CHAPTER #03

"RANDOM VARIABLE"

- It is a ~~real value~~ function that ~~is directly attached to the outcome of an experiment~~ associates a real no. with each element in Sample space.
- Exp → {HH, TT, HT, TH}

'x' → Random Variable

'Number of Heads'

$x = \underbrace{\{0, 1, 2\}}$ → To x ya to 1 hoga q k ek experiment mein shyad ek dafa head aaye ya 2 baar bhi aasakta hai ya 0 bar bhi (TT case).

ye ek esi real value ko show krrha hai jo no of heads ko represent krrhi hai in the ~~outcome~~ experiment → {HH, TT, HT, TH}

→ 'x' yaha par no. of heads ka random variable hai for expreiment of coin toss.

"Probability Distribution"

	x_1	x_2	x_3
X_i	0	1	2
$P(X_i)$	P_1 $\frac{1}{4}$	P_2 $\frac{2}{4}$	P_3 $\frac{1}{4}$
	0 wale case ki probab (TT)	1 wale case ki Prob (HT, TH)	2 wale case ki pro (HH)

4R 3B

Note: $P_1 + P_2 + P_3 = 1$

$P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

$P(X=2) = \frac{1}{4}$

" Types of Random Variable "

Discrete Random Variable (DRV):
random variable is said to be discrete if it take a finite no. of values.
Examples: In an experiment of tossing a pair of coins, if we define the random variable 'X' as the no. of heads obtained, then values of X are 0, 1 & 2 corresponding to outcomes TT, HT, TH & HH resp.
two dice rolled, Ball from bag. [in sab mein finite cases bnenge]

Continuous Random Variable (CRV):
is mein infinite hoga ya large data.
Example: The height of a person chosen at random from a population of 7000 persons lies b/w 140cm & 160cm.
Similarly age, weight etc are continuous variable.

Q: A coin is baised so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the prob distribtn - for no. of tails.

Prob of Tail = $1w = 1/4$
Prob of Head = $3w = 3/4$
Total probability = 1
$1w + 3w = 1$
$w = \frac{1}{4}$

X	0	1	2
P	$9/16$	$6/16$	$1/16$
	HH	HT+TH	(TT)
	$3/4 \times 3/4$	$\frac{3}{4}\frac{1}{4} + \frac{3}{4}\frac{1}{4}$	$\frac{1}{4} \times \frac{1}{4}$

$\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{6}$

Q: A shipment of 20 similar laptops computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 computers, find the probability distr. for the no. of deffetive. (koi 1 def) (jo do choose kre dono hi deffetive)

→ As no. of defective ye 0, 1 and 2 hoga.
Gya koi defective nhi nikla.

X	0	1	2
$P(X=x)$	$68/95$	$51/190$	$3/190$

we will find prob that none is deffetive — baaqi 17 bache usmein se 2 chose krne

$f(0) = P(X=0) = \frac{{}^{3}C_0 \, {}^{17}C_2}{{}^{20}C_2} = \frac{68}{95}$

3 defective mein se ek bhi choose nhi kra

$f(1) = P(X=1) = \frac{{}^{3}C_1 \, {}^{17}C_1}{{}^{20}C_2} = \frac{51}{190}$

$f(2) = P(X=2) = \frac{{}^{3}C_2 \, {}^{17}C_0}{{}^{20}C_2} = \frac{3}{190}$

Commulative Distribution Function: $F(x)$ of a discrete random variable X with probability distribution $f(x)$ is,

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t)$$

For above Question $F(2) = P(X \leq 2) = f(0) + f(1) + f(2)$

$= \frac{68}{95} + \frac{51}{190} + \frac{3}{190}$

Probability Mass Function: used to calculate probability of Discrete Random Variables. The set of ordered pair $(x, f(x))$ is a prob funct., prob mass funct, or prob distribution of discrete random variable X if for each possible outcome x, ① $f(x) \geq 0$ ② $\sum f(x) = 1$ ③ $P(X=x) = f(x)$.

$\frac{48}{52} \cdot \frac{48}{52}$

Q: Two cards are drawn sucessively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of no. of queens.

$P(X=0) = \frac{48}{52} \times \frac{48}{52}$

$P(X=1) = \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}$

$P(X=2) = \frac{4}{52} \times \frac{4}{52}$

X	0	1	2
$P(X=x)$			

• Probability Density Function:

The function $f(x)$ is a probability density function (PDF) for the continuous random var X, defined over the set of real no., if

1) $f(x) \geq 0$, for all $x \in \mathbb{R}$
2) $\int_{-\infty}^{+\infty} f(x)\, dx = 1$
3) $P(a<x<b) = \int_a^b f(x)\, dx$

Q: Is the function defined as follows a probability density function.

$$f(x) = \begin{cases} 0 & x<2 \\ (3+2x)/18 & 2<x<4 \\ 0 & x>4 \end{cases}$$

→ if it is PDF then $\int_{-\infty}^{+\infty} f(x)\, dx = 1$

$$\int_{-\infty}^{2} 0\, dx + \int_2^4 \frac{(3+2x)}{18}\, dx + \int_4^{+\infty} 0\, dx$$

$$0 + \frac{1}{18}\left(3x + \frac{2x^2}{2}\right)\Big|_2^4 + 0$$

$$0 + \frac{1}{18}\left[3(4)+(4)^2 - 3(2) + 2^2\right] + 0 = \frac{18}{18} = 1$$

0.01 5.18
0.05 3.20

Q: $f(x) = \begin{cases} x^2/3 & -1<x<2 \\ 0 & \text{elsewhere} \end{cases}$ (a) verify that it is density (b) Find $P(0<x\leq 1)$

(a) $= \int_{-\infty}^{+\infty} f(x)\, dx = \int_{-1}^{2} \frac{x^2}{3}\, dx = \frac{x^3}{9}\Big|_{-1}^{2} = \frac{8}{9} + \frac{1}{9} = 1$

(b) $P(0<x\leq 1) = \int_0^1 \frac{x^2}{3}\, dx = \frac{x^3}{9}\Big|_0^1 = \frac{1}{9}$

• Commulative Distribution Function: $F(x)$ for a CRV X with density function $f(x)$ is,

$$F(X) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt \quad \text{for } -\infty < x < \infty$$

$$P(a<x<b) = F(b) - F(a) \quad \& \quad f(x) = \frac{d\,F(x)}{dx}$$

Q: For the density function in above Qs, find $F(x)$ & use it to evaluate $P(0<x\leq 1)$.

For $-1<x<2$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-1}^{x} \frac{t^2}{3}\, dt = \frac{t^3}{9}\Big|_{-1}^{x} = \frac{x^3+1}{9}$$

Therefore,

$$F(x) = \begin{cases} 0 & x<-1 \\ \frac{x^3+1}{9} & -1<x<2 \\ 1 & x \geq 2 \end{cases}$$

$$P(0<x\leq 1) = F(1) - F(0) = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

$\int_{-1}^{x} \frac{x^2}{3} \quad \frac{x^3}{9} \quad \frac{1}{9}[x^3+1]$

Q: 3.5 Determine value of 'c' so that each of the following functions can serve as a probability distribution of the discrete random variable X:

$f(x) = c\binom{2}{x}\binom{3}{3-x}$ for $x = 0, 1, 2$

$\sum_{x=0}^{2} f(x) = 1 \Rightarrow \sum_{x=0}^{2} c\binom{2}{x}\binom{3}{3-x} = 1$

$c\sum_{x=0}^{2}\binom{2}{x}\binom{3}{3-x} = 1 \Rightarrow c\left(\binom{2}{0}\binom{3}{3} + \binom{2}{1}\binom{3}{2} + \binom{2}{2}\binom{3}{1}\right) = 1$

$c \cdot 10 = 1$

$\boxed{c = 1/10}$

Q 3.6: $f(x) = \begin{cases} \frac{20{,}000}{(x+100)^3} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$

(a) $P(x \geq 200) = 1 - P(x < 200)$

isi ko ese bhi kr sakte hai

$\int_{200}^{\infty} \frac{20{,}000}{(x+100)^3}dx$

$\lim_{t\to\infty}\int_{200}^{t} \frac{20{,}000}{(x+100)^3}dx$

$\lim_{t\to\infty} \frac{10{,}000}{(x+100)^2}\Big|_{200}^{t}$

$= 1/9$

$= 1 - \left[\int_{-\infty}^{0} 0\,dx + \int_{0}^{200} \frac{20{,}000}{(x+100)^3}dx\right]$

$= 1 - \left[\int_0^{200} (20000)(x+100)^{-3}dx\right]$

$= 1 - \left[20{,}000\int_0^{200}(x+100)^{-3}dx\right]$

$= 1 - \left[20{,}000\,\frac{(x+100)^{-2}}{-2}\right]_0^{200}$

$= 1 + \frac{10{,}000}{(x+100)^2}\Big|_0^{200}$

$P(x \geq 200) = \frac{1}{9}$

(b) Anywhere from 80 to 120 days

$P(80 \leq x \leq 120) = \int_{80}^{120}\frac{20{,}000}{(x+100)^3}dx = \left[-\frac{10{,}000}{(x+100)^2}\right]_{80}^{120}$

$= 0.1020$

Q 3.7: (a) less than 120 hours.

$P(x < \frac{120}{100}) = P(x < 1.2)$

$P(x < 1.2) = \int_0^{1.2} f(x)dx = \int_0^1 x\,dx + \int_1^{1.2}(2-x)\,dx$

$= \frac{x^2}{2}\Big|_0^1 + \left[2x - \frac{x^2}{2}\right]_1^{1.2}$

$P(x < 1.2) = 0.68$

(b) between 50 & 100 hrs.

$P(\frac{50}{100} < x < \frac{100}{100}) = P(0.5 < x < 1)$

$P(0.5 < x < 1) = \int_{0.5}^{1} f(x)\,dx = \int_{0.5}^{1} x\,dx = \frac{x^2}{2}\Big|_{0.5}^{1} = 0.375$

Q 3.9: $f(x) = \begin{cases} \frac{2}{5}(x+2) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$

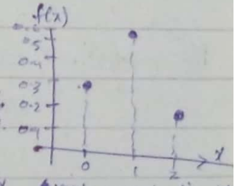
(a) $P(0 < x < 1) = \int_0^1 \frac{2}{5}(x+2)dx = \frac{2}{5}\int_0^1 (x+2)dx = \frac{2}{5}\left(\frac{x^2}{2} + 2x\right)\Big|_0^1$

$= 1$

(b) $P(0.25 < x < 0.5) = \int_{0.25}^{0.5}\frac{2}{5}(x+2)dx = \frac{2}{5}\left(\frac{x^2}{2} + 2x\right)\Big|_{0.25}^{0.5} = 0.2375$

Q 3.10:

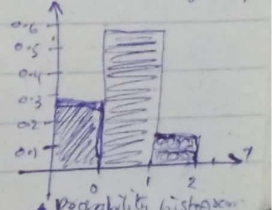
x	1	2	3	4	5	6
$f(x)$	1/6	1/6	1/6	1/6	1/6	1/6



prob mass function plot

Q 3.11:

x	0	1	2
$f(x)$	2/7	4/7	1/7
	$\frac{^2C_0\,^5C_3}{^7C_3}$	$\frac{^2C_1\,^5C_2}{^7C_3}$	$\frac{^2C_2\,^5C_1}{^7C_3}$



Probability histogram

Q. 3.138 : no. of ~~iffe~~ imperfections per 10 meter is DRV

so $F(x) = P(X \le x) = \sum_{t \le x} f(t)$

$F(0) = P(X \le 0) = f(0) = 0.41$

$F(1) = P(X \le 1) = f(0) + f(1) = 0.78$

$F(2) = P(X \le 2) = f(0) + f(1) + f(2) = 0.94$

$F(3) = 0.99$

$F(4) = 1$

x	0	1	2	3	4
$F(x)$	0.41	0.78	0.94	0.99	1

Q3.15:

$$F(x) = \begin{cases} 0 & x < 0 \\ f(0) & 0 \le x < 1 \\ f(0) + f(1) & 1 \le x < 2 \\ f(0) + f(1) + f(2) & 2 \le x \end{cases}$$

$$= \begin{cases} 0 & x < 0 \\ 2/7 & 0 \le x \le 1 \\ 6/7 & 1 \le x\ 2 \\ 1 & 2 \le x \end{cases}$$

$P(X=1) = 4/7$ (from prob dist table)

or $= P(X \le 1) - P(X \le 0) = F(1) - F(0) = \frac{6}{7} - \frac{2}{7} = \frac{4}{7}$

$P(0 < X \le 2) = F(2) - F(0) = \frac{5}{7}$

$P(0 \le X < 2)$ P(0

P(0) + P(1) P(1) + P(2) =

F(1) - F(0) + F(2) - F(1)

P(0) + P(1) = F(0) + F(1) - F(0)

Q3.18: $f(x) = 2(1+x)/27$

$P(X<4) = \int_2^4 f(x)dx = 2\int_2^4 (1+x)dx = \frac{2}{27}\left(x + \frac{x^2}{2}\right)\Big|_2^4 = \frac{16}{27}$

$P(3 \le X < 4) = \int_3^4 \frac{2(1+x)}{27}dx = \frac{2}{27}\left(x + \frac{x^2}{2}\right)\Big|_3^4 = \frac{1}{3}$

Q3.19: $f(x) = \frac{1}{2}$

$F(x) = P(X \le x) = \int_1^x \frac{1}{2}dt = \frac{1}{2}t\Big|_1^x = \frac{x}{2} - \frac{1}{2}$

$P(2 < X < 2.5) = F(2.5) - F(2) = \frac{2.5}{2} - \frac{1}{2} - \frac{2}{2} + \frac{1}{2} = \frac{1}{4}$

Q3.20: $F(x) = P(X \le x) = \int_2^x \frac{2}{27}(1+t)dt = \frac{2}{27}\left(t + \frac{t^2}{2}\right)\Big|_2^x$

$= \frac{2}{27}\left[x + \frac{x^2}{2} - 2 - \frac{4}{2}\right] = \frac{2}{27}\left(x + \frac{x^2}{2}\right)$

$P(3 \le X < 4) = F(4) - F(3) = \frac{2}{27}\left(4 + \frac{4^2}{2}\right) - \frac{2}{27}\left(3 + \frac{3^2}{2}\right) = \frac{1}{3}$

Q3.21: $f(x) = \begin{cases} k\sqrt{x} & 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$

$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^1 k\sqrt{x}\,dx = 1$

$\frac{2k}{3}x^{3/2}\Big|_0^1 = 1 \Rightarrow \frac{2k}{3}\left[1^{3/2} - 0^{3/2}\right] = 1$

$\boxed{k = 3/2}$

$F(x) = \int_0^x \frac{3}{2}\sqrt{t}\,dt = \frac{3}{2}\cdot\frac{2}{3}t^{3/2}\Big|_0^x$

$= \left[x^{3/2} - 0^{3/2}\right] \Rightarrow F(x) = x^{3/2}$

$P(0.3 < x < 0.6) = F(0.6) - F(0.3) = 0.6^{3/2} - 0.3^{3/2} = 0.3004$

$\frac{2}{6}\times\frac{2}{6}\times\frac{2}{6}$

3·26: $\boxed{\begin{matrix}4B\\2G\end{matrix}}$

X	0	1	2	~~3~~
f(x)	$\frac{4}{6}\times\frac{4}{6}\times\frac{4}{6}$	$\left(\frac{2}{6}\times\frac{4}{6}\times\frac{4}{6}\right)\times 3$	$\left(\frac{2}{6}\times\frac{2}{6}\times\frac{4}{6}\right)\times 3$	$1/27$
	BBB	GBB, BGB, BBG	GGB, GBG, BGG	
	$\frac{8}{27}$	$4/9$	$2/9$	

3·28 $f(x) = \begin{cases} 2/5 & , 23{\cdot}75 < x < 26{\cdot}25 \\ 0 & , \text{elsewhere.} \end{cases}$

a) verify valid density function!

$\int_{23\cdot75}^{26\cdot25} 2/5\, dx = 1 \Rightarrow \frac{2x}{5}\Big|_{23\cdot75}^{26\cdot25} = 1 \Rightarrow \frac{2}{5}\left(\frac{5}{2}\right) = 1$

b) $P(X<24) = \int_{23\cdot75}^{24} f(x)dx = \int_{23\cdot75}^{24} \frac{2}{5} dx = \frac{2x}{5}\Big|_{23\cdot75}^{24} = \frac{1}{10}$

c) $P(x>26) = \int_{26}^{26\cdot25} \frac{2}{5} dx = \frac{2x}{5}\Big| = 0{\cdot}1$ (it is not entirely zero)

"JOINT PROBABILITY"

1)

For Discrete Random Variable:

The function $f(x,y)$ is a joint probability distribution or probability mass function (PMF) of discrete RV X & Y if

1. $f(x,y) \geq 0$ for all (x,y)
2. $\sum_x \sum_y f(x,y) = 1$
3. $P(X=x, Y=y) = f(x,y)$

poora concept poorana wala hai sirf yaha 2 variables involved hai ek ki jaga.

Q: Two balls are drawn from a box containing $\boxed{3 \text{ blue}}$, $\boxed{2 \text{ red}}$ & $\boxed{3 \text{ green}}$ balls. If X is the no. of $\boxed{\text{blue}}$ balls selected & Y is the no. of $\boxed{\text{red}}$ balls selected find: joint probability function.

X → blue
Y → red

possible values of X → 0, 1, 2, ~~3~~

" " Y → 0, 1, 2 (q k 2 hi red balls hai)

So possible pairs would be → (0,0) (0,1), (0,2), (1,0), (1,1), ~~(1,2)~~, (2,0), ~~(2,1)~~, ~~(2,2)~~ (cross isliye kre hai q k total 2 hi balls nikalni hai!

(0,1) → iska matlab 0 blue & 1 red (or 0 red hai to matlab 1 green hi hogi)

(0,0) → 2 greens (bcz agr red blue dono zero hai to green hi hogi)

(2,0) → 2 blue only (no green no blue q k 2 hi nikalni thi or wo humne nikaal li)

3 → blue
2 → red
3 → green

$\frac{{}^{3}C_2}{{}^{8}C_2}$, $\frac{{}^{2}C_1\,{}^{3}C_1}{{}^{8}C_2}$, $\frac{{}^{3}C_2}{{}^{8}C_2}$

green me se choose karenge, blue no red; red, green; only red

$f(x,y)$		X = 0	X = 1	X = 2	Total
y	0	3/28	9/28	3/28	15/28 — h(0)
	1	3/14	3/14	0	3/7 — h(1)
	2	1/28	0	0	1/28 — h(2)
Total		5/14 — g(0)	15/28 — g(1)	3/28 — g(2)	1

→ Marginal probability of Y

→ marginal of X

$(0,1) \rightarrow \frac{{}^{2}C_1\,{}^{3}C_1}{{}^{8}C_2}$, $(1,1) \rightarrow \frac{{}^{2}C_1\,{}^{3}C_1}{{}^{8}C_2}$, $(2,1) \rightarrow 0$

red, green; red, blue

$(0,2) \rightarrow \frac{{}^{2}C_2}{{}^{8}C_2}$ → red

For Continuous Random Variable:

The function $f(x,y)$ is a joint density function of CRV variable X & Y if

1. $f(x,y) \geq 0$ for all (x,y)
2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$
3. $P[(X,Y)\in A] = \iint_A f(x,y)\,dx\,dy$, for any region A in xy plane

Q: $f(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$

(a) Verify condition 2.

pehle isko solve karenge

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^1 \boxed{\int_0^1 \frac{2}{5}(2x+3y)\,dx}\,dy$$

$$= \int_0^1 \left(\frac{2x^2}{5} + \frac{6xy}{5}\right)\Big|_{x=0}^{x=1} dy$$

yaha x ko integrate kar rahe hai or y ko as coefficient treat karenge.

now putting only x values

$$= \int_0^1 \left(\frac{2}{5} + \frac{6y}{5}\right) dy$$

$$= \left(\frac{2y}{5} + \frac{6y^2}{10}\right)\Big|_0^1$$

$$= \frac{2}{5} + \frac{3}{5}$$

$$= 1 \quad \text{verified!}$$

(b) Find $P[(X,Y)\in A]$, where $A = \{(x,y) \mid 0<x<\frac{1}{2}, \frac{1}{4}<y<\frac{1}{2}\}$

$$P(0<X<\tfrac{1}{2}, \tfrac{1}{4}<Y<\tfrac{1}{2}) = \int_{1/4}^{1/2}\int_0^{1/2} \frac{2}{5}(2x+3y)\,dx\,dy$$

$$= \int_{1/4}^{1/2} \left(\frac{2x^2}{5} + \frac{6xy}{5}\right)\Big|_{x=0}^{x=1/2} dy$$

$$= \int_{1/4}^{1/2} \left(\frac{1}{10} + \frac{3y}{5}\right) dy$$

$$= \left(\frac{y}{10} + \frac{3y^2}{10}\right)\Big|_{1/4}^{1/2}$$

$$= \frac{13}{160} \quad \text{Ans}$$

"MARGINAL DISTRIBUTIONS"

For Discrete Random Variable.

$g(x) = \sum_y f(x,y)$ and $h(y) = \sum_x f(x,y)$

x	g(x)
0	5/14
1	15/28
2	3/28

from prev question

y	h(y)
0	15/28
1	3/7
2	1/28

For Continuous Random Variable:

$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$ & $h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$

From prev example:

$\rightarrow y$ ki limits

$g(x) = \int_0^1 \frac{2}{5}(2x+3y)\,dy = \left(\frac{4xy}{5} + \frac{6y^2}{10}\right)_{y=0}^{y=1}$

$\boxed{g(x) = \frac{4x+3}{5}}$ for $0 \le x \le 1$, and $g(x) = 0$, elsewhere.

$\rightarrow x$ ki limits

$h(y) = \int_0^1 \frac{2}{5}(2x+3y)\,dx = \left(\frac{2x^2}{5} + \frac{6xy}{5}\right)\Big|_{x=0}^{x=1}$

$\boxed{h(y) = \frac{2(1+3y)}{5}}$ for $0 \le y \le 1$, and $h(y) = 0$ elsewhere

$$\boxed{P(a < X < b) = \int_a^b g(x)\,dx}$$

"CONDITIONAL DISTRIBUTION"

Let X & Y be two random variables discrete or cont.
The conditional distribution of the random variable Y given that X = x is

$$\boxed{f(y|x) = \frac{f(x,y)}{g(x)}}$$

Similarly, the conditional distribution of X given that Y = y

$$\boxed{f(x|y) = \frac{f(x,y)}{h(y)}}$$

If we wish to find the probability that the DRV X falls b/w a & b when it is known that DRV Y = y, we evaluate,

$$P(a < X < b \,|\, Y = y) = \sum_{a < x < b} f(x|y)$$

for Continuous RV,

$$P(a < X < b \,/\, Y = y) = \int_a^b f(x|y)\,dx$$

Example: Find the conditional distribution of X, given that Y = 1, & used it to determine $P(X=0/Y=1)$.

We need to find $f(x/y)$, where $y = 1$.

$$f(x/1) = \frac{f(x,1)}{h(1)}$$

For $h(1)$: from table we get the value $h(1) = 3/7$

Therefore,

$$f(x/1) = \frac{f(x,1)}{3/7} = \frac{7}{3} f(x,1), \quad x = 0, 1, 2.$$

Now,

$$f(0/1) = \frac{7}{3} f(0,1) = \frac{7}{3}\left(\frac{3}{14}\right) = \frac{1}{2}$$

$\hookrightarrow$ from table

$$f(1/1) = \frac{7}{3} f(1,1) = \frac{7}{3}\left(\frac{3}{14}\right) = \frac{1}{2}$$

$$f(2/1) = \frac{7}{3} f(2,1) = \frac{7}{3}(0) = 0$$

$\int\int ... = \int_0^1 5x^2y^2 \Big| dy = \int_0^1 5y^4\, dy = y^5$

the Conditional distribution of X given that $Y = 1$

x	0	1	2
$f(x\|1)$	$\frac{1}{2}$	$\frac{1}{2}$	0

$\Rightarrow P(X=0 / Y=1) = f(0/1) = \frac{1}{2}$

Iska matlab agr 2 main se ek pen red hai then to doosra blue nhi hoga uski probability $\frac{1}{2}$ hai.

Q: The joint density of random variable (X, Y) where X is the unit temperature change & Y is the proportion of Spectrum shift that a certain atomic particle produces, is

$$f(x,y) = \begin{cases} 10xy^2, & 0<x<y<1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the marginal densities $g(x)$, $h(y)$ & conditional density $f(y|x)$.

$$g(x) = \int_x^1 10xy^2\, dy = \frac{10}{3} xy^3 \Big|_{y=x}^{y=1} = \frac{10x(1-x^3)}{3}$$

$$h(y) = \int_0^y 10xy^2\, dx = 5x^2y^2 \Big|_{x=0}^{x=y} = 5y^4$$

Now, $$f(y|x) = \frac{f(x,y)}{g(x)} = \frac{10xy^2}{\frac{10}{3}x(1-x^3)} = \frac{3y^2}{1-x^3}, \quad 0<x<y$$

(b) Find the probability that the spectrum shifts more than half of the total obs., given that the temp is Increased by 0.25 unit.

$$P(Y > \tfrac{1}{2} \mid x = 0.25) = \int_{1/2}^{1} \frac{3y^2}{1-0.25^3}\, dy = \frac{8}{9}$$

hum y ko 1 se $\frac{1}{2}$ par integrate karenge or 0.25 value x ki put kardenge.

"Statistical Independence"

Q: $$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0<x<2,\ 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find $g(x)$, $h(y)$, $f(x|y)$ & $P(\frac{1}{4}<X<\frac{1}{2} \mid Y=\frac{1}{3})$

$$g(x) = \int_0^1 \frac{x(1+3y^2)}{4}\, dy = \frac{x}{4}(y+y^3)\Big|_{y=0}^{y=1} = \frac{x}{2}$$

$$h(y) = \int_0^2 \frac{x}{4}(1+3y^2)\, dx = \frac{x^2(1+3y^2)}{8}\Big|_{x=0}^{x=2} = \frac{1}{2}(1+3y^2)$$

Now,

$$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{\frac{x(1+3y^2)}{4}}{\frac{1}{2}(1+3y^2)} = \frac{x}{2}$$

Note: If $f(x|y)$ doesnot depend on y, as in this case then $f(x|y) = g(x)$ and $f(x,y) = g(x)h(y)$.

↳ it means that the outcome of random variable Y has no impact on outcome of RV X. X & Y are independent RV.

The random variable either disc or cont X & Y are said to be statistically independent if & only if

$f(x,y) = g(x)\, h(y)$

Q 3.37: $f(x,y) = cxy$, $x = 1,2,3$, $y = 1,2,3$

$\sum_x \sum_y f(x,y) = 1$

$\sum_x \sum_y cxy = 1$

$c \sum_x x \sum_y y = 1$

$c(1+2+3)(1+2+3) = 1$

$36c = 1$

$\boxed{c = 1/36}$

(b) $f(x,y) = c|x-y|$, $x = -2,0,2$, $y = -2,3$

$\sum_x \sum_y c|x-y|$

$c(|-2-(-2)| + |-2-(3)| + |0-(-2)| + |0-(3)| + |2-(-2)| + |2-(3)|)$

$c(0 + 5 + 2 + 3 + 4 + 1) = 1$

$\boxed{c = \frac{1}{15}}$

Q 3.38:

(1) $P(X \leq 2, Y = 1) = f(0,1) + f(1,1) + f(2,1)$

$= \frac{0+1}{30} + \frac{1+1}{30} + \frac{2+1}{30}$

$= 1/5$ Ans

(2) $P(X > 2, Y \leq 1) = f(3,0) + f(3,1)$

$= \frac{3+0}{30} + \frac{3+1}{30}$

$= 7/30$ Ans

(3) $P(X > Y) = f(1,0) + f(2,0) + f(2,1) + f(3,0) + f(3,1) + f(3,2)$

$= 3/5$

(4) $P(X + Y = 4) = f(3,1) + \cancel{f(1,3)} + f(2,2)$

$= 4/15$

Q 3.39: Oranges → 3 , Apples → 2 , Bananas → 3

Selected = 4 , X = no. of Oranges , Y = no. of Apples.

$X = 0, 1, 2, 3$, $Y = 0, 1, 2$

$f(0,0) = 0$ (bcz dono mein se ek to aayegi hi)

$f(0,1) = \frac{^3C_3 \, ^2C_1}{^8C_4}$

$f(0,2) = \frac{^3C_2 \, ^2C_2}{^8C_4}$

$f(1,0) = \frac{^3C_1 \, ^3C_3}{^8C_4}$

$f(1,1) = \frac{^3C_1 \, ^2C_1 \, ^3C_2}{^8C_4}$

$f(1,2) = \frac{^3C_1 \, ^2C_2 \, ^3C_1}{^8C_4}$

$f(2,0) = \frac{^3C_2 \, ^3C_2}{^8C_4}$

$f(2,1) = \frac{^3C_2 \, ^2C_1 \, ^3C_1}{^8C_4}$

$f(2,2) = \frac{^3C_2 \, ^2C_2}{^8C_4}$

$f(3,0) = \frac{^3C_3 \, ^3C_1}{^8C_4}$

$f(3,1) = \frac{^3C_3 \, ^2C_1}{^8C_4}$

$f(3,2) = 0$ (bcz total will be 5 & we need to select 4.)

f(x,y)		x = 0	x = 1	x = 2	x = 3	Row total
y	0	0	3/70	9/70	3/70	
	1	1/35	9/35	9/35	1/35	
	2	3/70	9/70	3/70	0	
Total						1

b) $P[(X,Y) \in A]$ where $A = \{(x,y) / x+y \le 2\}$

$= f(0,0) + f(0,1) + f(1,0) + f(2,0) + f(0,2) + f(1,1)$

$= 1/2$

3.40: $f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & , 0 \le x \le 1, 0 \le y \le 1 \\ 0 & , \text{elsewhere} \end{cases}$

(a) Marginal Density of X $g(x)$

$g(x) = \int_0^1 \frac{2}{3}(x+2y)\, dy = \frac{2}{3}(xy + y^2)\Big|_{y=0}^{y=1} = \frac{2}{3}(x+1)$

b) Marginal Density of y $h(y)$

$h(y) = \int_0^1 \frac{2}{3}(x+2y)\, dx = \frac{2}{3}\left(\frac{x^2}{2} + 2xy\right)\Big|_{x=0}^{x=1} = \frac{2}{3}(4y+1)$

c) $P(X < 1/2\}, 0 \le Y \le 1) = \int_0^1 \int_0^{0.5} f(x,y)\, dx\, dy$ $\quad \frac{2}{3}\left[\frac{1}{2} + 2y \cdot \frac{1}{8}\right]$

$= \int_0^1 \frac{2}{3}\left(\frac{x^2}{2} + 2xy\right)\Big|_{x=0}^{x=0.5} dy$ $\quad \frac{2}{3}\left[\frac{5}{8} + y\right]$

$= \frac{2}{3}\int_0^1 \left(\frac{1}{8} + y\right) dy = \frac{2}{3}\left(\frac{y^2}{2} + \frac{y}{8}\right)\Big|_{y=0}^{y=1}$

$= \frac{5}{12}$

3.41: $f(x,y) = \begin{cases} 24xy & , 0 \le x \le 1, 0 \le y \le 1, x+y \le 1 \\ 0 & , \text{elsewhere} \end{cases}$

X → cream
Y → toffee

(a) weight of box = 1 kg

cream + toffee + cordial = 1

cordial = $1 - X - Y$

$P(1 - X - Y > 1/2) = P(X + Y < 1/2)$

$1 - 4y + 4y^2$

$= \int_0^{1/2} \int_0^{1/2 - y} f(x,y)\, dx\, dy$

$= \int_0^{1/2} \int_0^{1/2 - y} 24xy\, dx\, dy = \int_0^{1/2} 12x^2 y \Big|_0^{1/2 - y} dy$

$= \int_0^{1/2} 12y\left(\frac{1}{2} - y\right)^2 dy = \int_0^{1/2} 3y(1 - 2y)^2\, dy$

$= \int_0^{1/2} 3y - 12y^2 + 12y^3\, dy$

$= \frac{3}{2}y^2 - 4y^3 + 3y^4 \Big|_0^{1/2}$

$= \frac{3}{2}\left(\frac{1}{4}\right) - 4 \cdot \frac{1}{8} + 3 \cdot \frac{1}{16}$

$= 1/16$

$x+y \le 1 \rightarrow y \le 1-x$

(b) $g(x) = \int_0^{1-x} 24xy\, dy = 12xy^2 \Big|_0^{1-x} = 12x(1-x)^2$

$= 12x(1 - 2x + x^2)$

$g(x) = 12x - 24x^2 + 12x^3$

(c) $P(Y < 1/8 \mid X = 3/4) = \int_0^{1/8} f(y \mid 3/4)$

$f(y \mid x) = \frac{f(x,y)}{g(x)} = \frac{24xy}{12x(1-x)^2}$

$P(Y < 1/8 \mid X = 3/4) = \int_0^{1/8} \frac{24(3/4) \cdot y}{12(3/4)(1 - 3/4)^2} = \int_0^{1/8} 32y$

$= 16y^2 \Big|_0^{1/8} = 16 \cdot \frac{1}{8^2}$

$= 1/4$

Q 3.42: $f(x,y) = \begin{cases} e^{-(x+y)} & , x>0, y>0 \\ 0 & , \text{else} \end{cases}$

Find $P(0<X<1 \mid Y=2) = \int_0^1 f(x|2) \rightarrow$

$f(x|y) = \dfrac{f(x,y)}{h(y)} \rightarrow ②$ → improper integral (calculus!)

$h(y) = \int_0^\infty f(x,y)\,dx = \int_0^\infty e^{-(x+y)}\,dx$

$= e^{-y} \lim_{t\to\infty} \int_0^t e^{-x}\,dx = e^{-y}\left[-e^{-x}\right]_0^t$

$= e^{-y}\left[-e^{-t} + e^0\right] = e^{-y}\left(-\dfrac{1}{e^t} + 1\right)$ $\quad \frac{1}{\infty}=0$

$h(y) = e^{-y}$

eq ②⇒

$f(x|2) = \dfrac{e^{-(x+2)}}{e^{-2}} = \dfrac{\cancel{e^{-2}}\cdot e^{-x}}{\cancel{e^{-2}}} = e^{-x}$

eq ①⇒

$= \int_0^1 e^{-x}$

$= -e^{-x}\Big|_0^1$

$= 1 - \dfrac{1}{e}$

Q 3.43: $f(x,y) = \begin{cases} 4xy & , 0<x<1, 0<y<1 \\ 0 & , \text{elsewhere} \end{cases}$

Find

(a) $P(0 \le X \le \frac{1}{2}$ and $\frac{1}{4} \le Y \le \frac{1}{2}) \rightarrow$ double integral karenge
$\int_0^1\int_0^1 4xy\,dx\,dy$

(b) $P(X<Y) = P(0 \le X<1 \text{ \& } X<Y<1) = \int_0^1 \int_x^1 4xy\,dy\,dx$

$= \int_x^1 2x^2 y \Big|_{x=0}^{x=1} dy = \int_x^1 2y\,dy$

$= y^2\Big|_x^1$

$= \int_0^1 2xy^2 \Big|_{y=x}^{y=1} dx = \int_0^1 2x - 2x^3\,dx$

$= x^2 - \dfrac{x^2}{2}\Big|_0^1$

$= \frac{1}{2}$

Q 3.44: $f(x,y) = \begin{cases} k(x^2+y^2) & , 30 \le x \le 50, 30 \le y \le 50 \\ 0 & , \text{else} \end{cases}$

(a) Find K

$\int_{30}^{50}\int_{30}^{50} k(x^2+y^2)\,dy\,dx = 1$

$\int_{30}^{50} k\left(x^2 y + \dfrac{y^3}{3}\right)\Big|_{30}^{50} dx = 1$

$\int_{30}^{50} k\left(50x^2 + 50^3/3 - 30x^2 - 30^3/3\right) dx = 1$

$\int_{30}^{50} k\left(20x^2 + 98000/3\right) dx = 1$

$k\left(\dfrac{20}{3}x^3 + \dfrac{98000x}{3}\right)\Big|_{30}^{50} = 1$

$\boxed{k = 3/392000}$

(b) Find $P(30 \le X \le 40 \text{ \& } 40 \le Y \le 50) = \int_{30}^{40}\int_{40}^{50} \frac{3}{392000}(x^2+y^2)\,dy\,dx$
$= 49/196$

(c) filled k liye 40 hona zaroori hai to underfilled → $X<40$, $Y<40$

$P(30 \le X \le 40 \text{ \& } 30 \le Y \le 40) \rightarrow \int_{30}^{40}\int_{30}^{40} \frac{3}{392000}(x^2+y^2) = \frac{37}{196}$

Q:3.49: (a)

x	1	2	3
g(x)	0.10	0.35	0.55

(b)

y	1	3	5
h(y)	0.20	0.50	0.30

(c) $f(y|x) = \frac{f(x,y)}{g(x)} = f(3|2) = \frac{f(2,3)}{g(2)} = \frac{0.1}{0.3}$

$\boxed{f(Y=3|X=2) = 2/7}$

Q 3.54: $f(1,5) \neq g(1)\,h(5)$ so not independent