# AI-503 Advanced Machine Learning

## Up until now...

- Classification
  - Predict discrete classes
- Regression
  - Predict continuous targets
- Feature Extraction/ Selection
  - Dimensionality reduction
- Clustering
  - Group data into clusters

## Ranking

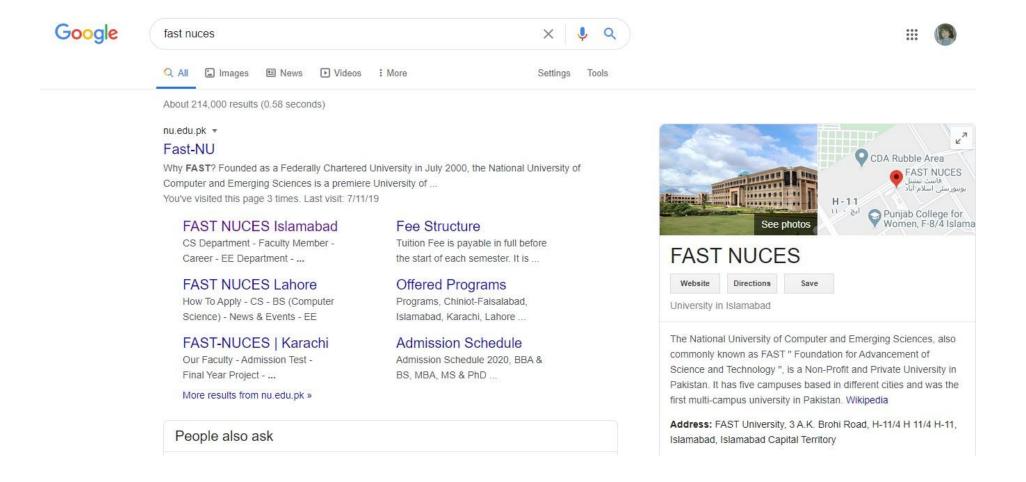
- Rank a set of examples
  - The example with a higher true rank produces a higher score as compared to the other examples
- Examples...

## Recommendation systems

- Netflix
- YouTube
- Amazon
- Facebook
- Twitter
- Journal recommendations

• ...

#### Information retrieval



## Drug discovery

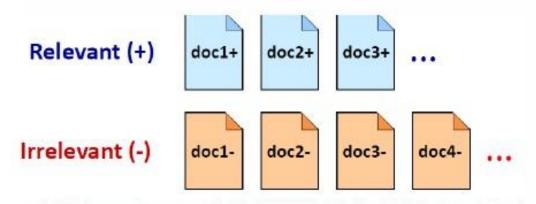
- Rank the given set of drugs in the decreasing order of their potency against a disease
- Test the most promising ones in the lab

## So how to develop models than can rank?

- SVMs
- Neural Networks
- Tree based methods

• ...

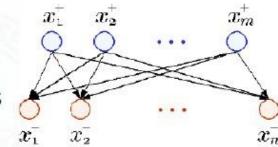
## Bipartite ranking



- Instance space: X
- Input:  $S = \{S_+, S_-\}$   $-S_+ = \{x_1^+, ..., x_m^+\}$  (positive examples)  $-S_- = \{x_1^-, ..., x_n^-\}$  (negative examples)
- Output: Ranking function  $f: X \to \mathbb{R}$

## Bipartite ranking vs classification & regression

- In classification we require
  - f(x) > 0 for positive examples
  - f(x) < 0 for negative examples



- In regression
  - f(x) should be as close as possible to the corresponding output
- The ranking function should be such that
  - f(x) > f(x') if instance x should rank higher than x'
    - Any given positive instance should rank higher than any negative instance

## Definition of error in ranking

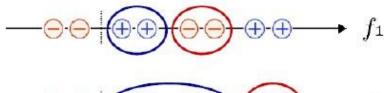
- If we have m positive instances and n negative instances, the number of possible 'pairings' becomes mn
  - Thus, the error for a given ranking function will be given by:

$$\widehat{\mathbf{er}}_S(f) = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n 1(f(x_i^+) < f(x_j^-))$$

### Ranking error

 What will be the classification and ranking errors of the following ranking functions?

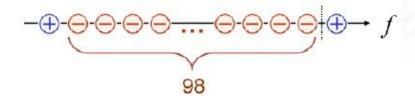
#### Example 1



Classification error = 
$$\frac{1}{4}$$
  
Ranking error =  $\frac{1}{4}$ 

Classification error = 
$$\frac{1}{4}$$
  
Ranking error =  $\frac{1}{2}$ 

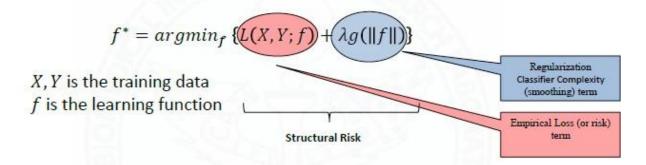
#### Example 2



Classification error = 
$$\frac{1}{100}$$
  
Ranking error =  $\frac{1}{2}$ 

## Ranking with SRM

SRM requires



 For ranking, we know the empirical error can be given by

$$\widehat{\mathbf{er}}_S(f) = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n \mathbf{1}(f(x_i^+) < f(x_j^-))$$

#### Ranking with SRM

 Find me a ranking function that minimizes the empirical error, possibly with some regularization, over some class of ranking functions

- Examples: 
$$f(x) = \langle w, x \rangle$$

Mathematically

$$\min_{f \in \mathcal{F}} \left[ \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell(f, x_i^+, x_j^-) + \lambda N(f) \right]$$

 $\ell(f, x_i^+, x_j^-)$  : convex upper bound on  $\mathbf{1}(f(x_i^+) < f(x_j^-))$ 

N(f) : regularizer

 $\lambda > 0$  : regularization parameter

 ${\mathcal F}$  : class of ranking functions

#### Ranking with SRM

- What should be loss function?
  - The ideal loss function would be:
    - Zero-one loss

$$1(f(x_i^+) < f(x_j^-))$$

- Incurs a loss of 1 only if the ranking is not proper
- Otherwise the loss is zero
- Problems?
  - » Non-convex
  - » Discontinous
- Hinge-Loss

$$\ell_{\text{hinge}}(f, x_i^+, x_j^-) = \left(1 - \left(f(x_i^+) - f(x_j^-)\right)\right)_+ \left[u_+ = \max(u, 0)\right]$$

### Bipartite Rank-SVM

$$min_{w,\xi>0} \lambda ||w||^2 + \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n \xi_{ij}$$

Such that, for all i = 1...m and j = 1...n

$$f(x_i) \ge f(x_j) + 1 - \xi_{ij}$$

#### K-partite ranking

Input: Training sample 
$$S=(S_1,S_2,\ldots,S_k)$$
:
$$S_k=(x_1^k,\ldots,x_{n_k}^k)\in X^{n_k} \quad \text{(examples of rating }k)$$

$$\vdots$$

$$S_2=(x_1^2,\ldots,x_{n_2}^2)\in X^{n_2} \quad \text{(examples of rating 2)}$$

$$S_1=(x_1^1,\ldots,x_{n_1}^1)\in X^{n_1} \quad \text{(examples of rating 1)}$$

$$Rating k$$

$$\vdots$$

$$Rating 2$$

$$doc1^k \quad doc2^k \quad doc3^k \quad \ldots$$

$$\vdots$$

$$Rating 2$$

$$doc1^2 \quad doc2^2 \quad doc3^2 \quad doc4^2 \quad \ldots$$

Output: Ranking function  $f: X \rightarrow \mathbb{R}$ 

#### Empirical error:

$$\widehat{\text{er}}_{S}(f) = \left(\frac{1}{\sum_{1 \le a < b \le k} n_{a} n_{b}}\right) \sum_{1 \le a < b \le k} \sum_{i=1}^{n_{b}} \sum_{j=1}^{n_{a}} (b - a) \, \mathbf{1}(f(x_{i}^{b}) < f(x_{j}^{a}))$$

#### K-partite Ranking with SRM

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

$$\min_{f \in \mathcal{F}} \left[ \left( \frac{1}{\sum_{1 \le a < b \le k} n_a n_b} \right) \sum_{1 \le a < b \le k} \sum_{i=1}^{n_b} \sum_{j=1}^{n_a} \ell(f, x_i^b, x_j^a, (b-a)) + \lambda N(f) \right]$$

where

 $\ell(f, x_i^b, x_j^a, (b-a))$  : convex upper bound on  $(b-a) \, 1(f(x_i^b) < f(x_j^a))$ 

N(f): regularizer

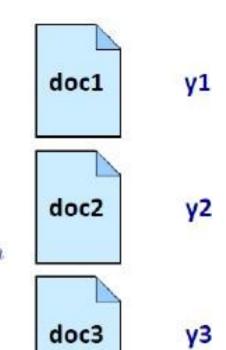
 $\lambda > 0$ : regularization parameter

F: class of ranking functions

## Ranking with Real-Valued Labels

- Instance space X
- ightharpoonup Real-valued labels  $Y = \mathbb{R}$
- ▶ Input: Training sample  $S = ((x_1, y_1), \ldots, (x_m, y_m)) \in (X \times \mathbb{R})^m$
- ▶ Output: Ranking function  $f: X \rightarrow \mathbb{R}$
- Empirical error:

$$\widehat{\text{er}}_S(f) = \frac{1}{\binom{m}{2}} \sum_{1 \le i < j \le m} |y_i - y_j| \, 1 \left( (y_i - y_j)(f(x_i) - f(x_j)) < 0 \right)$$



#### Real-Valued Ranking with SRM

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

$$\min_{f \in \mathcal{F}} \left[ \frac{1}{\binom{m}{2}} \sum_{1 \leq i < j \leq m} \ell(f, (x_i, y_i), (x_j, y_j)) \, + \, \lambda N(f) \right]$$

where

 $\ell(f,(x_i,y_i),(x_j,y_j))$  : convex upper bound on  $|y_i-y_j|\,\mathbf{1}\left((y_i-y_j)(f(x_i)-f(x_j))<0\right)$  N(f) : regularizer

 $\lambda > 0$ : regularization parameter

 $\mathcal{F}$ : class of ranking functions

### Example

k=3 relevance levels (1 = low, 2 = medium, 3 = high)

#### Each class has 2 documents:

#### Data

 $S_1$  (Rating 1):  $x_1^1, x_2^1$ 

 $S_2$  (Rating 2):  $x_1^2, x_2^2$ 

 $S_3$  (Rating 3):  $x_1^3, x_2^3$ 

Document	Rating	Score $f(x)$
$x_1^1$	1	1.0
$x_2^1$	1	2.0
$x_1^2$	2	2.5
$x_2^2$	2	2.2
$x_1^3$	3	2.4
$x_2^3$	3	1.5

- Ranking SVM maximizes the AUC score
  - Because it ensures that positive examples rank higher than negative examples
    - Ulf Brefeld, Tobias Scheffer. n.d. "AUC Maximizing Support Vector Learning."

#### SVMRank Tools



#### SVMrank



#### Support Vector Machine for Ranking

Author: Thorsten Joachums - thorsten@joachums.org -Cornell University
Department of Computer Science

> Version: 1.00 Date: 21.03.2009

#### Overview

#### Source Code

The program is free for scientific use. Please contact use, if you are planning to use the software for commercial purposes. The software must not be further distributed without prior permission of the author. If you use SVM\*\*\* in our scientific work, please cite as

T. Joachims, Training Linear SVMs in Linear Time, Proceedings of the ACM Conference on Knowledge Discovery and Data Mining (KDD), 2006. [Postscript (g2)] [PDF]

The implementation was developed on Linux with got, but compiles also on Solaris, Cygain. Windows (using MinGW) and Mac (after small modifications, see FAC). The source code is available at the following location:

http://download.joachims.org/sym\_rank/current/sym\_rank/tac.gz

https://www.cs.cornell.edu/people/tj/svm\_light/svm\_rank.html https://sourceforge.net/p/lemur/wiki/RankLib/ (Java)

22

### Other methods forranking

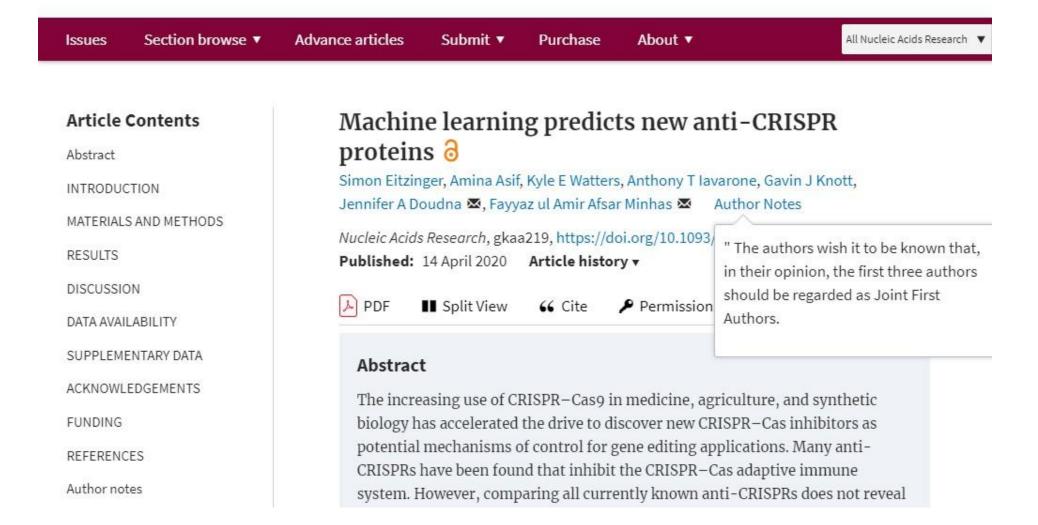
• <a href="https://en.wikipedia.org/wiki/Learning\_to\_rank#List\_of\_methods">https://en.wikipedia.org/wiki/Learning\_to\_rank#List\_of\_methods</a>

#### Issues

- The number of constraints becomes quadratic
- Efficient Algorithms Required

## Ranking ML case study

#### **Nucleic Acids Research**



#### References