22K-4187 → SYED SAMROZE

ASSIGNMENT # 02

QUESTION # 01

(i) undirected graph edges

Multiple edges : Yes
Loops : No
Type : undirected multigraph

(ii) undirected ~~graph~~ edges

Multiple edges : No
Loops : No
~~Undirected~~ Type : Undirected Simple graph

(iii) Undirected edges

Multiple edges : Yes
Loops : Yes (3)
Type : Undirected Pseudo graph

(iv) Directed edges

Multiple edges : Yes
Loop : Yes (2)
Type : Directed Pseudo graph

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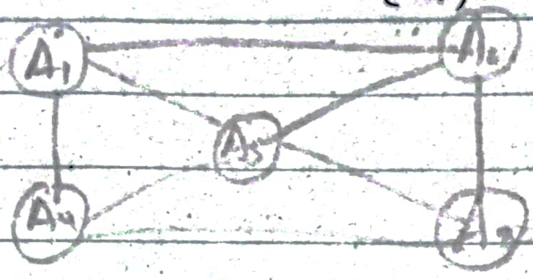
QUESTION # 02

(a)

(i)



0 2 4 6 8

$A_1 A_2$ 0, 2

$A_1 A_3$ —

$A_1 A_4$ 6 8

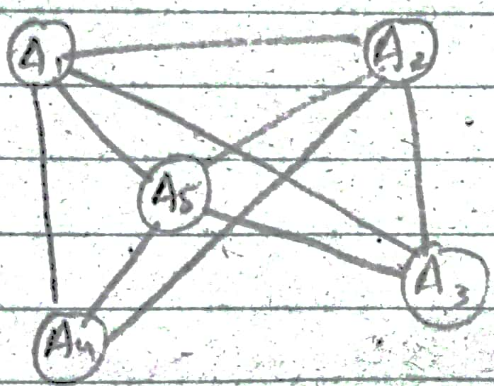
$A_1 A_5$ 0 8

~~$A_2 A_1$ 0 2~~

~~$A_2 A_3$ 1 3~~

~~$A_2 A_5$ 0 1~~

(ii)



(b)

~~As~~ this graph is Euler

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QUESTION # 3

(a)

(i) No. of vertices : 5

No. of edges : 13

deg(a) = 6 , deg(b) = 6, deg(c) = 6, deg(d) = 5

deg(e) = 3

N(a) = {a, b, e}

N(b) = {a, c, d, e}

N(c) = {b, c, d}

N(d) = {b, c, e}

N(e) = {a, b, d}

(ii) No. of vertices : 9

No. of edges : 12

deg(a) = 3, deg(b) = 2, deg(c) = 4, deg(d) = 0

deg(e) = 6 , deg(f) = 0 , deg(g) = 4, deg(h) = 2,

deg(i) = 3

N(a) = {c, e, i}

N(b) = {e, h}

N(c) = {a, e, g, i}

N(d) = ϕ

N(e) = {a, b, c, g}

N(f) = ϕ

N(g) = {c, e}

N(h) = {b, i}

N(i) = {a, c, h}

(b)

i i) No. of vertices : 5

No. of edges : 13

In-degree :

$deg^-(a) = 6$, $deg^-(b) = 1$, $deg^-(c) = 2$, $deg^-(d) = 4$

$deg^-(e) = 0$

(ii) Out-degree :

$deg^+(a) = 1$, $deg^+(b) = 5$, $deg^+(c) = 5$, $deg^+(d) = 2$, $deg^+(e) = 0$

(ii) No. of vertices : ~~8~~ 4

No. of edges : 8

In-degree :

$deg^-(a) = 2$, $deg^-(b) = 3$, $deg^-(c) = 2$, $deg^-(d) = 1$

Out-degree :

$deg^+(a) = 2$, $deg^+(b) = 4$, $deg^+(c) = 1$, $deg^+(d) = 1$

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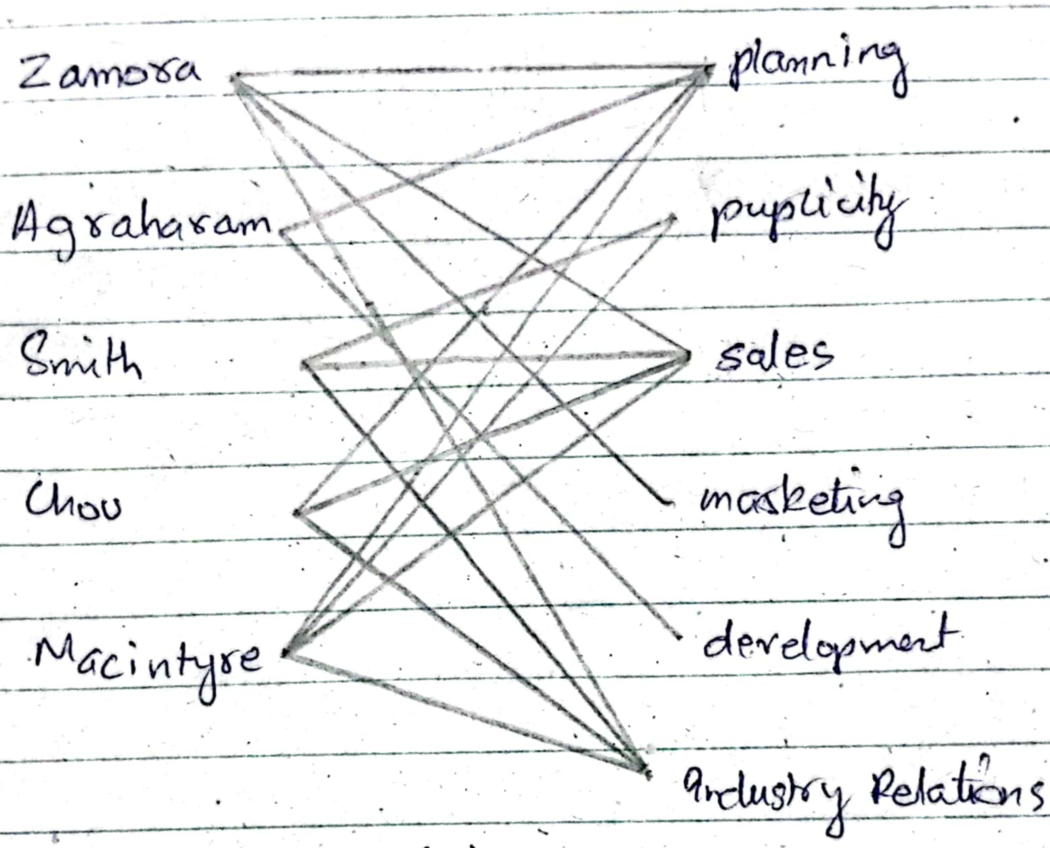
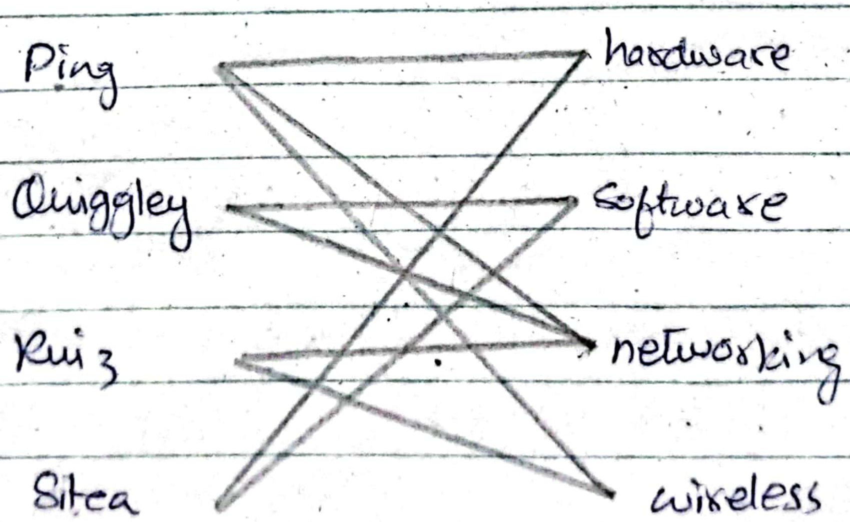
QUESTION # 4

(a)



(b)



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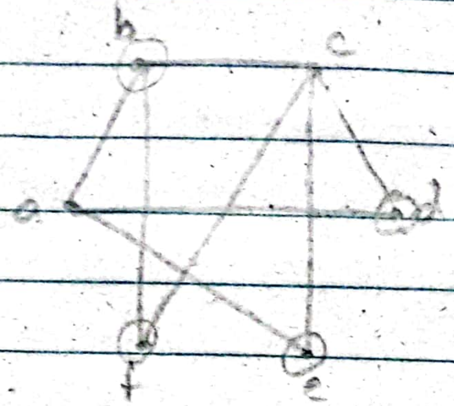
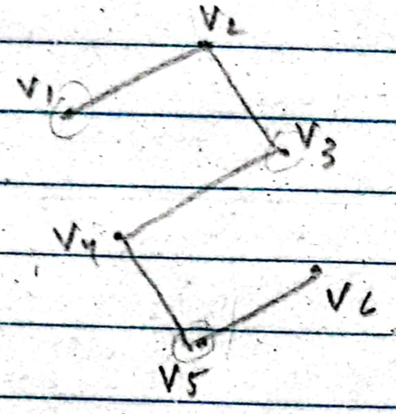
QUESTION #5

(i)

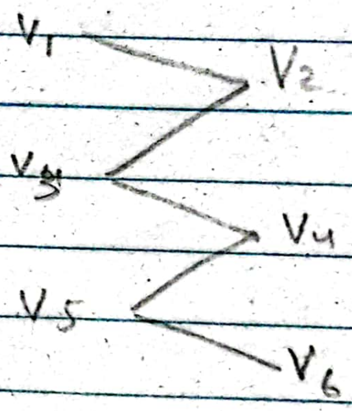


Not bipartite ∵ a is adjacent to b & f vertices

(ii)

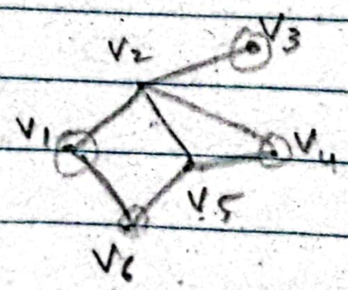


⇒



$A = \{V_1, V_3, V_5\}$, $B = \{V_2, V_4, V_6\}$

(iii)



Not bipartite.

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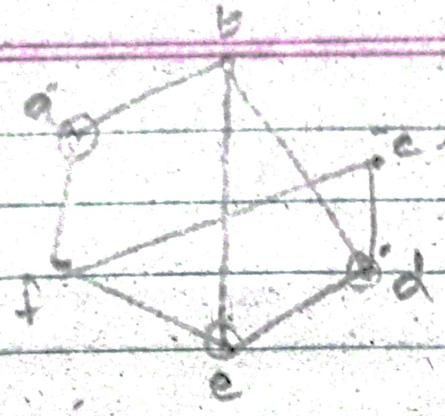
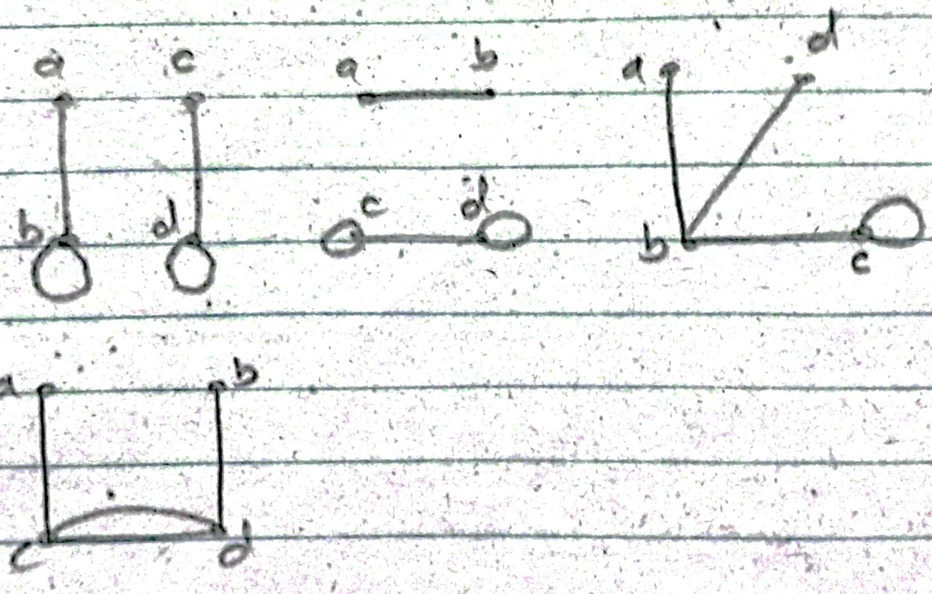


Not bipartite e & d are adjacent

QUESTION#6

(i) No such graph exist as, acc to handshaking theorem sum of degrees should be even but here it is odd $1+1+2+3=7$

(ii)



(iii) No simple graph exist with 4 vertice and degree 1, 1, 3 & 3.

QUESTION # 07

(a)

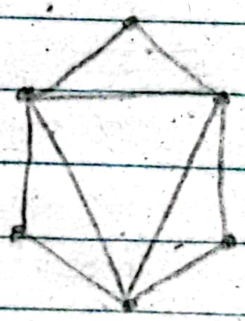
i (i) By using handshaking theorem,
No, it's not possible such that 15 vertices have degree 3 $\because (15 \cdot 3) \neq 2e$

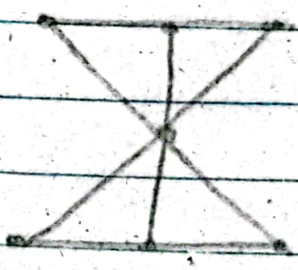
(b)

(ii) By using handshaking theorem,
ii Yes, possibe such that 4 vertices have degree 3 i.e $4 \cdot 3 = 2e$

QUESTION # 8

(a)

i) 

(ii) 

(b)

no. of edges = 10
deg = 4
no. of vertice = n = ?

$$4(n) = 2|E|$$
$$4n = 2(10) \Rightarrow \boxed{n = 5}$$

QUESTION #9

(a)

i) g Both graphs are isomorphs
$g(V_1) = W_2$ $g(V_2) = W_3$, $g(V_5) = W_4$
$g(V_3) = W_1$ $g(V_4) = W_5$

(ii) Both graphs are isomorphs
$g(V_1) = U_5$, $g(V_2) = U_2$, $g(V_3) = U_4$, $g(V_4) = U_3$
$g(V_5) = U_1$, $g(V_6) = U_6$

(iii) Both are isomorphs
$g(V_1) = U_5$, $g(V_2) = U_4$, $g(V_3) = U_3$, $g(V_4) = U_2$
$g(V_5) = U_7$, $g(V_6) = U_1$, $g(V_7) = U_6$

(iv) Graph G has only 1 vertex of deg 2 i.e U_2 but graph G' has two vertex of deg 2 i.e V_1 & V_3.

(b)

- A & R
- K & X
- Z & S , W & M
- F & T

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Question 10

	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(h)
a	2,a	4,a	1,a				
ad	2,a	4,a			6,d	5,d	
adb		4,a		3,b	6,d	5,d	
adbe		4,a			6,d	5,d	6,e
adbec					6,c	5,d	6,e
adbecg					6,c		6,e
adbecgf							6,e
adbecgfh							
adbecgfhk							
adbecgf hkl							
adbegf hkli							
adbegf hklix							
adbecgfhklixj							
adbecgfhkl ixjm							
adbecgfhkl ixjmn							
adbecgfhklixjmnp							
adbecgfghklixjmnpq							
adbecgfhklixj mnpqo							
adbecgfhklixjmnpqot							
adbecgfhklixjmnpqots							
adbecgfgklixjmnpqotsz							

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	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
adbecg			7,g										
adbecgf	9,f	10,f	7,g										
adbecgfh	9,f	10,f	7,g	7,h			14,h						
adbecgfhk	9,f	10,f		7,h		11,k	14,k			9,k			
adbecgf hkl	9,f	10,f			10,l	11,k	13,l			9,k			
adbegf hkli		10,f			10,l	11,k	13,l			9,k			
adbegf hklix		10,f			10,l	10,x	13,l		17,x			14,x	
adbecgfhklixj					10,l	10,x	13,l	12,m	17,x			14,x	
adbecgfhkl ixjm						10,x	13,l	12,m	17,x			14,x	
adbecgfhkl ixjmn							13,l	12,m	12,n			14,x	
adbecgfhklixjmnp							13,l		12,n		14,p	13,p	
adbecgfghklixjmnpq							13,l				14,p	13,p	
adbecgfhklixj mnpqo											14,p	13,p	
adbecgfhklixjmnpqot											14,p		21,t
adbecgfhklixjmnpqots													16,s

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(b)

i

	D(b)	(c)	D(d)	D(e)	D(f)	D(g)	D(z)
a	4,a	3,a					
ac			6,c	9,c			
acb			6,c	9,c			
acbd				7,d	11,d		
acbde					11,d	12,e	
acbdef						12,e	18,f
acbdefg							16,g
acbdefgz	4,a	3,a	6,c	7,d	11,d	12,e	16,g

ii

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QUESTION# 11

(i) Hamiltonian Circuits are:

ABCDA = 125 , ABDCA = 140 , ACBDA = 155

Therefore ABCDA is the minimum distance

(ii) Hamiltonian Circuits are:

ABDCA = 108 , ADBCA = 141 , ABCDA = 97

Therefore ABCDA is the minimum distance.

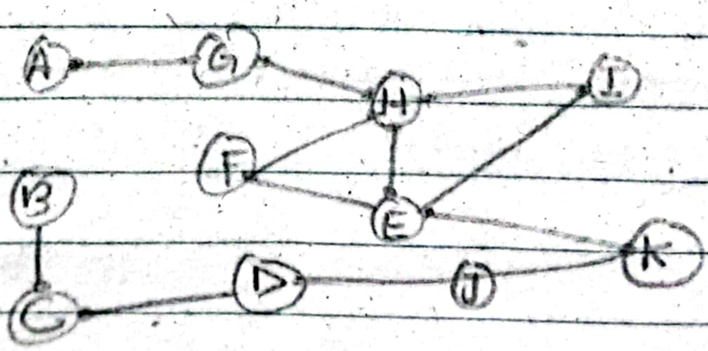
QUESTION# 12

(a)

Yes it can be done.

A → H → G → D → C → B → G → F → E

(b)



Path: A G H E I H F E K J D C B

QUESTION # 13

i (i) Hamiltonian Circuits: $V_0\ V_1\ V_5\ V_4\ V_7\ V_6\ V_2\ V_3\ V_0$

(ii) Hamiltonian Paths: $V_0\ V_1\ V_5\ V_4\ V_7\ V_6\ V_2\ V_3$

(ii) Hamiltonian Circuits: Doesnot Exist

Hamiltonian Path: ehgfebad

(iii) (iii) Hamiltonian Circuits: abcefgda

Hamiltonian Paths: abcefgda

QUESTION #14

(a)

(i) Euler Circuits: $\rightarrow V_4\ V_1\ V_2\ V_3\ V_4\ V_5\ V_2\ V_5\ V_4$

$\rightarrow e_8, e_1, e_4, e_5, e_7, e_3, e_2, e_6, e_8$

(ii). Euler Circuits donot exist because all vertices don't have even vertices.

(b)

(i) Euler path doesnot exist because u, f, e, w, h have odd deg.

(ii) Euler path: $u\ v_7\ v_0\ v_1\ u, v_2, v_3, v_4, v_2, v_6, v_4, w, v_6, v_5, w$.

QUESTION #15

(a)

(i)

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(ii)

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}
\begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

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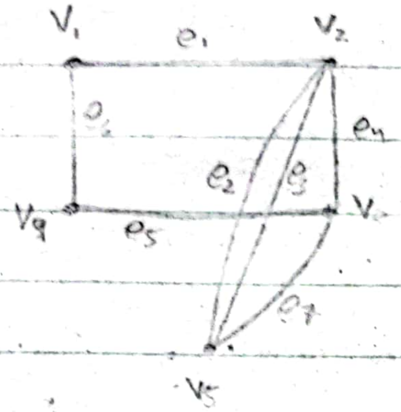
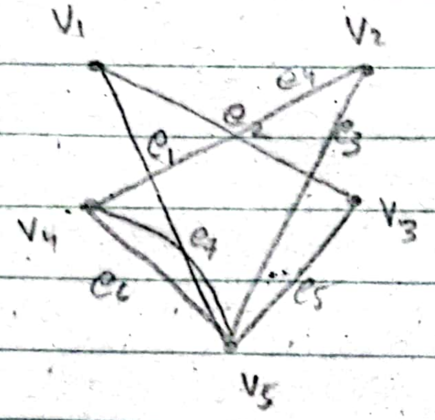
(b)

(i)



	e_1	e_2	e_3	e_4	e_5	e_6	e_7
v_1	1	0	0	0	0	1	0
v_2	1	1	1	1	0	0	0
v_3	0	0	0	1	1	0	1
v_4	0	0	0	0	1	1	0
v_5	0	1	1	0	0	0	1

ii



	e_1	e_2	e_3	e_4	e_5	e_6	e_7
v_1	1	1	0	0	0	0	0
v_2	0	0	1	1	0	0	0
v_3	0	1	0	0	1	0	0
v_4	0	0	0	1	0	1	1
v_5	1	0	1	0	1	1	1

QUESTION # 16

(i)

$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
a & 1 & 1 & 1 & 1 \\
b & 0 & 0 & 1 & 1 \\
c & 1 & 1 & 0 & 0 \\
d & 0 & 1 & 1 & 1
\end{array}
$$

a	a, b, c, d
b	c, d
c	a, b
d	b, c, d

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(ii)

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

a	b, d
b	a, c, d, e
c	b, c
d	a, e
e	d, e

(iii)

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix} \end{array}$$

a	b, d
b	a, c
c	b, d
d	a, c

(iv)

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \end{array}$$

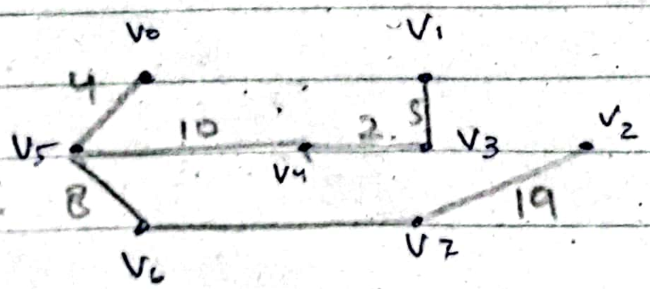
a	a, c, d
b	b, c, d
c	a, b, c
d	a, b, d

QUESTION # 17

(i) level of n is 3
(ii) level of a is 0
(iii) Height of tree is 5
(iv) U & V are childrens of n
(v) d is parent
(vi) k & l are siblings of j
(vii) m, s, t, x & y are desendants
(viii) a, b, c, d, e, f, h, i, k, m, n, o, v, t are internal no
(ix) v, n, h, d & a are ancestors
(x) j, l, q, r, s, x, y, g, p, u, w & z are the leaves

QUESTION # 18.

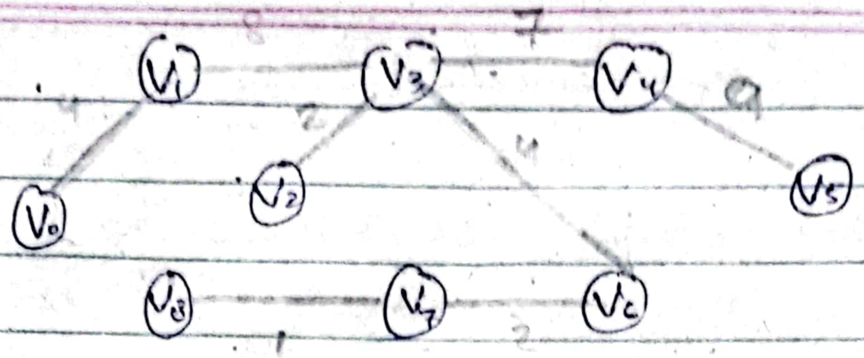
(i)



$(V_0, V_5), (V_5, V_6), (V_5, V_4), (V_4, V_3), (V_3, V_1), (V_6, V_7)$
(V_7, V_2)

MST = 61

(iii)

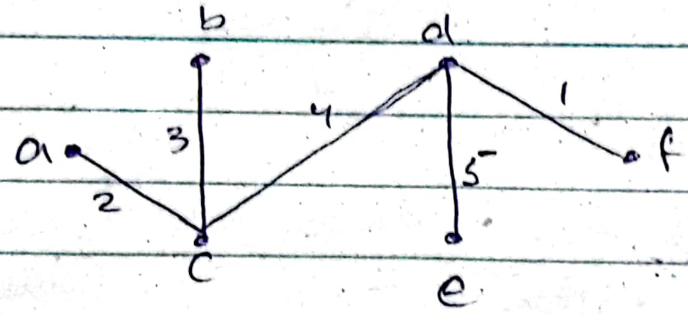


$(V_0, V_1), (V_1, V_3), (V_3, V_2), (V_3, V_6), (V_6, V_7)$
$(V_7, V_8), (V_3, V_4), (V_4, V_5)$

MST = 37

QUESTION#19

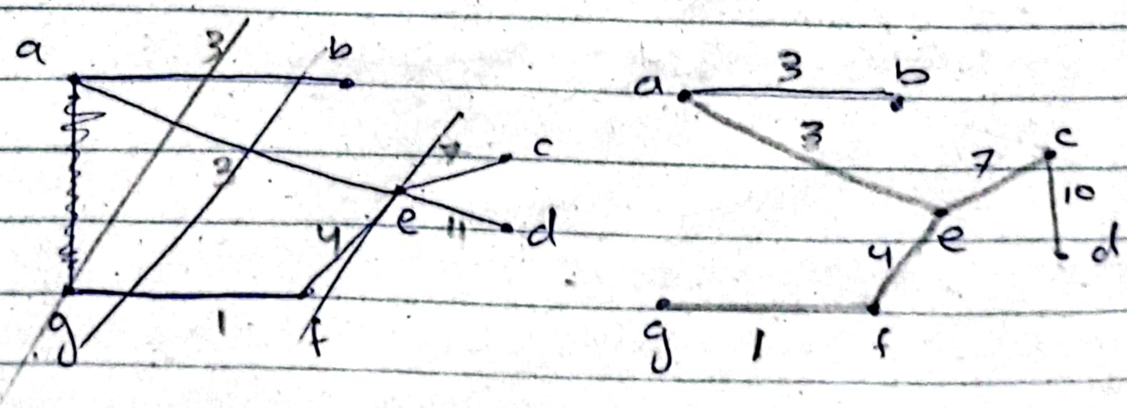
(i)



MST cost = 15

(d,f), (a,c), (b,c), (c,d), (d,e)

(ii)



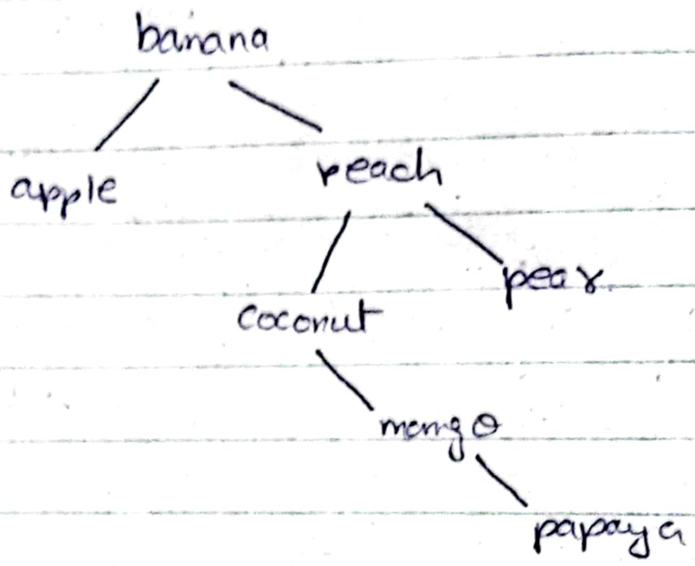
(g,f) (a,b), (a,e), (e,f), (e,c), (c,d)

MST cost = 28

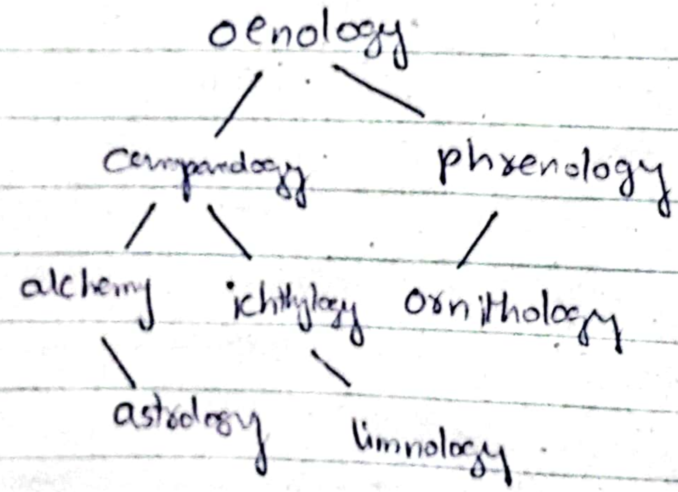
QUESTION #20

(a)

(i)

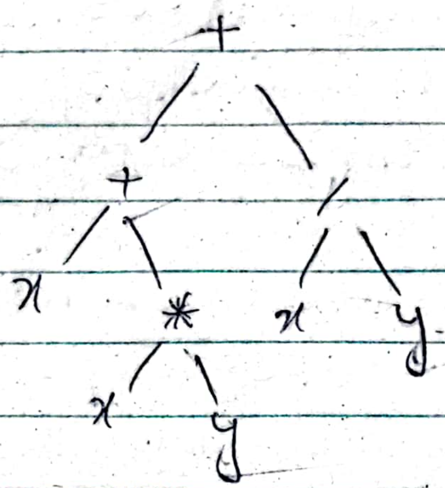


(ii)

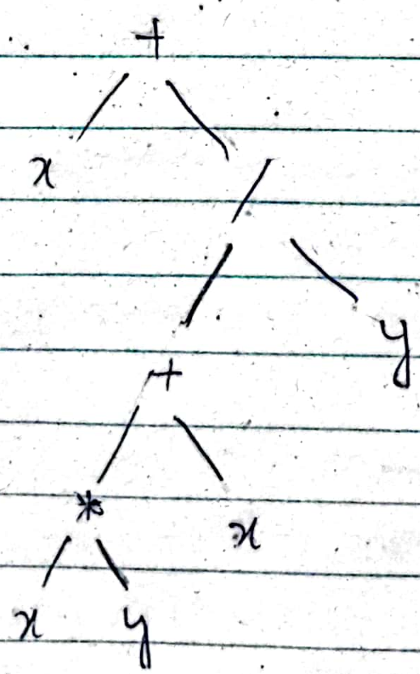


(g)

(i)



(ii)



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QUESTION # 21

(i) Preorder: a b e k l m f g n r s c d h o i j p q
Inorder: k e l m b f r n s g a c o h d i p j q
Postorder: k l m e f r s n g b c o h i p q j d a

(ii) Preorder: a b d e i j m n o c f g h k l p
Inorder: d b i e m j n o a f c g k h p l
Postorder: d i m n o j e b f g k p l h c a

QUESTION # 22

(a) no. of edges = 10000 - 1 = 9999

(b) no. of vertices = $mi + 1$

Here $m = 2$ as binary tree & $i = 1000$ (given)

no. of vertices = 2001

no. of edges = 2001 - 1

no. of edges = 2000

(c) no. of vertices = $mi + 1$

$m = 5$ as 5-m-ary tree , $i = 100$

no. of vertice = 5(100) + 1 = 501

~~no. of edges = no. of vertices - 1~~

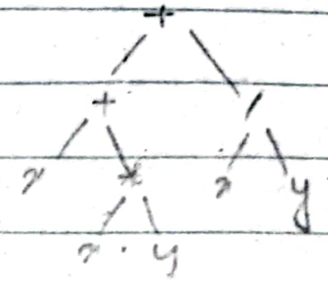
~~no. of edges = 500~~

QUESTION # 23

(a)

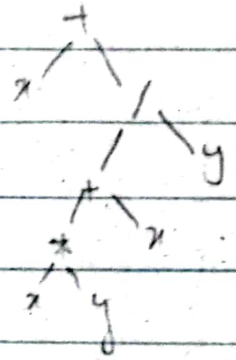
i) Prefix: $+ + x * x y / x y$

Postfix: $x x y * * x y / +$



ii) Prefix: $+ x / + * x y x y$

Postfix: $x x y * x + y / +$



(b)

(i) 4

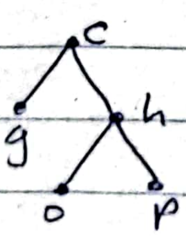
(ii) 3

QUESTION # 24

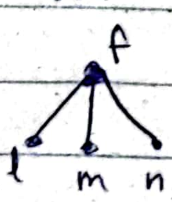
i) It is not a fully m-ary tree for any m bcz some of its internal nodes have 2 or 3 childrens.

ii) It is not a balanced m-ary tree bcz it has a leaf at level 2.

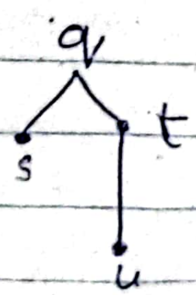
iii) (a)



(b)



(c)



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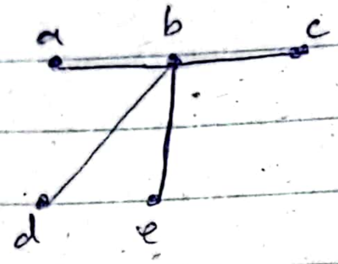
QUESTION # 25

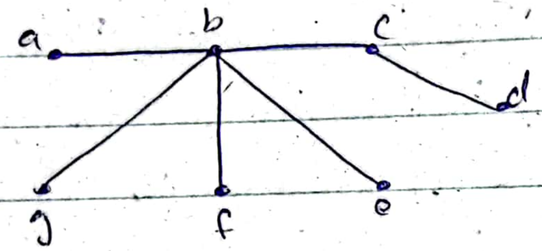
(i) (i)



(a,d) & (c,e) are removed

(ii)

(ii)

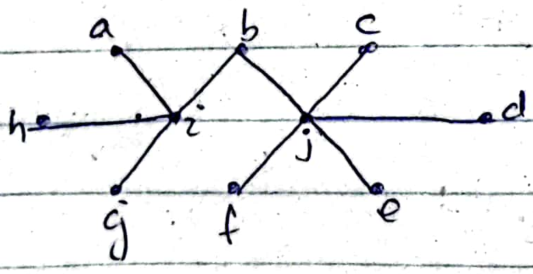


(a,g), (g,f), (f,e), (d,e), (c,e), (g,e), (g,d), (a,d)

(a) are removed.

(b)

(iii)

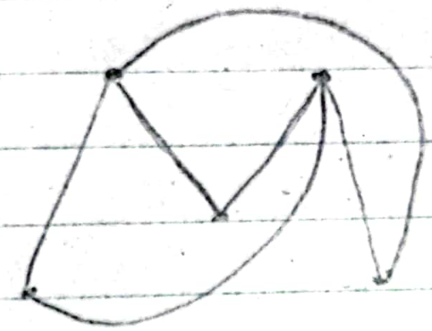


(a,b), (b,c), (c,d), (d,e), (e,f), (f,g), (g,h), (h,a)

(c) (i,j) are removed

QUESTION # 26

(a)



graph is planar

(b)

The graph is non-planar.

QUESTION # 27

R is Reflexive, $\rightarrow \{(a,a),(b,b),(c,c),(d,a)\} \in R$

R is not Symmetric $\rightarrow (a,c) \in R$ but $(c,a) \notin R$

R is not Antisymmetric $\rightarrow (b,c) \in R$ & $(c,b) \in R$

R is not Transitive $\rightarrow (a,d) \in R$, $(d,b) \in R$ but no (a,b)

R is not Irreflexive $\rightarrow (a,a) \in R$

R is not Assymmetric $\rightarrow (a,a) \in R$

QUESTION # 28

(a) $R = \{(0,0),(1,1),(2,2),(3,3)\}$

(b) $R = \{(1,3),(2,2),(3,1),(4,0)\}$

(c) $R = \{(1,0),(2,0),(2,1),(3,0),(3,1),(3,2),(4,0),(4,1)$
$(4,2),(4,3)\}$

(d) $R = \{(1,0),(2,0),(3,0),(4,0),(1,1),(1,2),(2,2),(1,3)$
$(3,3)$

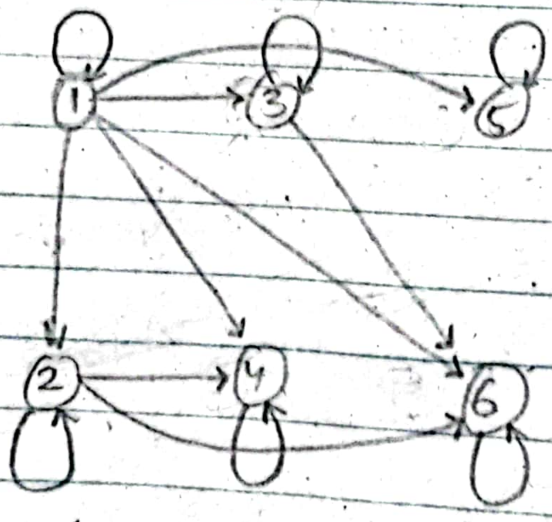
(e) $R = \{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(4,1),(4,3)$
$(0,1),\cancel{(0,2),(0,3)}\ (1,0)\}$

(f) $R = \{(1,2),(2,1),(2,2)\}$

Question # Q9

$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6)$
$(3,3),(3,6),(4,4),(5,5),(6,6)\}$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



QUESTION # 30

(a)

Reflexive: $(1,1) \notin R$ & $(4,4) \notin R$ → Not Reflex

Symmetric: $(2,4) \in R$ but $(4,2) \notin R$ → Not Symmetric

Antisymmetric: $(2,3) \in R \wedge (3,2) \in R$ & $3 \neq 2$ → Not Antisymmetric

Transitive: Yes Transitive.

(b)

Reflexive: Yes

Symmetric: Yes

AntiSymmetric: No because $(1,2) \in R$ & $(2,1) \in R$

Transitive: Yes

(c)

Reflexive: No

Symmetric: Yes bcz $(2,4) \in R$ & $(4,2) \in R$

AntiSymmetric: NO

Transitive: ~~Yes~~ Not bcz $(2,4)$ & $(4,2) \in R$ but $(2,2) \notin R$

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(d)

Reflexive: No
Symmetric: No bcz $(1,2) \in R$ but $(2,1) \notin R$
Antisymmetric: Yes bcz we have (a,b) but no (b,a)
Transitive: No $(1,2) \in R$ $(2,3) \in R$ but $(1,3) \notin R$

(e)

Reflexive: Yes
Symmetric: Yes
AntiSymmetric: Yes
Transitive: Yes

(f)

Reflexive: No
Symmetric: No $(1,4) \in R$ but $(4,1) \notin R$
Anti Symmetric: No → $(1,3) \in R$ & $(3,1) \in R$
transitive: $(1,3) \in R$ & $(3,1) \in R$ but $(1,1) \notin R$
No

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QUESTION # 31

(a)

Reflexive: No (a,a) ∈ R. means a taller than a. Not possibe

Symmetric: No (a,b) ∈ R ∧ (b,a) ∈ R means a taller than b & b taller than a not possible.

Antisymmetric: ~~Not~~ Yes if a>b & b>a than a=b

Transitive: Yes → a is taller than b & b taller than c. the c is taller than a.

Irreflexive: Yes

Assymmetric: Yes.

(b)

Reflexive : Yes → (a,a) ∈ R mean as ~~an~~ a & a are born on same day which is true.

Symmetric: Yes → If a and b born on sameday than b & a are also born on sameday

Antisymmetric: Not

Transitive : Yes → If a and b born on sameday, day a & c b born on same day then a & c also born on same day.

Irreflexive : No

Assymmetric : ~~Yes~~ No

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(c)

Reflexive: Yes → Every has the same first nam are themselves So (a,a) is True.

Symmetric: Yes → If a has first name as b. than b has first name as a.

Antisymmetric: No → as it allows (a,b) & (b,a).

Transitive: Yes → If a & b has same name, b &c has same name then definitely a &c has same name.

Assymetric: No as it is neither antisymmetric nor irreflexive.

Irreflexive: No

(d)

Reflexive: Yes → because a and a will have common grandparent.

Symmetric: Yes → If a & b have common grandparent then b &a have common grand parent.

Antisymmetric: No → If a &b have same grand parents and b &a have same grandparents than it is not necessary that a=b. a & b can be sibblings as well.

Transitive: ~~Y~~ NO → If a & b has same grandparent ~~and~~, b & c also have same grandparent ~~then~~ c & a

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Assymteric: NO ↳ As it is not antisymmetric & irreflexive

Irreflexive: NO

QUESTION # 32

(a) $A = \{1, 2, 3, 4\}$

$R = \{(a,b) \mid a = b\} \Rightarrow \{(1,1), (2,2), (3,3), (4,4)\}$

b) $A = \{1, 2, 3, 4\}$

$R = \{(1,3), (3,1), (2,4)\}$

↳ not symmetric bcz $(2,4) \in R$ but $(4,2) \notin R$

↳ not antisymmetric bcz $(1,3) \in R$ & $(3,1) \in R$

QUESTION # 33

$R_1 = \{(3,2), (3,1), (2,1)\}$

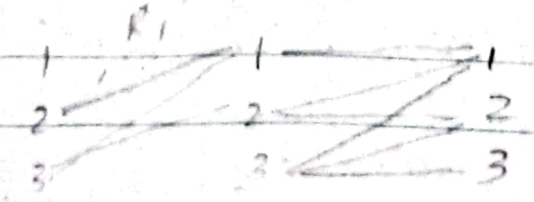
$R_2 = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$

$R_3 = \{(1,2), (1,3), (2,3)\}$

$R_4 = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

$R_5 = \{(1,1), (2,2), (3,3)\}$

$R_6 = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$



(2,1), (3,1), (3,2)

(a) $R2 \cup R4$

$R2 \cup R4 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

(b) $R3 \cup R6$

$R3 \cup R6 = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$

(c) $R3 \cap R6$

$R3 \cap R6 = \{(1,2), (1,3), (2,3)\}$

(d) $R4 \cap R6$

$R4 \cap R6 = \{(1,2), (1,3), (2,3)\}$

$R3 - R6$

$R_3 - R_6 = \{ \}$

$R6 - R3$

$R_6 - R_3 = \{(2,1), (3,1), (3,2)\}$

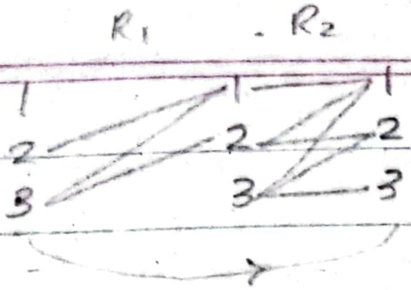
$R2 \oplus R6$

$R2 \oplus R_6 = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,2), (3,3)\}$

$R3 \oplus R5$

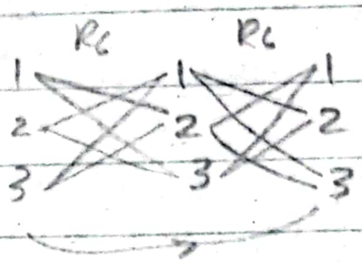
$R3 \oplus R_5 = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

4.87



R2 ∘ R1

$R_2 \circ R_1 = \{(2,1), (3,1), (3,2)\}$



R6 ∘ R6

$R_6 \circ R_6 = \{(1,1), (1,3), (1,2), (2,2), (2,3), (3,2), (3,3)\}$

QUESTION # 34(a)

(i) $\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{array}$ 3×3

(ii) $\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}$ 3×3

(iii) $\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \end{array}$ 3×3

(iv) $\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \end{array}$ 3×3

QUESTION # 34 (b)

(i) $R = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

(ii) $R = \{(1,2), (2,2), (3,2)\}$

(iii) $R = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$

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QUESTION # 35

(a)

Reflexivity: Yes → $l(a) = l(a)$, it follos that aRa for all strings a

Symmetric: Yes → 7 $l(a) = l(b)$ & $l(b) = l(a)$

Transitivity: Yes → if $l(a) = l(b)$ & $l(b) = l(c)$ the $l(c) = l(a)$ also holds.

part b at last page.

QUESTION # 36

(a)

(i) 1, 3, 7, 15, 32

(ii) 8.5, 7, 5.5, 4, 2.5

(iii) $-1, 1/4, -1/9, 1/16, -1/25$

(iv) $7, 10/3, 13/5, 16/7, 19/9$

(b)

(i) Arithemetic sequence

$a = -15$, $d = -7$

$a_n = a + (n-1)d$

$a_{11} = -15 + 10(-7) = -85$

QUESTION# 35(b)

Reflexivity ⇒ $a \equiv a \pmod m$ ∵ $a - a = 0$ is divisible by m

Symmetry ⇒ Suppose that $a \equiv b \pmod m$. Then $a-b$ is divisible by m, and so $a-b = km$, where k is an integer. It follows that $b-a = (-k)m$, so $b \equiv a \bmod (m)$.

Transivity ⇒ Suppose that $a \equiv b \pmod m$ and $b \equiv c \pmod m$. Then m divides both $a-b$ & $b-c$. Hence there are integers k & l with $a-b = km$ and $b-c = lm$.
Add both eqn,

$$a-c = (a-b)+(b-c) = km + lm$$

$$\therefore a-c = m(k+l)$$

Therefore $a \equiv c \pmod m$.

Therefore equivalence relation.

$4, 3, \frac{9}{4}$ $\quad \frac{9}{4} \div \frac{1}{3}$

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(ii) Arithemetic Sequence

$a = a - 42b \quad , d = 3b$

$a_{15} = a - 42b - 14(3b)$

$= a - 42b - 42b$

$\boxed{a_{15} = a}$

(iii) Geometric Sequence

$a = 4, \quad r = 3/4$

$a_{17} = ar^{n-1}$

$= 4(3/4)^{16}$

$\boxed{a_{17} = 0.040259}$

(iv) Geometric Sequence

$a = 32, \quad r = 1/2$

$a_9 = 32 \cdot (1/2)^8$

$\boxed{a_9 = 1/8}$

QUESTION # 37

(a)

(i) $T_3 = 10 \Rightarrow ar^2 = 10 \rightarrow ①$

$T_5 = 5/2 \rightarrow ar^4 = 5/2 \rightarrow ②$

Dividing both eqn

$$\frac{\cancel{a}r^4}{\cancel{a}r^2} = \frac{5/2}{10}$$

$$r^2 = \frac{1}{4}$$

$$r = \pm\frac{1}{2}$$

put 'r' in eq(1)

$$\text{eq(1)} \Rightarrow \quad a = \frac{10}{(1/2)^2}$$

$$\boxed{a = 40}$$

When $r = 1/2$

$40, 20, 10, 5, 5/2, \ldots$

When $r = -1/2$

$40, -20, -10, -5, -5/2, \ldots$

(ii) $T_5 = 8 \Rightarrow ar^4 = 8 \rightarrow ①$

$T_8 = 64/27 \Rightarrow ar^7 = -64/27.$

Dividing both eqn.

$r^3 = -8/27$

Taking cuberoot

$\boxed{r = -2/3}$

put in ①

$a = \dfrac{8}{(-2/3)^4} \Rightarrow \boxed{a = \dfrac{81}{2}}$

$81/2, -27, 18, -12, 8, -16/3, 32/9, -64/27, \ldots$

(b)

(i) $T_4 = 7 \Rightarrow a + 3d = 7 \rightarrow ①$

$T_{16} = 7 \Rightarrow a + 15d = 31 \rightarrow ②$

Subtracting eq ① & ②

$12d = 24 \Rightarrow \boxed{d = 2}$

$a + 3(2) = 7$

$\boxed{a = 1}$

$1, 3, 5, 7, 9, 11, 13, 15, \ldots$

(ii) $T_5 = 86 \Rightarrow a + 4d = 86 \rightarrow (1)$

$T_{10} = 146 \Rightarrow a + 9d = 146 \rightarrow (2)$

Subtracting

$5d = 60$

$\boxed{d = 12}$

eq(1) $\Rightarrow a + 4(12) = 86$

$\boxed{a = 38}$

$38, 50, 62, 74, 86, 98, \ldots$

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QUESTION # 38

(a)

We have a series,

$$259 + 266 + 273 + 280 + \cdots + 784$$

$a = 259$

$d = 7$

$T_n = 784$

For n:

$$T_n = a + (n-1)d$$

$$784 = 259 + (n-1)7$$

$$525 = (n-1)7$$

$$(n-1) = 75$$

$$\boxed{n = 76}$$

For S_n:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{76}{2}\{2(259) + (76-1)7\}$$

$$= \frac{76}{2}\{518 + 525\}$$

$$= \frac{76}{2}(1043)$$

$$\boxed{S_n = 39,634}$$

(b)

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}\{a + a + (n-1)d\}$

$S_n = \frac{n}{2}\{a + T_n\}$ $\quad$ T_n = last term

$a = \frac{1}{n}$, $T_n = \frac{n^2 - n + 1}{n}$

For d:

$T_n = a + (n-1)d$

$\frac{n^2 - n + 1}{n} = \frac{1}{n} + (n-1)d$

Multiply whole eqn by n

$n^2 - n + \cancel{1} = \cancel{1} + (n^2 - n)d$

$\cancel{(n^2 - n)} = \cancel{(n^2 - n)}d$

$\boxed{d = 1}$

$$\boxed{S_n = \frac{n^2 - n + 2}{2}}$$

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QUESTION # 39

(a)

$$\sum_{j=1}^{100} (1/j)$$

(b)

(i) $1+(-1)+1+(-1)+1 = 1$

(ii) $1+4+9+16+25 = 55$

QUESTION # 40

(i) $a_n = -2a_{n-1}\ ,\ a_0 = -1$

$a_1 = 2$

$a_2 = -4$

$a_3 = 8$

$a_4 = -16$

$a_5 = 32$

$a_6 = -64$

So,

$2, -4, 8, -16, 32, -64$

(b) $a_n = a_{n-1} + a_{n-2}$, $a_0 = 2, a = -1$

$a_2 = -3$

$a_3 = -2$

$a_4 = 1$

$a_5 = 3$

$a_6 = 2$

So,

$2, -1, -3, -2, 1, 3, 2,$

(c) $a_n = 3a_{n-1}^2$, $a_0 = 1$

$a_1 = 3$

$a_2 = 27$

$a_3 = 2187$

$a_4 = 14348907$

$a_5 = 617673396283947$

(d) $a_n = na_{n-1} + a_{n-2}^2$, $a_0 = -1$, $a_1 = 0$

$a_2 = + (-1)^2 = 1$

$a_3 = 3(1) + (0)^2 = 3$

$a_4 = 4(3) + (1)^2 = 13$

$a_5 = 5(13) + (3)^2 = 74$

$a_6 = 6(74) + (13)^2 = 613$

So,

$-1, 0, 1, 3, 13, 74, 613$

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QUESTION # 41

Propositional Logic:

→ Computer Programming ⇒ ~~to make~~ create decision making statements & conditions.

→ Artifical Intelligence ⇒ used to represent knowledge and reasoning.

Predicates & Quantifiers:

→ Database Querying ⇒ to retrieve ~~data~~ specific data from database.

→ Mathmatecal Proofs ⇒ essential for constructing proofs

Sets:

→ DataScience ⇒ for task like data dedeplication and clustering.

→ Venn Diagram ⇒ widely used in business and marketing to understand customer segments and market intersections.

Functions:

→ Engineering & Physics ⇒ functions are used to model and predict physical phenomena.

→ Economics ⇒ In economics, they are used to model supply & demand curve.

Relations:

→ Social Networks ⇒ used to represent connections b/w individual or entities

→ Database Managment ⇒ They define structure & connection between tables, allowing for efficent ~~data~~ storage.

Sequence & Series:

→ Financial Planning ⇒ for calculations related to investments, loans & retirement savings.

→ Statistics ⇒ employed in time series anaylsis ~~mode~~ to model and predict trends in data, such as stock prices, whether patterns etc.

Graph Theory:

→ Transportation Network ⇒ used in designing transportation network such as road & railway systems.

→ Social Network Recommender System ⇒ used to identify connection b/w users & suggest relevant content.

Trees:

→ Computer Science ⇒ used in data structure, such as binary Tree, decision tree for searching & sorting algo.

→ Evolutionary Biology ⇒ Phylogenetic trees are used in this field to represent relationships.