GENERAL EQN:

$x = A\cos(\omega t + \phi)$ → initial phase; $(\omega t + \phi)$ → phase

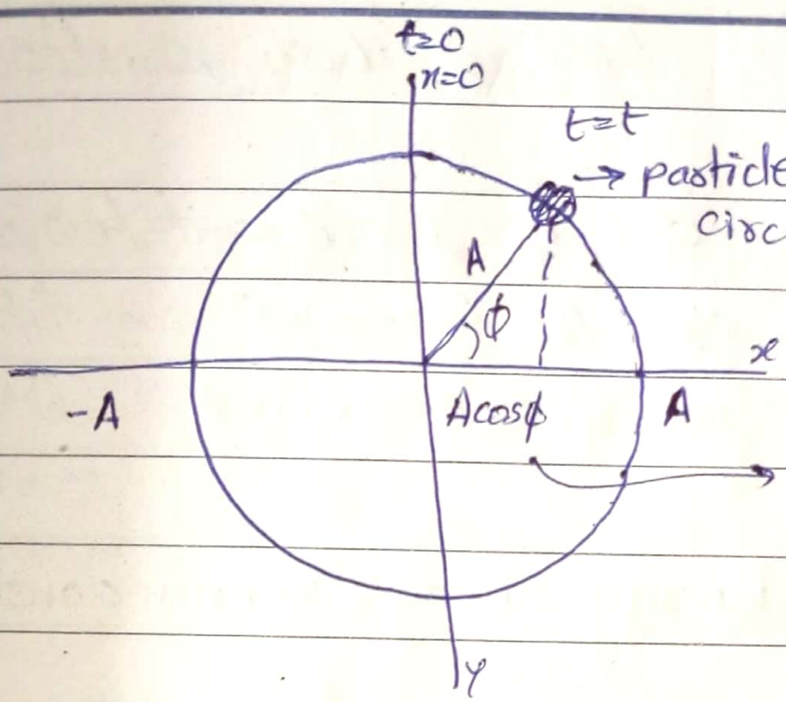
$$\boxed{T\ \text{period} = \frac{2\pi}{\omega}}$$

sin function repeats itself after 180 (π) or 360 (2π)

cos function repeats itself after 2π (180)

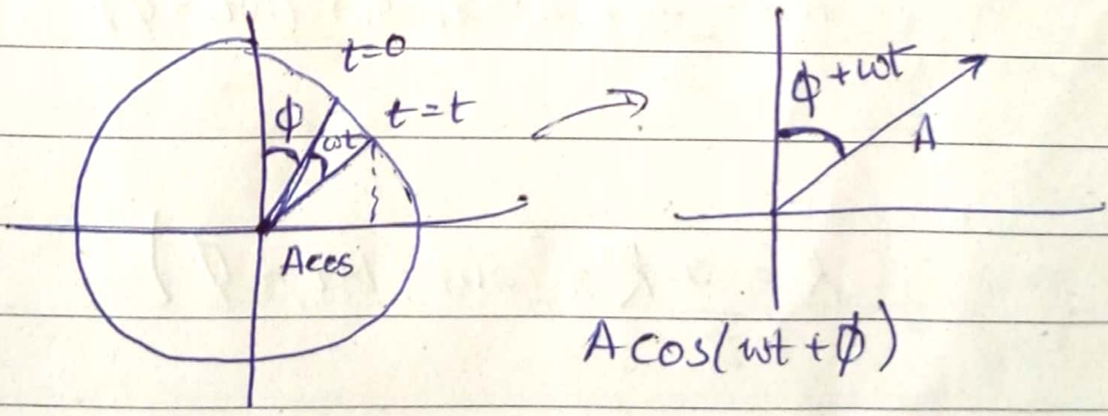
SHM & CIRCULAR MOTION:-

Date



$A\cos\phi$ &
$\phi = \omega t$
$\therefore A\cos(\omega t)$

SHM, therefore is the projection of uniform cir. on ~~a~~ a diameter of the circle in which the circular motion occurs. (particle ka projection motion on ~~SHM~~ the diameter is SHM)



$A\cos(\omega t + \phi)$

yaha t=0 par particle phele hi ϕ angle move kr chuka tha to jabhi time 't' mein ϕ add krdia.

ϕ is the angle from mean position to t=0.

ENERGY IN SHM

Date

In SHM there is both kinetic & P.E.

K.E is due to movement / motion

P.E is ~~due to~~ because in SHM energy remains conserved

riable

Total Energy always remain constant in SHM.

Kinetic Energy:

$K(t) = \frac{1}{2} mv^2 \quad \because v = -\omega x_m \sin(\omega t + \phi)$

$\therefore K = \frac{1}{2} m\omega^2 x_m^2 \sin^2(\omega t + \phi) \quad \because \omega^2 = \frac{k}{m}$

$k = \omega^2 m$

$\therefore K = \frac{1}{2} k x_m^2 \sin^2(\omega t + \phi)$

max at $x = 0$ mean position $\Big\}$ min at $x = \pm A$ extreme position

$K.E = \frac{1}{2} K(A^2 - x^2)$ → another formula

Total Energy = $\frac{1}{2} kA^2$

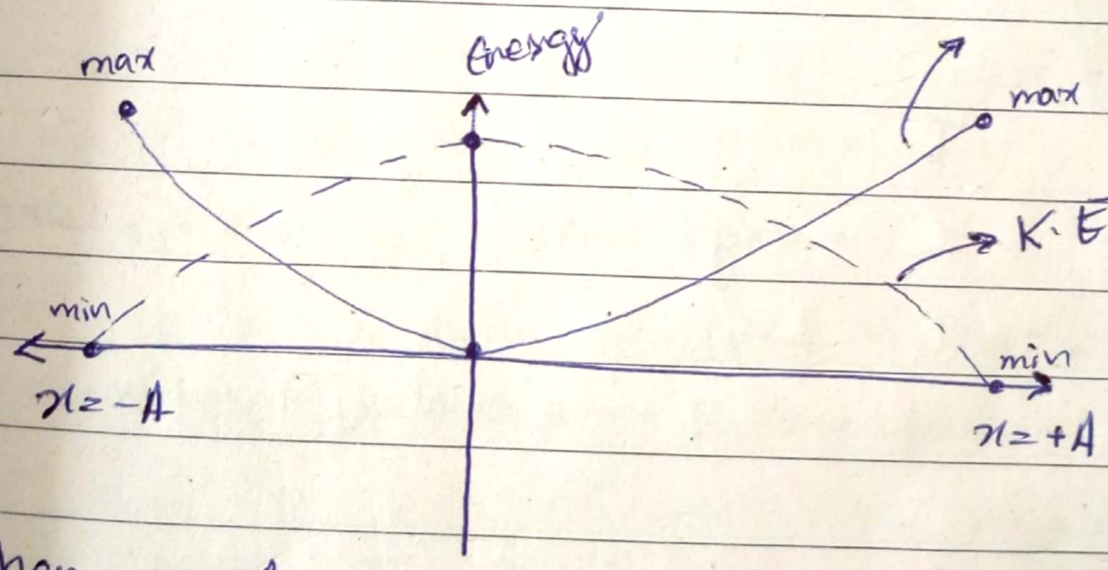
Date

POTENTIAL ENERGY:—

$\because x = x_m \cos$

$$U(t) = \frac{1}{2} kx^2 = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$$

Min	Max
$x = 0$	$x = \pm A$
mean position	extreme



When $x = +A$

$$P.E = \frac{1}{2} kA^2 = max$$

$$KE = \frac{1}{2} k(A^2 - A^2) = 0 = min$$

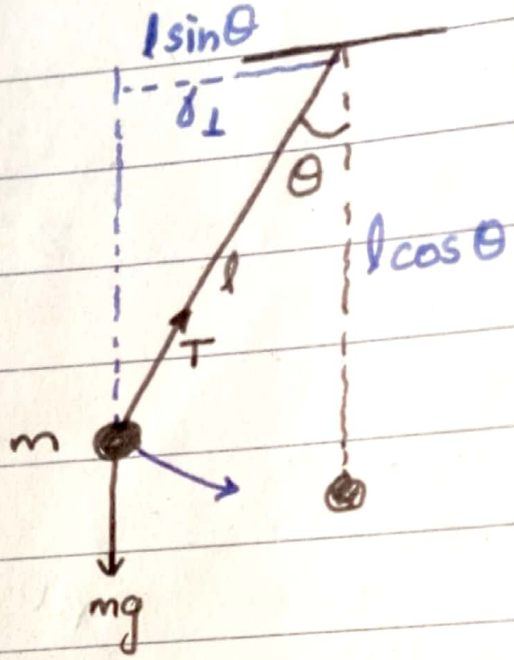
When $x = 0$

$$P.E = \frac{1}{2} k(0)^2 = 0 = min$$

$$K.E = \frac{1}{2} k(A^2 - 0^2) = max$$

SIMPLE PENDULUM

Date



$\tau = mg\, r_\perp$

$\tau = -mgl\sin\theta \qquad \because r_\perp = l\sin\theta$

$\because \tau = I\alpha$

where I = momemnt of inertia.

$I\alpha = -mgl\sin\theta$

$$\boxed{\alpha = \frac{-mgl\,\theta}{I}}$$ (if θ is very small then $\sin\theta = \theta$)

$\therefore$

$$\boxed{\omega = \sqrt{g/l}} \quad \text{or} \quad \omega = \sqrt{\frac{mgL}{I}} \quad \left(I = mL^2, \ \omega = \sqrt{\frac{mgL}{mL^2}}, \ \omega = \sqrt{\frac{g}{l}}\right)$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Page No.

in angular SHM $\rightarrow$ $\tau = -k\theta$

linear SHM $\rightarrow$ $F = -kx$

Date

- For lift rising up with an acceleration then

$$T = 2\pi\sqrt{l/g+a}$$

- For lift rising down with an acc.

$$T = 2\pi\sqrt{l/g-a}$$

DAMPED OSCILLATION

" When the motion of an oscillator is reduced by an external force, the oscillator and its motion are said to be damped.

$$F_d = -bv$$

where b is a damping constant that depends upon the characteristics of both vane & liquid

SI unit of b = kg/sec. The minus sign indicates that F_d opposes the motion.

According to 2nd law

$$F = -kx - bv$$

$$ma = -kx - bv$$

Subsiting $\frac{d^2x}{dt}$ for a & dx/dt for v

$$\frac{md^2x}{dt} + b\frac{dx}{dt} + kx = 0$$

solution of eqn

$$x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega' t + \phi) \rightarrow$$

x_m = amplitude

ω' = is angular frequency

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \text{ or}$$

If $b=0$ then there is no damping eqn reduces to $\omega = \sqrt{\frac{k}{m}}$ and eq ① also reduces to for a displacement of an undamped oscillator.

Energy of damped oscillator

$$E(t) \approx \frac{1}{2} k x_m^2 e^{-bt/m}$$

which tells us that amplitude, the mec energy decr. exponentially with time.

$x_m = \dfrac{F_0/m}{\omega^2 - \omega_d^2}$ max amplitude for forced oscillation

$b > 2m\omega_0$ (overdamped)
$b < 2m\omega_0$ (underdamped)
$b = 2m\omega_0$ (critical damped)

Date

If the damping constant is small but non-zero (so that $b \leq \sqrt{km}$), then $\omega' \approx \omega$

$b > \sqrt{km}$

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \quad \frac{k}{m} = \omega_0^2$$

natural frequency

" FORCED OSSCILATION "

$$\frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

If an external driving force with angular frequency ω_d acts on an oscillating systems with natural angular frequency 'ω', the system oscillates with angular frequency ω_d.

- ~~the velo~~ Such a forced oscillator oscillates at the angular frequency ω_d of the driving force and its displacement $x(t)$ is,

$$x(t) = x_m \cos(\omega_d t + \phi)$$

How large the displacement amplitude x_m depends upon complicated function ω_d & ω. The velocity amplitude v_m of the oscillation is greatest when $\boxed{\omega_d = \omega}$ this is called RESONANCE.

" WAVES "

Date

Equation of Wave,

$$y(x,t) = y_m \overbrace{\sin(\underbrace{kx - \omega t + \phi}_{\text{phase}})}^{\text{oscillating term}}$$

ϕ is ph[...] consta[...]

Where,

$y(x,t)$ = displacement → up and down.

y_m = Amplitude

k = Wave no.

x = position (distance covered by wave)

ω = Angular frequency

t = time

y is the displacement of particle at 'x' at time 't'
↳ position:

This eqn meems
wave is travelling along x-axis
particle are oscillating along y-axis

$\boxed{\omega = 2\pi f}$ $\boxed{k = \frac{2\pi}{\lambda}}$ → radian/meter or

Date

WAVE SPEED :-

$\boxed{V = f\lambda}$ & $\boxed{V = \frac{\omega}{k}}$ & $V = \frac{\lambda}{T}$

Agr $(\omega t - kx)$ mein x or t kay coefficent same sign kay hai to wave -ve x-axis mein jayegi and vice versa.

$(\omega t - kx)$ mein opp sign hai to iska matlab wave +ve ~~axis~~ x-axis par jayegi.

frequency = wave / time

- The phase of wave changes linearly with time.

- Transverse Velocity $u = -\omega y_m \cos(kx - \omega t)$

(means velocity in y-axis)

main eq ko derivate kiya treating 'x' as constant q/k y-axis par 'x' constant rhega.

- Transverese acc $= a_y = -\omega^2 y_m \sin(kx - \omega t)$

or $a_y = -\omega^2 y$

Date

A travelling wave is always in the following form,

$$y(x,t) = f(x \pm vt)$$

such functions are solution of the wave eqn.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial^2 t} \quad \text{(wave eqn)}$$

it is a linear partial differential eqn; when y_1 & y_2 are solutions, any linear combination of y_1 & y_2 (like $ay_1 + by_2$) is also a solution.

The principle of linear supperposition is valid only when the amplitude is small.

Date

SUPERPOSITION OF Waves:-

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

if $y_1(x,t) = y_m \sin(kx - \omega t)$

$y_2(x,t) = y_m \sin(kx - \omega t + \phi)$

then $y'(x,t) = \underbrace{[2y_m \cos \tfrac{1}{2}\phi]}_{\text{magnitude gives amplitude}} \underbrace{\sin(kx - \omega t + \tfrac{1}{2}\phi)}_{\text{Oscillating Term}}$

$V = \sqrt{\dfrac{T}{\mu}}$, T = tension

μ = linear density , m/l

v = velocity.