

Classification

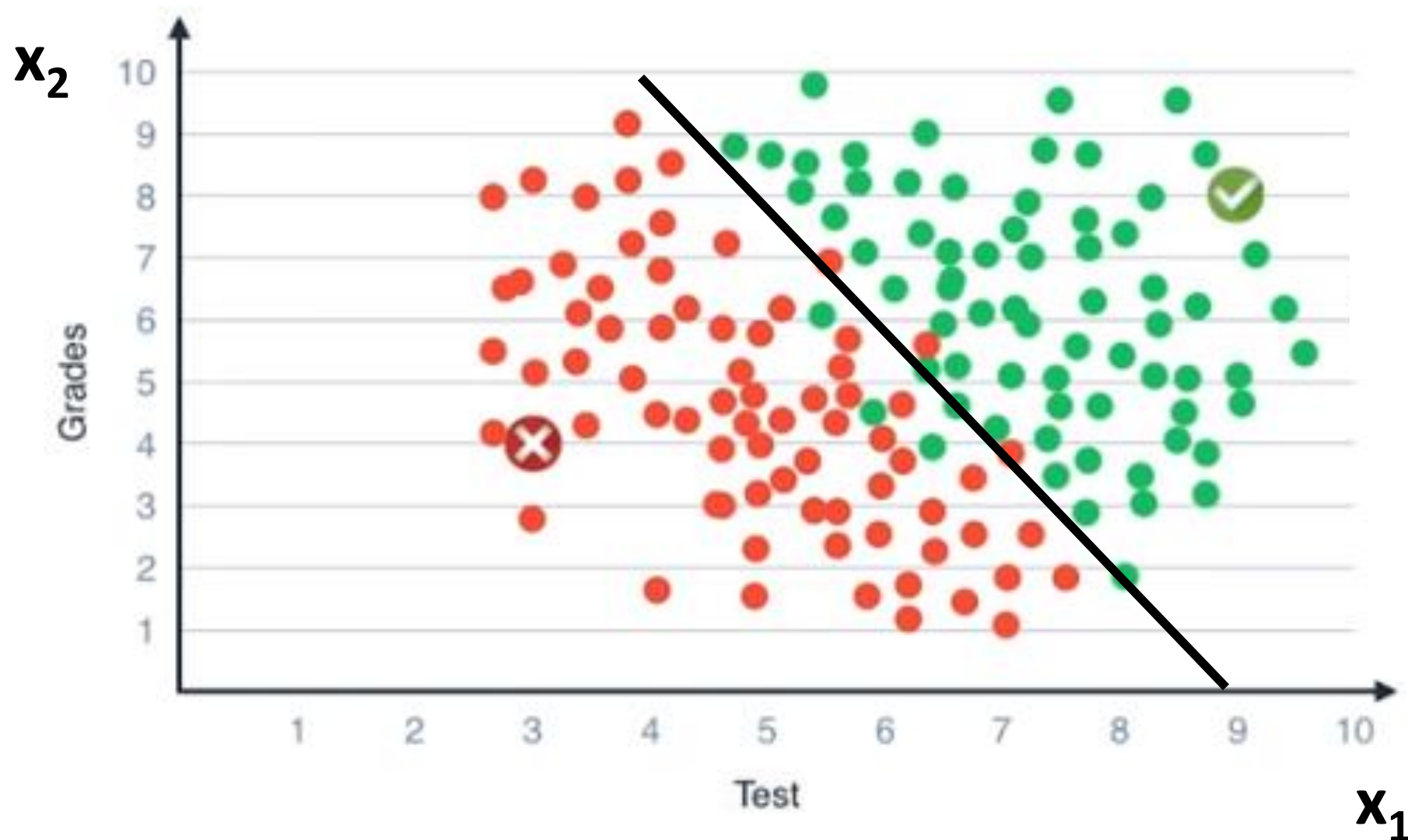


Test



Grades





BOUNDARY: A LINE $2x_1 + x_2 - 18 = 0$

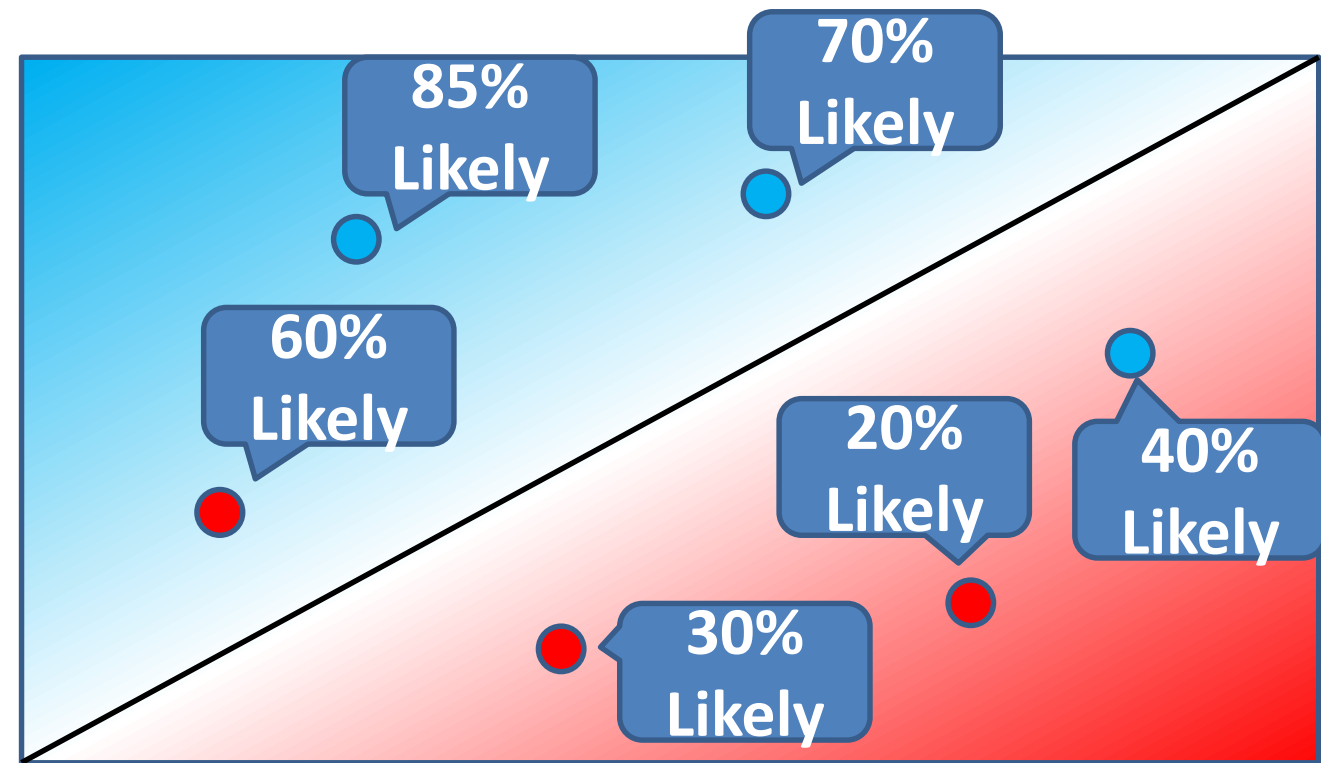
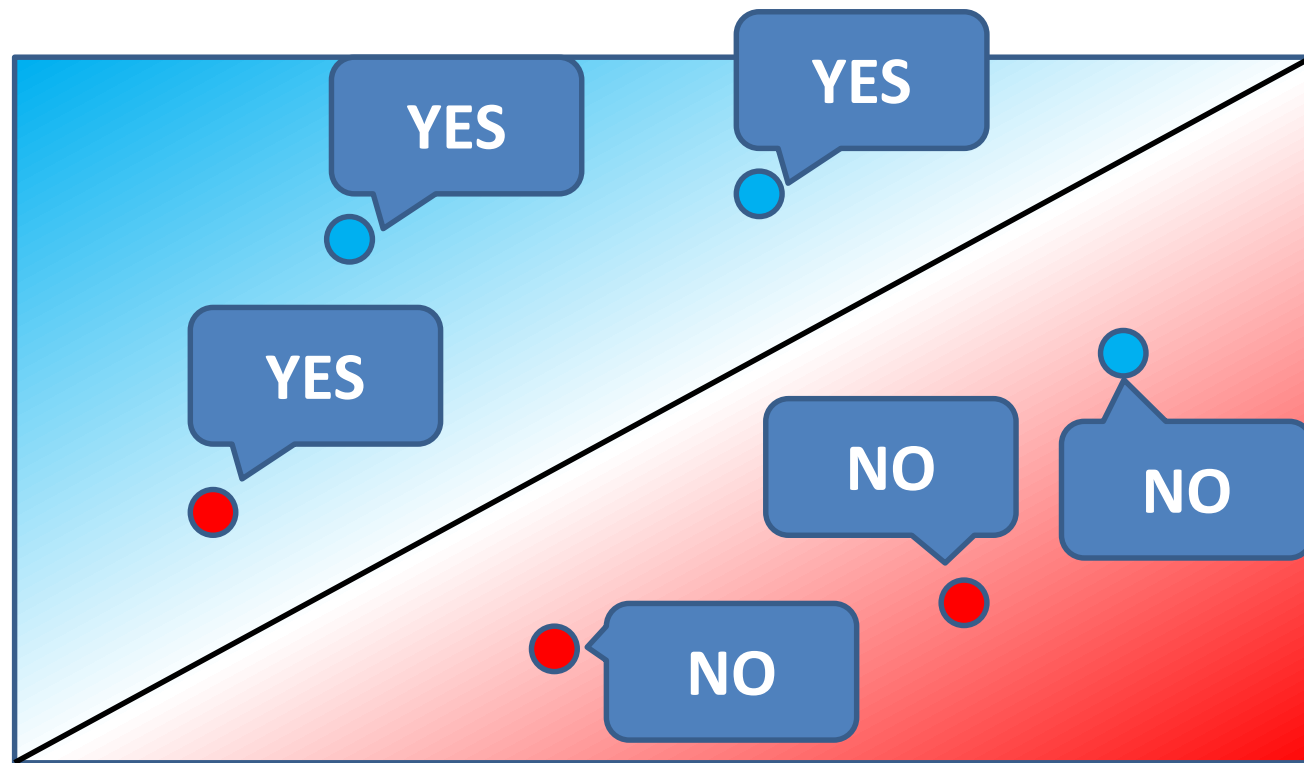
Score $= 2 * \text{Test} + \text{Grades} - 18$

PREDICTION: $\left\{ \begin{array}{l} \text{Score} \geq 0 \text{ } \mathbf{ACCEPT} \\ \text{Score} < 0 \text{ } \mathbf{REJECT} \end{array} \right\}$

Now that you know the equation for the line ($2x_1 + x_2 - 18 = 0$), and similarly the “score” ($2x_1 + x_2 - 18$), what is the score of the student who got 7 in the test and 6 for grades?

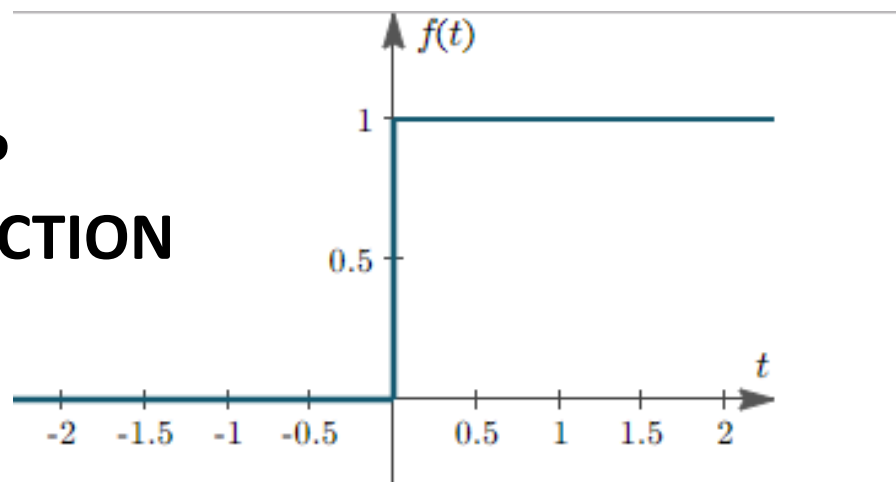


Discrete vs Continuous

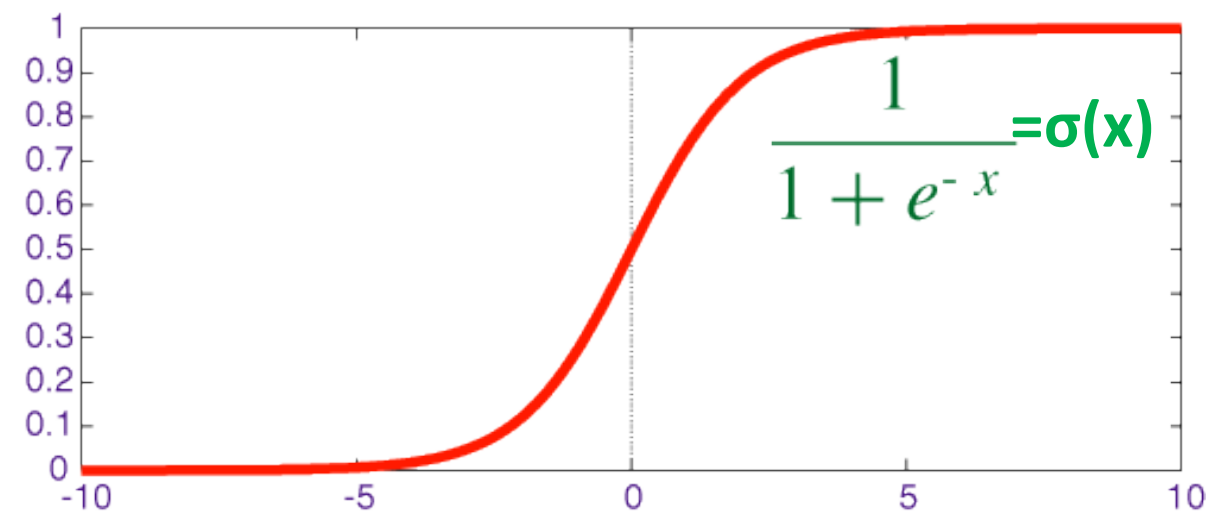


PREDICTION: $\left\{ \begin{array}{l} \text{Score} \geq 0 \text{ YES} \\ \text{Score} < 0 \text{ NO} \end{array} \right\}$

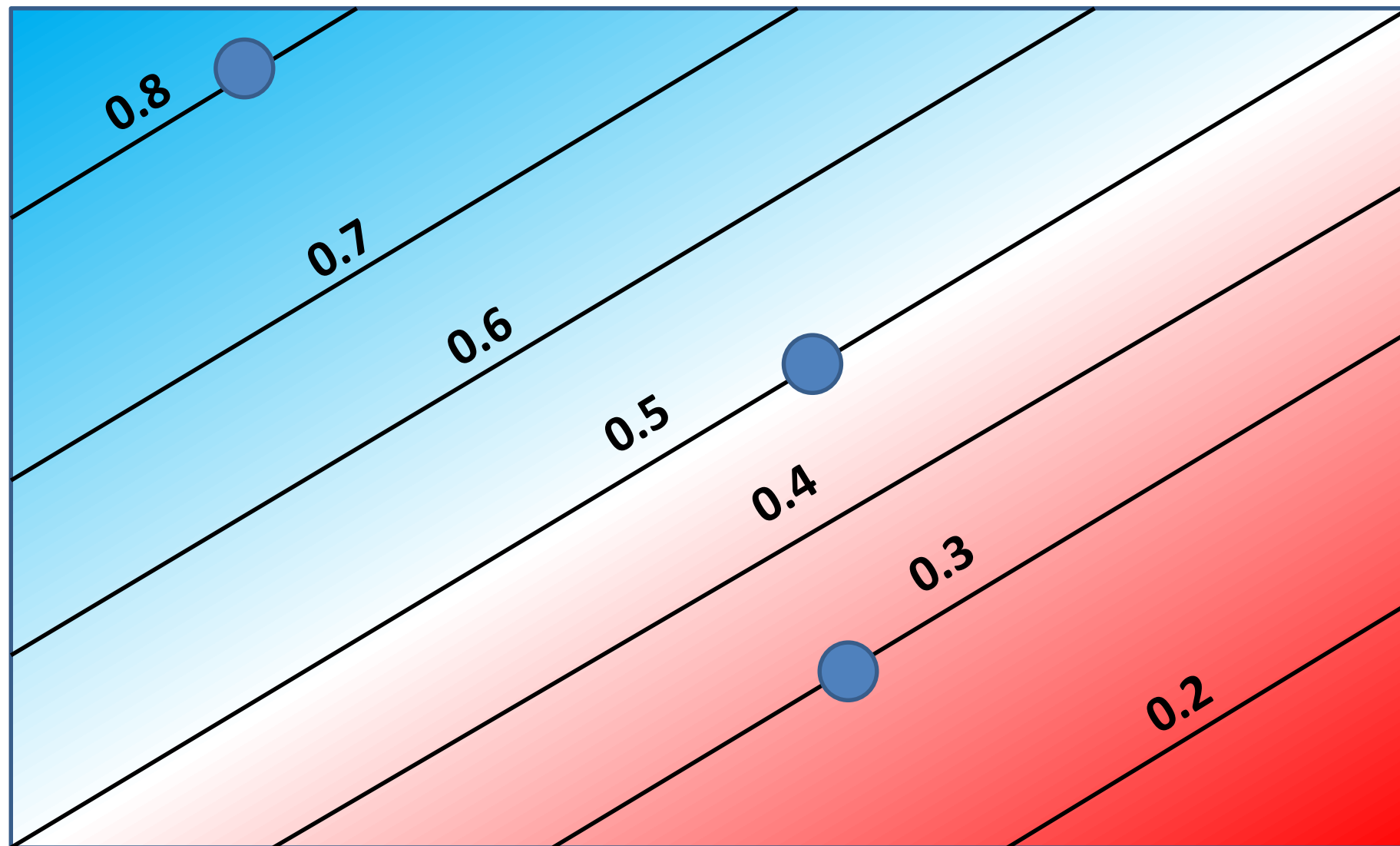
STEP
FUNCTION



SIGMOID FUNCTION

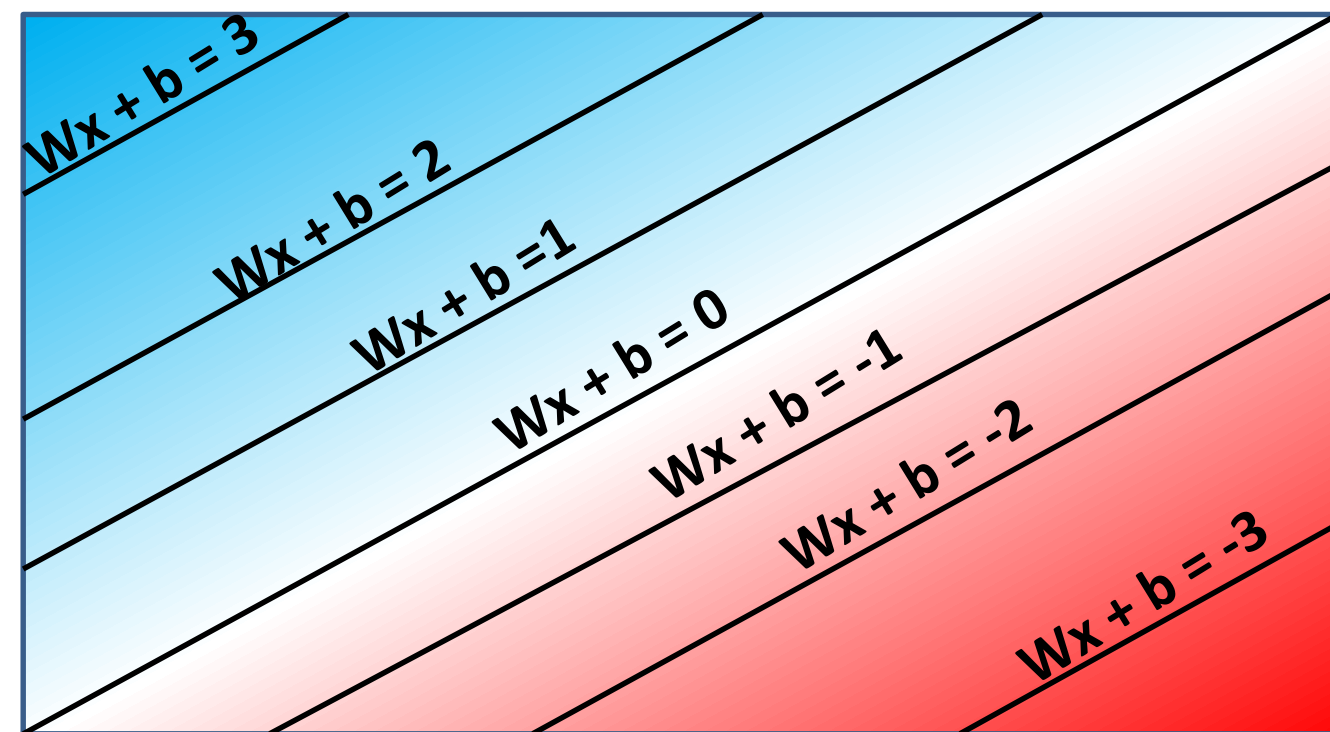


Predictions

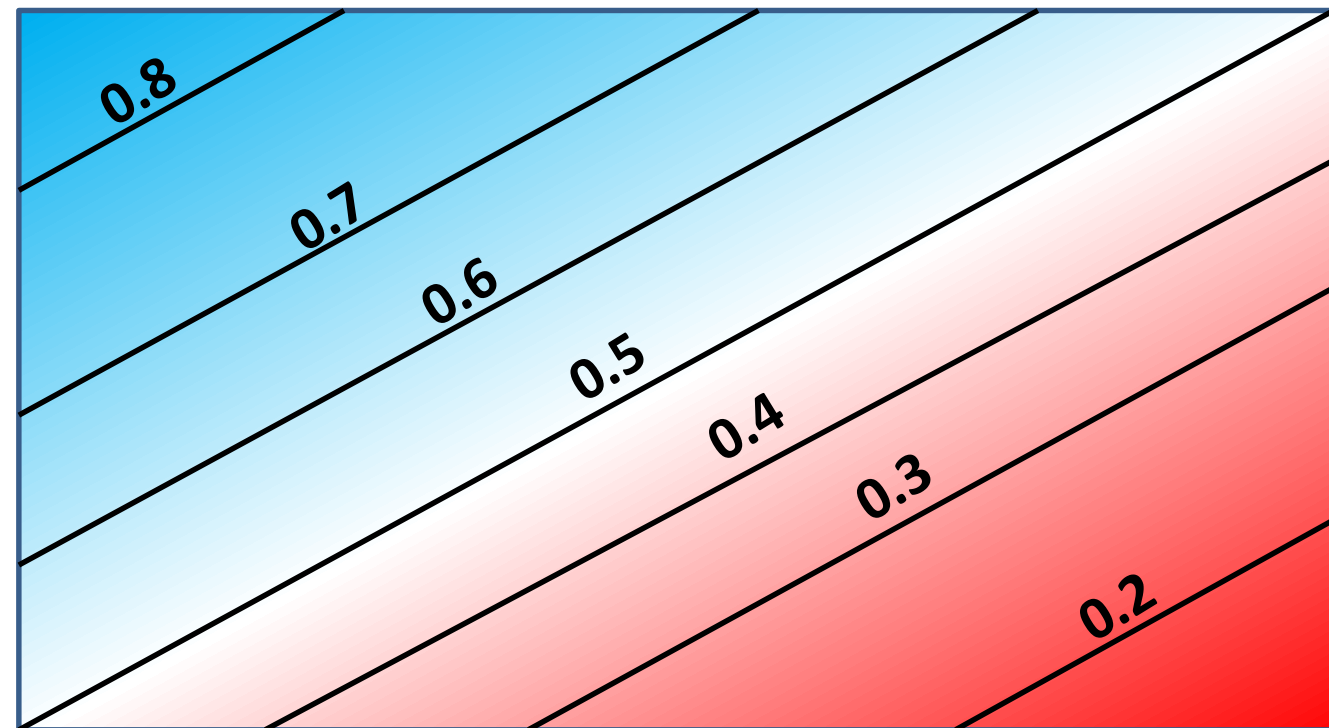


Predictions

σ

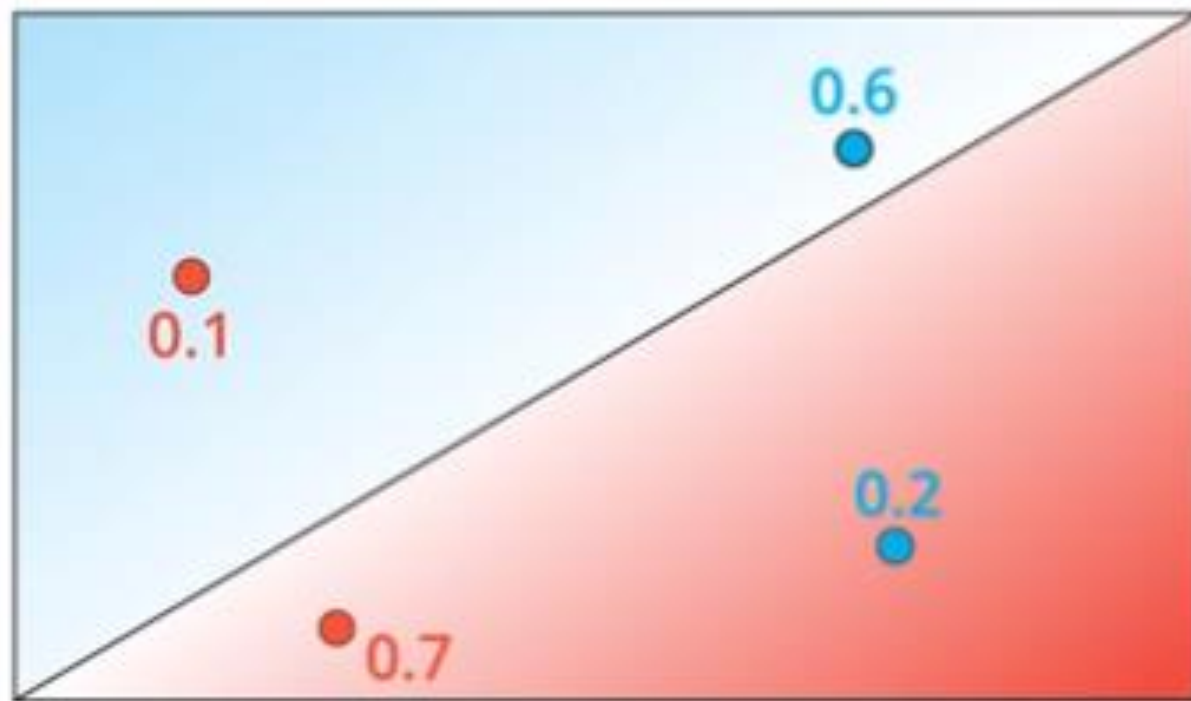


$Wx + b$

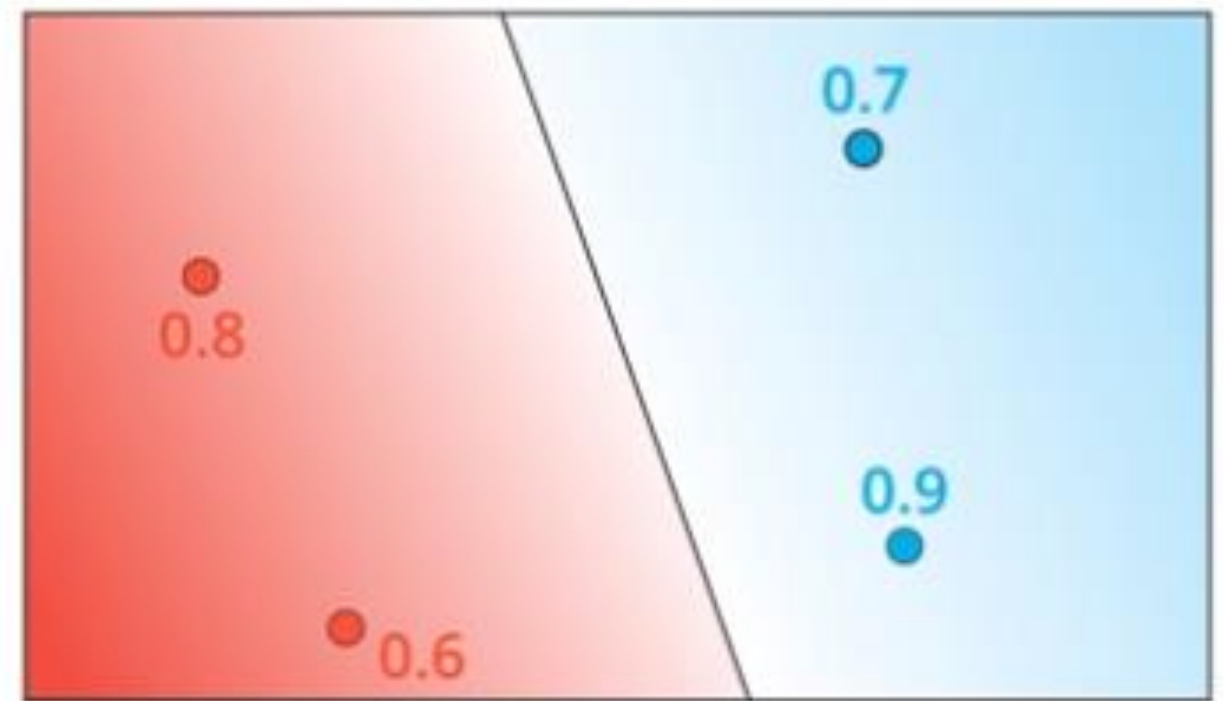


$\hat{y} = \sigma(Wx + b)$

Maximum Likelihood



$$0.6 * 0.2 * 0.1 * 0.7 = 0.0084$$



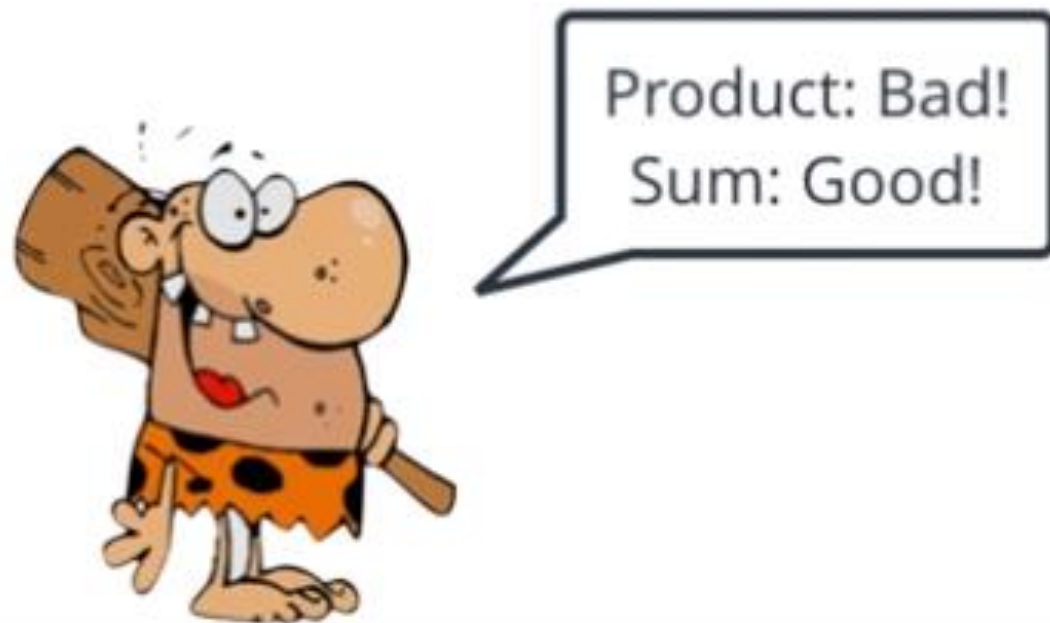
$$0.7 * 0.9 * 0.8 * 0.6 = 0.3024$$



Products of Probabilities

$$0.6 * 0.2 * 0.1 * 0.7 = 0.0084$$

$$0.7 * 0.9 * 0.8 * 0.6 = 0.3024$$



Quiz:
What function to use?

- sin ☐
- cos ☐
- log ☐
- exp ☐

Cross Entropy

$$0.6 * 0.2 * 0.1 * 0.7 = 0.0084$$

$$0.7 * 0.9 * 0.8 * 0.6 = 0.3024$$

$$\begin{array}{cccc} \ln(0.6) & + & \ln(0.2) & + & \ln(0.1) & + & \ln(0.7) \\ -0.51 & & -1.61 & & -2.3 & & -0.36 \end{array}$$

$$\begin{array}{cccc} \ln(0.7) & + & \ln(0.9) & + & \ln(0.8) & + & \ln(0.6) \\ -0.36 & & -0.1 & & -0.22 & & -0.51 \end{array}$$

$$\begin{array}{cccc} -\ln(0.6) & - & -\ln(0.2) & - & -\ln(0.1) & - & -\ln(0.7) \\ 0.51 & & 1.61 & & 2.3 & & 0.36 \end{array}$$

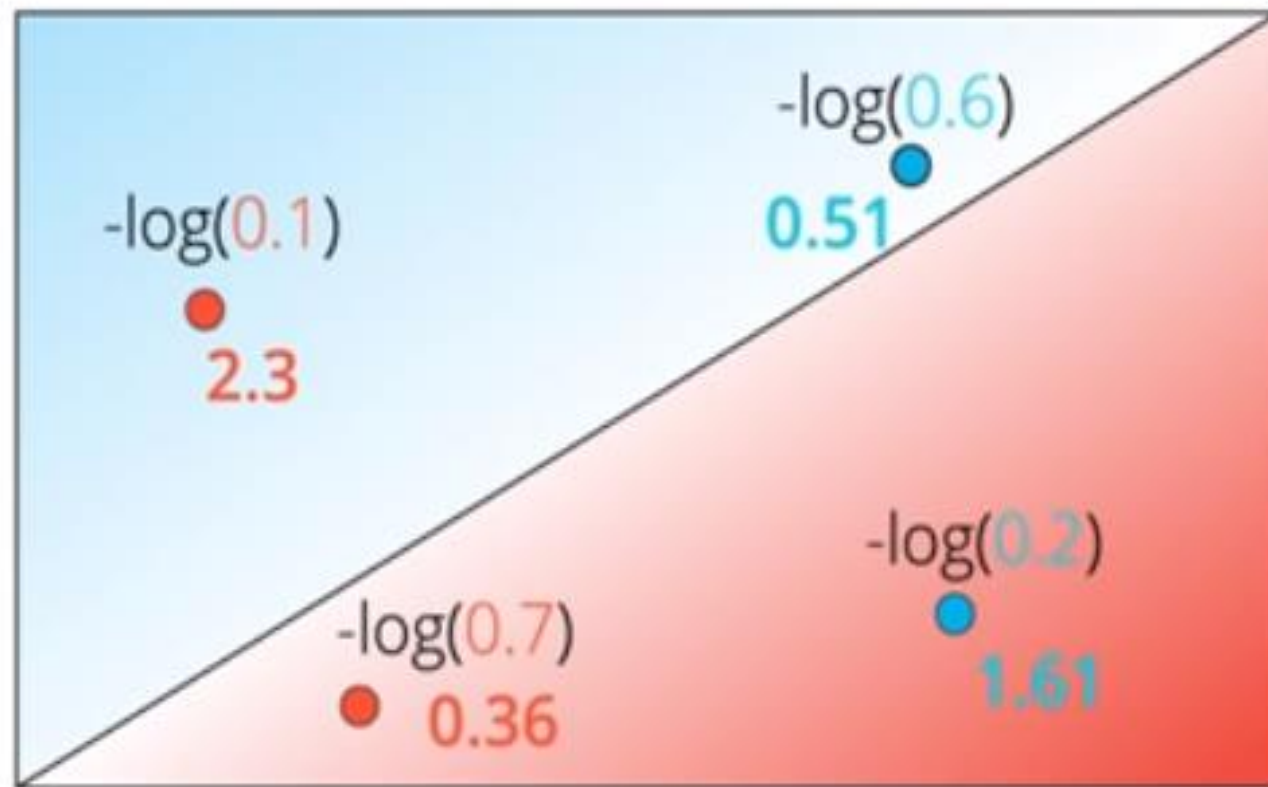
$$\begin{array}{cccc} -\ln(0.7) & - & -\ln(0.9) & - & -\ln(0.8) & - & -\ln(0.6) \\ 0.36 & & 0.1 & & 0.22 & & 0.51 \end{array}$$

4.8

1.2

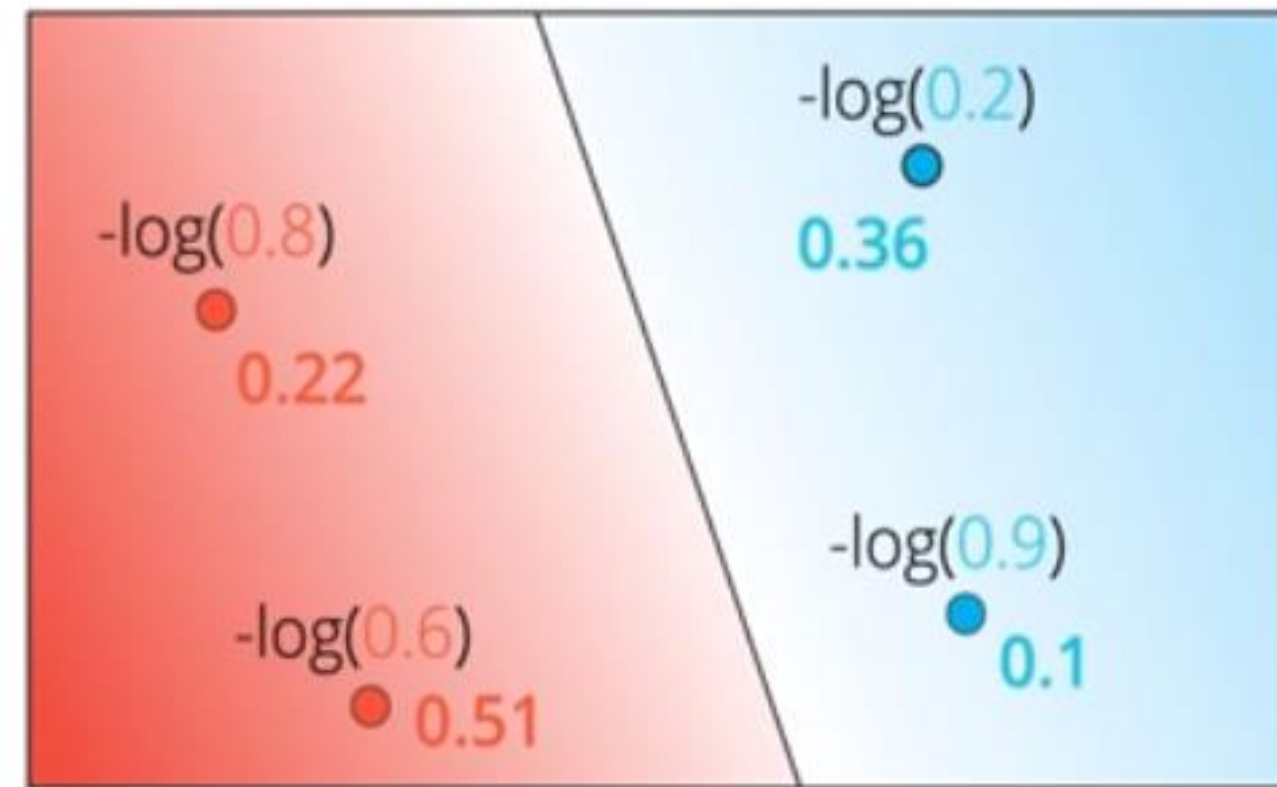
CROSS ENTROPY

Cross Entropy



$$0.6 * 0.2 * 0.1 * 0.7 = 0.0084$$

$$-\log(0.6) - \log(0.2) - \log(0.1) - \log(0.7) = 4.8$$

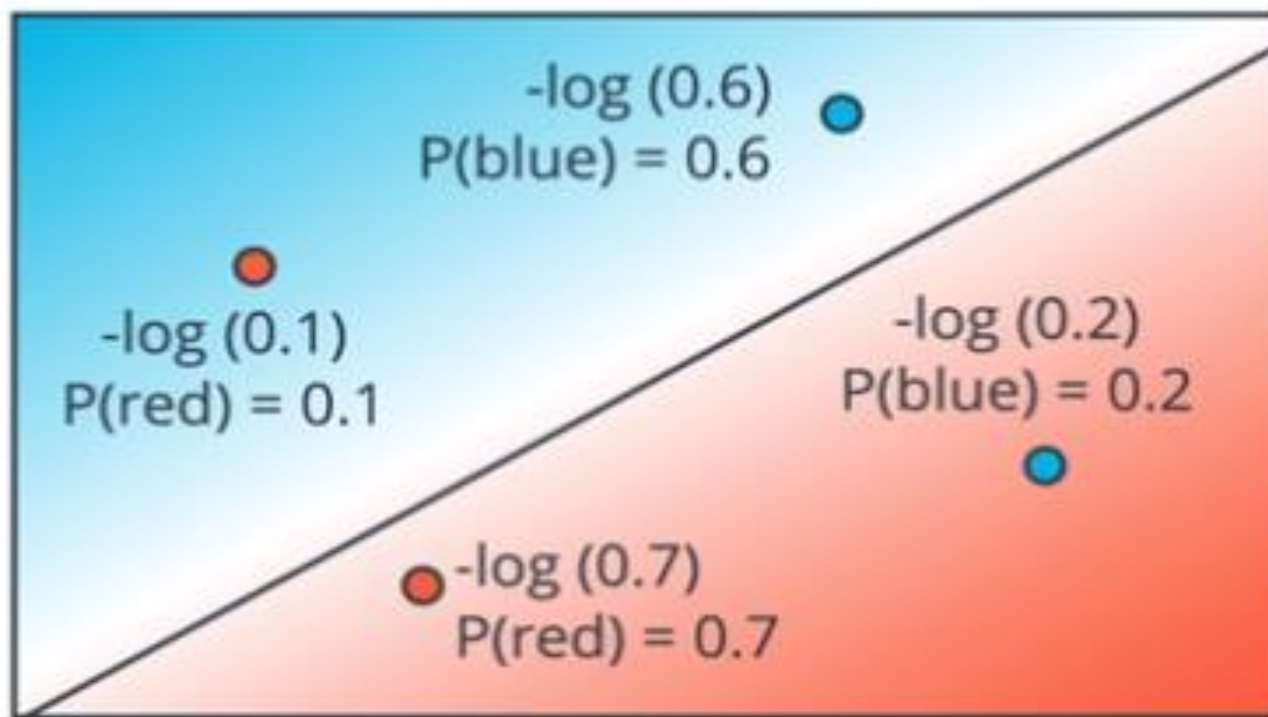


$$0.7 * 0.9 * 0.8 * 0.6 = 0.3024$$

$$-\log(0.7) - \log(0.9) - \log(0.8) - \log(0.6) = 1.2$$



Error Function



$$-\log(0.6) - \log(0.2) - \log(0.1) - \log(0.7) = 4.8$$

0.51 1.61 2.3 0.36

If $y = 1$

$$P(\text{blue}) = \hat{y}$$

$$\text{Error} = -\ln(\hat{y})$$

If $y = 0$

$$P(\text{red}) = 1 - P(\text{blue}) = 1 - \hat{y}$$

$$\text{Error} = -\ln(1 - \hat{y})$$

$$\text{Error} = -(1-y)(\ln(1-\hat{y})) - y\ln(\hat{y})$$

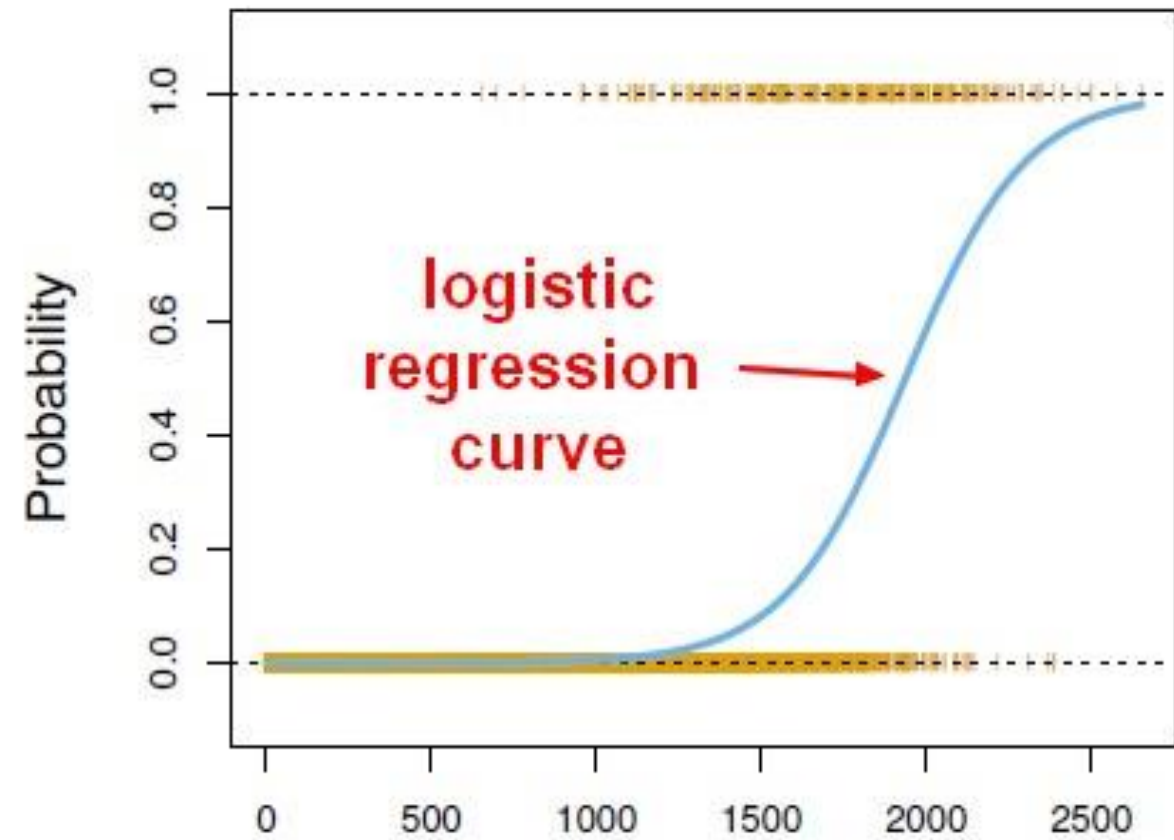
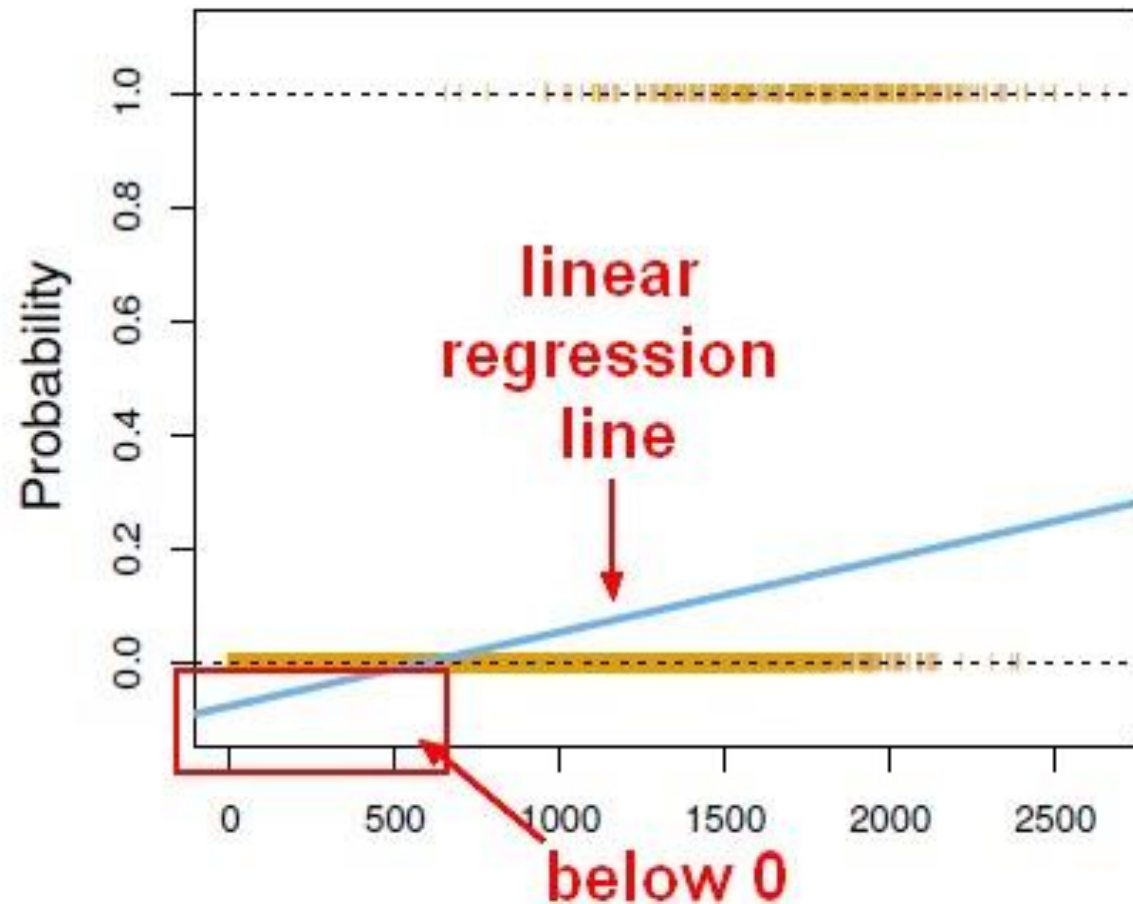
$$\text{Error Function} = -\frac{1}{m} \sum_{i=1}^m (1-y_i)(\ln(1-\hat{y}_i)) + y_i \ln(\hat{y}_i)$$

$$E(W,b) = -\frac{1}{m} \sum_{i=1}^m (1-y_i)(\ln(1-\sigma(Wx^{(i)}+b))) + y_i \ln(\sigma(Wx^{(i)}+b))$$



**MOUNT
ERRORIST**

Logistic Regression



$$\log \left(\frac{p(X)}{1 - p(X)} \right) = \beta_0 + \beta_1 X.$$

Model Evaluation

Confusion Matrix



10,000 PATIENTS

PATIENTS

DIAGNOSIS

	Diagnosed Sick	Diagnosed Healthy
Sick	1000 True Positives	200 False Negatives
Healthy	800 False Positives	8000 True Negatives

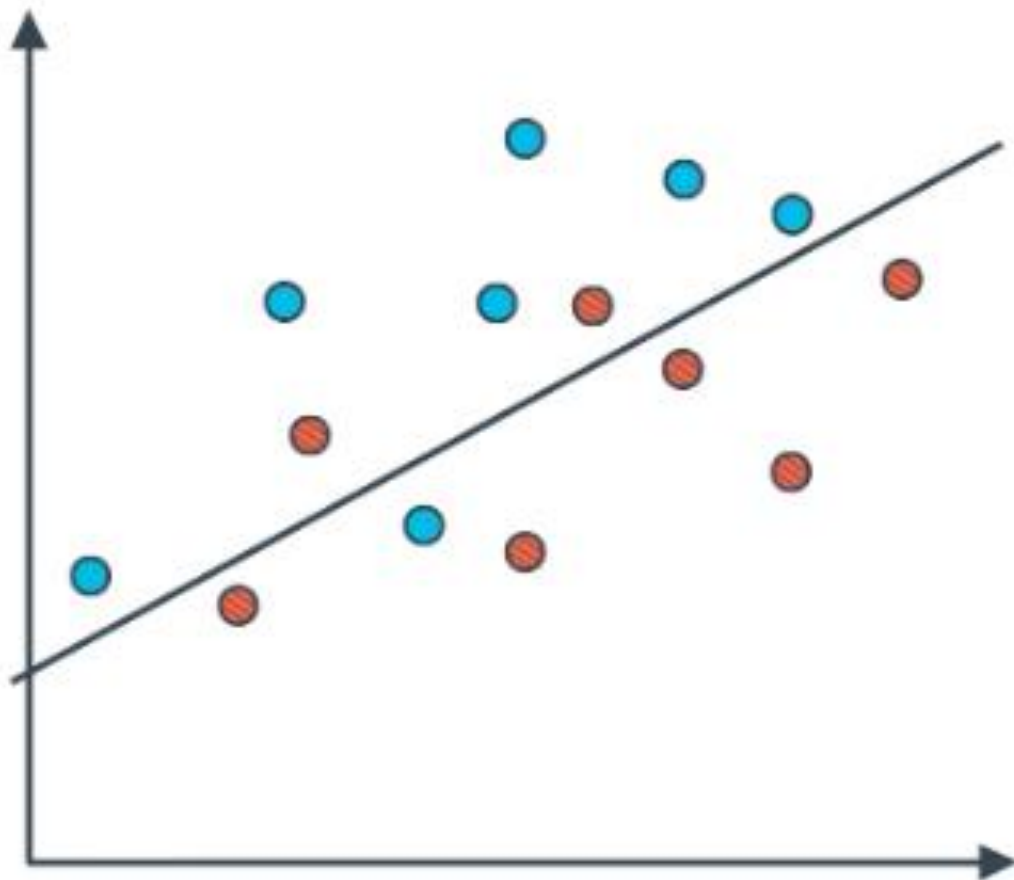
Confusion Matrix



1000 EMAILS

		SPAM	
		Spam Folder	Inbox
EMAIL	Spam	100 True Positives	170 False Negatives
	Not Spam	30 False Positives	700 True Negatives

Confusion Matrix



	Guessed Positive	Guessed Negative
Positive	6 True Positives	1 False Negatives
Negative	2 False Positives	5 True Negatives

TYPE-1 Error

TYPE-2 Error

In this image, the blue points are labelled positive, and the red points are labelled negative. Furthermore, the points on top of the line are predicted (guessed) to be positive, and the points below the line are predicted to be negative.

Accuracy

Out of total patients, how many identified correctly

	Diagnosed Sick	Diagnosed Healthy
Sick	1000 True Positives	200 False Negatives
Healthy	800 False Positives	8000 True Negatives

$$\frac{1000+8000}{1000+8000+200+800} = 90\%$$

Out of total emails, how many identified correctly

	Spam Folder	Inbox
Spam	100 True Positives	170 False Negatives
Not Spam	30 False Positives	700 True Negatives

$$\frac{100+700}{100+700+30+170} = 80\%$$

CREDIT CARD FRAUD






MODEL: ALL TRANSACTIONS ARE FRAUDULENT.

GREAT! NOW I'M CATCHING ALL OF THE FRAUDULENT TRANSACTIONS!

PROBLEM: I'M ACCIDENTALLY CATCHING ALL OF THE GOOD ONES!

WHICH ONE IS WORST?

Emails	Folder		
		SENT TO SPAM	SENT TO INBOX
	SPAM		 FALSE NEGATIVE
	NOT SPAM	 FALSE POSITIVE	

HOW WE EVALUATE?



Medical Model

FALSE POSITIVES OK

FALSE NEGATIVES NOT OK

OK IF NOT ALL ARE SICK
FIND ALL THE SICK PEOPLE

HIGH RECALL



Spam Detector

FALSE POSITIVES NOT OK

FALSE NEGATIVES OK

DON'T NECESSARILY NEED
TO FIND ALL THE SPAM
BETTER BE SPAM

HIGH PRECISION

Precision

Out of all patients diagnosed as sick, how many diagnosed sick correctly

	Diagnosed Sick	Diagnosed Healthy
Sick	1000 True Positives	200 False Negatives
Healthy	800 False Positives	8000 True Negatives

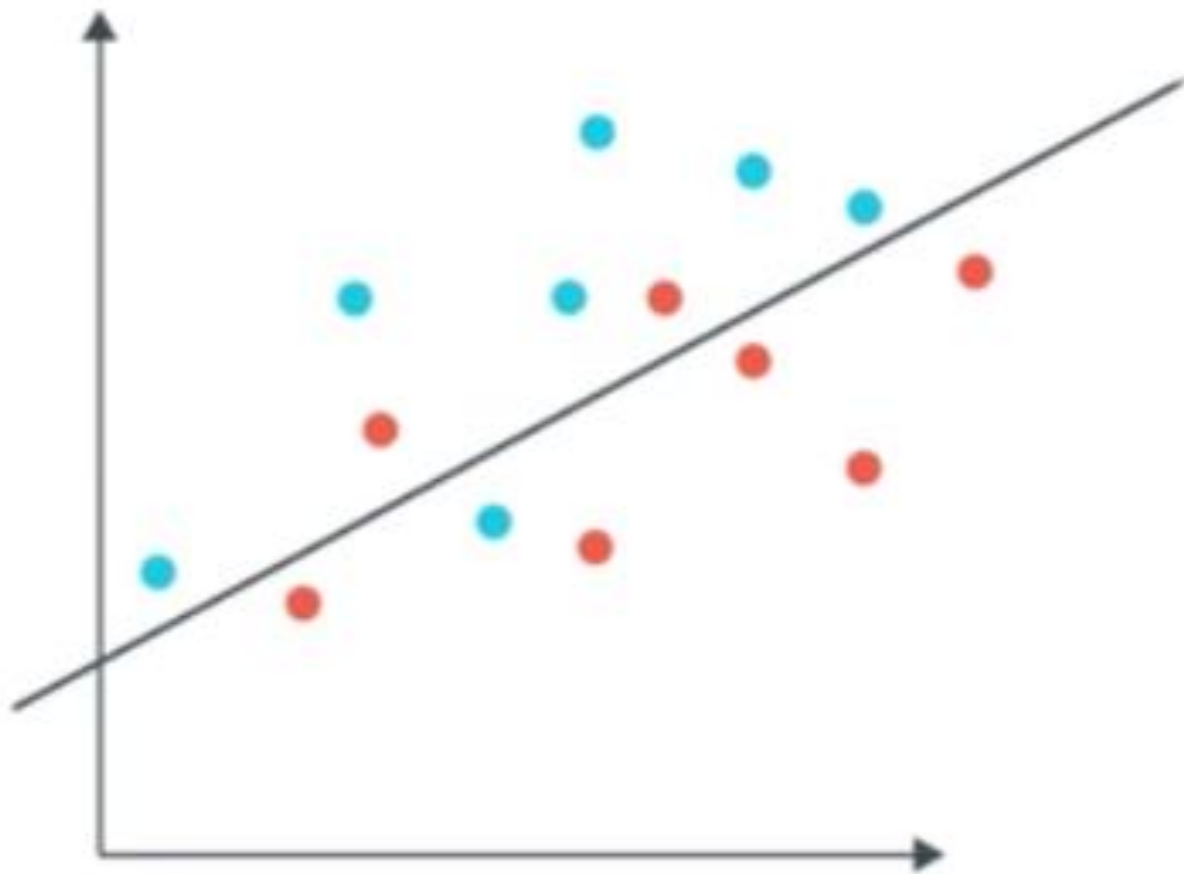
$$\frac{1000}{1000+800} = 55.6\%$$

Out of all emails sent to Spam folder, how many emails sent correctly

	Spam Folder	Inbox
Spam	100 True Positives	170 False Negatives
Not Spam	30 False Positives	700 True Negatives

$$\frac{100}{100+30} = 76.9\%$$

QUIZ



OUT OF THE POINTS WE HAVE
PREDICTED TO BE POSITIVE,
HOW MANY ARE CORRECT?

Recall

Out of all sick patients, how many were correctly diagnosed as sick

	Diagnosed Sick	Diagnosed Healthy
Sick	1000 True Positives	200 False Negatives
Healthy	800 False Positives	8000 True Negatives

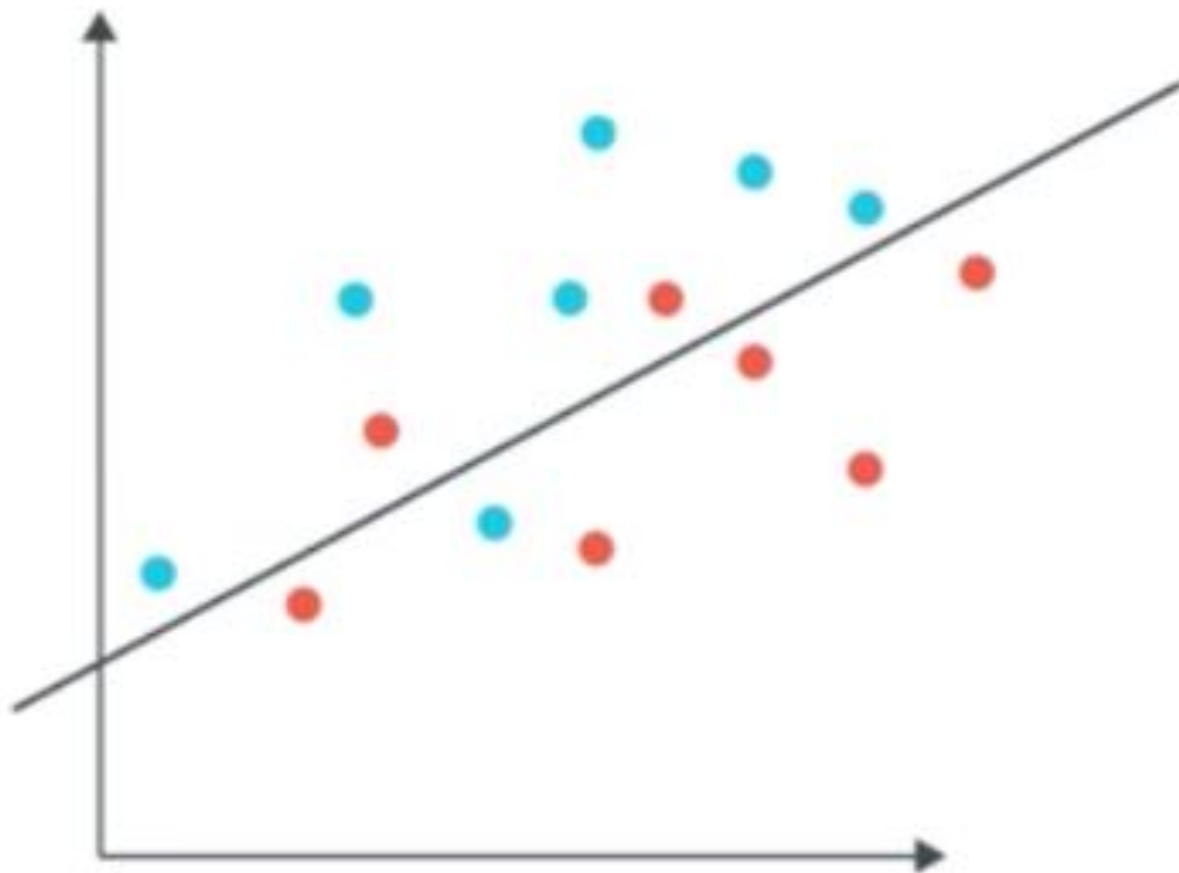
$$\frac{1000}{1000+200} = 83.3\%$$

Out of all spam emails, how many were correctly sent to spam folder

	Spam Folder	Inbox
Spam	100 True Positives	170 False Negatives
Not Spam	30 False Positives	700 True Negatives

$$\frac{100}{100+170} = 37\%$$

QUIZ



OUT OF THE POINTS
LABELLED "POSTIIVE,"
HOW MANY DID WE
CORRECTLY PREDICT?

Precision vs Recall



MEDICAL MODEL
PRECISION: 55.7%
RECALL: 83.3%

ONE SCORE?



SPAM DETECTOR
PRECISION: 76.9%
RECALL: 37%

F1 Score



MEDICAL MODEL
PRECISION: 55.7%
RECALL: 83.3%

AVERAGE: 69.5%

ONE SCORE?



SPAM DETECTOR
PRECISION: 76.9%
RECALL: 37%

AVERAGE: 56.95%

F1 Score



MODEL: ALL TRANSACTIONS ARE GOOD.

PRECISION = 100%

AVERAGE = 50%

RECALL = 0%

F1 Score



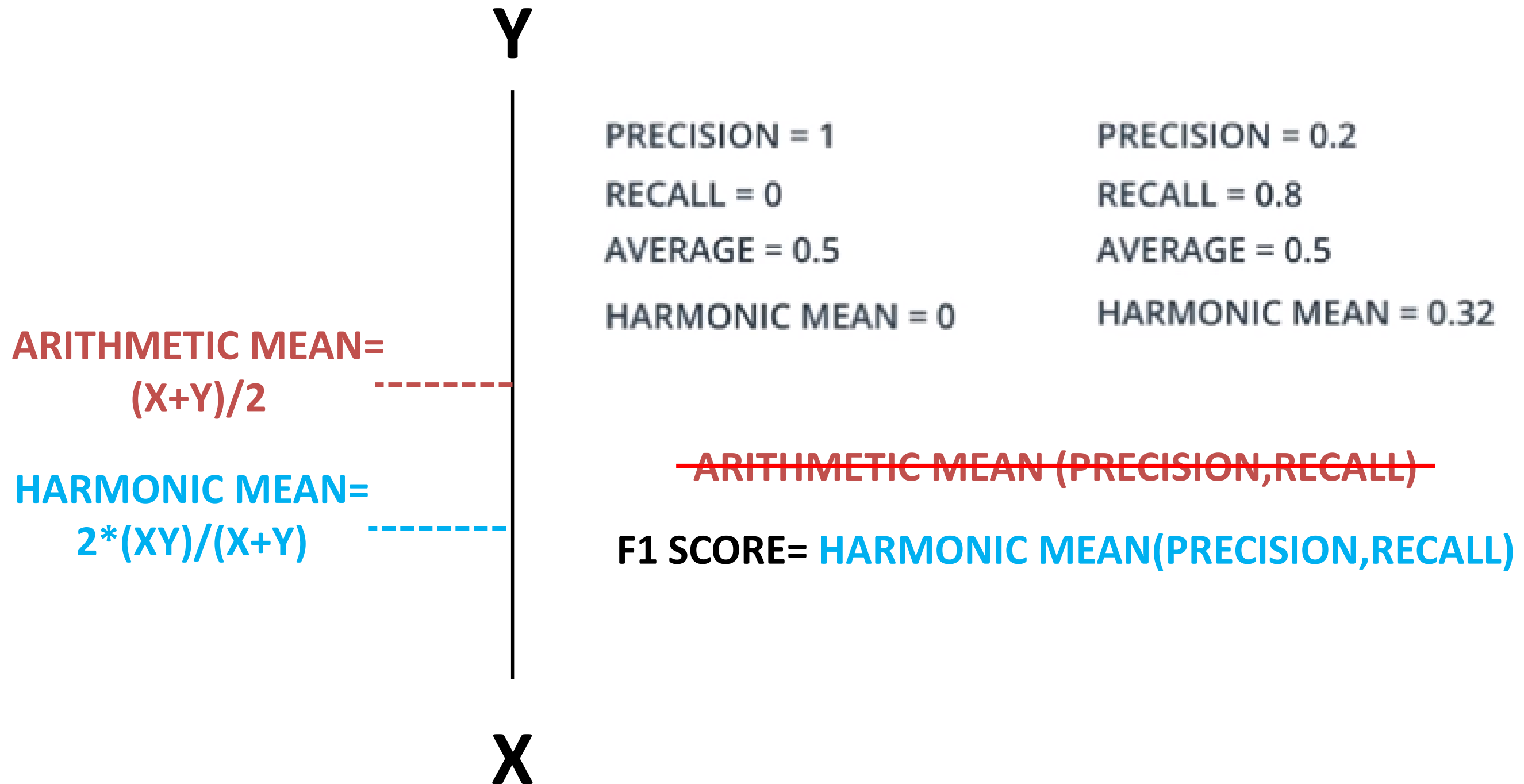
MODEL: ALL TRANSACTIONS ARE FRAUDULENT.

$$\text{PRECISION} = 472 / 284,807 = 0.16\%$$

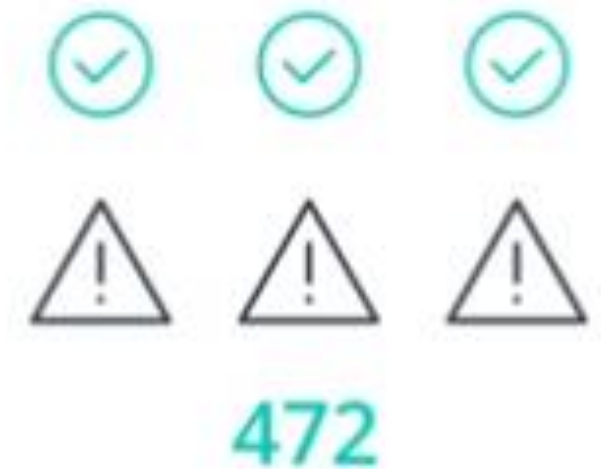
$$\text{RECALL} = 472 / 472 = 100\%$$

$$\text{AVERAGE} = 50.08\%$$

F1 Score



F1 Score



MODEL: ALL TRANSACTIONS ARE GOOD.

PRECISION = 100%

F_1 SCORE = 0

RECALL = 0%

Confusion Matrix

n=165	Predicted: NO	Predicted: YES
	Actual: NO	50
	Actual: YES	5

	Predicted: NO	Predicted: YES
Actual: NO	TN	FP
Actual: YES	FN	TP

- **true positives (TP):** These are cases in which we predicted yes (they have the disease), and they do have the disease.
- **true negatives (TN):** We predicted no, and they don't have the disease.
- **false positives (FP):** We predicted yes, but they don't actually have the disease. (Also known as a "Type I error.")
- **false negatives (FN):** We predicted no, but they actually do have the disease. (Also known as a "Type II error.")

Confusion Matrix

n=165	Predicted: NO	Predicted: YES	
Actual: NO	TN = 50	FP = 10	60
Actual: YES	FN = 5	TP = 100	105
	55	110	

Accuracy: Overall, how often is the classifier correct?
 $(TP+TN)/total = (100+50)/165 = 0.91$

Precision: When it predicts yes, how often is it correct?
 $TP/predicted\ yes = 100/110 = 0.91$

True Positive Rate: When it's actually yes, how often does it predict yes?
 $TP/actual\ yes = 100/105 = 0.95$
also known as "Sensitivity" or "Recall"

Confusion Matrix

n=165	Predicted: NO	Predicted: YES	
Actual: NO	TN = 50	FP = 10	60
Actual: YES	FN = 5	TP = 100	105
	55	110	

False Positive Rate: When it's actually no, how often does it predict yes?

$$\text{FP/actual no} = 10/60 = 0.17$$

Specificity: When it's actually no, how often does it predict no?

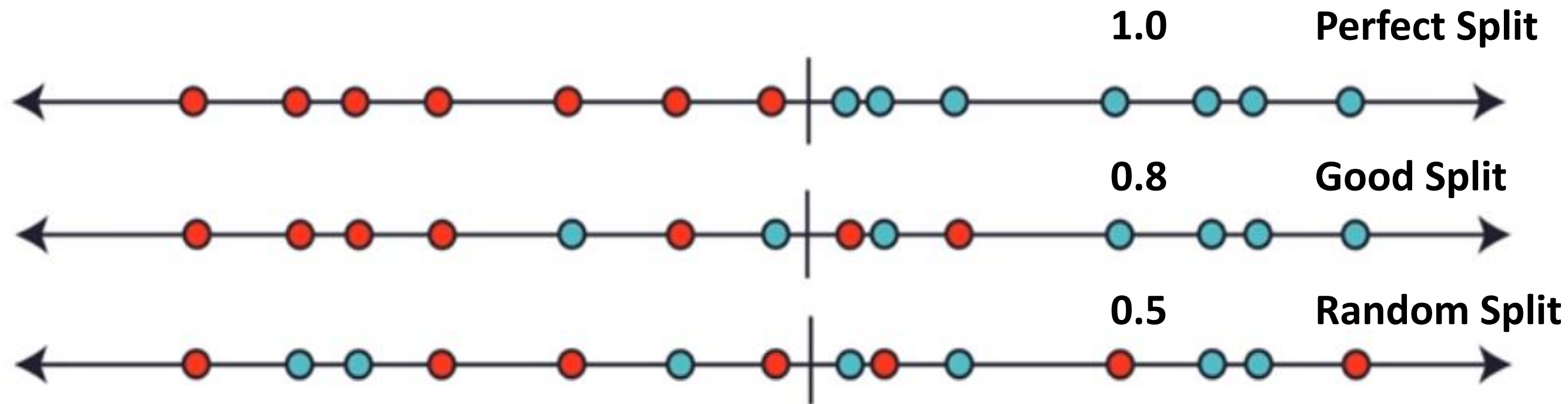
$$\text{TN/actual no} = 50/60 = 0.83$$

equivalent to 1 minus False Positive Rate

F1 Score: This is a weighted average of the true positive rate (recall) and precision.

$$2 * (\text{precision} * \text{recall}) / (\text{precision} + \text{recall})$$

Receiver Operating Characteristic



ROC Curve

$$\text{True Positive Rate} = \frac{\text{TRUE POSITIVES}}{\text{ALL POSITIVES}} = \frac{6}{7}$$

$$\text{False Positive Rate} = \frac{\text{FALSE POSITIVES}}{\text{ALL NEGATIVES}} = \frac{2}{7}$$



ROC Curve

$$\text{True Positive Rate} = \frac{\text{TRUE POSITIVES}}{\text{ALL POSITIVES}} = \frac{7}{7} =$$

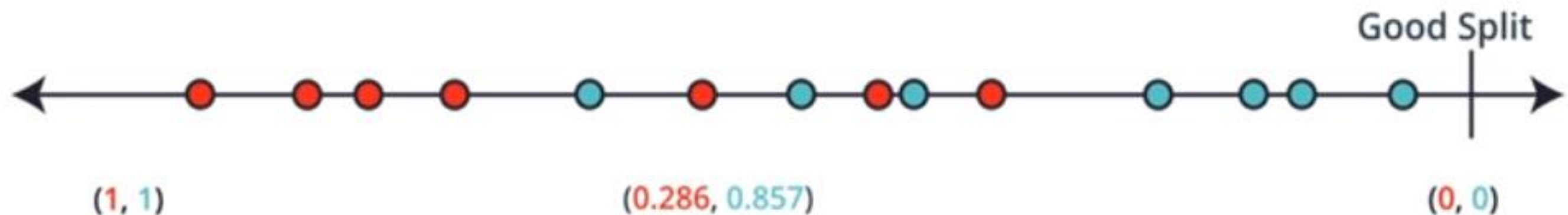
$$\text{False Positive Rate} = \frac{\text{FALSE POSITIVES}}{\text{ALL NEGATIVES}} = \frac{7}{7} =$$



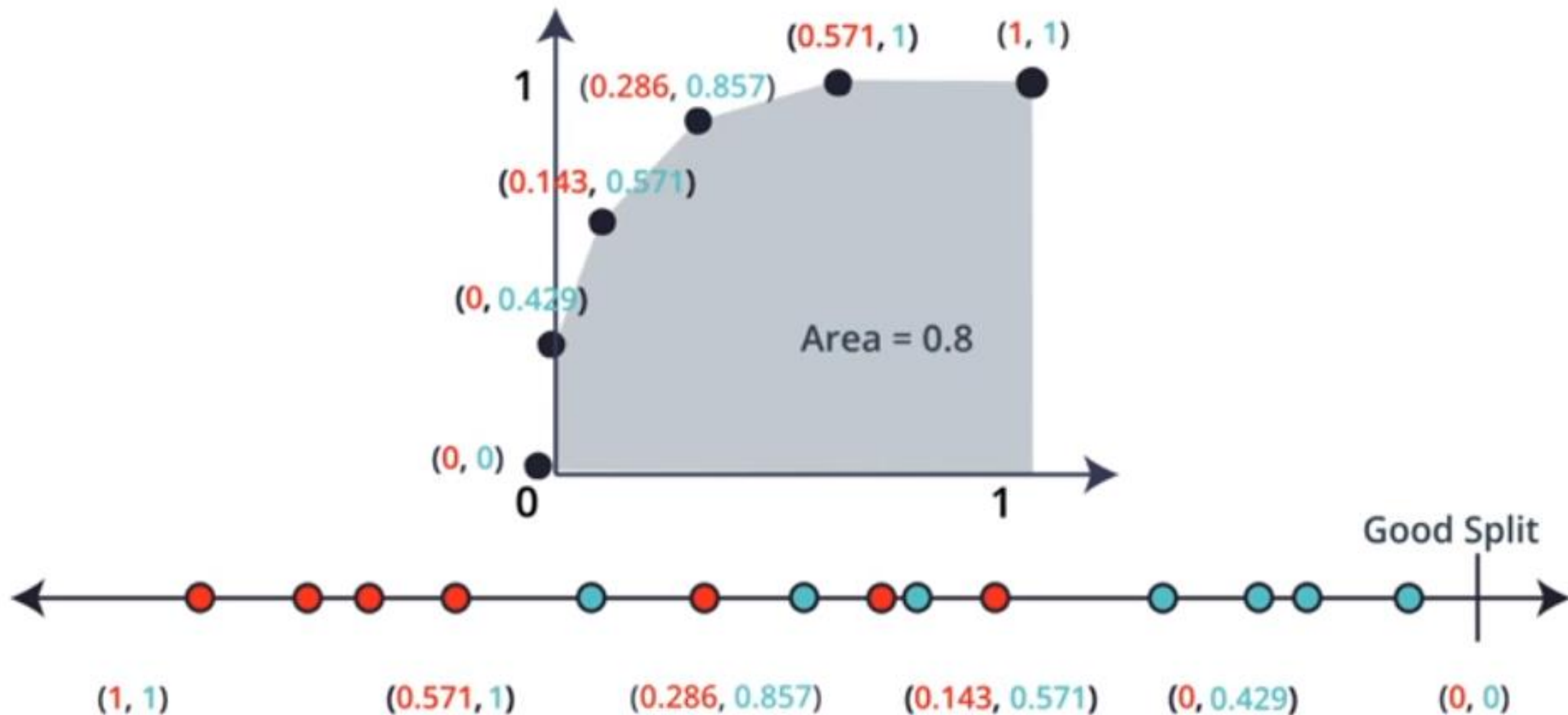
ROC Curve

$$\text{True Positive Rate} = \frac{\text{TRUE POSITIVES}}{\text{ALL POSITIVES}} = \frac{0}{7} =$$

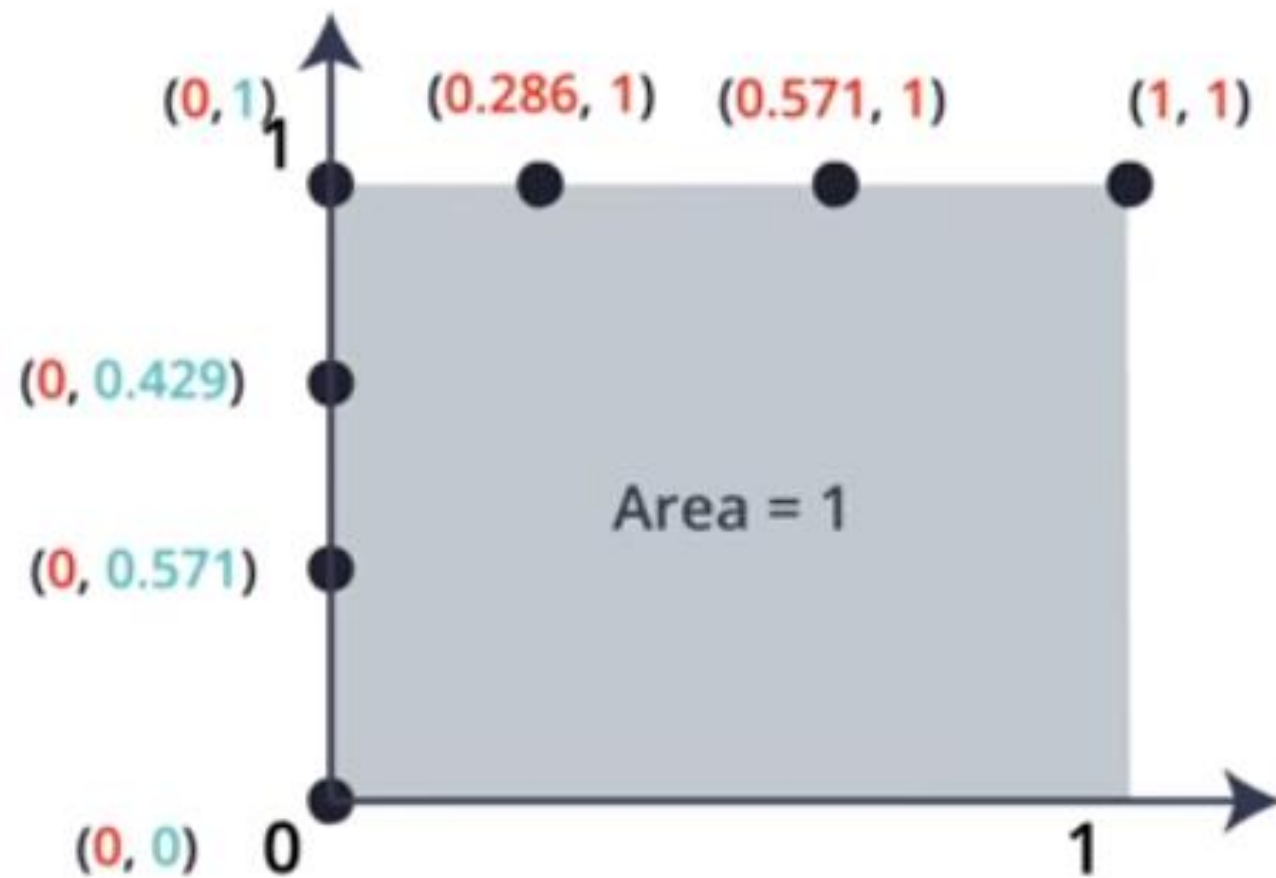
$$\text{False Positive Rate} = \frac{\text{FALSE POSITIVES}}{\text{ALL NEGATIVES}} = \frac{0}{7} =$$



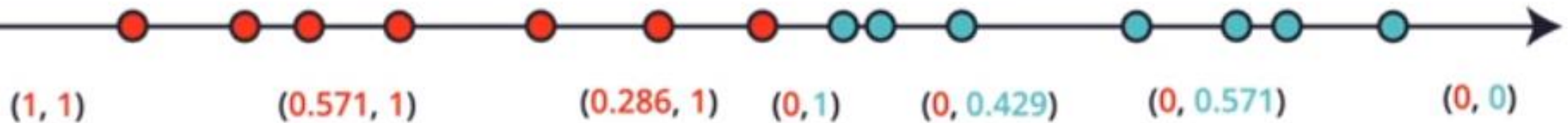
ROC Curve



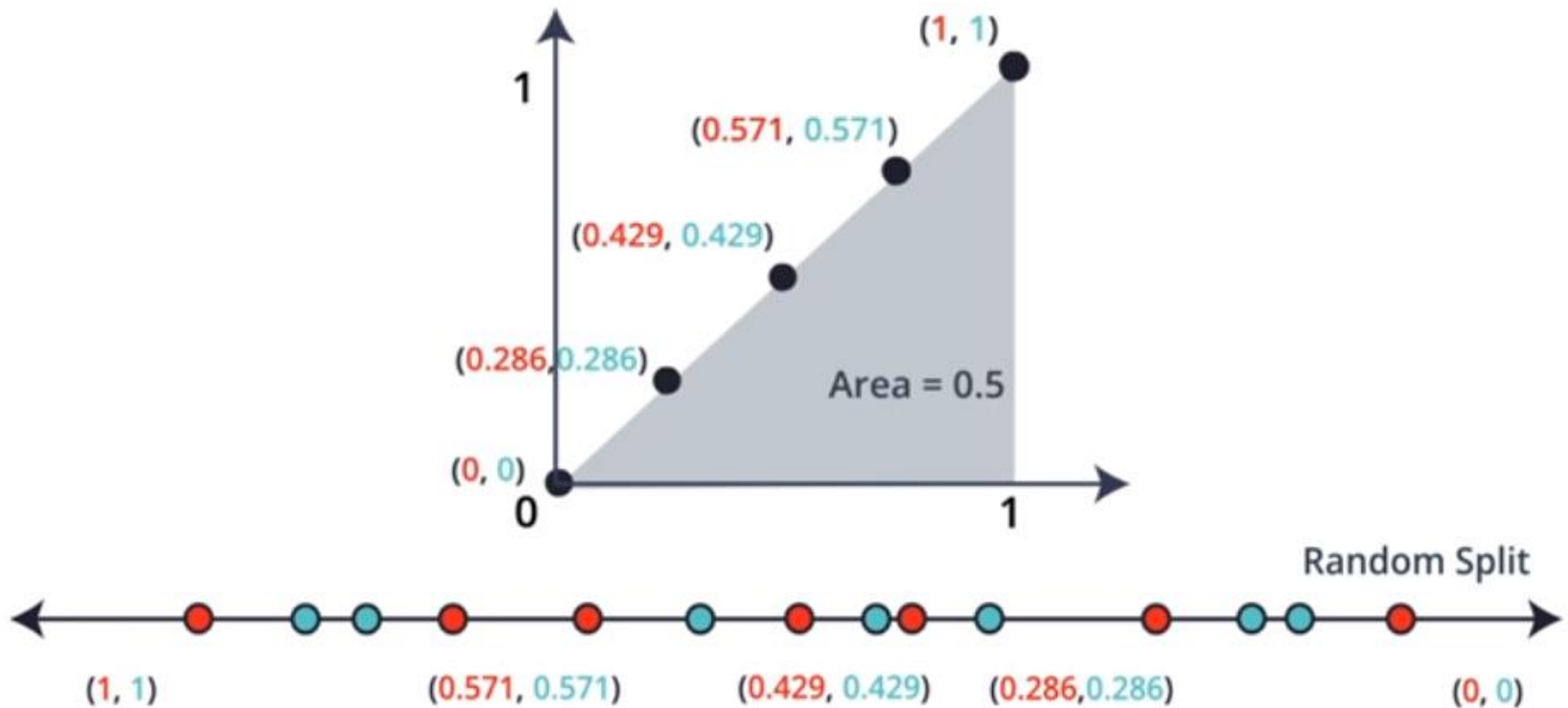
ROC Curve



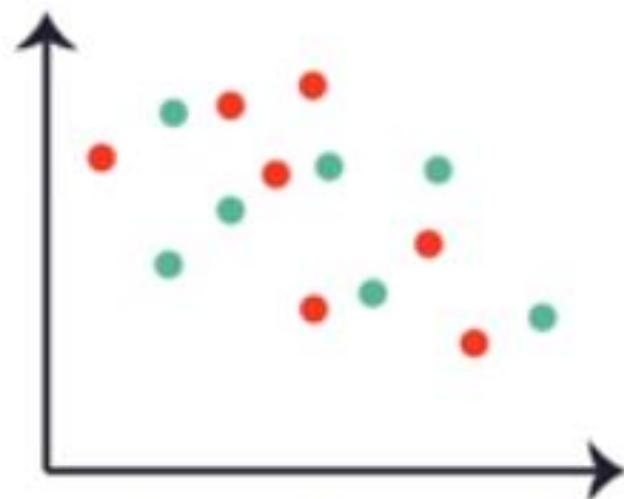
Perfect Split



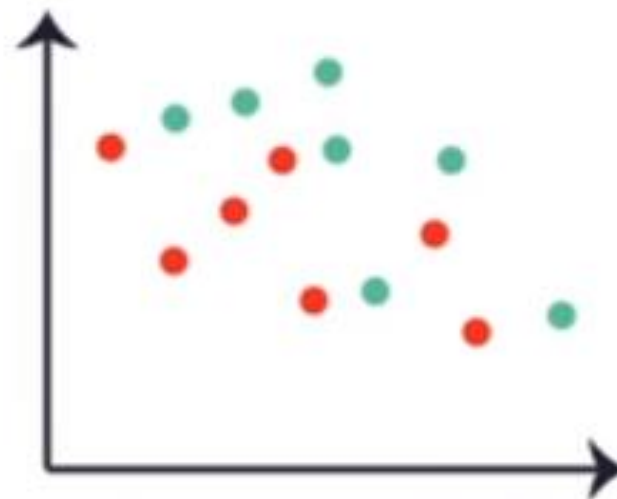
ROC Curve



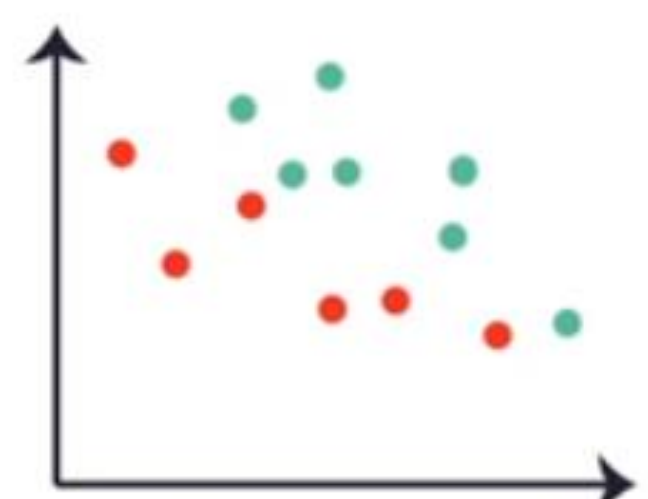
AREA UNDER ROC Curve



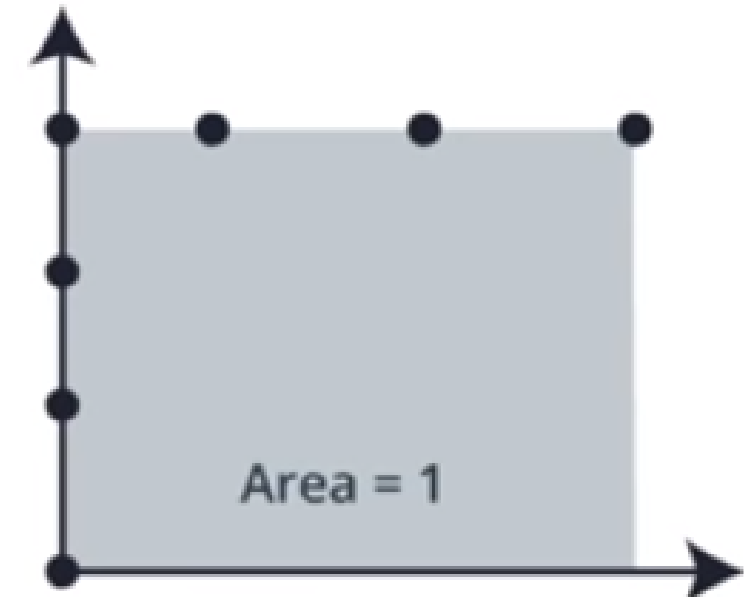
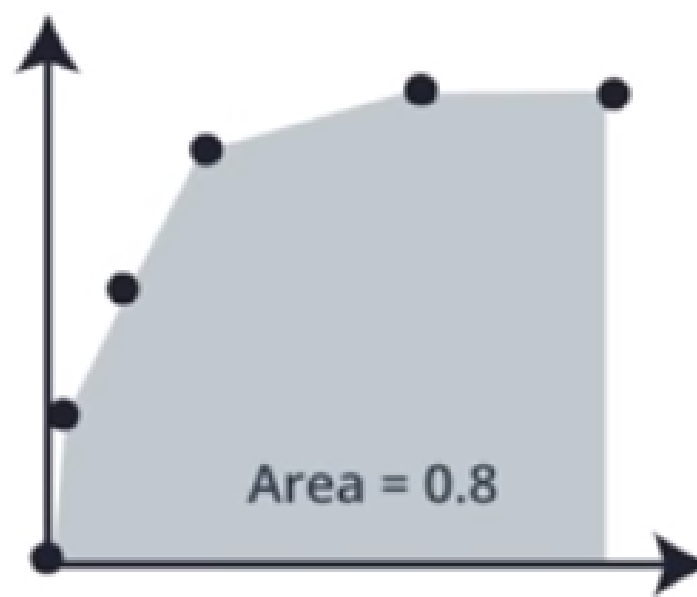
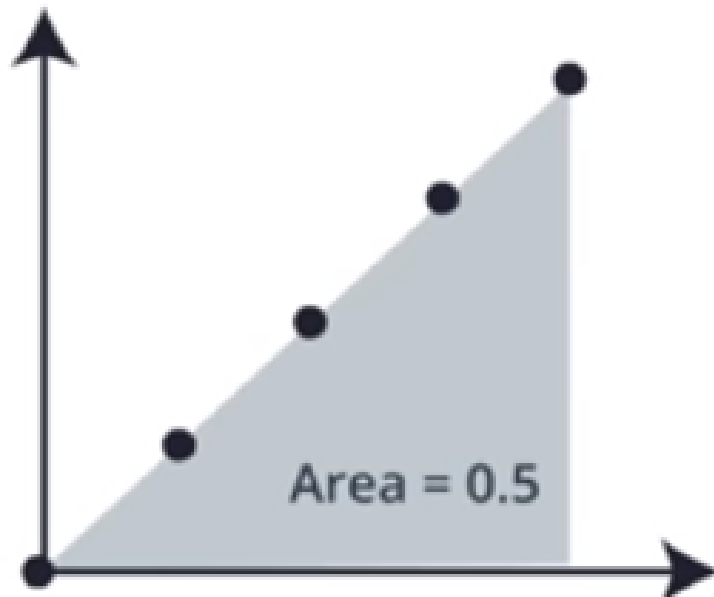
RANDOM SPLIT



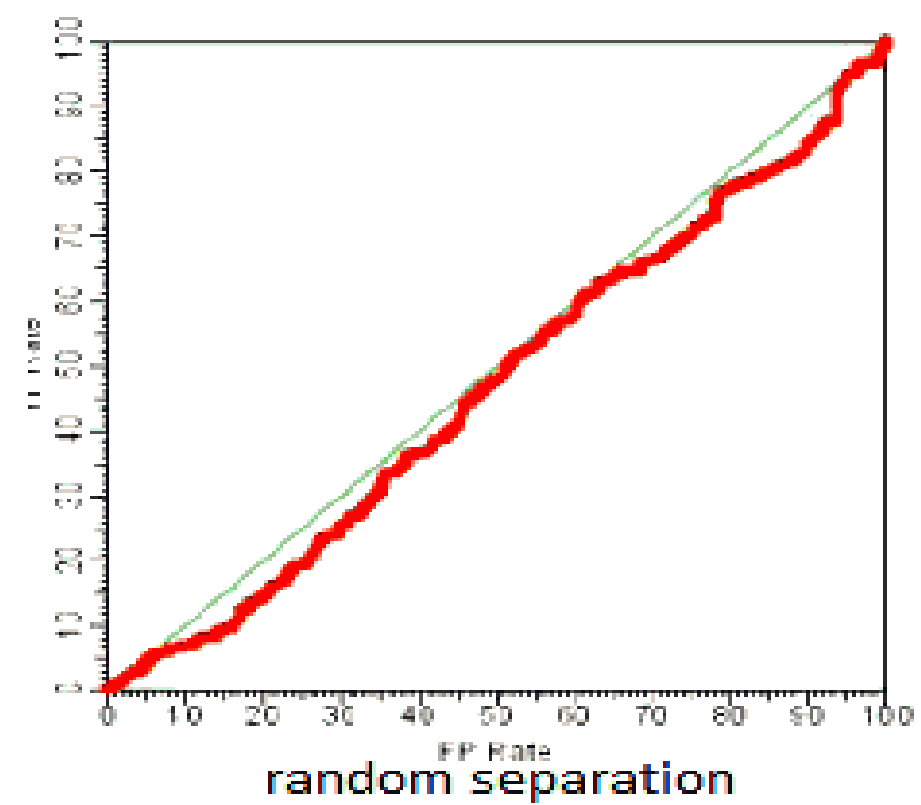
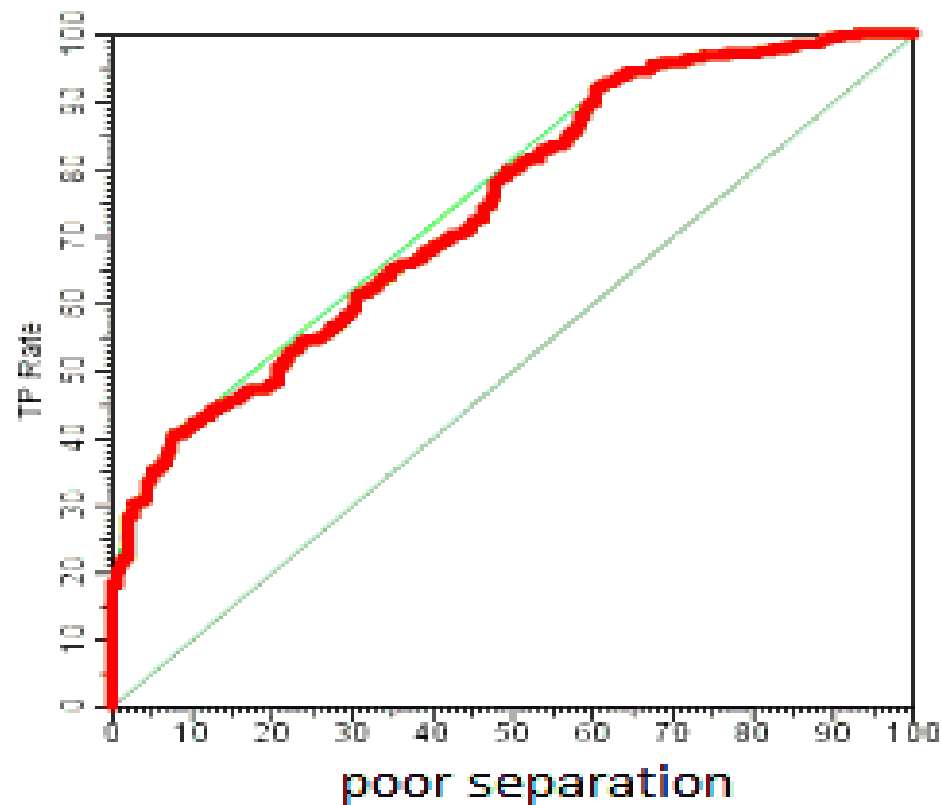
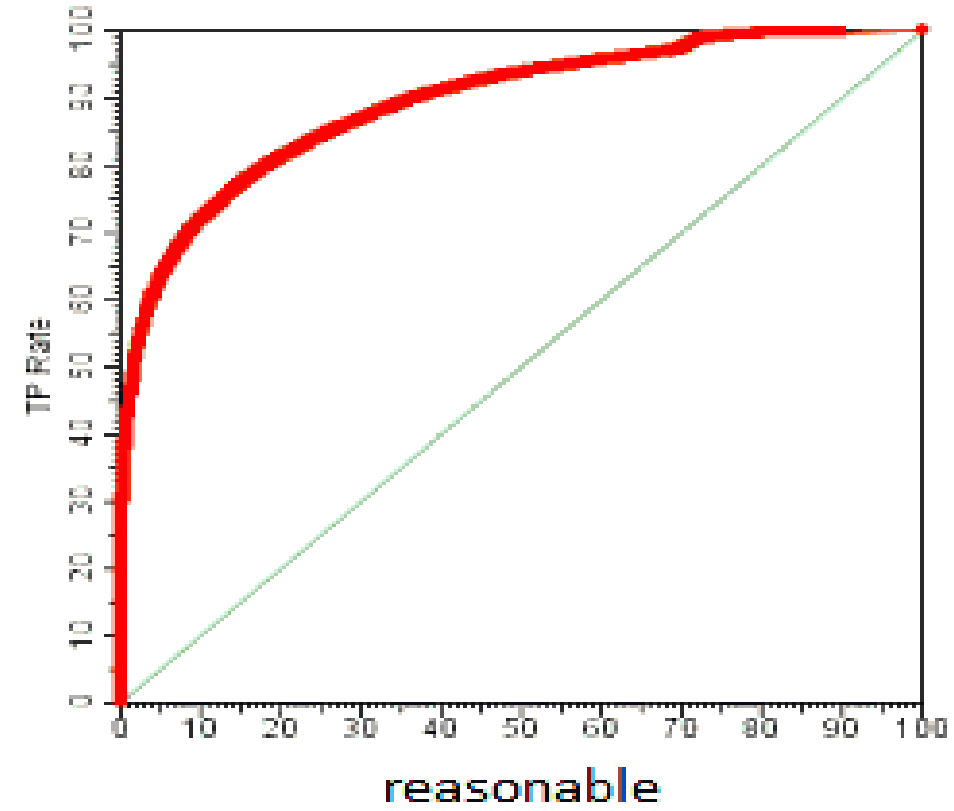
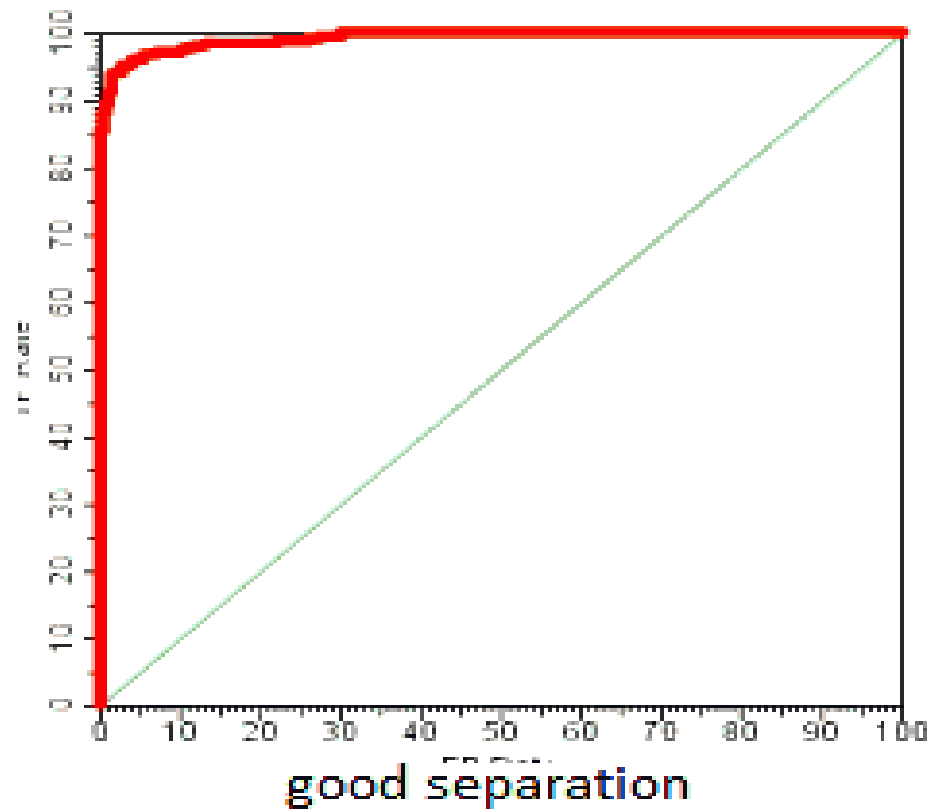
GOOD SPLIT



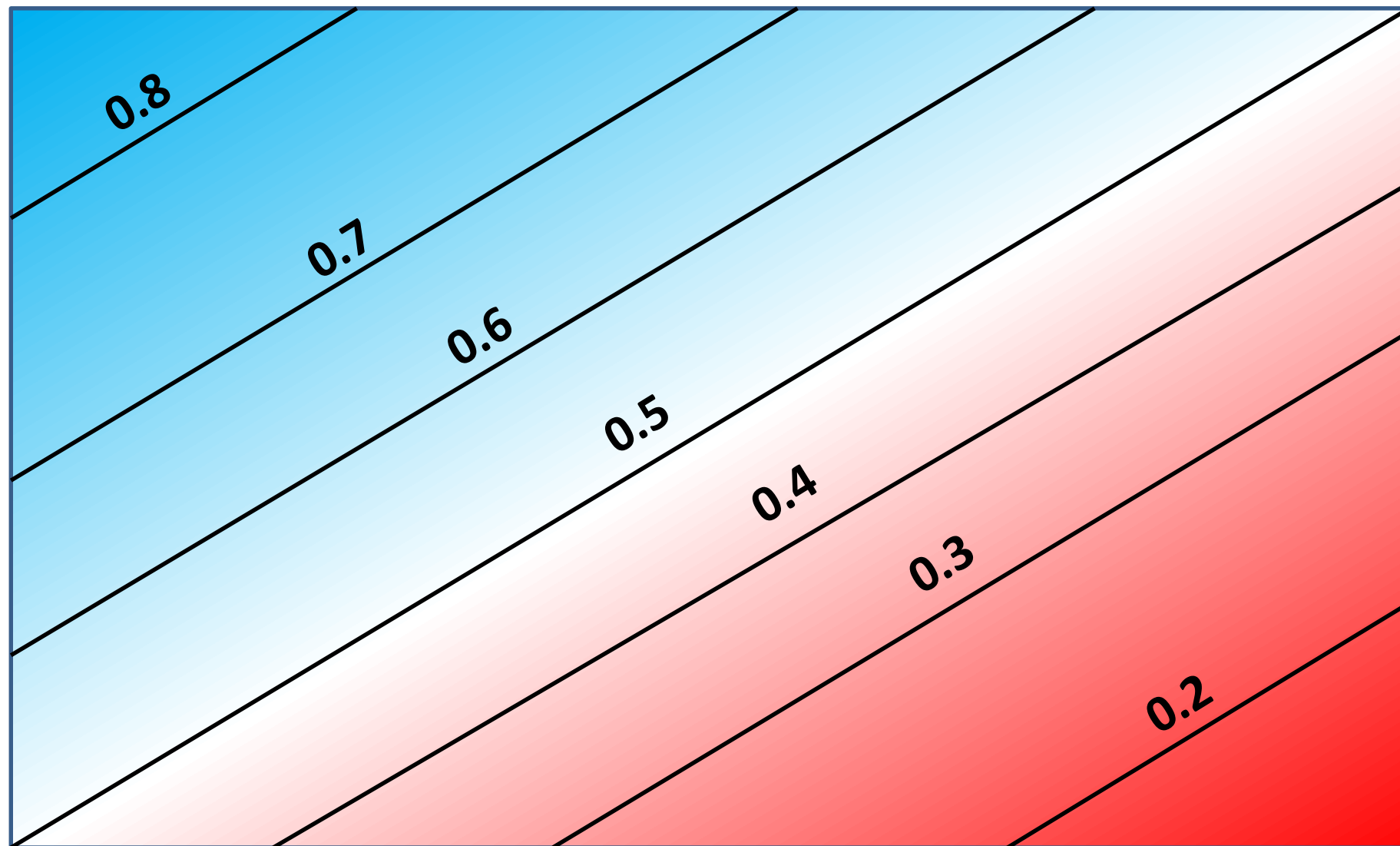
PERFECT SPLIT



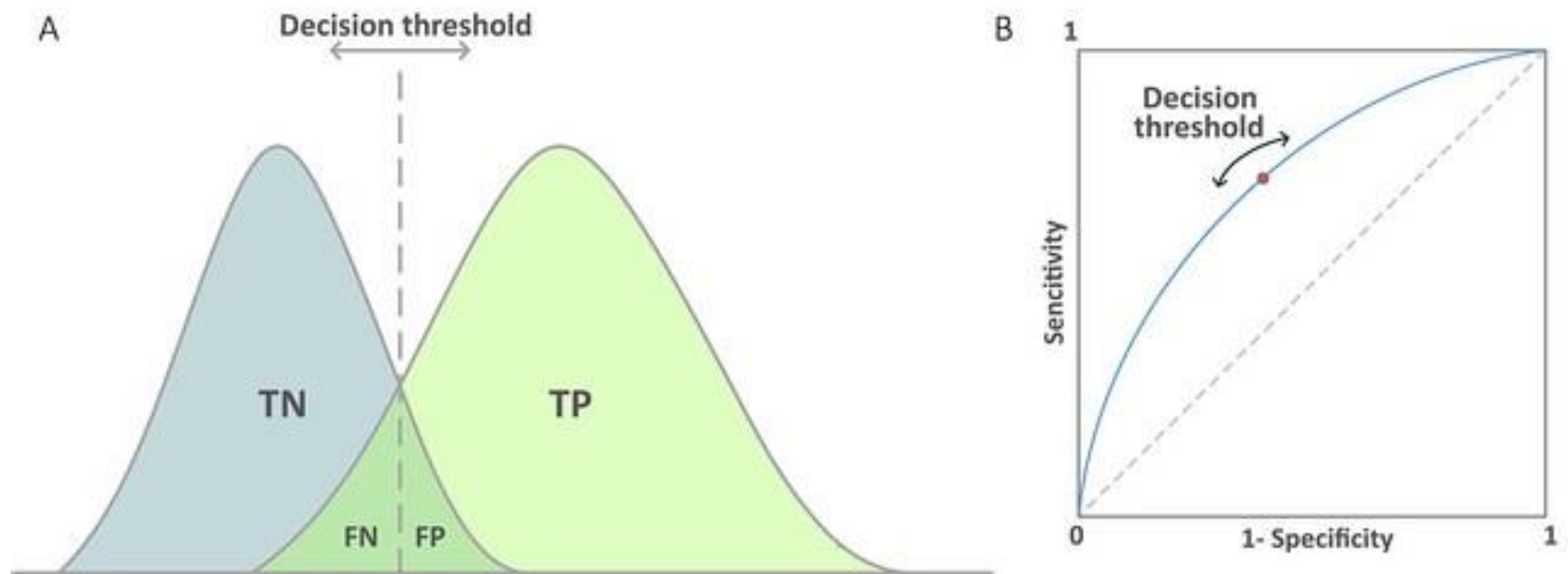
ROC AUC Curve



Classifier Decision Threshold



Classifier Decision Threshold



The goal is to outline how to move the decision threshold to in Figure A, reducing false negatives or reducing false positives as per domain knowledge

Let's Practice