Date

(Ex: 7.1)

A sq. matrix is said to orthogonal if

$$\boxed{A^{-1} = A^{T}}$$

or

$$\boxed{AA^{T} = A^{T}A = I}$$

$T_A : R^n \to R^n$ is an orthogonal transformation or an orthogonal operator if A is orthogonal

If A is orthogonal then

→ Row vector of A ~~are~~ form orthonormal set.

→ Col vector of A form orthonormal set.

Q#6 $A = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & -2/3 & 1/3 \\ -2/3 & -1/3 & 2/3 \end{bmatrix}$

$$AA^{T} = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & -2/3 & 1/3 \\ -2/3 & -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & -2/3 & -1/3 \\ 2/3 & 1/3 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

If row vector are ~~orthogonal~~ orthonormal set then,

$\langle r_1, r_2 \rangle = 0$

$\langle r_2, r_3 \rangle = 0$ & $\|r_1\| = \|r_2\| = \|r_3\| = 1$

$\langle r_1, r_3 \rangle = 0$

$r_1 = (1/3, 2/3, 2/3)$

$r_2 = (2/3, -2/3, 1/3)$

$r_3 = (-2/3, -1/3, 2/3)$

$\langle r_1, r_2 \rangle = 1/3(2/3) + 2/3(-2/3) + 2/3(1/3)$

$= \frac{2}{9} - \frac{4}{9} + \frac{2}{9} = 0$

$\langle r_2, r_3 \rangle = 2/3(-2/3) + -2/3(-1/3) + 1/3(2/3)$

$= \frac{-4}{9} + \frac{2}{9} + \frac{2}{9}$

$= 0$

$\langle r_1, r_3 \rangle = 0$

Date

(Ex: 7.2)

If A & B are sq. matrix then we say B is orthogonally similar to A if there is an orthogonal matrix P such that, $B = P^T A P$

→ If B is orthogonally similar to A then A is also orth. similar to B.

→ If A is orth. similar to some diagonal matrix "D" then

$$\boxed{D = P^T A P}$$

then we say A is orthogonally diagonaziable, and P orth. diagonalizes A.

Conditions For Orthogonal Diagonalization:

→ If A is a $n \times n$ matrix with real entries then,

(a) A is orthogonally diagonalizable

(b) A has an orthonormal set of eigen vectors

(c) A is symmetric.

Properties of Symmetric Matrix:

(a) The eigen values of A are all real entries

(b) Eigenvector from different eigen values/spaces are orthogonal

ORTHOGONALLY DIAGONA[LIZING] USING NxN SYMMETR[IC] MATRIX:

1. Find a basis of each eigen value of A

2. Apply Gram-Schmidt process to each of the bases to obtain an orthonormal basis for each eigen sp[ace]

Page No.

Date

Q: Form Matrix P whose col are the vectors constructed in (2). This matrix will orthogonally diagonalizes A, and the eigen values on the diagonal of $D = P^T A P$ will be in the same order as their corresponding eigen vectors in P.

Q#11 $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

Step#1:

$\det(\lambda I - A) = 0$

$\det \begin{bmatrix} \lambda-2 & 1 & 1 \\ 1 & \lambda-2 & 1 \\ 1 & 1 & \lambda-2 \end{bmatrix}$

$(\lambda-2)[(\lambda-2)^2 - 1] - 1[(\lambda-2)-1] + 1[1-\lambda+2] = 0$

$(\lambda-2)[\lambda^2 - 4\lambda + 3] - 1[\lambda-3]$

$[3-\lambda] = 0$

$\lambda^3 - 4\lambda^2 + 3\lambda - 2\lambda^2 + 8\lambda - 6 - \lambda + 3$

$+3 - \lambda = 0$

$\lambda^3 - 6\lambda^2 + 9\lambda = 0$

$\lambda(\lambda^2 - 6\lambda + 9) = 0$

$\lambda(\lambda-3)^2 = 0$

$\boxed{\lambda = 0}, \boxed{\lambda = 3}$

at $\lambda = 0$

$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} x = 0$

$2R_2 + R_1 \to R_1$

$-R_2 + R_3 \to R_3$

$\begin{bmatrix} 0 & -3 & 3 \\ 1 & -2 & 1 \\ 0 & 3 & -3 \end{bmatrix}$

$R_1 \div -3,\ R_3 \div 3$

$R_1 \leftrightarrow R_2$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$

Date

$-R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_3 = t$

$2R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_3$

$x_2 = x_3$

$x_3 = t$

$(x_1, x_2, x_3) = (t, t, t)$

$$P_1 = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Applying Gram-process as there is a single vector so normalizing it

$$V_1 = \frac{P_1}{\|P_1\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

at $\lambda = 3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} x = 0$$

$-R_1 + R_2 \to R_2$, $-R_1 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x = 0$$

$\therefore x_1 = -x_2 - x_3$

$x_2 = s$

$x_3 = t$

$(x_1, x_2, x_3) = (-s-t, s, t)$

$= s(-1, 1, 0) + t(-1, 0, 1)$

$$= s\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Appl — G —

Date

$V_3 = P_3$

$V_3 = P_3 - \frac{\langle P_3, V_2 \rangle}{\|V_2\|^2} \cdot V_2$

$V_3 = (-1, 0, 1) - \frac{(-1)(-1)}{\sqrt{2}^2} \cdot (-1, 1, 0)$

$= (-1, 0, 1) - \frac{(-1, 1, 0)}{2}$

$= (-1, 0, 1) - \left(\frac{-1}{2}, \frac{1}{2}, 0\right)$

$V_3 = \left(-\frac{1}{2}, -\frac{1}{2}, 1\right)$

$q_2 = \frac{V_2}{\|V_2\|} = \frac{(-1, 1, 0)}{\sqrt{2}}$

$= \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$q_3 = \frac{V_3}{\|V_3\|} = \frac{\left(-\frac{1}{2}, -\frac{1}{2}, 1\right)}{\sqrt{(-\frac{1}{2})^2 + (-\frac{1}{2})^2 + 1^2}}$

$= \left(\frac{-1}{2\sqrt{\frac{3}{2}}}, \frac{-1}{2\sqrt{\frac{3}{2}}}, \frac{1}{\sqrt{\frac{3}{2}}}\right)$

$q_3 = \left(\frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

Step #3.

$P = [q_1 \ q_2 \ q_3]$

$$P = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$$

$P^{-1}AP = P^{T}AP$

$$= \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ & & \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$$

Date

Q#13: $A = \begin{bmatrix} -7 & 24 & 0 & 0 \\ 24 & 7 & 0 & 0 \\ 0 & 0 & -7 & 24 \\ 0 & 0 & 24 & 7 \end{bmatrix}$

$\det \begin{bmatrix} \lambda+7 & -24 & 0 & 0 \\ -24 & \lambda-7 & 0 & 0 \\ 0 & 0 & \lambda+7 & -24 \\ 0 & 0 & -24 & \lambda-7 \end{bmatrix}$

$(\lambda+7)\begin{bmatrix} \lambda-7 & 0 & 0 \\ 0 & \lambda+7 & -24 \\ 0 & -24 & \lambda-7 \end{bmatrix}$

$+24\begin{bmatrix} -24 & 0 & 0 \\ 0 & \lambda+7 & -24 \\ 0 & -24 & \lambda-7 \end{bmatrix}$

$= \lambda+7\left[(\lambda-7)[(\lambda^2-49)-24^2]\right] + 24\left[-24[\lambda^2-49-24^2]\right]$

$= \lambda+7\left[(\lambda-7)(\lambda^2-625)\right] + 24\left[-24(\lambda^2-625)\right]$

$D = (\lambda+7)(\lambda-7)(\lambda^2-625) - 24^2(\lambda^2-625)$

$D = (\lambda^2-625)[(\lambda^2-49)-24^2]$

$= (\lambda^2-625)[\lambda^2-49-24^2]$

$= (\lambda^2-625)(\lambda^2-625)$

$= (\lambda^2-625)^2$

$\lambda^2-625=0$

$\lambda^2 = 625$

$\lambda = \pm 25$

at $\lambda = 25$

$\begin{bmatrix} 32 & -24 & 0 & 0 \\ -24 & 18 & 0 & 0 \\ 0 & 0 & 32 & -24 \\ 0 & 0 & -24 & 18 \end{bmatrix}$

$16R_2 + R_1 \to R_1$

$R_1 \leftrightarrow R_2 \quad R_2 \div -2$

$\begin{bmatrix} 0 & 264 & 0 & 0 \\ 1 & 9 & 0 & 0 \\ 0 & 0 & 32 & -24 \\ 0 & 0 & -24 & 18 \end{bmatrix}$

$R_1 \leftrightarrow R_2 \quad R_3 \div 32$

$R_2 \div 264$

$\begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & -24 & 18 \end{bmatrix}$

Page No.

$P^T A \cdot P$ ka jo ans hoga uske saare elements 0 honge or diagonal mein eigen values hongi hamesha.

Date

$24R_3 + R_4 \to R_4$

$$\begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-9R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 = 0$

$x_3 = -3/4\, s$

$x_4 = s$

(x_1, x_2, x_3, x_4)

$R_1 \div 32$

$$\begin{bmatrix} 1 & -3/4 & 0 & 0 \\ -24 & 18 & 0 & 0 \\ 0 & 0 & 32 & -24 \\ 0 & 0 & -24 & 18 \end{bmatrix}$$

$24R_1 + R_2 \to R_2$ $\quad R_3 \div 32$

$$\begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & -24 & 18 \end{bmatrix}$$

$24R_3 + R_4 \to R_4$

$$\begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -3/4 & 0 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 = s \;,\; x_4 = t$

$x_1 = 3/4\, s$

$x_2 = s$

$x_3 = 3/4\, t$

$x_4 = t$

$(x_1, x_2, x_3) = (3/4\, s, s, 3/4\, t, t)$

$$= s\begin{bmatrix} 3/4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 3/4 \\ 1 \end{bmatrix}$$

~~(x) by~~ ~~(x) by~~

(x) by 4 to reduce frac

$$= s\begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix}$$

Date

$V_1 = P_1 = (3,4,0,0)$

$V_2 = P_2 - \frac{\langle P_2, V_1\rangle}{\|V_1\|^2} V_1$

$= (0,0,3,4) - \frac{0}{\|V_1\|^2} \cdot V_1$

$V_2 = (0,0,3,4)$

$q_1 = \frac{V_1}{\|V_1\|} = \frac{(3,4,0,0)}{\sqrt{25}}$

$\boxed{q_1 = \left(\frac{3}{5}, \frac{4}{5}, 0, 0\right)}$

$\boxed{q_2 = \left(0, 0, \frac{3}{5}, \frac{4}{5}\right)}$

at $\lambda = -25$

$$\begin{bmatrix} -18 & -24 & 0 & 0 \\ -24 & -32 & 0 & 0 \\ 0 & 0 & -18 & -24 \\ 0 & 0 & -24 & -32 \end{bmatrix}$$

R.e

$$\begin{bmatrix} 1 & 4/3 & 0 & 0 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 = s, \quad x_4 = t$

$(x_1, x_2, x_3, x_4) = (-4/3\, s, s, -4/3\, t, t)$

$$= s\begin{bmatrix} -4/3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ -4/3 \\ 1 \end{bmatrix}$$

(x) by 3

$$= s\begin{bmatrix} -4 \\ 3 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ -4 \\ 3 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} -4 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \quad P_4 = \begin{bmatrix} 0 \\ 0 \\ -4 \\ 3 \end{bmatrix}$$

$V_3 = P_3 = (-4, 3, 0, 0)$

$V_4 = P_4 - \frac{\langle P_4, V_3\rangle}{\|V_3\|^2} \cdot V_3$

$= P_4 - 0$

$= (0, 0, -4, 3)$

$q_3 = (-4/5, 3/5, 0, 0)$

$q_4 = (0, 0, -4/5, 3/5)$

Date

$$P = \begin{bmatrix} 3/5 & 0 & -4/5 & 0 \\ 4/5 & 0 & 3/5 & 0 \\ 0 & 3/5 & 0 & -4/5 \\ 0 & 4/5 & 0 & 3/5 \end{bmatrix}$$

$$P^{T} A P = P^{T} A P$$

$$= \begin{bmatrix} 3/5 & 4/5 & 0 & 0 \\ 0 & 0 & 3/5 & 4/5 \\ -4/5 & 3/5 & 0 & 0 \\ 0 & 0 & -4/5 & 3/5 \end{bmatrix} \times \begin{bmatrix} -7 & 24 & 0 & 0 \\ 24 & 7 & 0 & 0 \\ 0 & 0 & -7 & 24 \\ 0 & 0 & 24 & 7 \end{bmatrix} \times \begin{bmatrix} & P & \end{bmatrix}$$

$$= \begin{bmatrix} -25 & 0 & 0 & 0 \\ 0 & -25 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix}$$

Ex: 7.3

Quadratic Form:

$a_1 x_1^2 + a_2 x_2^2 + \cdots a_n x_n^2 + 2a_{n+1} x_1 x_2 + 2a_{n+2} x_1 x_3 +$ (all possible terms $a_k x_i x_j$ & $i \neq j$)

the $x_i x_j$ terms are called cross product terms.

General Quadratic Form for R^3,

$a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2 + 2a_4 x_1 x_2 + 2a_5 x_1 x_3 + 2a_6 x_2 x_3$

this can be represented as,

$$[x_1 \;\; x_2] \begin{bmatrix} a_1 & a_3 \\ a_3 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = X^T A X$$

the diagonals are the coefficient of sq. terms and the rest are the ~~of~~ coeff. of ~~cross~~ half the cross product terms.

$$Q_A(x) = x^T A x$$

Known as Quadratic form associated with A.

Date

this is also written as,

$X^TAX = X \cdot AX = AX \cdot X$

If A is a diagonal matrix then it has no cross product terms.

The Principal Axes Theorem:

If A is a symmetric $n \times n$ matrix, then there is an orthogonal change of variable that transforms the quadratic X^TAX into a Quadratic form Y^TAY with no cross product terms. Specifically if P orthogonally diagonalizes A, then making the chge of variable $X = PY$ in the Quadratic form X^TAX which yield,

$$X^TAX = Y^TDY = \lambda_1 Y_1^2 + \lambda_2 Y_2^2 + \dots + \lambda_n Y_n^2$$

in which

$\lambda_1, \dots \lambda_n$ are eigen values of A.

$D = P^TAP$

Q#2 (a)

$$[x_1, x_2]\begin{bmatrix} 5 & 5/2 \\ 5/2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$(c)\ [x_1, x_2, x_3]\begin{bmatrix} 1 & -\cancel{8}\ 5/2 & 9/2 \\ -5/2 & 1 & 0 \\ 9/2 & 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q#7

$$[x_1, x_2, x_3]\begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & -2 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

$$\det\begin{bmatrix} \lambda-3 & -2 & 0 \\ -2 & \lambda-4 & 2 \\ 0 & 2 & \lambda-5 \end{bmatrix}$$

$\lambda-3[(\lambda-4)(\lambda-5)-4]+2[-2\lambda+10]$

$\lambda-3[\lambda^2-9\lambda+16]-4\lambda+2$

$\lambda^3-9\lambda^2+16\lambda-3\lambda^2+27\lambda-48$

$\lambda^3-12\lambda^2 \cancel{39\lambda} -28$

$\lambda = 1, 7, 4$

Date

$\lambda = 3$

$$\begin{bmatrix} -3 & -2 & 0 \\ -2 & -4 & 2 \\ 0 & 2 & -5 \end{bmatrix} x = 0$$

$R_2 \div -2$

$$\begin{bmatrix} -3 & -2 & 0 \\ 1 & 2 & -1 \\ 0 & 2 & -5 \end{bmatrix}$$

$3R_2 + R_1 \to R_1$

$$\begin{bmatrix} 0 & 4 & -3 \\ 1 & 2 & 1 \\ 0 & 2 & -5 \end{bmatrix}$$

$R_1 \div 4$

$$\begin{bmatrix} 0 & 1 & -3/4 \\ 1 & 2 & 1 \\ 0 & 2 & -5 \end{bmatrix}$$

$-2R_1 + R_2$

$\lambda = 1$

$$\begin{bmatrix} -2 & -2 & 0 \\ -2 & -3 & 2 \\ 0 & 2 & -4 \end{bmatrix}$$

$R_1 \div -2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & -3 & 2 \\ 0 & 2 & -4 \end{bmatrix}$$

$2R_1 + R_2 \to R_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & -4 \end{bmatrix}$$

$R_2 \times -1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -2 \\ 0 & 2 & -4 \end{bmatrix}$$

$-2R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$-R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

Date

$x_1 = -2s$

$x_2 = 2s$

$x_3 = s$

$(x_1, x_2, x_3) = (-2s, 2s, s)$

$$P_1 = s\begin{bmatrix}-2\\2\\1\end{bmatrix}$$

$V_1 = P_1$

$$q_1 = \frac{V_1}{\|V_1\|} = \frac{(-2, 2, 1)}{\sqrt{2^2+2^2+1}}$$

$$q_1 = \left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)$$

at $\lambda = 7$

$$\begin{bmatrix}4 & -2 & 0\\-2 & 3 & 2\\0 & 2 & 2\end{bmatrix}$$

$2R_2 + R_1 \to R_1$

$$\begin{bmatrix}0 & 4 & 4\\-2 & 3 & 2\\0 & 2 & 2\end{bmatrix}$$

$R_1 \div 4$, $R_3 \div 2$

$R_1 \leftrightarrow R_3$ $\quad R_2 \div -2$

$$\begin{bmatrix}0 & 1 & 1\\1 & -3/2 & -1\\0 & 1 & 1\end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$-R_2 + R_3 \to R_3$

$$\begin{bmatrix}1 & -3/2 & -1\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix}$$

$\frac{3}{2}R_2 + R_1 \to R_1$

$$\begin{bmatrix}1 & 0 & 1/2\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix}$$

$x_1 = -1/2\, s$

$x_2 = s$

$x_3 = s$

$(x_1, x_2, x_3) = (-\frac{1}{2}s, -s, s)$

$$= s\begin{bmatrix}-1/2\\-1\\1\end{bmatrix}$$

$$= s\begin{bmatrix}-1\\-2\\2\end{bmatrix}$$

$$q_2 = \frac{(-1, -2, 2)}{\sqrt{1^2+2^2+2^2}}$$

$$= \left(\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$$

Page No.

Date

at $\lambda = 4$

$q_3 = (2/3, 1/3, 2/3)$

$P = [q_1 \; q_2 \; q_3]$

$$P = \begin{bmatrix} -2/3 & -1/3 & 2/3 \\ 2/3 & -2/3 & 1/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$$

$X = PY$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} & P & \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$Q = Y^T D Y = [y_1 \; y_2 \; y_3] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$Q = y_1^2 + 7y_2^2 + 4y_3^2$

For D:

$$P^T A P = \begin{bmatrix} P \end{bmatrix}^T \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} P \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

Principal SubMatrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$\Rightarrow |1| \rightarrow$ 1st prine. sub matr.

$\Rightarrow \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix} \rightarrow$ 2nd "

$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 10 & 11 \end{vmatrix} \rightarrow$ 3rd

$\Rightarrow$ Overall matrix is 4th.

→ A is a positive definite iff determinant of every principal sub. matr is +ve

→ A is neg definite if the determinate alternates b/w -ve & +ve values starting from -ve.

→ If neither +ve nor -ve definite then indefinite. At least one +ve & one -ve det.

Page No.