CHAPTER # 05

"EULER METHOD"
(Ex: 5.1)

"Range Kutta First Order"

- In this method we get approximate value of Ordinary DE.
- Consider the given differential eqn

$\frac{dy}{dx} = f(x,y)$ with initial condition $y(x_0) = y_0$

to find $y(x_n) = y_n$

→ Acc to Euler Method,

$$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1}) \quad , n = 1, 2, 3, \ldots$$

where

h = width of differencing $= \frac{x_n - x_0}{n}$

↳ small values of h give more accurate answer.

n → jitne zyada intervals mein divide hoga utna chota stepsize yaani "h" milega

put $n=1$, $y_1 = y_0 + h f(x_0, y_0)$

$n=2$ $\quad y_2 = y_1 + h f(x_1, y_1)$ & so on...

Hence starting from y_0 we can get approximate values of $y_1, y_2 \ldots y_n$.

$y_1 = y_0 + 0.5[t_0 e^{3t_0} - 2y_0]$

Q: ~~$y' = y - t^2 + 1$~~
~~$0 \le t \le 2$, $y(0) = 0.5$~~

Q1(a): $y' = te^{3t} - 2y$

$0 \le t \le 1$, $y(0) = 0$

$h = 0.5$

i	t_i	y_i	$y(t_i)$
0	0	0	0
1	0.5	0	1.2042
2	1	1.2042	

$\boxed{N = 1/h}$

(d) $y' = \cos 2t + \sin 3t$

$0 \le t \le 1$, $y(0) = 1$

$h = 0.25$

$N = 1/h = 4$

i	t_i	y_i	
	0	1	1.25
	0.25	1.25	1.6398
2	0.5	1.6398	2.024
3	0.75	2.0242	
4	1	2.2365	

Q2(c): $y' = -y + ty^{1/2}$

$2 \le t \le 3$, $y(2) = 2$

$h = 0.25$ $N = 1/0.25 = 4$

i	t_i	y_i
0	2	2
1	2.25	2.207
2	2.5	2.49898
3	2.75	2.8546
4	3	3.40819

Q5: $y' = -5y + 5t^2 + 2t$

(d) $0 \le t \le 1$, $y(0) = 1/3$

$h = 0.1$ $N = 10$

i	t_i	y_i
0	0	1/3
1	0.1	0.191667
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	
9	0.9	
10	1	

Runge-Kutta 2

→ Consider the ordinary differential eqn,

$$\frac{dy}{dx} = f(x,y)$$

with initial condition $y(x_0) = y_0$

To find $y(x$

MODIFIED EULER METHOD:

~~MIDPOINT METHOD:~~

ye wohi purana wala euler ka formula hai

$$y_{i+1} = y_i + \frac{h}{2}\left[f(t_i, y_i) + f(t_{i+1}, \underbrace{w_i + hf(t_i, y_i)}_{y^*_{i+1}})\right]$$

MIDPOINT METHOD METHOD:

$$y_{i+1} = y_i + h f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2} f(t_i, y_i)\right)$$

Q: Given $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$, find $y(0.02)$ & $y(0.04)$ By Euler modified Method.

$x_0 = 0$ $\quad y_0 = 1$

$x_1 = 0.02$ $\quad y_1^* =$ $\quad y_1 =$

$x_2 = 0.04$ $\quad y_2^* =$ $\quad y_2 =$

$h = 0.02$ $(x_1 - x_0)$

ye do purane wale eulerse nikalenge.

$n = 0$:

$y_1^* = y_0 + h f(x_0, y_0)$

$= 1 + 0.02(0^2 + 1^2)$

$y_1^* = 1.02$

$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^*)\right]$

$= 1 + \frac{0.02}{2}\left[(0^2 + 1) + ((0.02)^2 + (1.02$

$\boxed{y_1 = 1.0202}$

$y_i + \frac{0.2}{2}[y_i - 0.01i^2 + 1 + y_i + 0.2y_i - 0.08i^2 + 0.2 - t_{i+1}$

$y_i + 0.1y_i - 0.004i^2 + 0.1 + 0.1y_i + 0.02y_i - 0.008i^2 + 0.02$

~~$1.22y_i + 0.22$~~

at $n = 1$,

$y_2^* = y_1 + h f(x_1, y_1)$

$= 1.0202 + 0.02(0.002^2 + 1.0202)$

$= 1.0406$

$y_2 = y_1 + \frac{h}{2}[f(x_1,y_1) + f(x_2, y_2^*)]$

$= 1.0202 + \frac{0.02}{2}[((0.02)^2 + 1.0202) + (0.04^2 + 1.0406)]$

$\boxed{y_2 = 1.0408}$

$y_0 + \frac{0.5}{2}[0 \times e^{3\times0} - 2\times y_0 + 0.5 \times e^{3\times0.5} - 2[y_0 + 0.5(0\times e^{3\times0} - 2y_0$

Q: Use modified $\boxed{\text{Euler Method}}$ to approximate the sol.

(a) $y' = te^{3t} - 2y \quad 0 \le t \le 1$

$y(0) = 0 \quad h = 0.5$

$x_0 = 0 \quad y_0 = 0$

$x_1 = 0.5 \quad y_1 =$

$x_2 = 1 \quad y_2 =$

at $N = 0$;

$y_1^* = y_0 + 0.5 f(x_0, y_0)$

$y_1^* = 0$

$y_1 = y_0 + \frac{0.5}{2}[f(x_0,y_0) + f(t_1, y_1^*)$

$= 0.560211$

at $N = 1$

$y_2^* = y_1 + 0.5[0.5 \times e^{3\times0.5} - 2(\ldots$

$y_2^* = 1.1204227$

$y_2 = y_1 + \frac{0.5}{2}[f(x_1,y_1) + f(x_2, y_2^*)$

$= 5.3014$

(c): $y' = 1 + y/t \quad 1 \le t \le 2$

$y(1) = 2$, $h = 0.25$

$t_0 = 1 \qquad y_0 = 2$

$t_1 = 1.25$

$t_2 = 1.5$

$t_3 = 1.75$

$t_4 = 2$

eqn to put on calculator

$y_1 = y_0 + \frac{0.25}{2}\left[1 + \frac{y_0}{t_0} + 1 + \frac{1}{t_1}\left[y_0 + 0.25\left(1 + \frac{y_0}{t_0}\right)\right]\right]$

$\boxed{y_1 = 2.775}$

→ ab same eqn calculator mei rhegi bs y_0 ki jaga ANS or t_1 ki jaga t_2 & t_0 ki ~~jag t_1~~.

→ $y_2 = 3.600833$

$y_3 = 4.4688294$

$y_4 = 5.37286$

Q2: $y' = t^{-2}(\sin 2t - 2ty)$

(d) $1 \le t \le 2$

$y(1) = 2$, $h = 0.25$

$t_0 = 1 \quad y_0 = 2$

$t_1 = 1.25 \quad y_1 =$

$t_2 = 1.5 \quad y_2 =$

$t_3 = 1.75 \quad y_3 =$

$t_4 = 2 \quad y_4 =$

$= y_0 + \frac{0.25}{2}\left[t_0^{-2}(\sin 2t_0 - 2t_0 y_0) + t_1^{-2}(\sin 2t_1 - 2t_1(y_0 + 0.25(t_0^{-2}\sin 2t_0 - 2t_0 y_0)\right.$

$y_1 = 1.416075$

$y_2 = 1.0310111$

$y_3 = 0.75226668$

$y_4 = 0.54324500.$

Q: Use [Midpoint Method]

Q1: $y' = 1 + (t-y)^2 \quad 2 \le t \le 3$

(b) $y(2) = 1$, $h = 0.5$

$x_0 = 2$, $y_0 = 1$

$x_1 = 2.5$ $\quad y_1 =$

$x_2 = 3$ $\quad y_2 =$

$$y_i = y_0 + 0.5\left[1 + \left(t_0 + \tfrac{h}{2} - \left(y_0 + \frac{h}{2}\left(1 + (t_0 - y_0)^2\right)\right)\right)^2\right]$$

$$\boxed{\begin{aligned} y_1 &= 1.78125 \\ y_2 &= 2.45506 \end{aligned}}$$

Q3: $1 + y/t + (y/t)^2$, $1 \le t \le 3$

(b) $y(1) = 0$, $h = 0.2$

t_i	y_i	t_i	y_i
1	0	2.6	
1.2	0.21983	2.8	
1.4		3.0	
1.6			
1.8			
2			
2.2			
2.4			

HEUN'S METHOD :

$$y_{i+1} = y_i + \frac{h}{4}\left[f(t_i, y_i) + 3f\left(t_i + \frac{2h}{3}, y_i + \frac{2h}{3} f\left(t_i + \frac{h}{3}, w_i + \frac{h}{3} f(t_i, w_i)\right)\right)\right]$$

RANGE-KUTTA ORDER 4 :

$$k_1 = hf(t_i, y_i)$$

$$k_2 = hf\left(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(t_i + \frac{h}{2}, y_i + \frac{1}{2}k_2\right)$$

$$k_4 = hf(t_{i+1}, y_i + k_3)$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$