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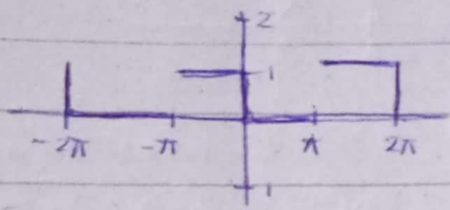
ASSIGNMENT #03

—(ACTIVITY QUESTIONS)—

< Q#01 >

$$f(x) = \begin{cases} 1 & -\pi < x < 0 \\ 0 & 0 < x < \pi \end{cases}$$

(a)



$P = \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \rightarrow (1)$$

For a_0:

$$a_0 = \frac{1}{P} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} dx$$

$$= \frac{1}{\pi} \left[x \right]_{-\pi}^{0}$$

$$\boxed{a_0 = 1}$$

For a_n:

$$a_n = \frac{1}{P} \int_{-\pi}^{\pi} f(x) \cos nx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{0} \cos nx \, dx$$

$$= \frac{1}{\pi} \left[\frac{\sin nx}{n} \right]_{-\pi}^{0}$$

$$\boxed{a_n = 0}$$

For b_n:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} \sin nx \, dx$$

$$= \frac{1}{\pi} \left[\frac{-1}{n} \cos nx \right]_{-\pi}^{0}$$

$$b_n = \frac{1}{\pi}\left[\frac{-1}{n} + \frac{(-1)^n}{n}\right]$$

$$\boxed{b_n = \frac{1}{\pi n}\left[(-1)^n - 1\right]}$$

Now using values in eq ①

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{\pi n}\left[(-1)^n - 1\right] \sin nx$$

(b) Putting values of 'n' we get

$$f(x) = \frac{1}{2} - \frac{2}{\pi}\sin x - \frac{2}{3\pi}\sin 3x - \frac{2}{5\pi}\sin 5x - \frac{2}{7\pi}\sin 7x + \cdots]$$

$$f(x) = \frac{1}{2} - \frac{2}{\pi}\left[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \cdots\right] \to ②$$

~~Now put $x = \pi/2$~~

$f(\pi/2) = 0$

(C) now puting $x = \pi/2$ in eq ②

$$0 = \frac{1}{2} - \frac{2}{\pi}\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\right]$$

$$\cancel{-}\frac{1}{2} = \cancel{-}\frac{2}{\pi}\left[1 - 1/3 + 1/5 - 1/7 + 1/9 - \cdots\right]$$

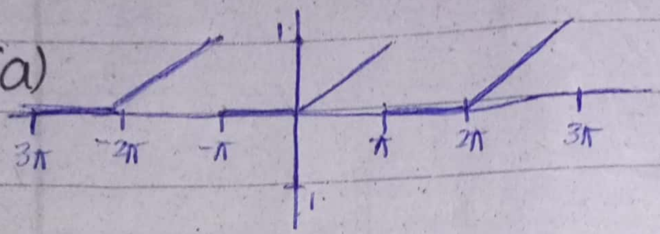
$$\boxed{\pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9 - \cdots\cdots}$$

Proved!

<Q#02>

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

(a)



$P = \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

For a_0:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} x\, dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2}\right]$$

$$\boxed{a_0 = \frac{\pi}{2}}$$

For a_n:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} x \cos nx\, dx$$

$$= \frac{1}{\pi}\left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$\boxed{a_n = \frac{1}{n^2\pi}\left((-1)^n - 1\right)}$$

For b_n:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} x \sin nx \, dx$$

$$= \frac{1}{\pi}\left[-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi(-1)^n}{n}\right]$$

$$\boxed{b_n = \frac{(-1)^{n+1}}{n}}$$

putting values in eq ①

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{1}{n^2\pi}\left[(-1)^n - 1\right]\cos nx + \frac{(-1)^{n+1}}{n}\sin nx\right)$$

(b) putting values of n,

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right] +$$

$$\left[\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + \cdots\right] \rightarrow ②$$

(c) $f(0) = \frac{f(0^+) + f(0^-)}{2} = \frac{x+0}{2} = \frac{x}{2} = \frac{(0)}{2} = 0$

putting $x=0$ in eq ②

$$0 = \frac{\pi}{4} - \frac{2}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right]$$

$$\cancel{-}\frac{\pi}{4} = \cancel{-}\frac{2}{\pi}\left[1 + 1/3^2 + 1/5^2 + 1/7^2 + \cdots\right]$$

$$\boxed{\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots}$$

Proved!

$f(\pi/2) = \pi/2$

$$\pi/2 = \frac{\pi}{4} + 1 - 0 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$$

$$\frac{\pi}{2} - \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Hence proved!

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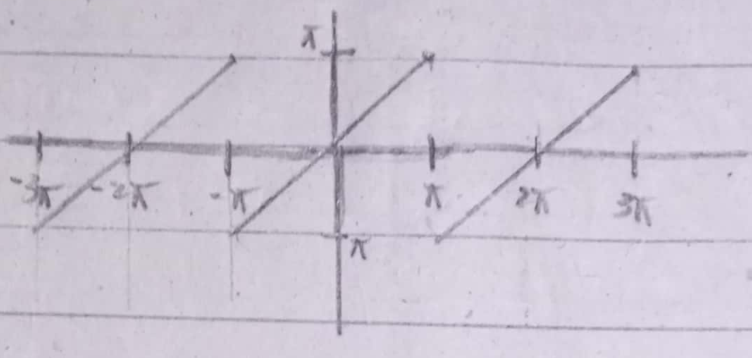
<(Q#06)>

$f(x) = x \quad -\pi < x < \pi$



$p = \pi$

$f(x) = a_0/2 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \rightarrow (1)$

For a_0 :

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \, dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{\pi}$$

$$\boxed{a_0 = 0}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^2}{2}\right]$$

For a_n :

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{x\cancel{\sin nx}}{n} + \frac{\cos nx}{n^2}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\cancel{\frac{(-1)^n}{n^2}} - \cancel{\frac{(-1)}{n^2}}\right]$$

$$\boxed{a_n = 0}$$

For b_n :

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx$$

$$= \frac{1}{\pi}\left[\frac{-x\cos nx}{n} + \cancel{\frac{\sin nx}{n^2}}\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi(-1)^n}{n} - \frac{\pi(-1)^n}{n}\right]$$

$$= \frac{1}{\cancel{\pi}}\left[\frac{-2\cancel{\pi}(-1)^n}{n}\right]$$

$$\boxed{b_n = \frac{2(-1)^{n+1}}{n}}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n}\sin nx$$

(b) putting values of n,

$$f(x) = 2\sin x - \frac{2}{2}\sin 2x + \frac{2}{3}\sin 3x + \cdots$$

$$= 2\left[\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right] \hookrightarrow (2)$$

(c) $f(\pi/2) = \pi/2$

putting $x = \pi/2$ in eq (2)

$$\frac{\pi}{2} = 2\left[1 - \cancel{\frac{0}{2}} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right]$$

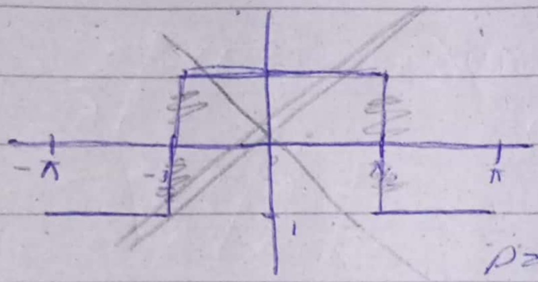
$$\boxed{\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots}$$

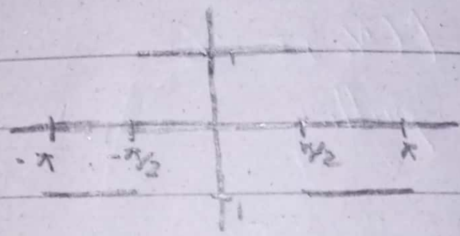
Proved.

(Assignmen Questions)

<Q#01>

$$f(x) = \begin{cases} -1 & -\pi < x < -\pi/2 \\ 1 & -\pi/2 < x < \pi/2 \\ -1 & \pi/2 < x < \pi \end{cases}$$





$p = \pi$

As the function is even so Cosine series,

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

For a_0:

$$a_0 = \frac{2}{p}\int_0^p f(x)\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} dx - \frac{2}{\pi}\int_{\pi/2}^{\pi} dx\right]$$

$$= \frac{2}{\pi}\left[x\Big|_0^{\pi/2} - x\Big|_{\pi/2}^{\pi}\right]$$

$$= \frac{2}{\pi}\cdot\frac{\pi}{2} - \frac{2\pi}{\pi} + \frac{2}{\pi}\cdot\frac{\pi}{2}$$

$$= 1 - 2 + 1 \qquad \boxed{a_0 = 0}$$

q_n ...

For a_n :

$$a_n = \frac{2}{P}\int_0^P f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2}\cos nx\,dx - \int_{\pi/2}^{\pi}\cos nx\,dx\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{n}\sin nx\Big|_0^{\pi/2} - \frac{1}{n}\sin nx\Big|_{\pi/2}^{\pi}\right]$$

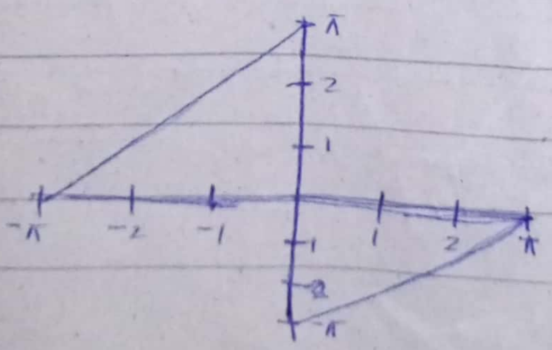
$$= \frac{2}{n\pi}\sin\frac{n\pi}{2} + \frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$$\boxed{a_n = \frac{4}{n\pi}\sin\frac{n\pi}{2}}$$

$$\boxed{f(x) = \sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\cos nx}$$

<Q#02>

$$f(t) = \begin{cases} t+\pi & -\pi < t < 0 \\ t-\pi & 0 < t < \pi \end{cases}$$



as the function is odd So Sine Series

$$\therefore \quad f(x) = \sum_{n=1}^{\infty} b_n \sin nt$$

$$b_n = \frac{2}{p}\int_0^p f(t)\sin nx \, dt$$

$$= \frac{2}{\pi}\int_0^\pi (t-\pi)\sin nt \, dt$$

$$= \frac{2}{\pi}\left[-\frac{(t-\pi)\cos nt}{n} + \cancel{\frac{\sin nt}{n^2}}^{\,0}\right]_0^\pi$$

$$= -\frac{2}{\pi}\left[\frac{\pi}{n}\right]$$

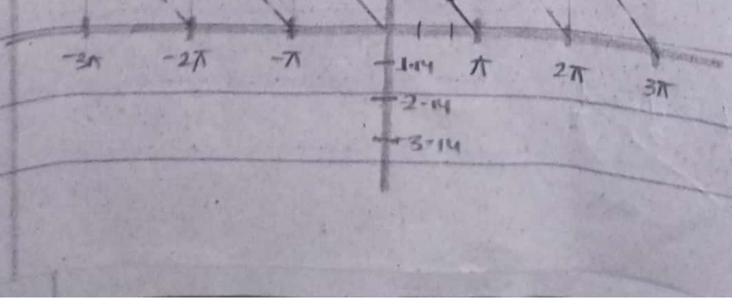
$$\boxed{b_n = \frac{-2}{n}}$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{-2}{n}\sin nt}$$

<Q#03>

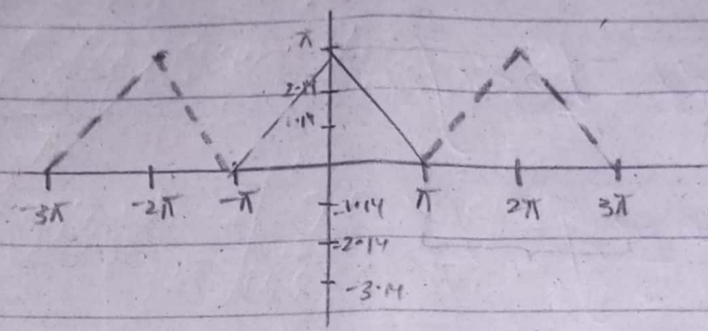
$$f(x) = \pi - x \; ; \quad 0 < x < \pi$$

(a) (i)

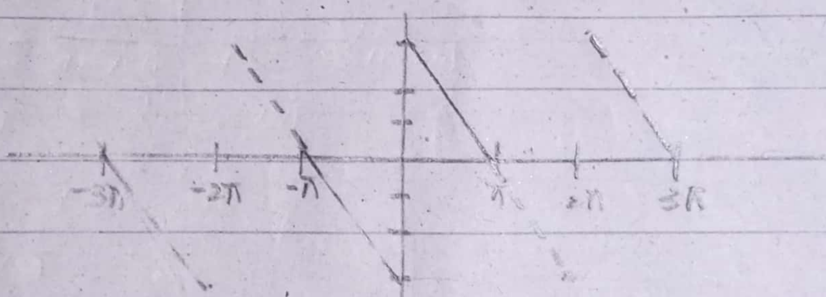


even

(iii) ~~odd~~ 2π periodic extension



(iii) ODD 2π periodic extension



(b) FOURIER COSINE SERIES:-

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x$$

$L = \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

FOR a_0:

$$a_0 = \frac{2}{P} \int_0^P f(x)\, dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}(\pi - x)\,dx$$

$$= \frac{2}{\pi}\left[\pi\int_0^{\pi} dx - \int_0^{\pi} x\,dx\right]$$

$$= \frac{2}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi^2 - \frac{\pi^2}{2} - 0\right]$$

$$= \frac{\cancel{2}}{\pi}\cdot\frac{\pi^2}{\cancel{2}}$$

$$\boxed{a_0 = \pi}$$

For a_n:

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}(\pi - x)\cos nx\,dx$$

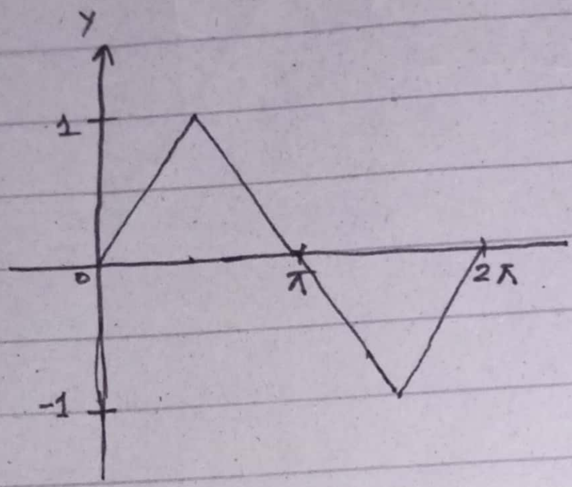
$$= \frac{2}{\pi}\left[\frac{(\pi - x)\sin nx}{n} - \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\frac{(-1)^n}{n^2} + \frac{1}{n^2}\right]$$

$$\boxed{a_n = \frac{2\left[(-1)^{n+1} + 1\right]}{n^2\pi}}$$

$$\boxed{f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{n^2\pi}\left[(-1)^{n+1} + 1\right]\cos nx}$$

<Q# 04>



$1 - \frac{2x}{\pi} + \frac{2}{\pi}\frac{\pi}{2}$

$2 -$

$$f(x) = \begin{cases} 2x/\pi & , 0 < x < \pi/2 \\ 2 - \frac{2x}{\pi} & , \pi/2 < x < \pi \end{cases}$$

$L = \pi$

∵ it is an odd function so Sine Series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{L}\int_0^L f(x) \sin nx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \frac{2x}{\pi} \sin nx + \int_{\pi/2}^{\pi} \left(2 - \frac{2x}{\pi}\right) \sin nx \right]$$

$$b_n = \frac{2}{\pi}\left[\frac{2}{\pi}\left(-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right)\Bigg|_0^{\pi/2} - \left(2-\frac{2x}{\pi}\right)\frac{\cos nx}{n}\Bigg|_{\pi/2}^{\pi}\right.$$

$$\left. - \frac{2}{n^2\pi}\sin nx\Bigg|_{\pi/2}^{\pi}\right]$$

$$= \frac{2}{\pi}\left[\cancel{-\frac{2}{\pi}\cdot\frac{\pi}{2}\cdot\frac{1}{n}\cos\frac{n\pi}{2}} + \frac{2}{n^2\pi}\sin\frac{n\pi}{2} \cancel{+\frac{1}{n}\cos\frac{n\pi}{2}}\right.$$

$$\left. + \frac{2}{n^2\pi}\sin\frac{n\pi}{2}\right]$$

$$= \frac{2}{\pi}\left[\frac{4}{n^2\pi}\sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$f(x) = \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}\sin nx$$

putting values of n, and $x = \theta$

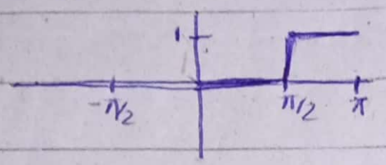
$$f(x) = \frac{8}{\pi^2}\left[\sin\theta - \frac{1}{3^2}\sin 3\theta + \frac{1}{5^2}\sin 5\theta - \cdots\right]$$

Hence Proved

<Q#05>

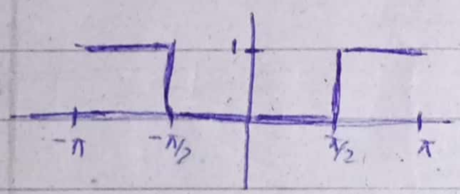
$$f(t) = \begin{cases} 0 & ; 0 < t < \pi/2 \\ 1 & , \pi/2 < t < \pi \end{cases}$$

$L = \pi$



$L = \pi$

For cosine series:



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

For a_0:

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_{\pi/2}^{\pi} dx \quad \because \int_{\pi/2}^{0} 0\,dx \to 0$$

$$= \frac{2}{\pi}[x]_{\pi/2}^{\pi} \qquad \boxed{a_0 = 1}$$

~~a_n~~

For a_n:

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

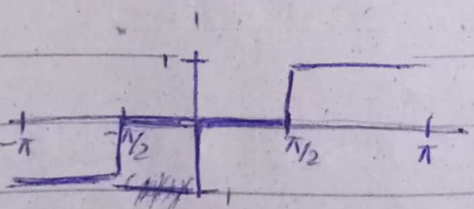
$$= \frac{2}{\pi}\left[\cancel{\int_0^{\pi/2} 0\cdot\cos nx\,dx} + \int_{\pi/2}^{\pi}\cos nx\,dx\right]$$

$$= \frac{2}{\pi}\left[\frac{\sin nx}{n}\right]_{\pi/2}^{\pi}$$

$$\boxed{a_n = \frac{-2}{n\pi}\sin\frac{n\pi}{2}}$$

$$\boxed{f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{-2}{n\pi}\sin\frac{n\pi}{2}\cos nx}$$

For Sine Series:



$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{l}\int_0^{l} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{2}{\pi}\int_{\pi/2}^{\pi}\sin nx\,dx$$

$$= \frac{2}{\pi}\left[\frac{-\cos nx}{n}\right]_{\pi/2}^{\pi}$$

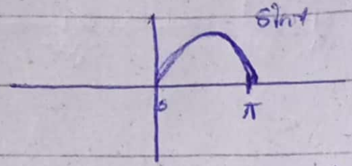
$$= \frac{2}{\pi}\left[-\frac{(-1)^n}{n} + \cos\frac{n\pi}{2}\right]$$

$$\boxed{b_n = \frac{2}{\pi}\left[\frac{(-1)^{n+1}}{n} + \cos\frac{n\pi}{2}\right]}$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi}\left[(-1)^{n+1} + \cos\frac{n\pi}{2}\right] \sin nx}$$

(Q#06)

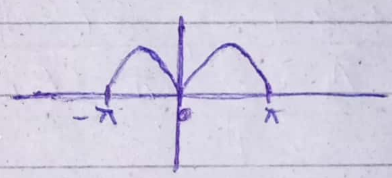
$f(x) = \sin x \qquad 0 < x < \pi$



$L = \pi$

For Cosine Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$



For a_0:

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin x\,dx$$

$$= \frac{2}{\pi}\left[-\cos x\right]_0^{\pi}$$

$$= \frac{-2}{\pi}\left[-1-1\right]$$

$$\boxed{a_0 = \frac{4}{\pi}}$$

For a_n:

$$a_n = \frac{2}{L}\int_0^L f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin nx \cos nx\,dx$$

$$\because \sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$= \frac{2}{\pi}\cdot\frac{1}{2}\left[\int_0^{\pi}\sin(1+n) + \int_0^{\pi}\sin(1-n)\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{-1}{1+n}\cos(1+n)x - \frac{1}{(1-n)}\cos(1-n)x\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{(-1)^n}{1+n} + \frac{(-1)^n}{1-n} + \frac{1}{1+n} - \frac{1}{1-n}\right]$$

$$= \frac{1}{\pi}\left[\frac{(1-n)(-1)^n + (1+n)(-1)^n}{(1+n)(1-n)} + 1+n+1-n\right]$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n + (-1)^n + 2}{(1-n^2)}\right]$$

$$\boxed{a_n = \frac{2[(-1)^n + 1]}{\pi(1-n^2)}}$$

For a_1:

$$a_1 = \frac{2}{\pi}\int_0^{\pi} \sin(1)x \cos(1)x \, dx$$

$$a_1 = \frac{2}{\pi}\int \frac{1}{2} \sin 2x$$

$$= \frac{1}{\pi}\int \sin 2x \, dx$$

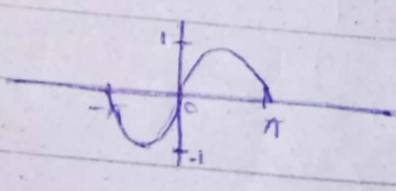
$$= \frac{1}{2\pi}\int \sin 2x$$

$$= \frac{1}{2\pi}\left[-\cos 2x\right]_0^{\pi}$$

$$\boxed{a_1 = 0}$$

$$\boxed{f(x) = \frac{2}{\pi} + \sum_{n=2}^{\infty} \frac{2[(-1)^n + 1]}{\pi(1-n^2)} \cos nx}$$

For Sine Series:



$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{L}\int_0^{L} f(x) \sin nx$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin x \sin nx \, dx$$

$$= \frac{1}{\pi} \quad \because \sin\alpha \sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} \cos(1-n)x \, dx - \int_0^{\pi} \cos(1+n)x \, dx\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{1}{1-n} \sin(1-n)x - \frac{1}{1+n} \sin(1+n)x\right]_0^{\pi} = 0$$

$$\boxed{f(x) = 0}$$

< Q#07 >

$$f(x) = \begin{cases} x+\pi & -\pi < x < 0 \\ \pi - x & 0 < x < \pi \end{cases}$$

fourier series,

$$f(x) = \frac{\pi}{2} + \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{\cos(2m-1)x}{(2m-1)^2},$$

$$f(0) = \frac{f(0^+)+f(0^-)}{2} = \frac{\pi+\pi}{2} = \frac{2\pi}{2} = \pi$$

$\therefore$

$$\pi = \frac{\pi}{2} + \frac{4}{\pi}\left[\frac{\cos(0)}{(2(1)-1)^2} + \frac{\cos(0)}{(2(2)-1)^2} + \frac{\cos(0)}{(2(3)-1)^2} + \cdots\right]$$

$$\frac{\pi}{2} = \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

$\boxed{\text{Sum is } \frac{\pi^2}{8}}$

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