22K-4187
SYED SAMROZE ALI

ASSIGNMENT #01

QUESTION #01

(a) Not Proposition

(b) Proposition ⟶ False

(c) Not Proposition ~~⟶ False~~

(d) Proposition ⟶ True

(e) Not Proposition

QUESTION #02

(a) Let, p = A student gets A in Discrete
q = his weight total is ≥ 95%.

then,

$$p \leftrightarrow q$$

(b) Let, p = Alice is smart.
q = Alice is honest

then,

$$p \wedge (\neg p \wedge q)$$

(c) Let p = $\sqrt{30}$ is greater than 6
q = $\sqrt{30}$ is less than 5

$$p \oplus q$$

(d) Let, p = Sam had pizza last Night
q = Chris finished her homework
Then

$$p \wedge q$$

(e) Let, p = Chris finished her homework
q = Pat watched the news this morning
Then

$$p \oplus q$$

QUESTION # 03

(a) $2 \geq 5$ and 8 is an even integer. (F)
(b) If $2 \leq 5$ and 8 is an even integer then 11 is a prime number. (T)
(c) If $2 \leq 5$ then, 8 is an even integer or 11 is not a prime number. (T)
(d) If $2 \geq 5$ the 8 is not an even integer.

(e)

Let, p = Smartphone A has more RAM than B
q = Smartphone B has more RAM than A

$p \leftrightarrow q$ = FALSE

Question #05

(a)

Let, p = Quixote Mechia has the largest revenue
p = FALSE

(b)

Let, p = Nadir Software has the lowest net profit
q = Acme Computer had the largest annual rev.
$p \wedge q$ = TRUE

(c)

Let, p = Acme Computer had largest net profit
q = Quixote media had largest net profit.
$p \vee q$ = TRUE

(d)

Let, p = Quixote has smallest net profit
q = Acme computer had largest revenue
$p \rightarrow q$ = TRUE

(e)

p = Nadir software had smallest profit
q = Acme Computer had the largest annual rev.
$p \leftrightarrow q$ = TRUE

QUESTION #04

(a)

Let, p = Smartphone B has the most RAM of these three smartphone

p = TRUE

(b)

Let, p = Smartphone C has more ROM

q = Smartphone C has a higher resolution camera than Smartphone B.

$p \vee q$ = TRUE

(c)

Let, p = Smartphone B has more RAM than Smartph

q = " " " " ROM " "

r = " " " " higher resolution "

$p \wedge q \wedge r$ = FALSE as r is False

(d)

Let, p = Smartphone B has more RAM than C

q = " " " " ROM " "

r = " " " " higher resolution camera than C

$(p \wedge q) \rightarrow r$ = FALSE as r is False

QUESTION #06

Let p = You keep you textbook
q = It will be useful reference in your future course.

⇒ If will be useful reference in your future course whe you keep your textbook.

⇒ If will be useful reference in your future whenere you keep your textbook.

⇒ Keeping your textbook implies that it will be a usef reference for in your future course

⇒ Keeping your textbook is sufficient for usefull refer in your future course

⇒ If you keep your text book then it will be useful reference in your future course.

QUESTION #07

$0 \geq x \geq 4$ $x < 0$ or $x > 4$

$-2 > x \geq 5$ $x \leq -2$ or $x \geq 5$

The fan is not slow, and it is not very hot.

Akram is fit or Ahmed is not injured

Zia is healthy or Zia is wealthy or Zia is wise

QUESTION #08

(a) Jan is not rich or not happy

(b) Carlos will not bicycle and not run tomorro

(c) The fan is not slow and it is not very ho

(d) Akram is not unfit or saleem is not in

QUESTION #09

(a) $(p \wedge q) \vee (\neg p \wedge q) \equiv q$

$q \wedge (p \vee \neg p)$

$q \wedge T$

q

Hence Proved!

(b) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg p \wedge (p \vee \neg q)$

$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$F \vee (\neg p \wedge \neg q)$

$\neg p \wedge \neg q$

Hence proved!

$(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

$(p \wedge (p \vee \neg q)) \vee (p \wedge q)$

$\therefore p \vee (p \wedge q)$ $\quad \because p \wedge (p \wedge q) \equiv p$ absorbtion law

p $\quad$ again using absorbtion law

$p (\wedge q) \rightarrow (p \rightarrow q) \equiv T$

$p \wedge q \rightarrow (\neg p \vee q)$ $\quad \because$ implication law

$\neg(\neg p \wedge q) \vee (\neg p \vee q)$ $\quad$ again applying implicatio

$(\neg p \vee \neg q) \vee (\neg p \vee q)$

$(\neg p \vee \neg p) \vee (\neg q \vee q)$ $\quad$ ' ceasing similar elements

$\neg p \vee (T)$ $\quad \because p \vee p \equiv p \;\&\; \neg p \vee p \equiv T$

T $\quad \neg p \vee T \equiv T$

QUESTION #10

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$p \vee q$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$(p \vee q) \rightarrow r$
F	F	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	T	F	T	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	T	T	F	F
T	F	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	T	T	T	T	T	T	T

Logically equivalent!

p	q	r	s	p→q	r→s	p→r	q→s	(p→q)→(r→s)	(p→r)	
F	F	F	F	T	T	T	T	T	T	
F	F	F	T	T	T	T	T	T	T	
F	F	T	F	T	F	T	T	F	T	
F	F	T	T	T	T	T	T	T	T	
F	T	F	F	T	T	T	F	T	F	
F	T	F	T	T	T	T	T	T	T	
F	T	T	F	T	F	T	F	F	F	
F	T	T	T	T	T	T	T	T	T	
T	F	F	F	F	T	F	T	T	T	
T	F	F	T	F	T	F	T	T	T	
T	F	T	F	F	F	T	T	T	T	
T	F	T	T	F	T	T	T	T	T	
T	T	F	F	T	T	F	F	T	T	
T	T	F	T	T	T	F	T	T	T	
T	T	T	F	T	F	T	F	F	F	
T	T	T	T	T	T	T	T	T	T	

Not Logically Equivalent!

$\neg(t \vee s)$ $\neg s$ $\neg t \vee (s \vee p)$

$\neg t \wedge \neg s$ $\neg s$

QUESTION # 11

$(t \rightarrow (s \vee p)) \rightarrow ((\neg s \vee k) \wedge \neg k)$

We have the conclusion of $t \rightarrow (s \vee p)$ i.e

$\neg s \vee k$

$\neg k$ Using Disjunctive Syllogism

$\therefore \neg s$

QUESTION # 12

(a)

let, p: Alice is a mathematics major

q: Alice is a Computer Science Major

$$\frac{p}{\therefore p \vee q}$$ (Addition)

(b)

let, P: Jessy is a mathematics Major

q: " " Computer Science Major

$$\frac{p \wedge q}{\therefore p}$$ (Simplification)

(c)

Let,

p = It is raining

q = Pool will be closed

$$\begin{array}{l} p \to q \\ \underline{p} \\ \therefore q \end{array}$$ (Modes Ponens)

(d)

Let,

p = It snows today

q = the University will be closed

$$\begin{array}{l} p \to q \\ \underline{\neg q} \\ \therefore \neg p \end{array}$$ (Modes Tollens)

(e)

Let, p = I go swimming

q = I will stay in Sun too long

r = I will sun burn

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ p \to r \end{array}$$ (Hypothetical Syllogism)

QUESTION #13

(a)

Let;

p = Today is Tuesday.
q = I have a test in Mathematics
γ = " " " Economics
s = My Economic Professor is Sick

$$p \rightarrow (q \vee \gamma)$$
$$s \rightarrow \neg\gamma$$
$$\frac{p \wedge s}{\therefore q}$$

$p \wedge s$ (simplification)
$\therefore p$

$p \rightarrow (q \vee \gamma)$
p (Modes Ponens)
$\therefore q \vee \gamma$

$p \wedge s$ (simplification)
$\therefore s$

$s \rightarrow \neg\gamma$
s (Modes Ponens)
$\neg\gamma$

From 2nd & 4th step we get:

$q \vee \gamma$
$\neg\gamma$ (Disjunction Syllogism)
$\therefore q$

VALID

(b)

Let

p = Ali is a lawyer
q = He is ambitous
r = Ali is an early riser
s = He ~~doesn't~~ like chocolate

P_1 : $p \to q$
P_2 : $r \to \neg s$
P_3 : $q \to r$
C_1 : $\therefore p \to \neg s$

$\Rightarrow$ From P_1 & P_3

$p \to q$
$q \to r$
C_2 : $\therefore p \to r$

$\Rightarrow$ From C_2 & P_2

$p \to r$
$r \to \neg s$
$\therefore p \to \neg s$ ✓

VALID

QUESTION #14

) Not a proposition
) Proposition → True
) Proposition → True
) Not a proposition
) Proposition → False
) Proposition → True

QUESTION #15

) True as we can take $x = \sqrt{2}$
) False
) True
) True

QUESTION #16

$\exists x (P(x) \wedge Q(x))$
$\exists x (P(x) \wedge \neg Q(x))$
$\forall x (P(x) \vee Q(x))$
$\neg \exists x (P(x) \vee Q(x))$

QUESTION #17

(a) There is some student in your class who has sent ~~some~~ an email-message to some stude in your class.

(b) There is some student in your class who has sent a message to all student in the class

(c) Every student in your class has sent an email message to some student in your class.

(d) ~~There is a student who has recieved an email by everyone in the class.~~
Every student in the class has been se
(e) a email by at least one student in the class.

(e) Every student in class has sent an email to every one in the class.

QUESTION #18

A is a subset of B:
A is a subset of B all elements in A {1, 2, 3} are also in B.

A is ~~not~~ a proper subset of B because as A is a subset of B and B contains few more elements from A.

(a) 2, 3, 5, 7

{ }, {2}, {3}, {5}, {7}, {2, 3}, {2, 5}, {2, 7}
{3, 5}, {3, 7}, {5, 7}, {2, 3, 5}, {2, 3, 7}
{3, 5, 7}, {2, 5, 7}

(b) 011

Subsets of L:
{}, {1}, {2}, {3}, {1, 2}, {1, 3}
{2, 3}, {1, 2, 3}

Power Set of L:
{ {}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} }

QUESTION (c)

The number of elements : $2^{(k)}$

(d)

$N = \{11, 13, 17, 19\}$

No of elements in power set $N = 2^4 = 16$

So Cardinality of ~~set~~ power set of $N = 16$

QUESTION #19

$E = \{1, 2, 4\}$, $F = \{a, b, c, d\}$,

$E \times F = \{(1,a), (1,b), (1,c), (1,d), (2,a), (2,b), (2,d), (4,a), (4,b), (4,c), (4,d)\}$

(a)

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$,

$A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$, $C = \{4, 5, 6, 7$

$(A - (A \cap B)) \cap (B - (A \cap B)) = \phi$

$A \cap B = \{2, 5\}$

$A - (A \cap B) = \{1, 2, 4, 5\} - \{2, 5\}$

$= \{1, 4\}$

$B-(A \cap B) = \{2,3,5,6\} - \{2,5\}$

$= \{3,6\}$

$(A-(A \cap B)) \cap (B-(A \cap B)) = \{1,4\} \cap \{3,6\}$

$= \{\}$

Hence Proved!

(b)

$(A-B) \cup (A \cap B) = A$

$A-B = \{1,2,4,5\} - \{2,3,5,6\}$

$= \{1,4\}$

$A \cap B = \{1,2,4,5\} \cap \{2,3,5,6\}$

$= \{2,5\}$

$(A-B) \cup (A \cap B) = \{1,4\} \cup \{2,5\}$

$= \{1,2,4,5\}$

$= A$

Hence Proved!

(c)

$(A-B)-C = (A-C)-B$

Taking L.H.S:

$(A-B)-C$:

$A-B = \{1, 2, 4, 5\} - \{2, 3, 5, 6\}$

$A-B = \{1, 4\}$

$(A-B)-C = \{1, 4\} - \{4, 5, 6, 7\}$

$= \{1\}$

Taking R.H.S:

$(A-C)-B$:

$(A-C) = \{1, 2, 4, 5\} - \{4, 5, 6, 7\}$

$(A-C) = \{1, 2\}$

$(A-C)-B = \{1, 2\} - \{2, 3, 5, 6\}$

$= \{1\}$

Hence Proved!

(d)

$(\bar{B} \cup (\bar{B} - A)) = B$

Taking L.H.S:

$\bar{B} = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 3, 5, 6\}$

$= \{1, 4, 7, 8\}$

$\bar{B} - A = \{1, 4, 7, 8\} - \{1, 2, 4, 5\}$

$= \{7, 8\}$

$(\bar{B} \cup (\bar{B} - A)) = \{1, 4, 7, 8\} \cup \{7, 8\}$

$= \{1, 4, 7, 8\}$

Dis aproved

(A-C)(B-C)
(AB-AC-CB+CC)

QUESTION #20

(a) $(A-(A\cap B))\cap(B-(A\cap B))=\phi$

$\because A-B=A\cap B'$

$\therefore (A\cap(A\cap B)')\cap[B\cap(A\cap B)']$

$(A\cap(A'\cup B'))\cap[B\cap(A'\cup B')]$ Distributive

$[(A\cap A')\cup(A\cap B')]\cap[(B\cap A')\cup(B\cap B')]$

$[\phi\cup(A\cap B')]\cap[(B\cap A')\cup\phi]$

$(A\cap B')\cap(B\cap A')$

$(A\cap A')\cap(B\cap B')$ Commutative

$\phi\cap\phi$

ϕ Hence Proved

(b) $(A-B)\cup(A\cap B)=A$

$(A\cap B')\cup(A\cap B)$

$A\cap(B'\cup B)$

$A\cap(U)$

A Hence Proved!

(c) $(A-B)-C=(A-C)-B$

Taking L.H.S

$(A\cap B')-C$

$(A\cap B')\cap C'$

Taking R.H.S

$(A\cap B)-B$ $\because A-B=A\cap B'$

$(A\cap C')\cap B'$

$(A\cap B')\cap C'$ commutative

L.H.S = R.H.S

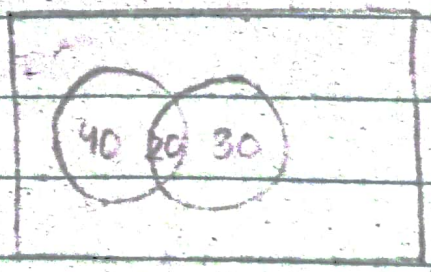
d) $(B' \cup (B' - A) = B$

$(B' \cup (B' \cap A')$ absorbation law

(B') · Disproved!!

QUESTION #201

(a)



$= 200 - (40 + 30 - 20)$

$= 150$ potatoes

$A = 44$

$P = 35$

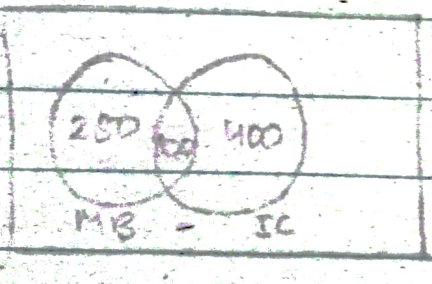
$B = 38$

$PA = 13$

$PB = 21$

$BA = 16$

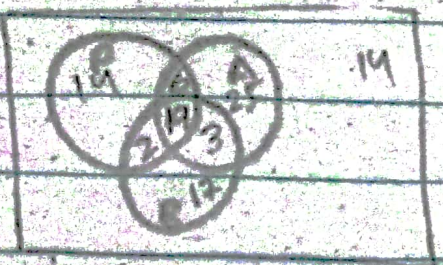
(b)



Filling flavours = 550

Those who liked none = 1000 − 550 = 450.

(d)



Total = A + P + B − (A∩P + A∩B + P∩B) + A∩P∩B

= 44 + 35 + 38 − (21 + 16 + 13) + 14

Total = 100

QUESTION # 22

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Taking R.H.S:

$= \{(a,b,c) \mid (a \in A \wedge b \in B) \wedge (a \in A \wedge c \in C)\}$

$= \{(a,b,c) \mid a \in A \wedge b \in B \wedge \{a \in A. \wedge c \in C\}\}$

In logical equivalence, $p \wedge q \wedge p \wedge r = p \wedge q \wedge r$

$\therefore = \{(a,b,c) \mid a \in A \wedge b \in B \wedge c \in C\}$

$= \{(a,b,c) \mid a \in A \wedge (b \in B \wedge c \in C)\}$

$= \{(a,b,c) \mid a \in A \wedge (b \in B \wedge c \in C)\}$

$= A \times (B \cap C)$

Hence proved!

QUESTION #23

(a)

Domain: $\{1,3,5,7\}$, Co-domain: $\{2,4,6,8\}$

Range: $\{4,2,6,8\}$

Injective: ✓

Surjective: ✓

Bijective: ✓

Inverse: $f^{-1}(4)=1$, $f^{-1}(2)=3$, $f^{-1}(6)=5$, $f^{-1}(8)=7$.

(b)

Domain: {1, 3, 5, 7}, Co-domain: {2, 4, 6, 8}
Range: {4, 8, 6}
Injective: X
Surjective: X
Bijective: X
Inverse: Not exist

(c)

Domain: {1, 3, 5, 7} Co-domain: {2, 4, 6, 8}
Range: {8, 4, 6}
Injective: X
Surjective: X
Bijective: X
Inverse: Not exist

(d)

Domain: {1, 3, 5, 7} Co-domain: {2, 4, 6, 8}
Range: {6, 2, 4, 8}
Injective: ~~X~~ ✓
Surjective: ✓ Bijective: ✓
Inverse: $f^{-1}(6) = 1$, $f^{-1}(2) = 3$, $f^{-1}(4) = 5$, $f^{-1}(8) = 7$

Question # 24

(a)

$$fog(x) = f(g(x))$$
$$= f(3x+4)$$
$$= 4(3x+4)+3$$
$$fog(x) = 12x + 19$$

$$gof(x) = g(f(x))$$
$$= g(4x+3)$$
$$= 3(4x+3)+4$$
$$gof(x) = 12x + 13$$

(b)

$f(x)$ & $g(x)$ both are linear functions

(c)

→ Checking if it's 1 to 1. [for $f(x)$]

$f(a) = f(b) \rightarrow a = b$

$4a + 3 = 4b + 3$

$4a = 4b + 1$

$a = b$ ✓

→ Checking [for g(x)]

$g(a) = g(b) \rightarrow a = b$

$3a + 4 = 3b + 4$

$3a = 3b$

$a = b$ ✓

→ Checking For Onto: [for f(x) & g(x)]

As both f & g are defined from the set of integers to the set of integers, which means they cover all integers,

So they are Bijective. ✓

→ Inverse:

$f^{-1}(x) = (x-3)/4$

$g^{-1}(x) = (x-4)/3$

QUESTION #25

(a)

(i) for $S = \{-2, -1, 0, 1, 2, 3\}$

$f(-2) = \lfloor (-2)^2/2 \rfloor = 2$

$f(-1) = \lfloor (-1)^2/2 \rfloor = 0$

$f(0) = \lfloor 0/2 \rfloor = 0$

$f(1) = \lfloor 1^2/2 \rfloor = 0$

$f(2) = \lfloor 2^2/2 \rfloor = 2$

$f(3) = \lfloor 3^2/2 \rfloor = 4$

(ii) for $S = \{0, 1, 2, 3, 4, 5\}$

$f(0) = \lfloor 0^2/2 \rfloor = 0$

$f(1) = \lfloor 1^2/2 \rfloor = 0$

$f(2) = \lfloor 2^2/2 \rfloor = 2$

$f(3) = \lfloor 3^2/2 \rfloor = 4$

$f(4) = \lfloor 4^2/2 \rfloor = 8$

$f(5) = \lfloor 5^2/2 \rfloor = 12$

(iii) for $S = \{1, 5, 7, 11\}$

$f(1) = \lfloor 1^2/2 \rfloor = 0$

$f(5) = \lfloor 5^2/2 \rfloor = 12$

$f(7) = \lfloor 7^2/2 \rfloor = 24$

$f(11) = \lfloor 11^2/2 \rfloor = 60$

(iv) for $S = \{2, 6, 10, 14\}$

$f(2) = \lfloor 2^2/2 \rfloor = 2$

$f(6) = \lfloor 6^2/2 \rfloor = 18$

$f(10) = \lfloor 10^2/2 \rfloor = 50$

$f(14) = \lfloor 14^2/2 \rfloor = 98$

(b)

(i) 4

(ii) 7

(iii) -3

(iv) -8

(v) 3

(vi) -1

(vii) 5

(viii) 6

(c)

def $x = 2.5$

Now $\lfloor -x \rfloor = -\lfloor x \rfloor \Rightarrow \lfloor -2.5 \rfloor = -\lfloor 2.5 \rfloor$

(c).

Let $x = 2.5$

Then, $\lfloor -x \rfloor = -\lceil x \rceil$

$\lfloor -2.5 \rfloor = -\lceil 2.5 \rceil$

$-3 = -3$ Proved!

Now $\lceil -x \rceil = -\lfloor x \rfloor$

$\lceil -2.5 \rceil = -\lfloor 2.5 \rfloor$

$-2 = -2$ proved!