22K-4187
SYED SAMROZE ALI

ASSIGINMENT #03

Ex: 6.1

QUESTION # 01

(a) $\langle u, v \rangle = 2(1)(3) + 3(1)(2) = 12$

$\langle kv, w \rangle = 2((3)(3))(0) + 3((3)(2))(-1) = -18$

$\langle u + v, w \rangle = 2(1+3)(0) + 3(1+2)(-1) = -9$

$\| v \| = \langle v, v \rangle^{1/2} = [2(3)(3) + 3(2)(2)]^{1/2} = \sqrt{30}$

$d(u,v) = \| u - v \| = \langle (-2,-1), (-2,-1) \rangle^{1/2} = \sqrt{11}$

$\| u - kv \| = \langle (-8,-5), (-8,-5) \rangle^{1/2} = \sqrt{203}$

QUESTION # 09

If $u = U$ & $v = V$ then, $\langle u, v \rangle = Tr(U^T V)$.

$= Tr\left(\begin{bmatrix} 1 & 13 \\ 10 & 2 \end{bmatrix}\right) = 3$

Ex: 6-2

QUESTION #03

$$\cos\theta = \frac{\langle p, q\rangle}{\|p\|\,\|q\|} = \frac{(-1)(2)+(5)(4)+(2)(-9)}{\sqrt{(-1)^2+5^2+2^2}\ \sqrt{2^2+4^2+(-9)^2}}$$

$$\boxed{\cos\theta = 0}$$

QUESTION # 07

(a)

Orthogonal : $\langle u, v\rangle = -4+6-2 = 0$

(b)

Not Orthogonal : $\langle u, v\rangle = -2-2-2 = -6 \neq 0$

(c)

Orthogonal : $\langle u, v\rangle = (a)(-b)+(b)(a) = 0$

Ex: 6-3

QUESTION# 05

Coloumn Vector:

$U_1 = (1, 0, -1)$, $U_2 = (2, 0, 2)$, $U_3 = (0, 5, 0)$

Checking Orthogonality:

$\langle U_1, U_2 \rangle = 2 + 0 - 2 = 0$

$\langle U_1, U_3 \rangle = 0 + 0 + 0 = 0$

$\langle U_2, U_3 \rangle = 0 + 0 + 0 = 0$

For Orthonormal Basis:

$$\frac{U_1}{\|U_1\|} = \frac{1}{\sqrt{1+0+1}} \cdot (1, 0, -1) = \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$$

$$\frac{U_2}{\|U_2\|} = \frac{1}{\sqrt{4+0+4}} \cdot (2, 0, 2) = \left(\frac{2}{2\sqrt{2}}, 0, \frac{2}{2\sqrt{2}}\right)$$

$$\frac{U_3}{\|U_3\|} = \frac{1}{\sqrt{0+25+0}} \cdot (0, 5, 0) = (0, 1, 0)$$

Orthonormal Basis $\rightarrow \left\{ \left(1/\sqrt{2}, 0, -1/\sqrt{2}\right), \left(2/2\sqrt{2}, 0, 2/2\sqrt{2}\right), (0, 1, 0) \right\}$

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QUESTION # 27

$V_1 = U_1 = (1, -3)$

$V_2 = U_2 - \frac{\langle U_2, V_1 \rangle}{\|V_1\|^2} V_1 = (2,2) - \frac{2-6}{10}(1,-3)$

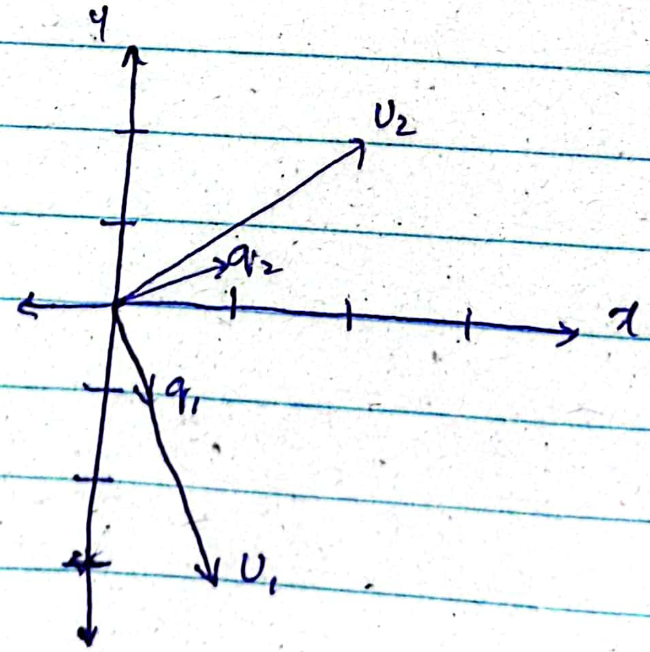
$= (2,2) - \left(\frac{-2}{5}, \frac{6}{5}\right)$

$= \left(\frac{12}{5}, \frac{4}{5}\right)$

An orthonormal basis is formed by the vectors

$q_1 = \frac{V_1}{\|V_1\|} = \frac{1}{\sqrt{10}}(1,-3) = \left(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right)$

$q_2 = \frac{V_2}{\|V_2\|} = \frac{1}{\sqrt{\frac{144}{25} + \frac{16}{25}}}\left(\frac{12}{5}, \frac{4}{5}\right) = \left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$



Ex: 7-1

QUESTION # 01

(a)

$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = I \quad \& \quad A^TA = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = I$$

therefore A is an orthogonal matrix; $A^{-1} = A^T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b)

$$AA^T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = I \quad \& \quad A^TA = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} =$$

therefore A is an orthogonal matrix; $A^{-1} = A^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

QUESTION # 03

(a)

$\|r_1\| = \sqrt{0^2 + 1^2 + (1/\sqrt{2})^2} = \sqrt{3/2} \neq 1$ so the matrix is not orthogonal.

(b)

$$AA^T = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} = I$$ and

$$A^T A = \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{bmatrix} = I$$ therefore A is an orthogonal matrix;

$$A^{-1} = A^T = \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$$

Ex: 7.2

QUESTION # 01

$$\begin{vmatrix} \lambda-1 & -2 \\ -2 & \lambda-4 \end{vmatrix} = \lambda^2 - 5\lambda$$

The characteristic eqn → $\lambda^2 - 5\lambda = 0$

Eigen value → $\lambda = 0$, $\lambda = 5$

⇒ Both Eigen space are one-dimensional

QUESTION # 05

$$\begin{vmatrix} \lambda-4 & -4 & 0 & 0 \\ -4 & \lambda-4 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{vmatrix} = \lambda^4 - 8\lambda^3 = \lambda^3(\lambda-8)$$

Characteristic eqn → $\lambda^3(\lambda-8) = 0$

Eigen values → $\lambda = 0$, $\lambda = 8$

- The eigenspace for $\lambda = 0$ is three-dimensional
- The eigenspace for $\lambda = 8$ is one dimensional

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Exs 7.3

QUESTION#01

(a) $3x_1^2 + 7x_2^2 = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(b) $4x_1^2 - 9x_2^2 - 6x_1x_2 = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(c) $9x_1^2 - x_2^2 + 4x_3^2 + 6x_1x_2 - 8x_1x_3 + x_2x_3 = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 9 & 3 & -4 \\ 3 & -1 & \frac{1}{2} \\ -4 & \frac{1}{2} & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

QUESTION #03

$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2x^2 + 5y^2 - 6xy$

Ex: 7.5

QUESTION #01

$$\bar{A} = \begin{bmatrix} -2i & i+1 \\ 4 & 3-i \\ 5-i & 0 \end{bmatrix} \text{ therefore } A^* = \bar{A}^T = \begin{bmatrix} -2i & 4 & 5-i \\ 1+i & 3-i & 0 \end{bmatrix}$$

QUESTION #06

(a)

$(A)_{12} = 1+i$ doesnot equal $(A^*)_{12} = 1-i$

(b)

$(A)_{33} = 2+i$ doesnot equal $(A^*)_{33} = 2-i$