22K-4187 → SYED SAMROZE ALI

ASSIGNMENT #03

QUESTION #01

(a)	$q = 2$	$r = 5$
(b)	$q = -11$	$r = 10$
(c)	$q = 34$	$r = 7$
(d)	$q = 77$	$r = 0$
(e)	$q = 0$	$r = 10$
(f)	$q = 0$	$r = 5$
(g)	$q = -1$	$r = 2$
(h)	$q = 4$	$r = 0$

QUESTION #02

(a)

(i)	$q = -1$	$r = -102$
(ii)	$q = -99$	$r = 0$
(iii)	$q = 10$	$r = 309$
(iv)	$q = 123$	$r = 333$

(b)

(i) $80 \not\equiv 5 \pmod{17} \rightarrow 80-5=75$ & 75 not divisible by 17

(ii) $103 \not\equiv 5 \pmod{17} \rightarrow 103-5=98$ & 98 " " " 17

(iii) $-29 \equiv 5 \pmod{17} \rightarrow -29-5=-34$ & -34 divisible by 17

QUESTION #03

(a)

(i) Yes $\rightarrow \gcd(11,5)=1, \gcd(11,15)=1, \gcd(11,19)=1$

(ii) No $\rightarrow \gcd(14,15)=1, \gcd(14,21)=7, \gcd(15,21)=3$

(iii) Yes $\rightarrow \gcd(12,17)=\gcd(12,31)=\gcd(12,37)=\gcd(17,31)=$
$\gcd(17,37)=\gcd(31,37)=1$

(iv) Yes $\rightarrow \gcd(7,8)=\gcd(7,9)=\gcd(7,11)=\gcd(8,9)=\gcd(8,11)$
$\gcd(9,11)=1$

(b)

(i) $88 \rightarrow 2^3 \times 11$

(ii) $126 \rightarrow 2 \times 3^2 \times 7$

(iii) $729 \rightarrow 3^6$

(iv) $1001 \rightarrow 7 \times 3 \times 11$

(v) $1111 \rightarrow 11 \times 101$

(vi) $909 \rightarrow 3^2 \times 101$

Question # 4

gcd(144, 89)

$144 = 89(1) + (55)$	$55 = 1(144) - 1(89) \rightarrow 9$
$89 = 55(1) + (34)$	$34 = 1(89) - 1(55) \rightarrow 8$
$55 = 34(1) + (21)$	$21 = 1(55) - 1(34) \rightarrow 7$
$34 = 21(1) + (13)$	$13 = 1(34) - 1(21) \rightarrow 6$
$21 = 13(1) + (8)$	$8 = 1(21) - 1(13) \rightarrow 5$
$13 = 8(1) + (5)$	$5 = 1(13) - 1(8) \rightarrow 4$
$8 = 5(1) + (3)$	$3 = 1(8) - 1(5) \rightarrow 3$
$5 = 3(1) + (2)$	$2 = 1(5) - 1(3) \rightarrow 2$
$3 = 2(1) + (1) \rightarrow$	$1 = 1(3) - 1(2) \rightarrow 1$
$2 = 1(2) + (0)$	

$1 = 1(3) - 1(2)$

$= 1(3) - 1[1(5) - 1(3)]$

$= 1(3) - 1(5) + 1(3)$

$1 = -1(5) + 2(3)$

$1 = -1(5) + 2[1(8) - 1(5)]$

$1 = 2(8) - 3(5)$

$1 = 2(8) - 3[1(13) - 1(8)]$

$1 = -3(13) + 5(8)$

$1 = -3(13) + 5[1(21) - 1(13)]$

$1 = 5(21) - 8(13)$

$1 = 5(21) - 8[1(34) - 1(21)]$

$1 = -8(34) + 13(21)$

$1 = -8(34) + 13[1(55) - 1(34)]$

$1 = 13(55) - 21(34)$

$1 = 13(55) - 21[1(89) - 1(55)]$

$1 = -21(89) + 34(55)$

$1 = -21(89) + 34\{1(144) - 1(89)\}$

$\boxed{1 = 34(144) - 55(89)}$

gcd(1001, 100001)

$100001 = 1001(99) + (902) \rightarrow 902 = 1(100001) - 99(1001) \rightarrow ③$

$1001 = 902(1) + (99) \rightarrow 99 = 1(1001) - 1(902) \rightarrow ②$

$902 = 99(9) + (11) \rightarrow 11 = 1(902) - 9(99) \rightarrow ①$

$99 = 11(9) + (0)$

①⇒ $11 = 1(902) - 9(99)$

$11 = 1(902) - 9[1(1001) - 1(902)]$

$11 = 1(902) - 9(1001) + 9(902)$

$11 = -9(1001) + 10[1(100001) - 99(1001)]$

$\boxed{11 = 10(100001) - 999(1001)}$

QUESTION # 5
(a)

$55x = 34 \pmod{89}$

$\gcd(89, 55) = ?$

$89 = 55 \cdot 1 + 34 \qquad 34 = 1(89) - 1(55)$

$55 = 34 \cdot 1 + 21 \qquad 21 = 1(55) - 1(34)$

$34 = 21 \cdot 1 + 13 \qquad 13 = 1(34) - 1(21)$

$21 = 13 \cdot 1 + 8 \qquad 8 = 1(21) - 1(13)$

$13 = 8 \cdot 1 + 5 \qquad 5 = 1(13) - 1(8)$

$8 = 5 \cdot 1 + 3 \qquad 3 = 1(8) - 1(5)$

$5 = 3 \cdot 1 + 2 \rightarrow 2 = 1(5) - 1(3)$

$3 = 2 \cdot 1 + 1 \rightarrow 1 = 1(3) - 1(2) \rightarrow (1)$

$2 = 2 \cdot 1 + 0$

eq (1)

$1 = 1(3) - 1[1(5) - 1(3)]$

$1 = -1(5) + 2[1(8) - 1(5)]$

$1 = 2(8) - 3[1(13) - 1(8)]$

$1 = -3(13) + 5[1(21) - 1(13)]$

$1 = 5(21) - 8[1(34) - 1(21)]$

$1 = -8(34) + 13[1(55) - 1(34)]$

$1 = 13(55) - 21[1(89) - 1(55)]$

$1 = -21(89) + 34(55)$

$\bar{a} = 34$

$34 \times 55 \equiv 34 \times 34 \pmod{89}$

$x \equiv 1156 \pmod{89}$

$\boxed{x = 88}$

(b)

~~gcd~~ $89x \equiv 2 \pmod{232}$

$232 = 89(2) + 54 \rightarrow 54 = -2(89) + 1(232)$

$89 = 54(1) + 35 \rightarrow 35 = -1(54) + 1(89)$

$54 = 35(1) + 19 \rightarrow 19 = -1(35) + 1(54)$

$35 = 19(1) + 16 \rightarrow 16 = -1(19) + 1(35)$

$19 = 16(1) + 3 \rightarrow 3 = -1(16) + 1(19)$

$16 = 3(5) + 1 \rightarrow 1 = -5(3) + 1(16)$

$3 = 1(3) + 0$

$1 = (73)(89) + (232)(-28)$

So inverse is 73

$89 \times 73 \equiv 2 \times 73 \pmod{232}$

$x \equiv 146 \pmod{232}$

$\boxed{x = 146}$

QUESTION #6

(a)

(i) $M = m_1 \times m_2 \times m_3 = 210$

$$M_1 = \frac{M}{m_1} = \frac{210}{5} = 42$$

$$M_2 = \frac{M}{m_2} = \frac{210}{6} = 35$$

$$M_3 = \frac{M}{m_3} = \frac{210}{7} = 30$$

Also by simple inspection we can see that
$y_1 = 3$ is an inverse of $M_1 = 42$ modulo 5
$y_2 = 5$ is an inverse of $M_2 = 35$ modulo 6
$y_3 = 4$ is an inverse of $M_3 = 30$ modulo 7

$$X = (a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3) \bmod m$$

$$X = 836 \pmod{210} = 206$$

(ii) $M = m_1 \times m_2 \times m_3 \times m_4 = 330$

$$M_1 = \frac{M}{m_1} = \frac{330}{2} = 165$$

$$M_2 = \frac{M}{m_2} = \frac{330}{3} = 110$$

QUESTION # 7

(a)

$17 = 2(8) + 1 \rightarrow 1 = 1(17) - 8(2)$

$8 = 1(8) + 0$

So, $-8 + 17 = 9$

$\boxed{\bar{a} = 9}$

(b)

$89 = 34(2) + 21 \rightarrow 21 = 1(89) - 2(34)$

$34 = 21(1) + 13 \rightarrow 13 = 1(34) - 1(21)$

$21 = 13(1) + 8 \rightarrow 8 = 1(21) - 1(13)$

$13 = 8(1) + 5 \rightarrow 5 = 1(13) - 1(8)$

$8 = 5(1) + 3 \rightarrow 3 = 1(8) - 1(5)$

$5 = 3(1) + 2 \rightarrow 2 = 1(5) - 1(3)$

$3 = 2(1) + 1 \rightarrow 1 = 1(3) - 1(2)$

$2 = 1(2) + 0$

$1 = 1(3) - 1[1(5) - 1(3)]$

$1 = -1(5) + 2[1(8) - 1(5)]$

$1 = 2(8) - 3[1(13) - 1(8)]$

$1 = -3(13) + 5[1(21) - 1(13)]$

$1 = 5(21) - 8[1(34) - 1(21)]$

$1 = -8(34) + 13[1(89) - 2(34)]$

$1 = 13(89) - 34(34)$

So, $-34 + 89 = 55$

$\boxed{\bar{a} = 55}$

(c)

$233 = 144(1) + 89 \rightarrow 89 = 1(233) - 1(144)$

$144 = 89(1) + 55 \rightarrow 55 = 1(144) - 1(89)$

$89 = 55(1) + 34 \rightarrow 34 = 1(89) - 1(55)$

$55 = 34(1) + 21 \rightarrow 21 = 1(55) - 1(34)$

$34 = 21(1) + 13 \rightarrow 13 = 1(34) - 1(21)$

$21 = 13(1) + 8 \rightarrow 8 = 1(21) - 1(13)$

$13 = 8(1) + 5 \rightarrow 5 = 1(13) - 1(8)$

$8 = 5(1) + 3 \rightarrow 3 = 1(8) - 1(5)$

$5 = 3(1) + 2 \rightarrow 2 = 1(5) - 1(3)$

$3 = 2(1) + 1 \rightarrow 1 = 1(3) - 1(2)$

$2 = 1(2) + 0$

$\gcd(233, 144) = 1 = 89(144) + (-55)(233)$

So,

$\boxed{\bar{a} = 89}$

(d)

$1001 = 200(5) + 1 \rightarrow 1 = 1(1001) - 5(200)$

$200 = 1(200) + 0$

So $-5 + 1001 = 996$

$\boxed{\bar{a} = 996}$

QUESTION # 08

(a)

(i) S T O P P O L L U T I O N

18 19 14 15 15 14 11 11 20 19 8 14 13

After function

22 23 18 19 19 18 15 15 24 23 12 18 17

W X S T T S P P Y X M S R

(ii) S T O P P O L L U T I O N

18 19 14 15 15 14 11 11 20 19 8 14 13

After function:

13 14 9 10 10 9 6 6 15 14 3 9 8

N O J K K J G G P O D J I

(b)

(i) "SURRENDER NOW"

(ii) "BE MY FRIEND"

QUESTION # 9

(i) $5^{2003} \bmod 7$

$5^{6} \equiv 1 \bmod 7$

$5^{2003} = (5^{6})^{333} \cdot 5^{5} \bmod 7$

$\equiv (1)^{333} \cdot 5^{5} \bmod 7$

$= 5^{5} \bmod 7$

$= \boxed{3}$

(ii) $5^{2003} \bmod 11$

$5^{10} \equiv 1 \bmod 11$

$5^{2003} = (5^{10})^{200} \cdot 5^{3} \bmod 11$

$= (1)^{200} \cdot 5^{3} \bmod 11$

$= 5^{3} \bmod 11$

$= \boxed{4}$

(iii) $5^{2003} \bmod 13$

$5^{12} \equiv 1 \bmod 13$

$= (5^{12})^{166} \cdot 5^{11} \bmod 13 = 5^{11} \bmod 13 = \boxed{8}$

QUESTION#10

(a)

Encrypted ~~Cod~~ Message:

$f(p) = (p+3) \bmod 26$

"LORYH GLVFUHWH PDWKHPDWLFV"

(b)

(i) Decrypted Message:
"MID TWO ASSIGNMENT"

(ii) Decrypted Message:
"FAST NUCES UNIVERSITY".

QUESTION # 11

(a)

(i) 034567981 mod 97 = 91
(ii) 183211232 mod 97 = 57
(iii) 220195744 mod 97 = 21
(iv) 987255335 mod 97 = 5

(b)

(i) 104578690 mod 101 = 58
(ii) 432222187 mod 101 = 60
(iii) 372201919 mod 101 = 32
(iv) 501338753 mod 101 = 3

QUESTION # 12

$x_1 = (4 \times 3 + 1) \bmod 7 = 6$
$x_2 = (4 \times 6 + 1) \bmod 7 = 4$
$x_3 = (4 \times 4 + 1) \bmod 7 = 3$
$x_4 = (4 \times 3 + 1) \bmod 7 = 6$
$x_5 = (4 \times 6 + 1) \bmod 7 = 4$
~~$x_6 = (4 \times$ mod 7 =~~ Sequence: 6, 4, 3, 6, 4, 3 - - -

QUESTION # 13

(a)

(i) $3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 3x_7 + x_8 + 3x_9 + x_{10} + 3x_{11} + x_{12}$
$\equiv 0 \mod 10$

$3(7) + 3 + 3(2) + 3 + 3(2) + 1 + 3(8) + \cancel{3}(4) + 3(4) + 3 + 3(4) + x_{12}$
$\equiv 0 \mod 10$

$95 + x_{12} \equiv 0 \mod 10$

$\therefore \boxed{x_{12} = 5}$

$\therefore 95 + 5 \equiv 0 \mod 10$

$100 \equiv 0 \mod 10$

(ii) $6(3) + 3 + 6(3) + 2 + 3(3) + 9 + 9(3) + 1 + 3(3) + 4 + 6(3) + x_{12} \equiv 0 \mod 10$

$118 + x_{12} \equiv 0 \mod 10$

$\boxed{\text{Check digit} = 2}$

(b)

(i) $0(3) + 3 + 6(3) + 0 + 0(3) + 0 + 2(3) + 9 + 1(3) + 4 + 5(3) + 2 \equiv 0 \mod 10$

$60 \equiv 0 \mod 10$

Valid!

(ii) $0(3)+1+2(3)+3+4(3)+5+6(3)+7+8(3)+9+0(3)+3 \equiv 0 \bmod 10$

$88 \not\equiv 0 \bmod 10$

Not Valid!

QUESTION # 14

(a)

~~$1(0)+2(0)+3(7)+4(1)+5(1)+6(9)+7(8)+8(8)+9(1)+$~~ ~~$0 \pmod{11}$~~

~~$213 + x_{10} \equiv 0 \bmod 11$~~

$x_{10} \equiv 1(0)+2(6)+3(7)+4(1)+5(1)+6(9)+7(8)+8(8)+9(1) \bmod 11$

$= 213 \bmod 11$

$\boxed{x_{10} = 4}$

(b)

$x_{10} = 1(0)+2(3)+3(2)+4(1)+5(5)+6(0)+7(0)+8(Q)+9(1) \bmod 11$

$= 8Q + 50 \bmod 11$

$\to x_{10} = 8$

$8 = 8Q + 6 \bmod 11$ $\to 50 \bmod 11 = 6$

Subtract 6

$2 = 8Q \bmod 11$

$8Q \bmod 11 \equiv 2$

Finding inverse of $8 \bmod 11$

(i) $11 = 8(1) + 3$ $\quad 3 = 1(11) - 1(8)$

$8 = 3(2) + 2$ $\quad 2 = 1(8) - 2(3)$

$3 = 2(1) + 1$ $\quad \rightarrow 1 = 1(3) - 1(2)$

$2 = (1)(2) + 0$

$1 = 1(3) - 1[1(8) - 2(3)]$

$1 = -1(8) + 3[1(11) - 1(8)]$

$1 = 3(11) - 4(8)$

$-4 + 11$

(ii)

$\bar{a} = 7$

$7 \cdot 8Q \bmod 11 = 7 \cdot 2 \bmod 11$

$Q \bmod 11 = 14 \bmod 11$

$Q \bmod 11 = 3$

$\boxed{Q = 3}$

QUESTION # 15

$n = 43 \times 59 = 2537$

$e = 13$

$k = (42)(58) = 2436$

A T T A C K

00 19 19 00 02 10

$M_1 = 0019$

$C = M_1^e \bmod n$

$= (0019)^{13} \bmod 2537$

$= (19)^{10} \cdot 19^3 \bmod 2537$

$= (19)^{10} \cdot 1785 \bmod 2537$

$= (19^5)^2 \cdot 1785 \bmod 2537$ $\quad 19^5 \bmod 2537 = 2524$

$= (2524)^2 \cdot 1785 \bmod 2537$

$= 169 \cdot 1785 \bmod 2537$ $\quad (2524)^2 \bmod 2537 = 169$

$C = 2299$

$C = 1900^{13} \bmod 2537$

$2537 \overline{)1900}$
2537
-637

$= (-637)^{13} \bmod 2537)$

$= (-637)^{10} \cdot (-637)^3 \bmod 2537$

$= (-637)^{10} \cdot (-219) \bmod 2537$

$= ((-673^3))^3 \cdot (-637) \cdot (-219) \bmod 2537$

$= (-219)^3 \cdot (-637) \cdot (-219) \bmod 2537$

$= (-219)^3 \cdot 2505 \bmod 2537$

$= -279 \cdot 2505 \bmod 2537$

$= -1220 \bmod 2537$

$\boxed{C = 1317}$

$C = (0210)^{13} \bmod 2537$

$= ((210)^4)^3 \cdot 210 \bmod 2537$ $\quad (210)^4 \bmod 2537 = 1614$

$= (1614)^3 \cdot 210 \bmod 2537$ $\quad 7 (1614)^3 \bmod 2537 = 2535$

$= 2535$

$\boxed{C = 2117}$

QUESTION # 16

(a)

no. of floors = 27
no. of offices in each floor = 37
Total no. of offices = $27 \times 37 = 999$

(b)

12 M F
/ /
3 3

No. of colors = 12
No. of genders = 2
No. of sizes = 3
So $12 \times 2 \times 3 = 72$ different shirts.

QUESTION # 17

(a)

$26 \times 26 \times 26 = 17576$

(b)

$26 \times 25 \times 24 = 15,600$

QUESTION #18

(a)

→ there are 16 place values for Hex digits. so $16^{10} + 16^{28} + 16^{58}$ different WEP key are possible.

QUESTION #~~28~~ 26

at $n=5$

$2^n-1 = 2^5-1 = 32-1 = 33$ (Not prime)

at $n=6$

$2^n-1 = 2^6-1 = 64-1 = 65$ (Not prime)

at $n=7$

$2^n-1 = 128-1 = 127$ (prime)

Proved!

QUESTION # 29

(b)

If $p \mid a$ then,

$$\frac{a}{p} = k \Rightarrow a = pk$$

If $p \mid (a+1)$ then

$$\frac{a+1}{p} = l \Rightarrow a+1 = pl$$

it follows that

$$1 = (a+1) - a$$

$$1 = pk - pl$$

$$1 = p(k-l) \qquad k-l \in \mathbb{Z}$$

This implies that $p \mid 1$.

But the only integer divisor of 1 are 1 & -1 and $\because p$ is prime > 1. This is a contradiction. Hence supposition is false, & given statement is true.

QUESTION # 29

(a)

Let $\sqrt{a+b} = \sqrt{a} + \sqrt{b} \rightarrow ①$

Sq. b/s

$\cancel{a}+\cancel{b} = \cancel{a} + 2\sqrt{a}\sqrt{b} + \cancel{b}$

$0 = 2\sqrt{ab}$

$0 = \sqrt{ab}$

Sq. b/s

$0 = ab$

If we want to satisfy this condition than any one of a or b must be zero

if we let $a=0$

eq ① ⇒

$\sqrt{0+b} = \sqrt{0} + \sqrt{b}$

$\sqrt{b} = \sqrt{b}$

if $b=0$

$\sqrt{a+0} = \sqrt{a} + \sqrt{0}$

$\sqrt{a} = \sqrt{a}$

Hence proved!

(b)

The contrapositive is

$x \leq 1$ && $x \geq -1$ then $|x| \leq 1$ for $\forall$ $\mathbb{Z}$

let

$x \leq 1$ && $x \geq -1$

$x \leq 1$ && $x \geq -1$

$-1 \leq x \leq 1$.

So,

$|x| \leq 1$

therefor

$|x| > 1$

QUESTION#30

(a)

let,

$n = 13$ then

$n+2 = 13+2 = 15$ (not a prime)

(b)

Using Contradiction method.

let set of prime no. is finite

then all prime nos can be written as,

$p_1 = 2,\ p_2 = 3,\ p_3 = 5 \cdots p_n$

Conside the integer

$N = p_1 \cdot p_2 \cdot p_3 \cdots p_n + 1$

Then $N > 1$. Since any integer greater than 1 is divisible by some prime no. p, $p_1 p_2 \cdots p_n$

thus.

$p \mid (p_1, p_2, p_3 \cdots p_n)$

But then

$p \nmid (p_1, p_2, p_3 \cdots p_n + 1)$

So,

$p \nmid N$

Thus $p \mid N$ & $p \nmid N$, which is contradiction.

Hence theorem is true.

QUESTION # 31.

(a)

Contradiction,

If n & m are odd then n+m is also odd.

$n = 2k+1$

$m = 2l+1$

$n+m = 2k+1 + 2l+1$

$= 2k+2l+2$

$= 2(k+l+1)$; let $k+l+1 = r$

$n+m = 2r$

∴ n+m is even which is a contradiction to our supposition so theorem is true.

(b)

Contrapositive,

For all integers m & n if any one of n is odd or even then m+n is odd.

$n = 2k$

$m = 2l+1$

$m+n = 2k+2l+1$

$mn = 2(k+l)+1$

$mn = 2r+1$

The contrapositive is true so theorem also true.

Question # 32

(a)

Let $6-7\sqrt{2}$ is rational

then,

$$6-7\sqrt{2} = \frac{a}{b} \quad \text{where } b \neq 0$$

$$7\sqrt{2} = 6 - \frac{a}{b}$$

$$7\sqrt{2} = \frac{6b-a}{b}$$

$$\sqrt{2} = \frac{6b-a}{7b}$$

As a & b are 'Z' so $6b-a$ & $7b$ also Z

& quotient is $\sqrt{2}$ which is rational.

Therefore it contradicts the fact that $\sqrt{2}$ is irrational

Hence theorem is true.

(b)

$\sqrt{2}+\sqrt{3}$ is rational.

$$\sqrt{2}+\sqrt{3} = a/b$$

Sq. b/s

$$2+3+2\sqrt{2}\sqrt{3} = \frac{a^2}{b^2}$$

$$2\sqrt{6} = \frac{a^2}{b^2} - 5$$

$$\sqrt{6} = \frac{a^2-5b^2}{2b^2}$$

Since the quotient is $\sqrt{6}$ which is a rational and this contradicts the fact that $\sqrt{6}$ is irrational. So theorem true.

QUESTION # 33

(a)

For P(1):

$$n^2 = [n(n+1)(2n+1)]/6$$

$$1 = [1(1+1)(2(1)+1)]/6$$

$$1 = 1$$

For $P(k)$:

$$1^2+2^2+3^2+\dots+(k)^2 = [k(k+1)(2k+1)]/6$$

For $P(k+1)$

$$1^2+2^2+3^2+\dots+(k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

Now

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$$

$$= (k+1)\left[\frac{k(2k+1) + 6(k+1)}{6}\right]$$

$$= \frac{(k+1)}{6}\left[2k^2+k+6k+6\right]$$

$$= \frac{(k+1)}{6}\left[2k^2+7k+6\right]$$

$$= \frac{(k+1)}{6}\left[(k+2)(2k+3)\right]$$

$$= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$$

Proved!!

(b)

For $P(0)$:

$$2^0 = 2^{0+1} - 1$$
$$1 = 2 - 1$$
$$\boxed{1 = 1}$$

For $P(k)$:

$$1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1$$

For $P(k+1)$:

$$1 + 2 + 2^2 + \cdots + 2^{k+1} = 2^{(k+1)+1} - 1$$
$$= (1 + 2 + 2^2 + \cdots + 2^k) + 2^{k+1}$$
$$= 2^{k+1} - 1 + 2^{k+1}$$
$$= 2 \cdot 2^{k+1} - 1$$
$$= 2^{k+1+1} - 1$$

Proved!

(c)

For $P(1)$:

$$1^3 = \frac{1^2(1+1)^2}{4}$$
$$\boxed{1 = 1}$$

For $P(k)$:

$$1^3 + 2^3 + 3^3 + \cdots k^3 = \frac{k^2(k+1)^2}{4}$$

For $P(k+1)$

$$1^3+2^3+3^3+\cdots+(k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$$

$$\frac{k^2(k+1)^2}{4} + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$$

$$k^2(k+1)^2 + 4(k+1)^3 = (k+1)^2(k+2)^2$$

Dividing b/s by $(k+1)^2$

$$k^2 + 4(k+1) = (k+2)^2$$

$$k^2 + 4k + 4 = k^2 + 4k + 4$$

Hence proved//

QUESTION # 34

① COMBINATION

→ Team formation : Creates diverse project teams
→ lottery systems : Ensures fair number selection

② Permutations:

→ Password Security : Enhance unique password generation
→ Genetic Code : Models trait variations.

③ BIONOMIAL THEOREM

→ Stock Analysis : Models Stock price movement.
→ Probability Distribution : Describes experiment outcomes

④ PROOF METHODS

→ Cryptography : Ensures protocal security
→ Software verification : Validates algo. correction

⑤ MATHMETICAL INDUCTION

→ Algo. Analysis : Checks Algo. validity
→ Number theory : Proves properties for integers.