Ex: 1.8

Date

Linear Transformation:

$T: R^n \to R^n$

If T is a function with domain R^n, codomain R^m, then we say that T is a Transformation from R_n to R_m.

T: variable → Eq

T: domain → co-domain.

Q1, Find the domain & Co-domain.

(a) $w_1 = 4x_1 + 5x_2$

$w_2 = x_1 - 8x_2$

$T: R^2 \to R^2$

Domain = 2 (bcz 2 variable)

Co-domain = 2 (bcz 2 eqn)

(b) $w_1 = 5x_1 - 7x_2$

$w_2 = 6x_1 + x_2$

$w_3 = 2x_1 + 3x_2$

$T: R^2 \to R_3$

Domain = 2

Co-domain = 3

Q: Find the domain & co-domain.

(a) $\begin{bmatrix} 3 & 1 & 2 \\ 6 & 7 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$T: R^3 \to R^2$

Domain = 3

Co-domain = 2

Q: Find the domain & co-domain.

(a) $T(x_1, x_2, x_3) = (4x_1 + x_2, x_1 +$

$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_1 + x_2 \\ x_1 + x_2 \end{bmatrix}$

$T: R^3 \to R^2$

domain = 3

Co-d~ = 2

Q: Find Standard Matrix

$W_1 = 2x_1 - 3x_2 + x_3$

$W_2 = 3x_1 + 5x_2 - x_3$

$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

St. Matrix = $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix}$

b) $W_1 = x_1$

$W_2 = x_1 + x_2$

$W_3 = x_1 + x_2 + x_3$

$W_4 = x_1 + x_2 + x_3 + x_4$

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

c) $W_1 = 7x_1 + 2x_2 - 8x_3$

$W_2 = -x_2 + 5x_3$

$W_3 = 4x_1 + 7x_2 - x_3$

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} = \begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{bmatrix}$$

Q: Find Stand. Matrix

$T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, \; x_2 + x_3, \; -x_1)$

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7x_1 + 2x_2 - x_3 + x_4 \\ x_2 + x_3 \\ -x_1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$T(x_1, x_2, x_3, x_4) = (x_4, x_1, x_3, x_2, x_1 - x_4)$

$$= \begin{bmatrix} 0+0+0+x_4 \\ x_1+0+0+0 \\ 0+0+x_3+0 \\ 0+x_2-x_4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

Q. Find St. Matrix $T: R^3 \to R^4$

$W_1 = 3x_1 + 5x_2 - x_3$

$W_2 = 4x_1 - x_2 + x_3$

$W_3 = 3x_1 + 2x_2 - x_3$

Compute $T(-1, 2, 4)$ by directly subsitution in the eqn and then by matrix Multiplication.

St. $$\begin{bmatrix} 3x_1 + 5x_2 - x_3 \\ 4x_1 - x_2 + x_3 \\ 3x_1 + 2x_2 - x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

St. $$\begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$$

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$T(x) = St.\ Matrix \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$T\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

$=$

Q. Find the St-Matrix & Compute $T(x)$.

$T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3,\ x_2 + x_3, 0)$

$x = (2, 1, -3)$

$$St: \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -0 \\ -2 \\ 0 \end{bmatrix}$$

Q. Find $T_A(x)$

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}; \quad x = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

$T_A(x) = Ax$

$$T_A(x) = \begin{bmatrix} 3 \\ 13 \end{bmatrix}$$

Q: The image of the St. Matrix basis vector $T: R^3 \to R^3$

Find the St. Matrix & $T(x)$

$$T(e_1) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \quad T(e_2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T(e_3) = \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}, \quad X = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$S.Matrix = \begin{bmatrix} + \\ T(e_1) \cdot T(e_2) \cdot T(e_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 3 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

Exe: 1.10

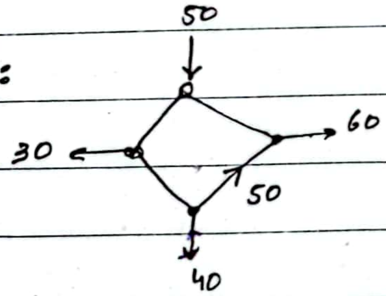
Application Of LA (System)

$$= \begin{bmatrix} 1 & 0 & 4 \\ 3 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$$

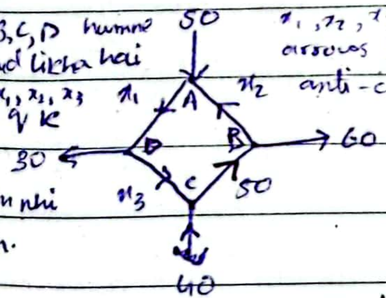
- Network Analysis
- Polynomial Interpolation

Q:

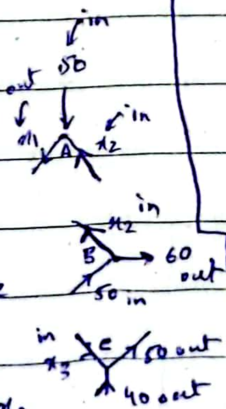


Find Flowrate & direction.

A, B, C, D humne khud likha hai
or x_1, x_2, x_3 bhi q ke
given nhi tha.



x_1, x_2, x_3 wale arrows ki direction anti-clockwise hogi



Node	In		Out
A	x_2+50	=	x_1
B	50	=	$60+x_2$
C	x_3+40	=	50
D	x_1	=	$30+x_3$

$\therefore x_1 - x_2 = 50 \rightarrow (1)$

$\boxed{x_2 = -10}$

$\boxed{x_3 = 10}$

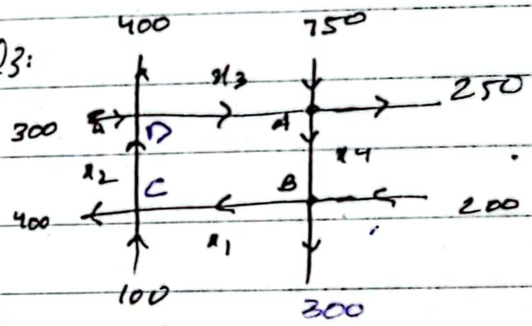
$x_1 - x_3 = 30 \rightarrow (2)$

eq (1) ⇒ put x_2

$x_1 + 10 = 50$

$\boxed{x_1 = 40}$

Q3:



Node	In		Out
A	$x_3 + 750$	=	$x_4 + 250$
B	$x_4 + 200$	=	$x_1 + 300$
C	$x_1 + 100$	=	$x_2 + 400$
D	$x_2 + 300$	=	$x_3 + \cancel{300} + 400$

$x_3 - x_4 = -500$

$x_1 - x_4 = -100$

$x_1 - x_2 = 300$

$x_2 - x_3 = 100$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & -1 & -500 \\ 1 & 0 & 0 & -1 & -100 \\ 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & -1 & 0 & 100 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -100 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 - R_3 \rightarrow R_3$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & -1 & -500 \\ 1 & 0 & 0 & -1 & -100 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 1 & -1 & 0 & 100 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -100 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 1 & -1 & 0 & 100 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -100 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 1 & -1 & 0 & 100 \end{array}\right]$$

$R_2 - R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -100 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 0 & 1 & -1 & -500 \end{array}\right]$$

$x_1 - x_4 = -100$

$x_2 - x_4 = -400$

$x_3 - x_4 = -500$

let $x_4 = t$

$x_1 = -100 + t$

$x_2 = -400 + t$

$x_3 = -500 + t$

where

$t > 500$ for smooth flow {no variable should be +ve}

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LINEAR TRANSFORMATION

$f : R^n \rightarrow R^m$

Domain (for R^n), Co-domain (for R^m)

$\overset{n=2}{\;} \quad \overset{m=1}{\;}$

$f : R^2 \rightarrow R^1$

matlab ek esa function jo k do value input mein lega lekn ans ek ayega

$\underbrace{f(x,y)}_{\text{2 inputs}} = x^2 + y^2$

$f(1,2) = 1 + 4$

$= 5 \rightarrow$ one ans

$f : R^n \rightarrow R^m$ means function 'f' is transforming or maping from R^n to R^m

$n=3 \quad m=1$

$f : R^3 \rightarrow R \; ; \; f(x,y,z) = x+y+z$

now,

$\underbrace{f(1,3,1)}_{\text{3 input}} = 1 + 3 + 1$

$= \underbrace{5}_{\text{1 output}}$

- So if $m = 1$ then functions are known as transformation
- If $m = n$ then functions are known as Operators

eg: $n=2 \quad m=2$

$f : R^2 \rightarrow R^2 \; ; \; \omega_1 = x + y$

$\omega_2 = x^2 + y^2$

If $x = 1$ & $y = 1$

then, $\omega_1 = 2$, $\omega_2 = 2$ two ans

$m=2$ hai to functions 2 hai.

$\Rightarrow$ Linear transformations are those which arise from Linear systems.

$\omega_1 = a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n$

$\omega_2 = a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n$

$\vdots$

$\omega_m = a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n$

$\downarrow$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

T_A means T is multiplication by A.

Date

or more briefly as

$W = Ax$

$T_A : R^n \to R^m \quad \to (1)$

In situations where domain & co-domain are not necessary then we write (1) as

$W = T_A(x)$

or

$x \xrightarrow{T_A} W$

→ **ZERO TRANSFORMATION** (From R^n to R^m)

A transformation $T_0 : R^n \to R^m$ defined by $T_0(x) = \underset{A}{0}X = \underset{A}{0}$ is called zero transformation

$0 \Rightarrow m \times n$ zero matrix

$x \in R^n$

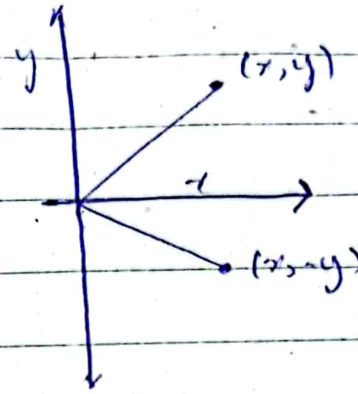
→ **IDENTITY OPERATOR:**

An operator $T_I : R^n \to R^m$ is denoted by $T_I(x) = IX = X$ is called Identity Operator

REFLECTION OPERATORS:

$T : R^2 \to R^2$

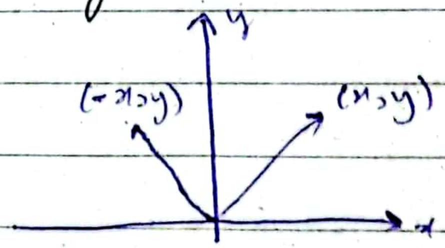
At x-axis:



St. Matrix = $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$w_1 = x + 0y \qquad w_2 = 0x - y$

At y-axis



St. Matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$w_1 = -x + 0y \qquad w_2 = 0x + y$

$T(x,y) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

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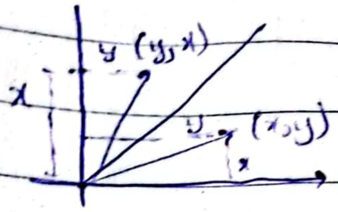
At line $y=x$



St. Matrix = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

At xy-plane : $T: R^3 \to R^3$

$w_1 = x, \; w_2 = y, \; w_3 = -z$

$$T(x,y,z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

↳ St. Matrix

At xz-plane :

$w_1 = x, \; w_2 = -y, \; w_3 = z$

$$T(x,y,z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

↳ St. Matrix

At yz-plane :

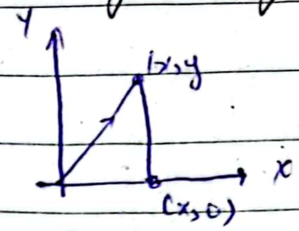
$w_1 = -x, \; w_2 = y, \; w_3 = z$

$$T(x,y,z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

PROJECTION OPERATOR :

$T: R^2 \to R^2$

- Orthogonal Proj. on X-axis

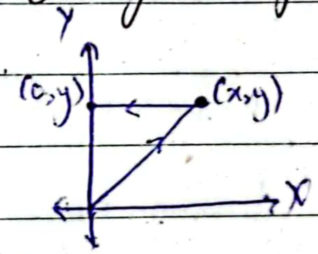


$w_1 = x$

$w_2 = 0$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

- Orthogonal Proj. on Y-axis



$w_1 = 0$

$w_2 = y$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$T: R^3 \to R^3$

- Orthogonal Proj. on XY-plane.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$w_1 = x$

$w_2 = y$

$w_3 = 0$

- Orthogonal Proj. on YZ-plane

$$(x,y,z) = \underbrace{\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}}_{\text{St. Matrix}}\begin{bmatrix}x\\y\\z\end{bmatrix}$$

- Orthogonal Proj. on XZ-plane

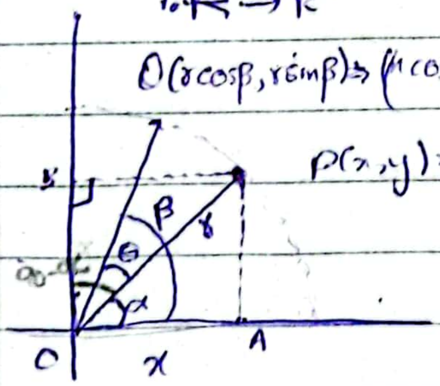
$$(x,y,z) = \underbrace{\begin{bmatrix}1&0&0\\0&0&0\\0&0&1\end{bmatrix}}_{\text{St. Matrix}}\begin{bmatrix}x\\y\\z\end{bmatrix}$$

ROTATION OPERATOR:

$T: R^2 \to R^2$

$O(r\cos\beta, r\sin\beta) \Rightarrow (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$

$P(x,y) \Rightarrow P(r\cos\alpha, r\sin\alpha)$



In ΔOAP

$\cos\alpha = \frac{x}{r}$

$\boxed{x = r\cdot\cos\alpha}$

In ΔOBP

$\cos(90-\alpha) = \frac{y}{r}$

$\sin\alpha = y/r$ $\quad \because \cos(90-\theta) = \sin\theta$

$\boxed{y = r\cdot\sin\alpha}$

Now,

$P(x,y) = (r\cos\alpha, r\sin\alpha)$

$\therefore O(x,y) = (r\cos\beta, r\sin\beta)$

$$r\cos\beta = r\cdot\cos(\alpha+\theta)$$
$$= r[\cos\alpha\cos\theta - \sin\alpha\sin\theta]$$
$$= \underbrace{r\cos\alpha}_{x}\cos\theta - \underbrace{r\sin\alpha}_{y}\sin\theta$$

$$\boxed{r\cos\beta = x\cos\theta - y\sin\theta}$$

$$r\sin\beta = r\sin(\alpha+\theta)$$
$$= r[\sin\alpha\cos\theta + \cos\alpha\sin\theta]$$
$$= \underbrace{r\sin\alpha}_{y}\cos\theta + \underbrace{r\cos\alpha}_{x}\sin\theta$$

$$\boxed{r\sin\beta = y\cos\theta + x\sin\theta}$$

$\therefore O(r\cos\beta, r\sin\beta) = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$

Eqns:

$w_1 = x\cos\theta - y\sin\theta$

$w_2 = x\sin\theta + y\cos\theta$

Matrix:

$$\underbrace{\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}}_{\text{St. Matrix}}\begin{bmatrix}x\\y\end{bmatrix}$$

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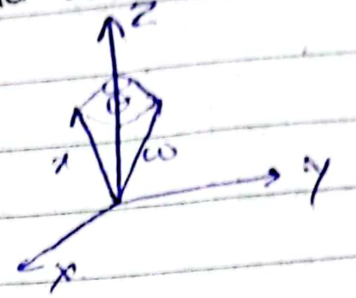
• $T: R^3 \to R^3$

"An operator that rotates each vector in R^3 about some rotation axis through a fixed angle "θ" is called rotation operator in R^3."

Rotation about z-axis:



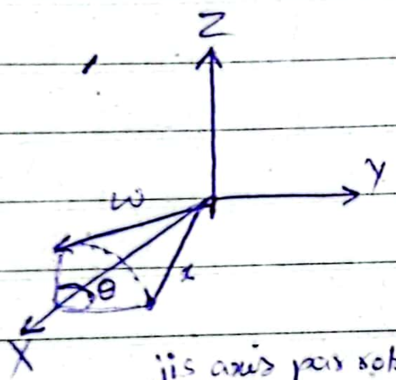
$w_1 = x\cos\theta - y\sin\theta$

$w_2 = x\cos\theta + y\sin\theta$

$w_3 = z$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• Rotation about x-axis.



jis axis par rotation ~~hai uski eqn~~ same rhegi

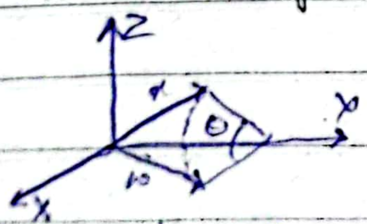
$w_1 = x$

$w_2 = y\cos\theta - z\sin\theta$

$w_3 = y\sin\theta + z\cos\theta$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation about y-axis:



$w_1 = z\sin\theta + x\cos\theta$

$w_2 = y$

$w_3 = z\cos\theta - x\sin\theta$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$\overline{(A \cup B \cap C)}^{x} - (A \cup C \cap B).$

$X \cap (Y)'$

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Q: Find the quadratic polynomial whose graph the point (1,1), (2,2) and (3,5)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 9 & 5 \end{bmatrix}$$

→ Gauss Jordan Echelon
Reduced Echelon form

Polynomial Interpolation:

$y = a_0 + a_1 x + a_2 x^2 + \cdots a_{n-1} x^{n-1}$

quadratic polynomial,

$\boxed{y = a_0 + a_1 x + a_2 x^2}$ — ①

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Cubic Polynomial,

$\boxed{y = a_0 + a_1 x + a_2 x^2 + a_3 x^3}$

$a_0 = 2$
$a_1 = -2$
$a_2 = 1$

eq(i) ⇒ $\boxed{y = 2 - 2x + x^2}$

at (1, 1)

eq(i) ⇒ $1 = a_0 + a_1 + a_2 \rightarrow$ ②

at (2, 2)

$2 = a_0 + 2a_1 + 4a_2 \rightarrow$ ③

at (3, 5)

$5 = a_0 + 3a_1 + 9a_2 \rightarrow$ ④

$y = a_0 + a_1(3) + a_2(3)^2$

Argument Matrix,

Q: Find the cube polynomia (1,1), (2,2), (3,5), (0,2)

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

at (1,1)

$1 = a_0 + a_1 + a_2 + a_3$

at (2,2)

$2 = a_0 + 2a_1 + 4a_2 + 8a_3$

at (3,5)

$5 = a_0 + 3a_1 + 9a_2 + 27a_3$

$2 = a_0$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 2 \\ 1 & 3 & 9 & 27 & 3 \\ 1 & 0 & 0 & 0 & 2 \end{array}\right]$$

$R_1 \leftrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 1 & 2 & 4 & 8 & 2 \\ 1 & 3 & 9 & 27 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

$-R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 2 & 4 & 8 & 0 \\ 1 & 3 & 9 & 27 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

$-R_1 + R_4 \rightarrow R_4$

$-R_1 + R_3 \rightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 2 & 4 & 8 & 0 \\ 0 & 3 & 9 & 27 & 1 \\ 0 & 1 & 1 & 1 & -1 \end{array}\right]$$

$R_2 \div 2$

$R_3 \div 3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 9 & 1/3 \\ 0 & 1 & 1 & 1 & -1 \end{array}\right]$$

$-R_2 + R_3 \rightarrow R_3$

$-R_2 + R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1/3 \\ 0 & 0 & -1 & -3 & -1 \end{array}\right]$$

$R_3 + R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1/3 \\ 0 & 0 & 0 & 2 & -2/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1/3 \\ 0 & 0 & 0 & 1 & -1/3 \end{array}\right]$$

$-4R_1 + R_2 \to R_2$

$-5R_4 + R_3 \to R_3$

$(-1,-1), (0,1), (1,3), (4,-1)$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & 4/3 \\ 0 & 0 & 1 & 0 & 2/3 \\ 0 & 0 & 0 & 1 & -1/3 \end{array}\right]$$

$-2R_3 + R_2 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2/3 \\ 0 & 0 & 0 & 1 & -1/3 \end{array}\right]$$

CHAPTER #2

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Q: Using Row Reduction to Evaluate a Determinant

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\det(A) = (-1)\begin{vmatrix} 3 & -6 & 9 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{vmatrix}$$

Augmented matrix nhi hai is waja se (-1) kraa hai.

Taking 3 as common.
divide nhi kr sakte as it is ~~not an augmented~~.

$$\det(A) = (-1)(3)\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{vmatrix}$$

$-2R_1 + R_3$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 10 & -5 \end{bmatrix}$$

$-10R_2 + R_3$

$$= (-3)\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -55 \end{vmatrix}$$

Taking -55 as common.

$$= (-3)(-55)\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (-3)(-55)(1)(1)(1)$$

$$= 165$$

Q2: Using cofactor expansion Find determinant.

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

Interchanging R_1 to R_2

$$\det(A) = (-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}$$

Cofactor wale mein 1st element $\begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$ (-2) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$
1 hoga or aake neeche waale sab
zero. Bare matrix ko chote matrix mein
break krte hai.

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$-2R_1 + R_2$

Or Using Co-factor

$= (-1)\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

$\begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}$

$R_2 \leftrightarrow R_1$

$= (-1)\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

$(-1)\begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 5 & -2 & 6 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}$

$-R_1 + R_3 \to R_3$

$-2R_1 + R_2 \to R_2$

$-3R_1 + R_2 \to R_2$

$-2R_1 + R_3 \to R_3$

$-3R_1 + R_4 \to R_4$

$= (-1)\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 8 & 0 \end{bmatrix}$

$= (-1)\begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$

$= (-1)(-4-2)$

$= (-1)(-6)$

$= 6$

$(-1)\begin{bmatrix} -1 & 1 & 3 \\ 0 & 3 & 3 \\ 1 & 8 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_3$

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$(-1)\begin{bmatrix} 1 & 8 & 0 \\ 0 & 3 & 3 \\ -1 & 1 & 3 \end{bmatrix}$

$R_1 + R_3 \to R_3$

$(-1)\begin{bmatrix} 1 & 8 & 0 \\ 0 & 3 & 3 \\ 0 & 9 & 3 \end{bmatrix}$

$(-1)\begin{bmatrix} 3 & 3 \\ 9 & 3 \end{bmatrix}$

$= (9 - 27)$

$\det(A) = -18$

(ii) $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$= (-1)\begin{bmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ 5 & -3 & 6 \end{bmatrix}$

$-2R_1 + R_2 \to R_2$

$-5R_1 + R_3 \to R_3$

$= (-1)\begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 0 & -13 & -14 \end{bmatrix}$

$(-1)\begin{bmatrix} -5 & -5 \\ -13 & -14 \end{bmatrix}$

$(-1)(5)$

$= -5$

Q: Verify that:

$\det(A) = \det(A^T)$

① $A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ $A^T = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$

$\det(A) = -8 - 3$

$= -11$

$\det(A^T) = -8 - 3$

$= -11$

agle wale ka bhi -5 ayega!