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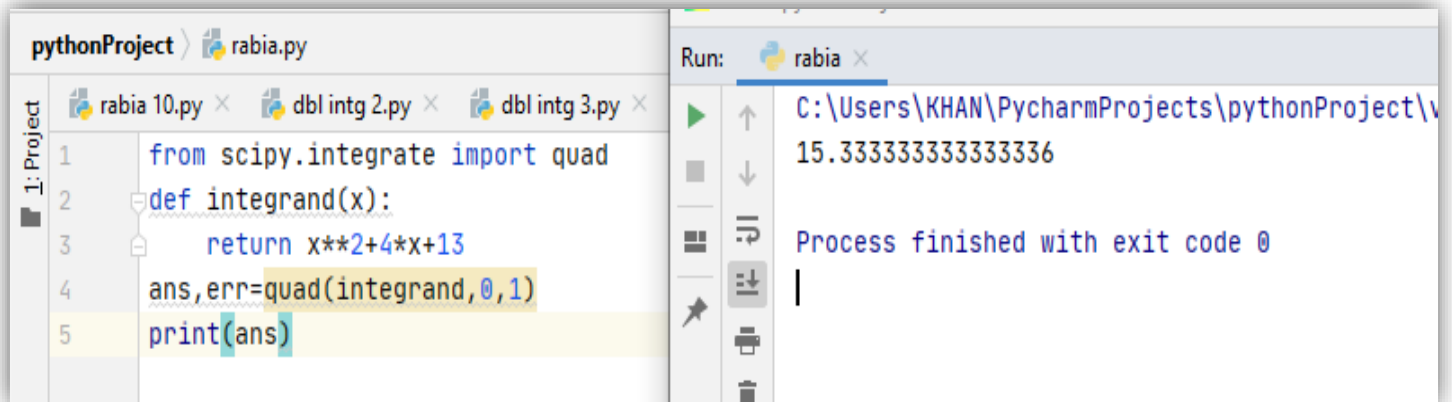
Department : mathematics

Teacher : Syed Umaid Ahmad

19-sep-2020

Simple Integration Question:

Question 1:



The screenshot shows a Python IDE with a file named `rabia.py`. The code defines a function `integrand(x)` that returns $x^2 + 4x + 13$ and then uses `quad` to integrate it from 0 to 1. The output of the script is displayed in the Run window.

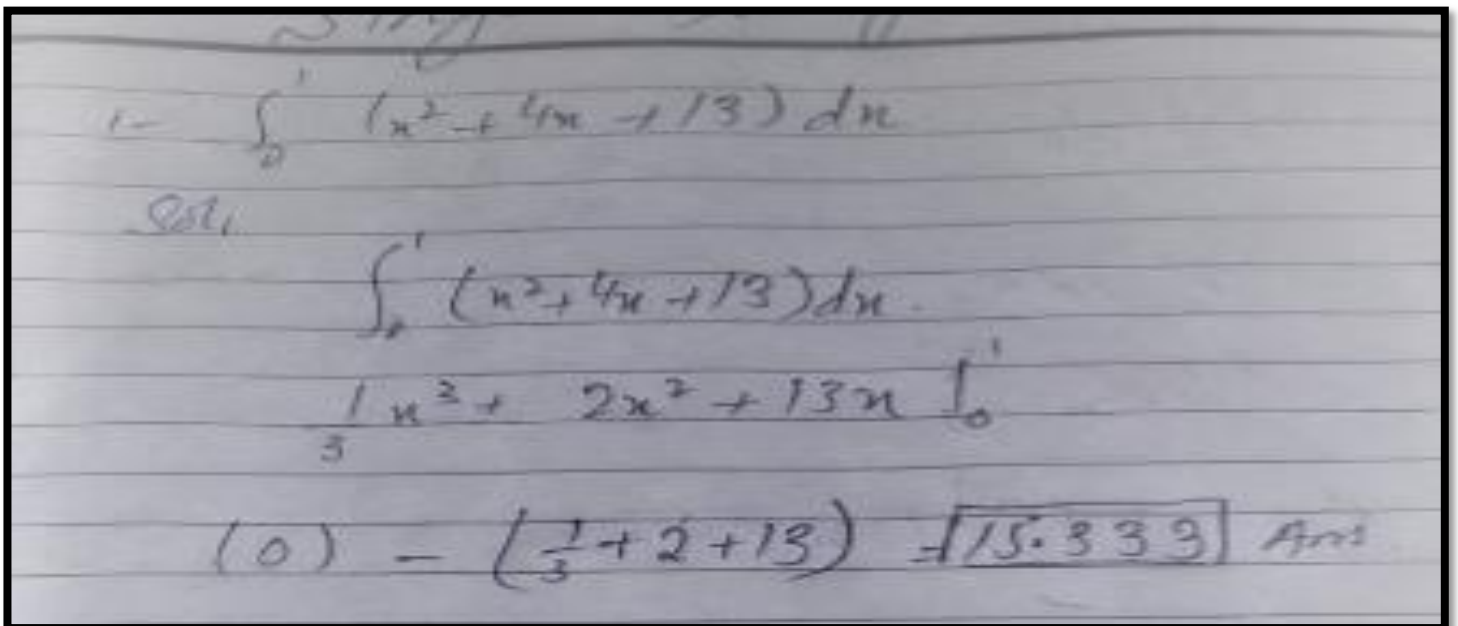
```
pythonProject > rabia.py
1 from scipy.integrate import quad
2 def integrand(x):
3     return x**2+4*x+13
4 ans,err=quad(integrand,0,1)
5 print(ans)
```

Run: rabia x

C:\Users\KHAN\PycharmProjects\pythonProject\w
15.333333333333336

Process finished with exit code 0

Manual solution:



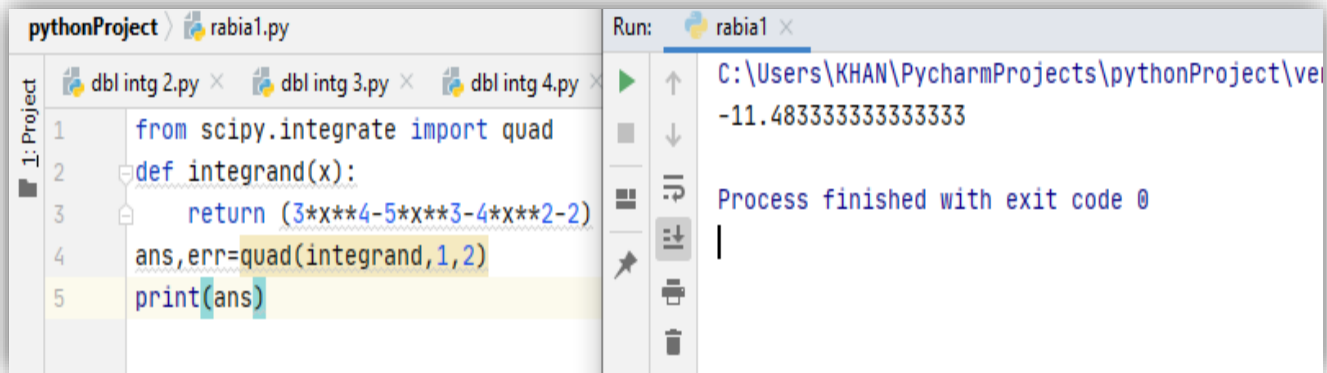
The handwritten solution shows the integral of the quadratic function $x^2 + 4x + 13$ from 0 to 1. The antiderivative is calculated as $\frac{1}{3}x^3 + 2x^2 + 13x$, and the definite integral is evaluated at the limits 0 and 1, resulting in $\frac{1}{3} + 2 + 13 = 15.333$.

$$\int_0^1 (x^2 + 4x + 13) dx$$

Sol:

$$\int_0^1 (x^2 + 4x + 13) dx$$
$$\left[\frac{1}{3}x^3 + 2x^2 + 13x \right]_0^1$$
$$(0) - \left(\frac{1}{3} + 2 + 13 \right) = 15.333 \text{ Ans}$$

Question 2 :



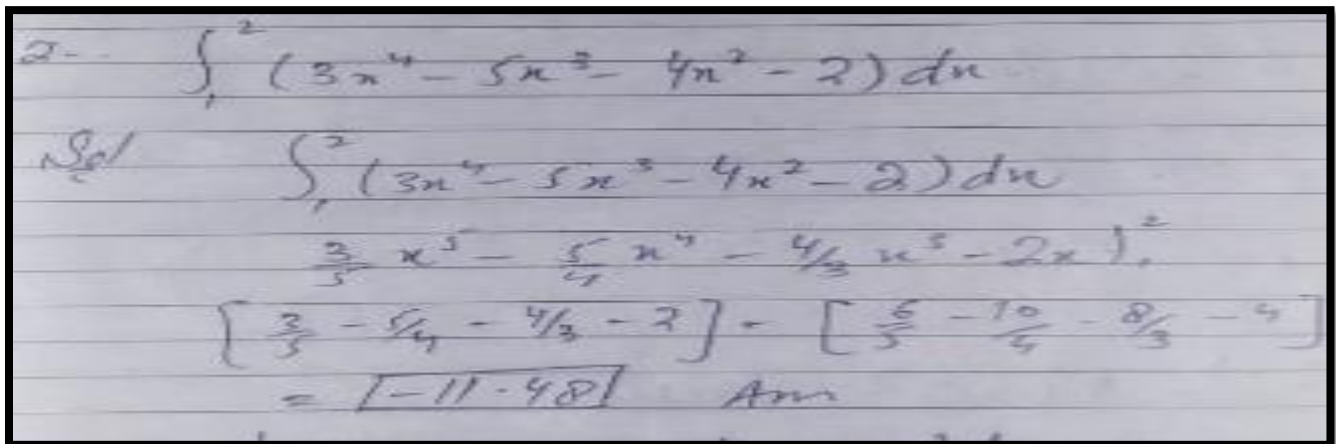
The screenshot shows a PyCharm IDE with a project named 'pythonProject'. The file 'rabia1.py' is open and contains the following Python code:

```
1 from scipy.integrate import quad
2 def integrand(x):
3     return (3*x**4-5*x**3-4*x**2-2)
4 ans,err=quad(integrand,1,2)
5 print(ans)
```

The 'Run' tab on the right shows the output of the script:

```
C:\Users\KHAN\PycharmProjects\pythonProject\venv\Scripts\python.exe C:\Users\KHAN\PycharmProjects\pythonProject\venv\Scripts\python.exe C:\Users\KHAN\PycharmProjects\pythonProject\venv\Scripts\python.exe -11.483333333333333
Process finished with exit code 0
```

Manual solution :

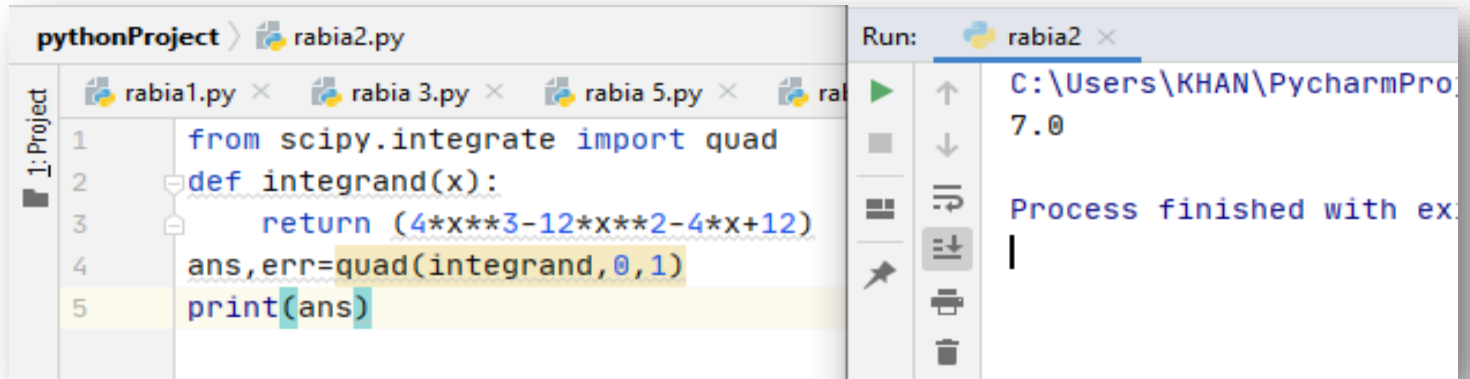


2- $\int_1^2 (3x^4 - 5x^3 - 4x^2 - 2) dx$

Sol $\int_1^2 (3x^4 - 5x^3 - 4x^2 - 2) dx$

$$\left[\frac{3}{5} x^5 - \frac{5}{4} x^4 - \frac{4}{3} x^3 - 2x \right]_1^2$$
$$\left[\frac{3}{5} - \frac{5}{4} - \frac{4}{3} - 2 \right] - \left[\frac{6}{5} - \frac{10}{4} - \frac{8}{3} - 2 \right]$$
$$= [-11.48] \quad \text{Ans}$$

Question 3:



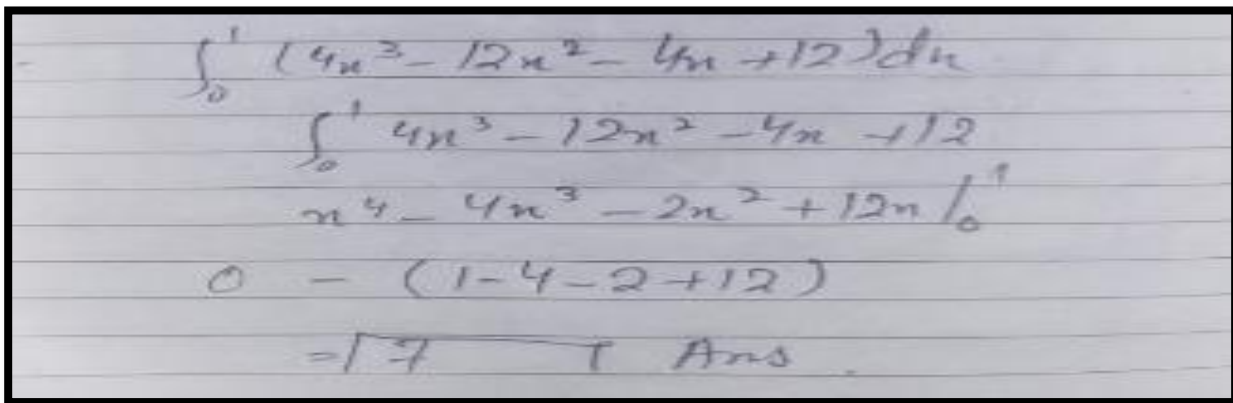
The screenshot shows a PyCharm IDE window with a project named 'pythonProject'. The file 'rabia2.py' is open and contains the following code:

```
1 from scipy.integrate import quad
2 def integrand(x):
3     return (4*x**3-12*x**2-4*x+12)
4 ans,err=quad(integrand,0,1)
5 print(ans)
```

The 'Run' console on the right shows the output of the script:

```
Run: rabia2 x
C:\Users\KHAN\PycharmPro
7.0
Process finished with ex
|
```

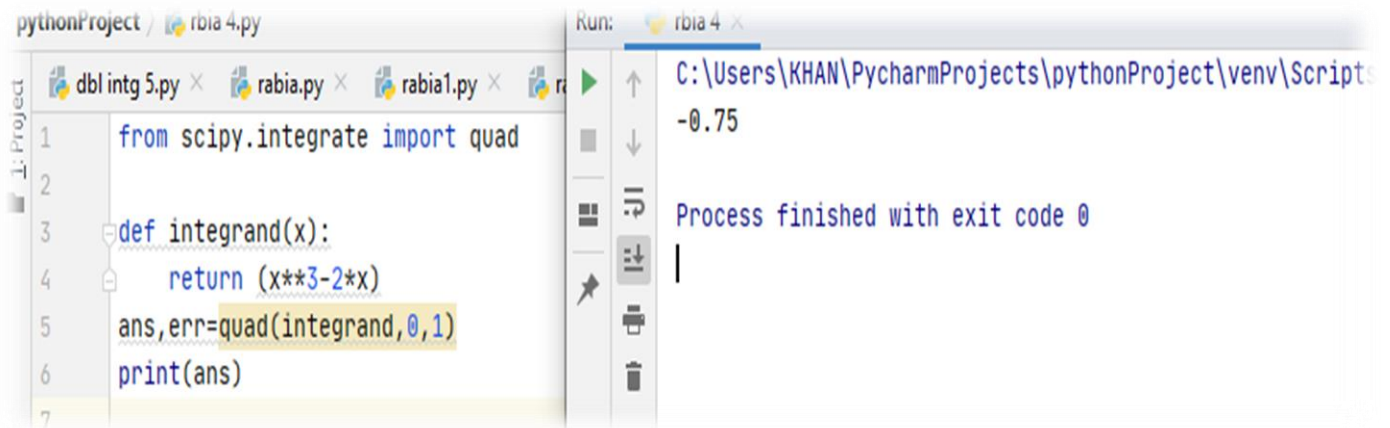
Manual solution :



The handwritten solution shows the following steps:

$$\int_0^1 (4x^3 - 12x^2 - 4x + 12) dx$$
$$\int_0^1 4x^3 - 12x^2 - 4x + 12$$
$$x^4 - 4x^3 - 2x^2 + 12x \Big|_0^1$$
$$0 - (1 - 4 - 2 + 12)$$
$$= 7 \text{ Ans.}$$

Question 4:



The screenshot shows a PyCharm IDE with a project named 'pythonProject'. The file 'rbia 4.py' is open, containing the following Python code:

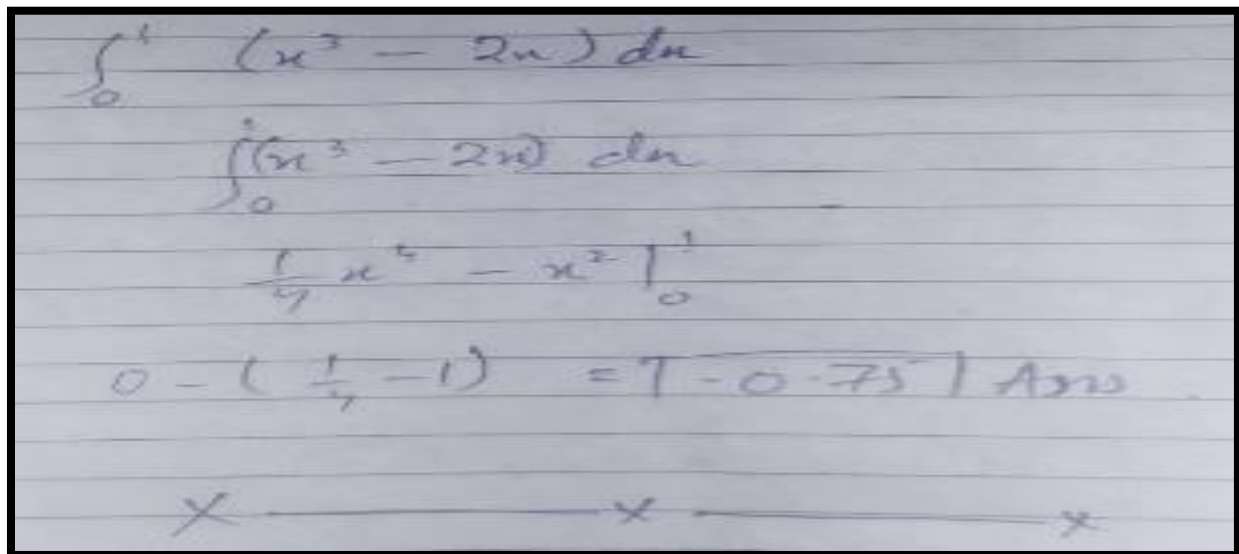
```
1 from scipy.integrate import quad
2
3 def integrand(x):
4     return (x**3-2*x)
5 ans,err=quad(integrand,0,1)
6 print(ans)
7
```

The 'Run' console on the right shows the output of the script:

```
Run: rbia 4 x
C:\Users\KHAN\PycharmProjects\pythonProject\venv\Scripts
-0.75

Process finished with exit code 0
```

Manual solution :

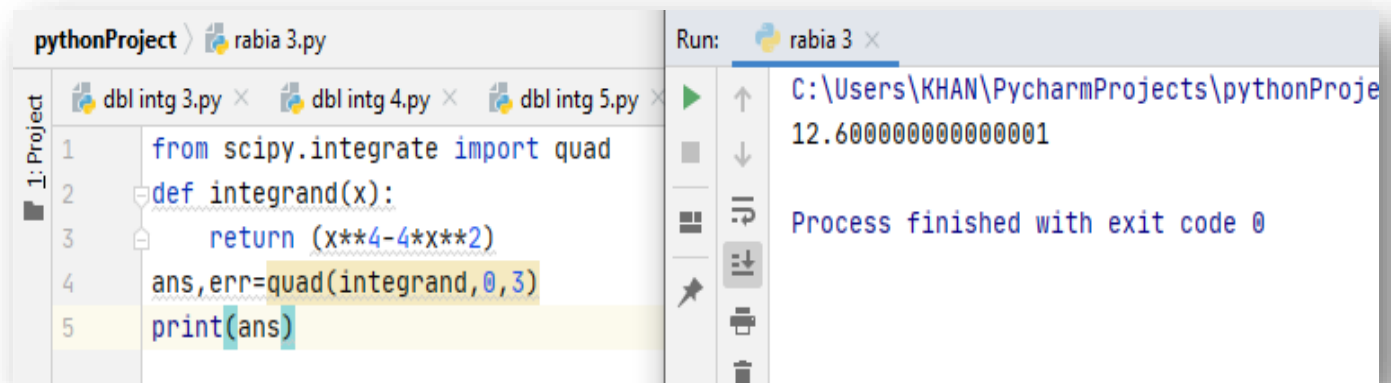


The handwritten solution shows the following steps:

$$\int_0^1 (x^3 - 2x) dx$$
$$\int_0^1 (x^3 - 2x) dx$$
$$\left[\frac{1}{4} x^4 - x^2 \right]_0^1$$
$$0 - \left(\frac{1}{4} - 1 \right) = 7 - 0.75 \text{ | Ans}$$

Below the calculations, there are three 'x' marks connected by a horizontal line.

Question 5:



The screenshot shows a PyCharm IDE with a project named 'pythonProject'. The file 'rabia 3.py' is open and contains the following Python code:

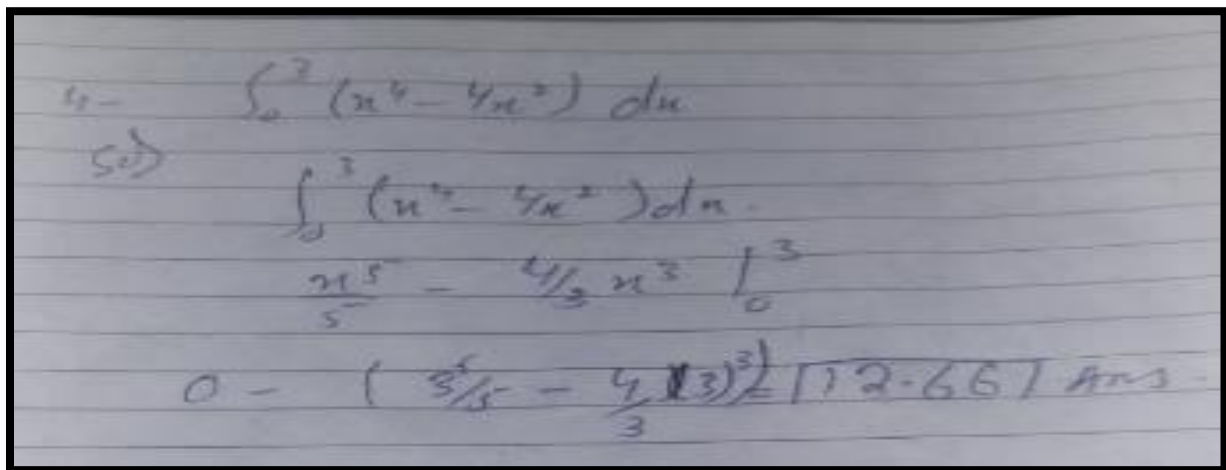
```
1 from scipy.integrate import quad
2 def integrand(x):
3     return (x**4-4*x**2)
4 ans,err=quad(integrand,0,3)
5 print(ans)
```

The 'Run' window on the right shows the output of the script:

```
C:\Users\KHAN\PycharmProjects\pythonProje
12.600000000000001

Process finished with exit code 0
```

Manual solution:

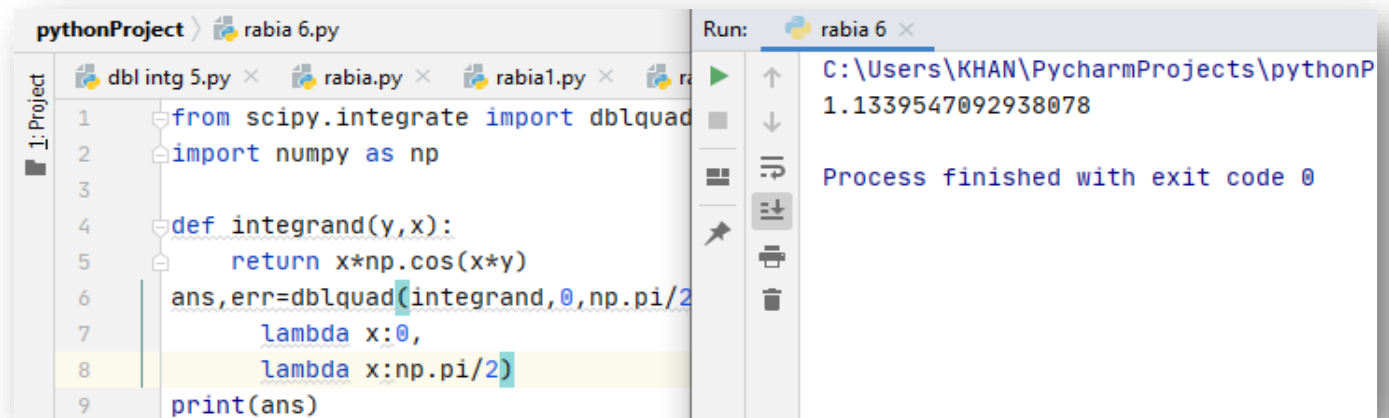


The handwritten solution shows the following steps:

$$\begin{aligned} & \int_0^3 (x^4 - 4x^2) dx \\ \text{So} & \int_0^3 (x^4 - 4x^2) dx \\ & \frac{x^5}{5} - \frac{4}{3}x^3 \Big|_0^3 \\ & 0 - \left(\frac{3^5}{5} - \frac{4}{3}(3)^3 \right) = 12.66 \text{ Ans.} \end{aligned}$$

Double Integration Questions:

Question 1:

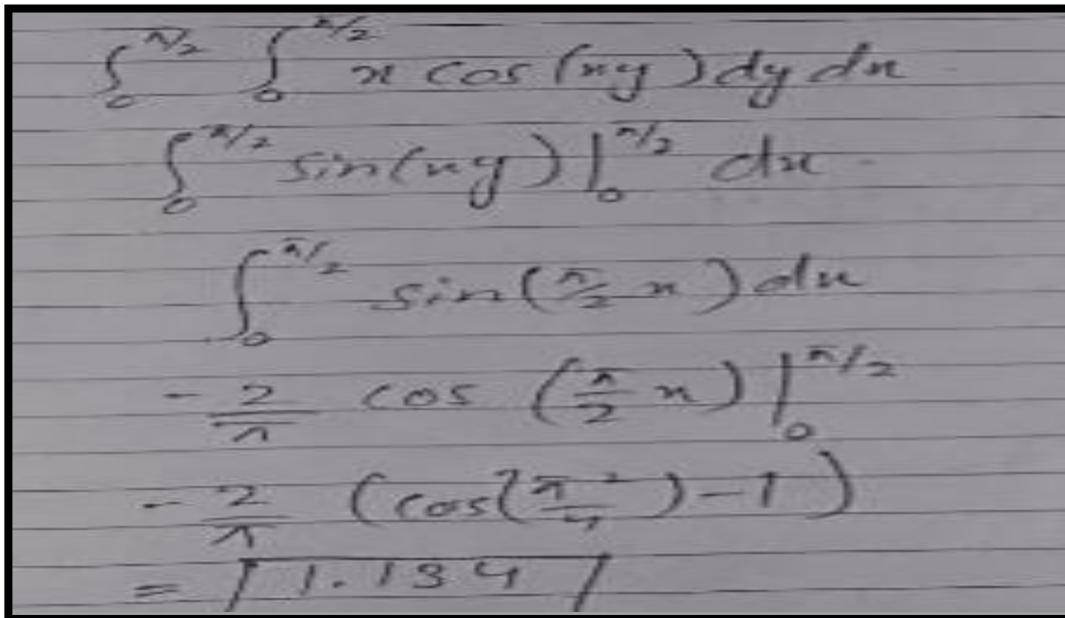


The screenshot shows a PyCharm IDE with a Python project named 'pythonProject'. The file 'rabia 6.py' is open, containing the following code:

```
1 from scipy.integrate import dblquad
2 import numpy as np
3
4 def integrand(y,x):
5     return x*np.cos(x*y)
6
7 ans,err=dblquad(integrand,0,np.pi/2,
8                 lambda x:0,
9                 lambda x:np.pi/2)
10 print(ans)
```

The 'Run' console on the right shows the output: 'C:\Users\KHAN\PycharmProjects\pythonP' followed by the numerical result '1.1339547092938078'. Below the output, it states 'Process finished with exit code 0'.

Manual:



The handwritten manual calculation shows the following steps:

$$\int_0^{\pi/2} \int_0^{\pi/2} x \cos(xy) dy dx$$
$$\int_0^{\pi/2} \sin(xy) \Big|_0^{\pi/2} dx$$
$$\int_0^{\pi/2} \sin\left(\frac{\pi}{2} x\right) dx$$
$$= -\frac{2}{\pi} \cos\left(\frac{\pi}{2} x\right) \Big|_0^{\pi/2}$$
$$= -\frac{2}{\pi} \left(\cos\left(\frac{\pi^2}{4}\right) - 1\right)$$
$$= |1.134|$$

Question 2:

```
rabia 6.py x rabia 7.py x dbl intg 1.py x
1 from scipy.integrate import dblquad
2
3 def integrand(y,x):
4     return x*y**2
5 ans,err=dblquad(integrand,0,2,
6                 lambda x:0,
7                 lambda x:x/2)
8 print(ans)
9
```

C:\Users\KHAN\PycharmProjects\pythonF
0.2666666666666667

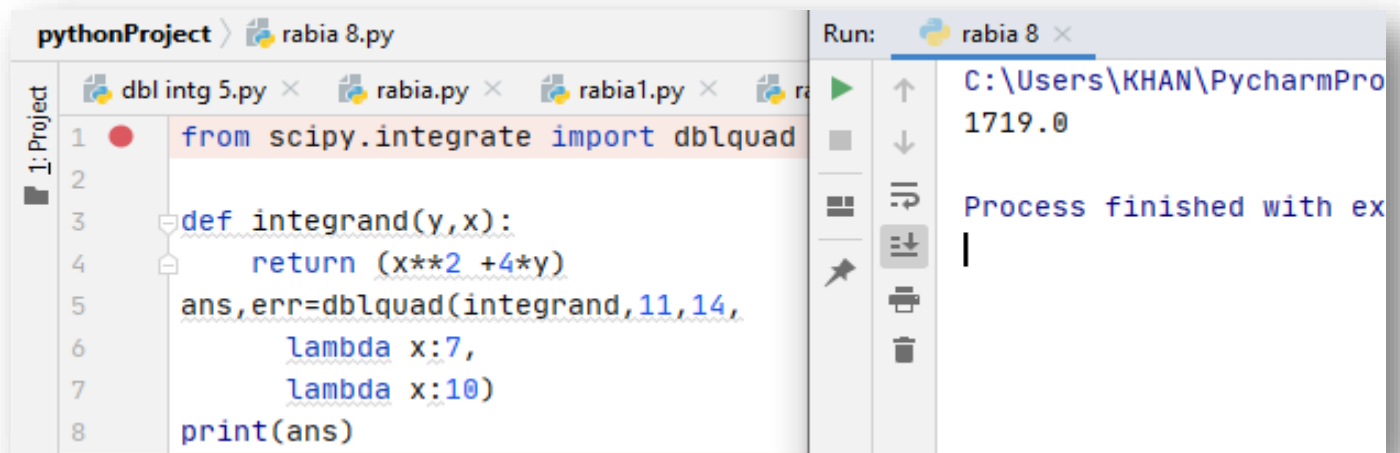
Process finished with exit code 0

Manual:

$$\int_0^2 \int_0^{x/2} xy^2 \, dy \, dx$$
$$\int_0^2 \left(\frac{xy^3}{3} \right) \bigg|_0^{x/2} dx$$
$$\int_0^2 \left(\frac{x^4}{24} \right) dx$$
$$\left(\frac{x^5}{120} \right) \bigg|_0^2$$
$$= \frac{32}{12}$$
$$\boxed{= 0.266}$$

Ans

Question 3:

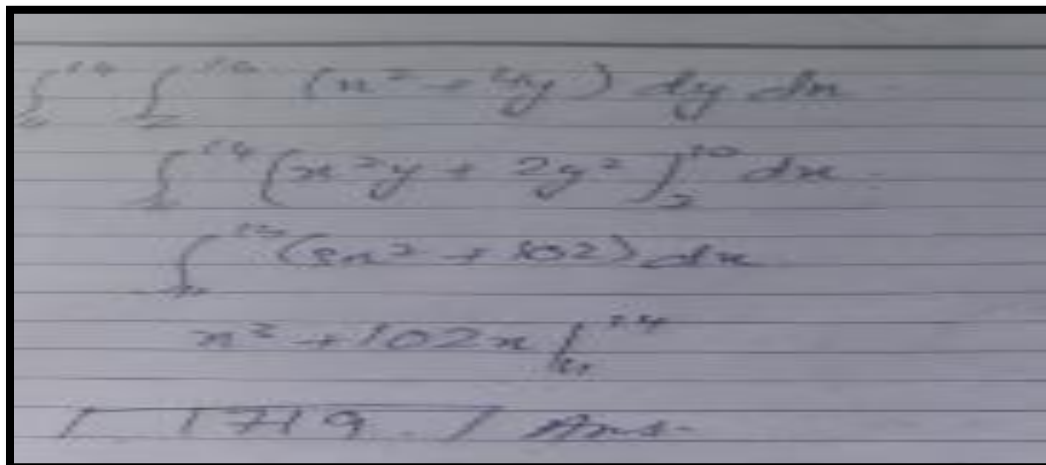


The screenshot shows a Python IDE with a file named 'rabia 8.py'. The code defines a function 'integrand(y,x)' that returns 'x**2 + 4*y'. It then uses 'dblquad' from 'scipy.integrate' to calculate a double integral. The first integral is over 'y' from 2 to 14, and the second is over 'x' from 7 to 10. The result is printed as 'ans'.

```
1 from scipy.integrate import dblquad
2
3 def integrand(y,x):
4     return (x**2 + 4*y)
5 ans,err=dblquad(integrand,11,14,
6                 lambda x:7,
7                 lambda x:10)
8 print(ans)
```

The Run console shows the output: '1719.0' and 'Process finished with exit code 0'.

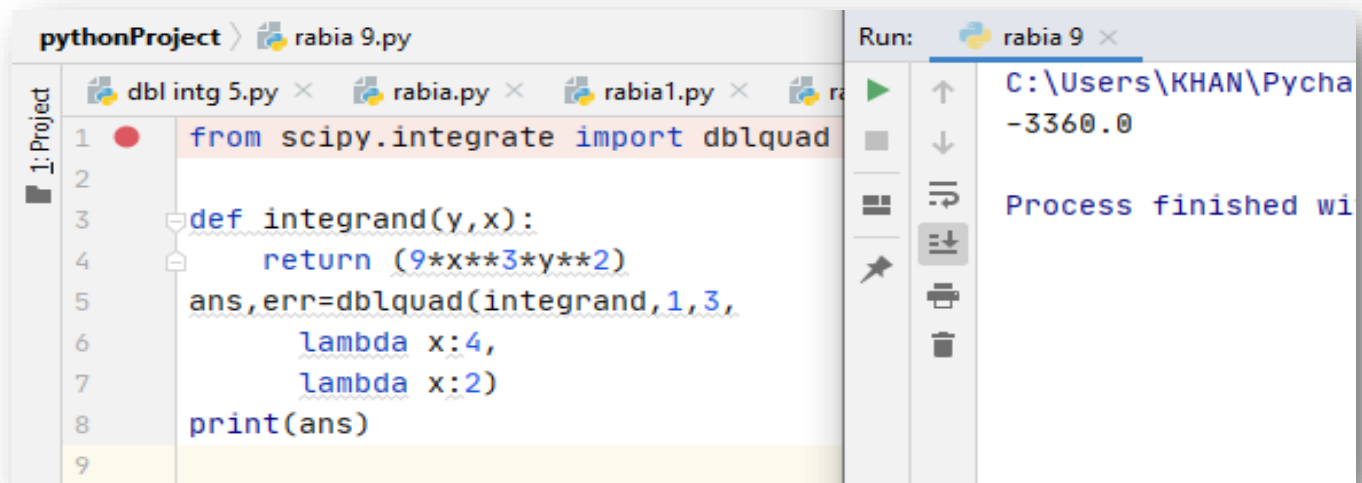
Manual:



The handwritten work shows the manual calculation of the double integral. It starts with the integral expression, then integrates with respect to 'y' first, followed by integration with respect to 'x'. The final result is 1719.

$$\int_7^{10} \int_2^{14} (x^2 + 4y) dy dx$$
$$\int_7^{10} \left(x^2 y + 2y^2 \right) \Big|_2^{14} dx$$
$$\int_7^{10} (8x^2 + 102) dx$$
$$\left(\frac{8x^3}{3} + 102x \right) \Big|_7^{10}$$
$$1719 \text{ Ans.}$$

Question 4:

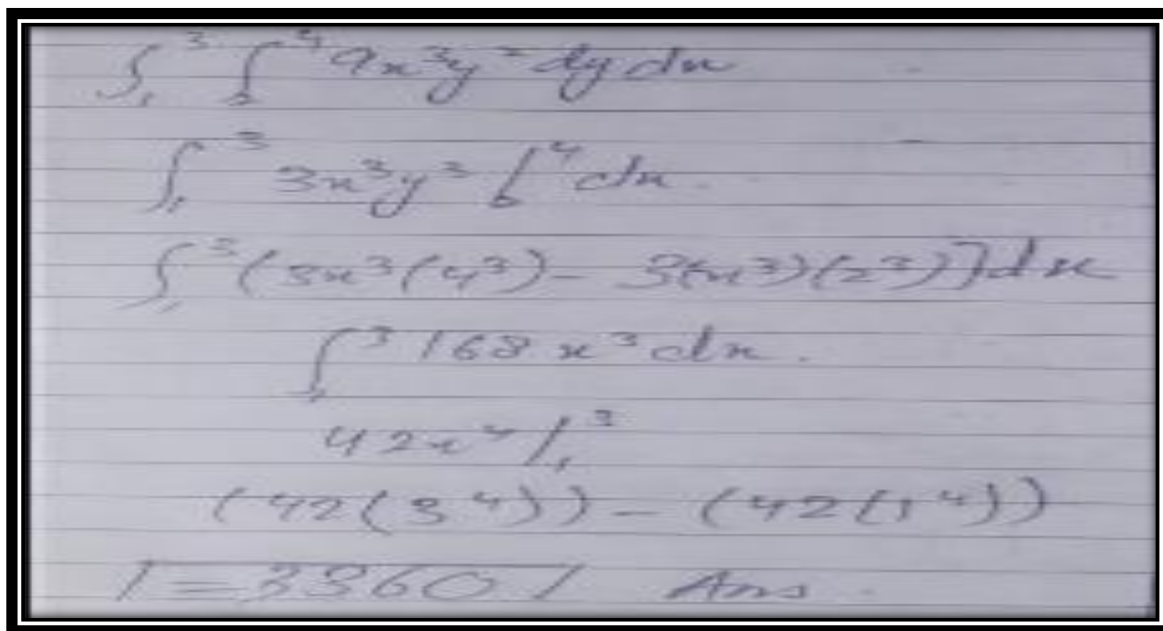


The screenshot shows a Python IDE with a project named 'pythonProject'. The file 'rabia 9.py' is open, containing the following code:

```
1 from scipy.integrate import dblquad
2
3 def integrand(y,x):
4     return (9*x**3*y**2)
5 ans,err=dblquad(integrand,1,3,
6                 lambda x:4,
7                 lambda x:2)
8 print(ans)
9
```

The 'Run' panel on the right shows the output: 'C:\Users\KHAN\Pycha' and '-3360.0'. Below the output, it says 'Process finished wi'.

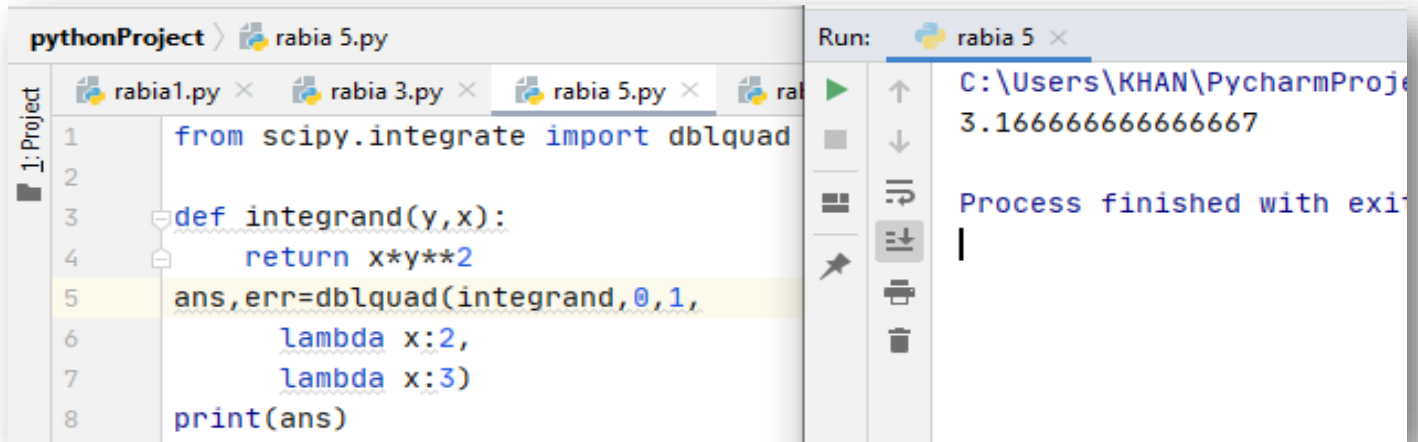
Manual:



The handwritten manual solution shows the following steps:

$$\int_1^3 \int_2^4 9x^3y^2 dy dx$$
$$\int_1^3 3x^3y^2 \Big|_2^4 dx$$
$$\int_1^3 (3x^3(4^2) - 3x^3(2^2)) dx$$
$$\int_1^3 168x^3 dx$$
$$42x^4 \Big|_1^3$$
$$(42(3^4)) - (42(1^4))$$
$$I = 3360 \text{ Ans.}$$

Question 5:



The screenshot shows a PyCharm IDE window titled 'pythonProject' with a file named 'rabia 5.py'. The code in the editor is as follows:

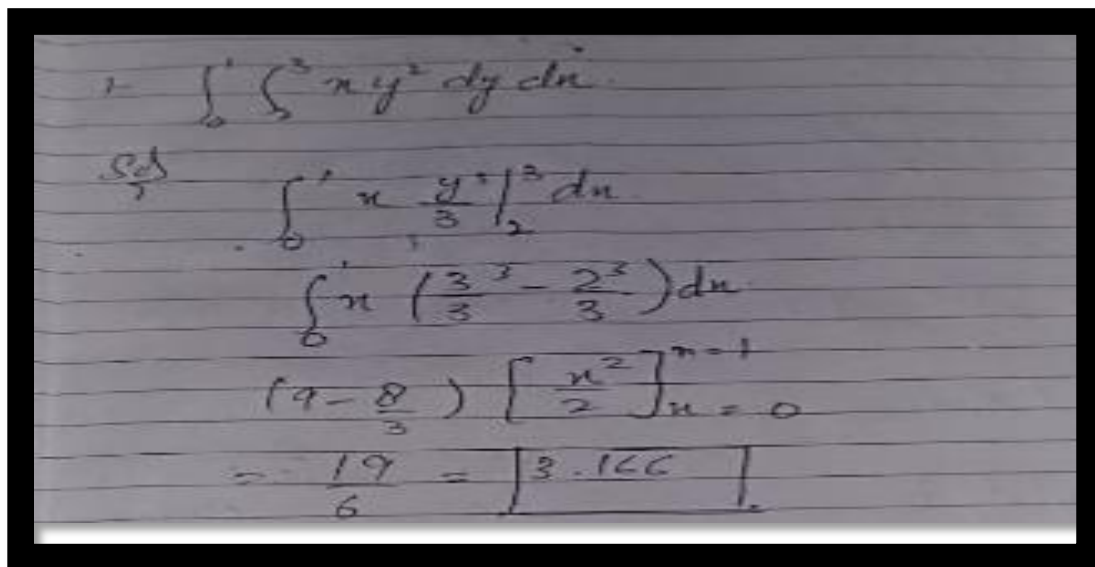
```
1 from scipy.integrate import dblquad
2
3 def integrand(y,x):
4     return x*y**2
5 ans,err=dblquad(integrand,0,1,
6                 lambda x:2,
7                 lambda x:3)
8 print(ans)
```

The 'Run' console on the right shows the output of the script:

```
Run: rabia 5 x
C:\Users\KHAN\PycharmProje
3.166666666666667

Process finished with exit
|
```

Manual:



The handwritten manual calculation shows the following steps:

$$I = \int_0^1 \int_2^3 xy^2 dy dx$$
$$\text{Sol} \int_0^1 x \left[\frac{y^3}{3} \right]_2^3 dx$$
$$\int_0^1 x \left(\frac{3^3}{3} - \frac{2^3}{3} \right) dx$$
$$\left(9 - \frac{8}{3} \right) \left[\frac{x^2}{2} \right]_{x=0}^{x=1}$$
$$= \frac{19}{6} = 3.166$$