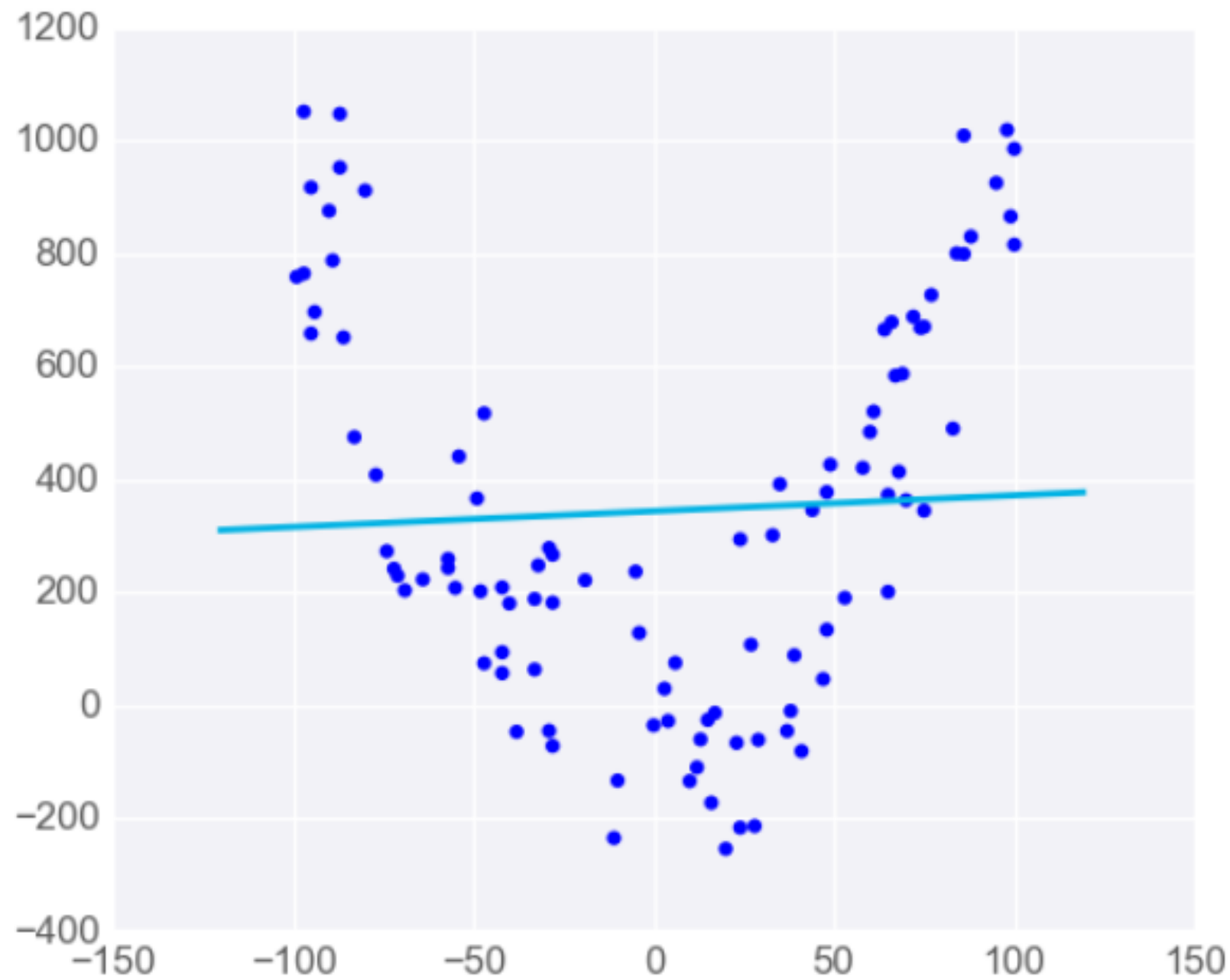
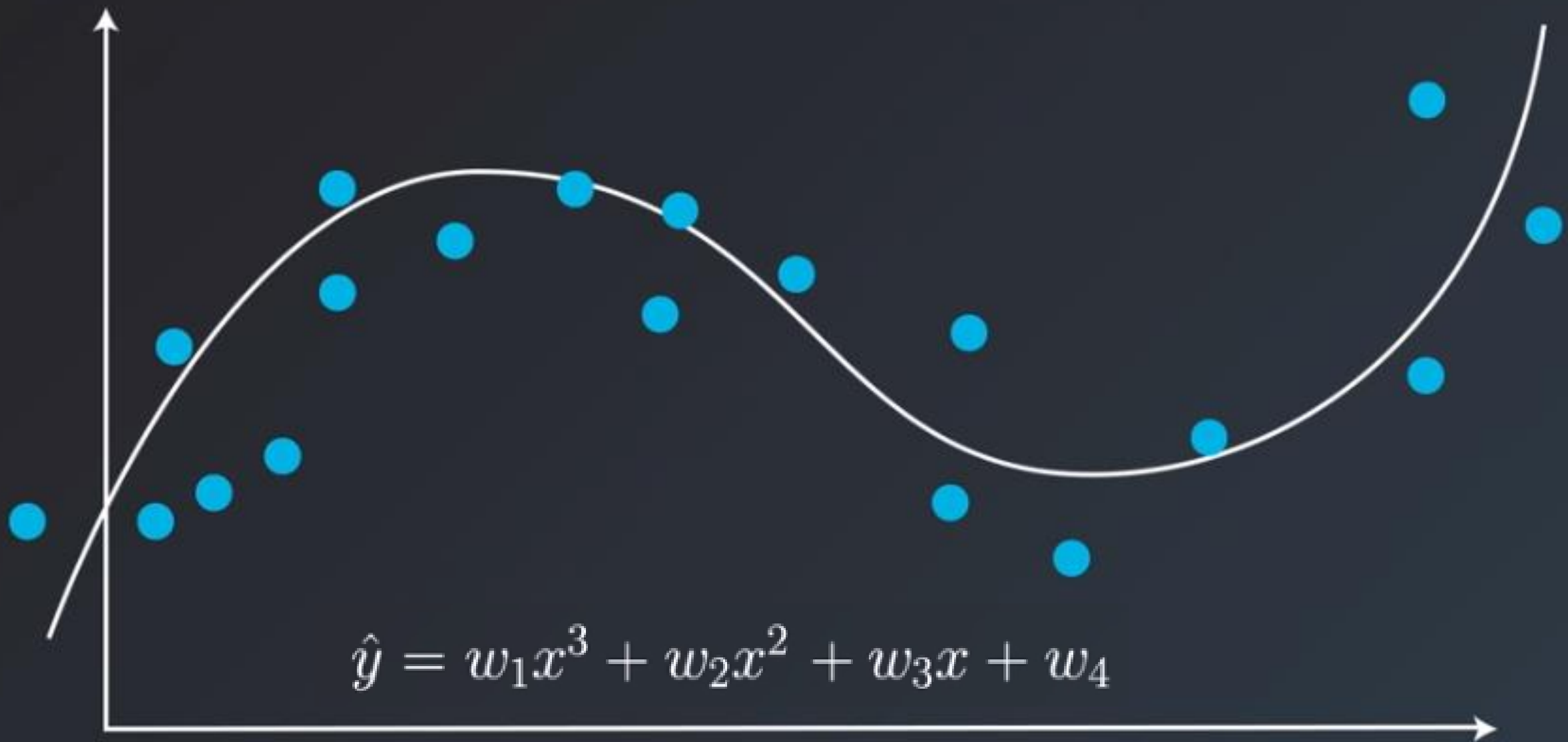


Polynomial Regression

Linear Regression Works Best When the Data is Linear



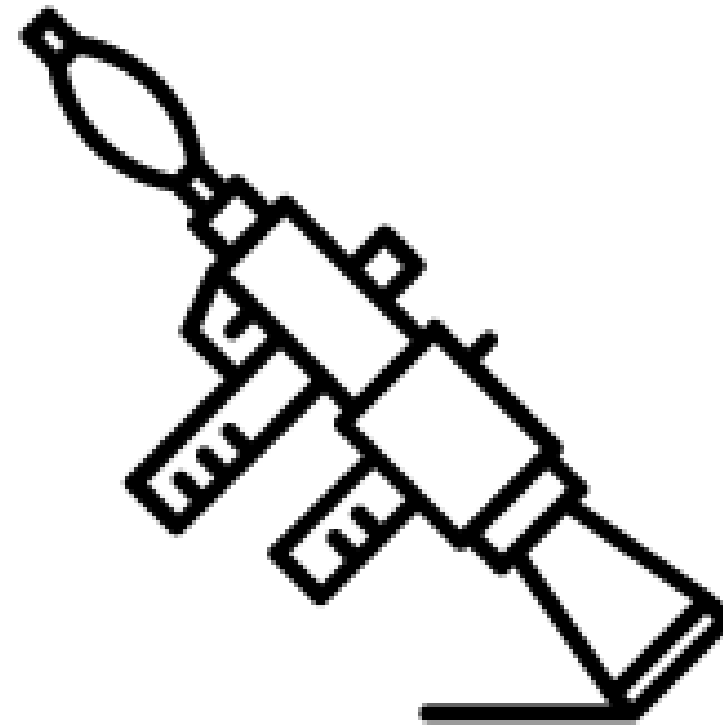
Polynomial Regression



Model Selection



Simple Problem



Complex Solution

Model Selection

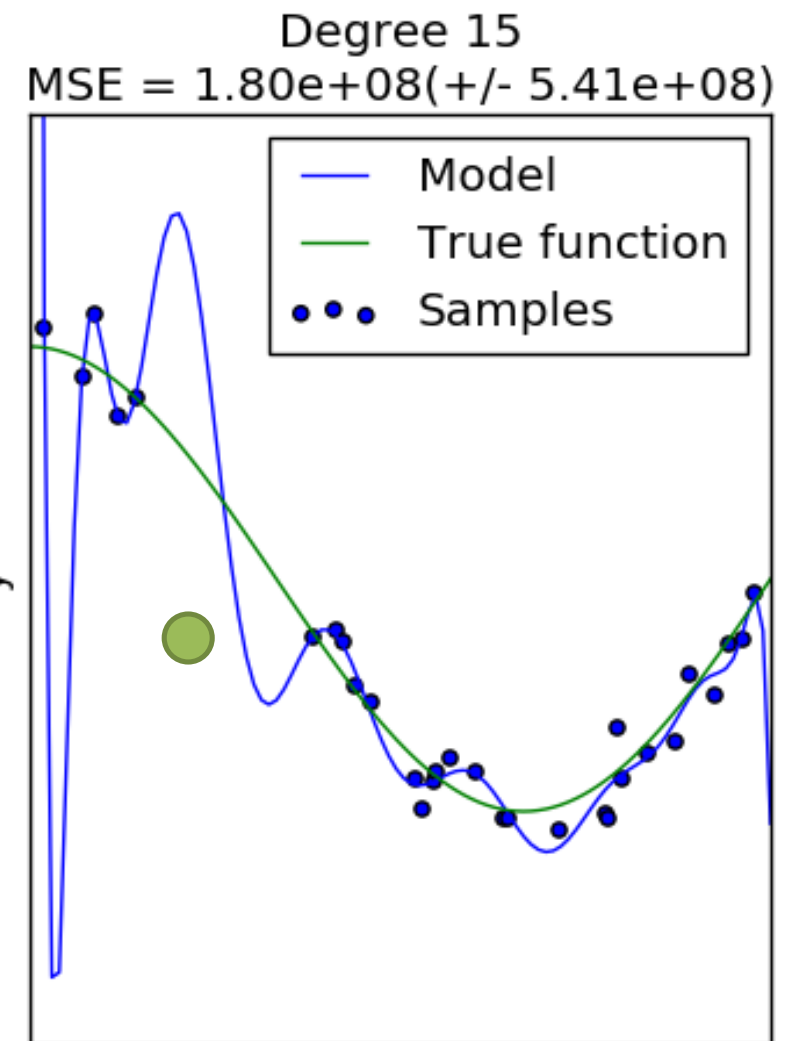
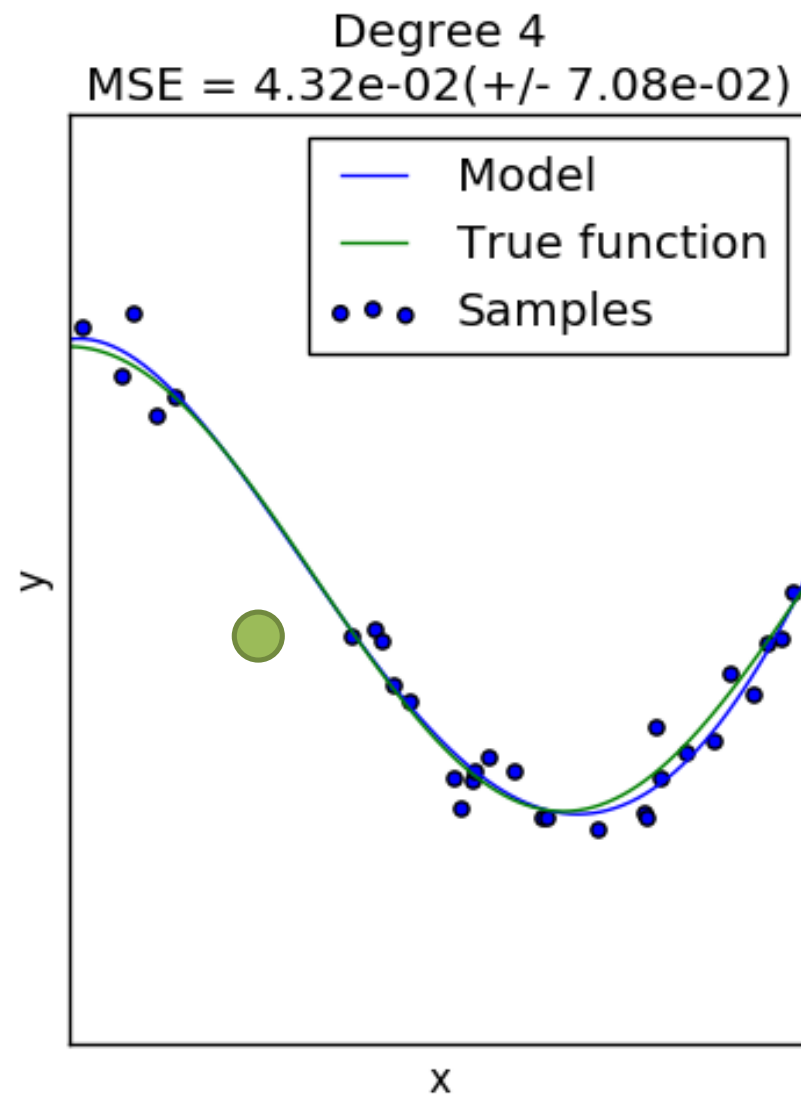
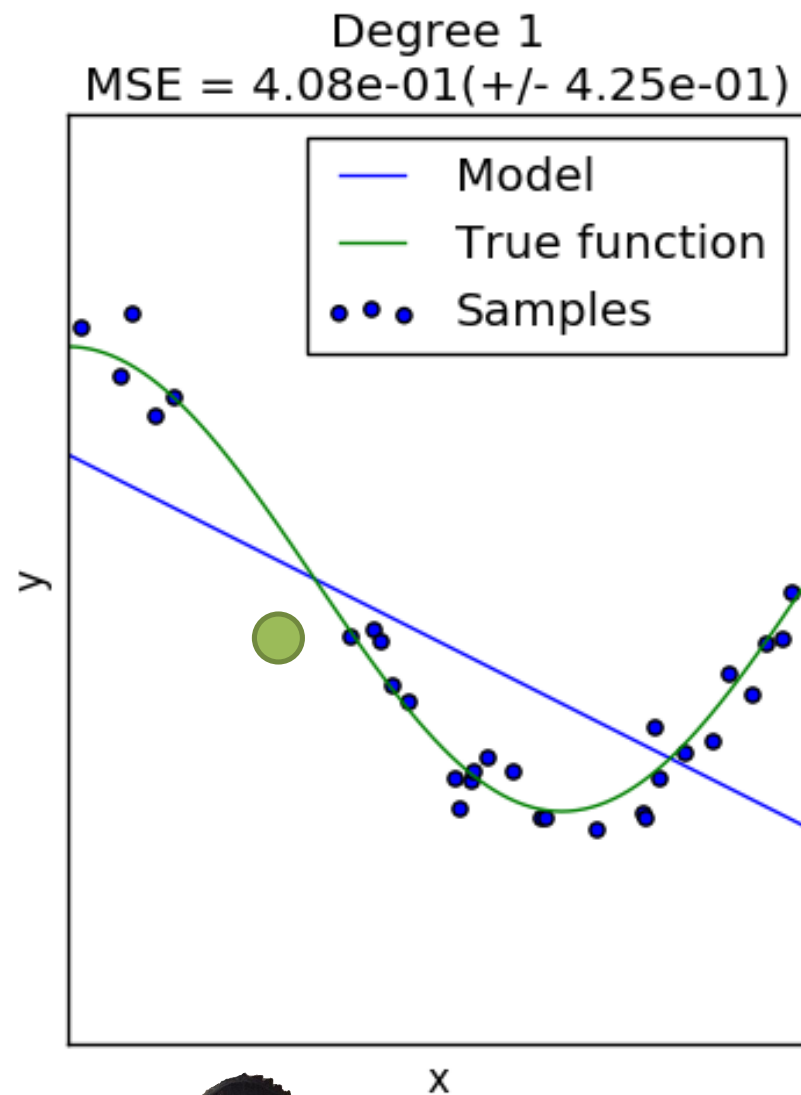


Complex Problem

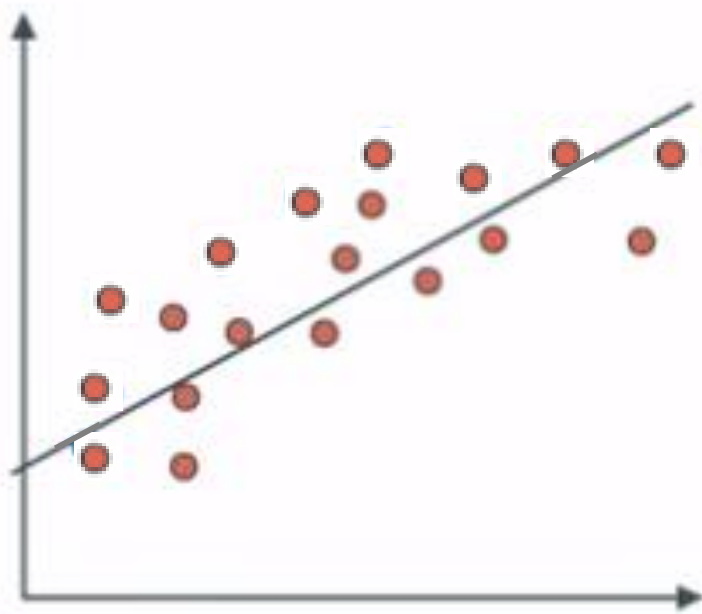


Simple Solution

Under-fitting & Over-fitting

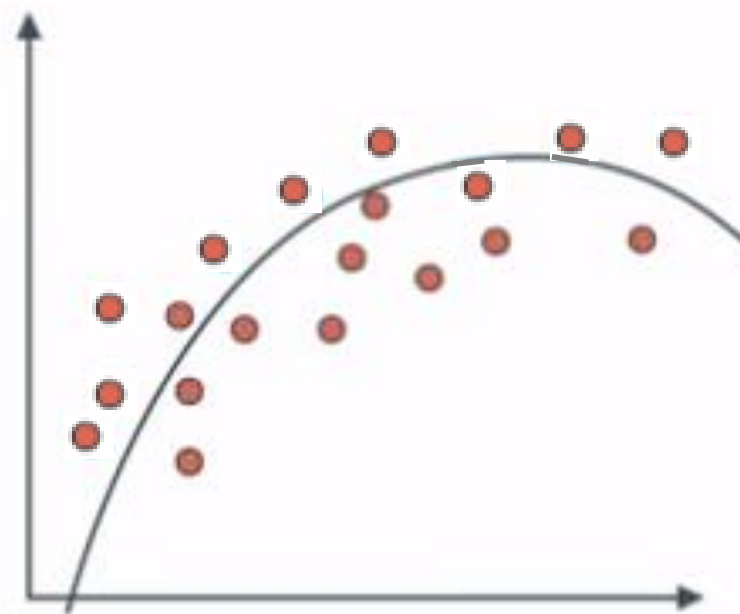


Model Complexity



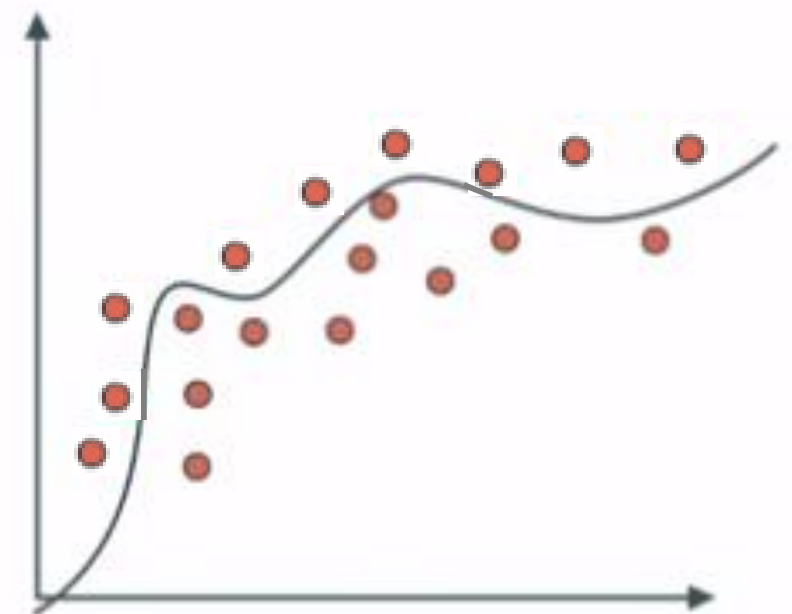
HIGH BIAS

Degree = 1



POLYNOMIAL

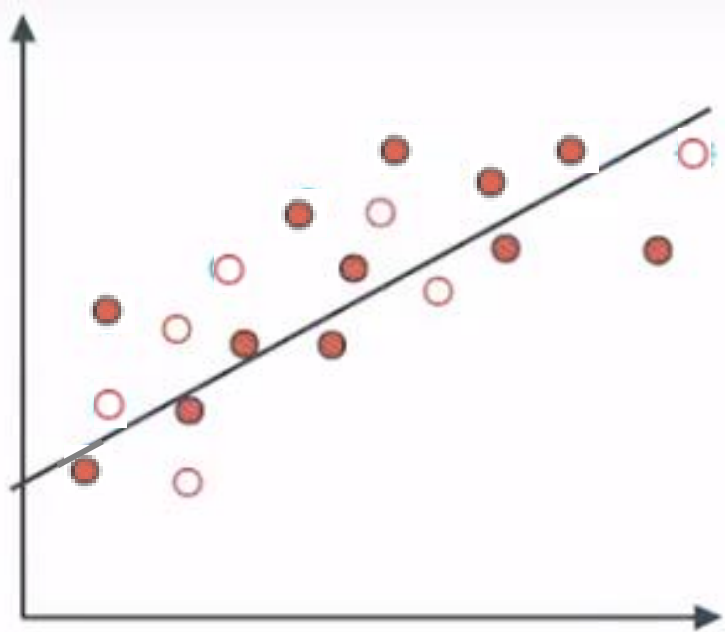
Degree = 2



HIGH VARIANCE

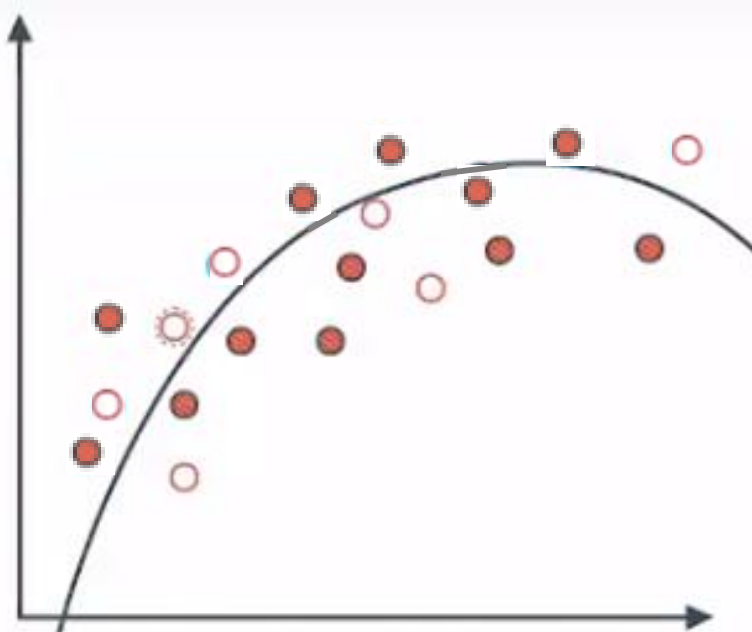
Degree = 6

Model Complexity



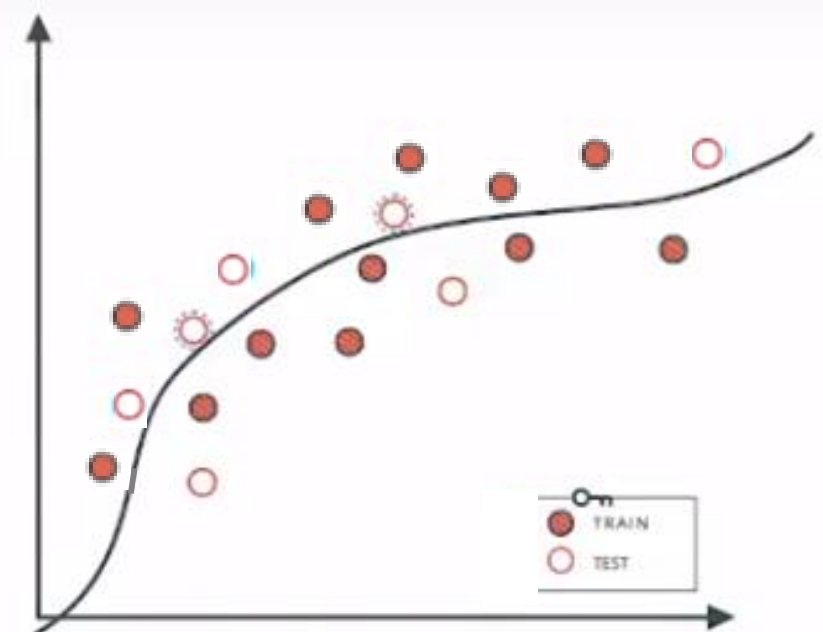
LINEAR MODEL

Degree = 1



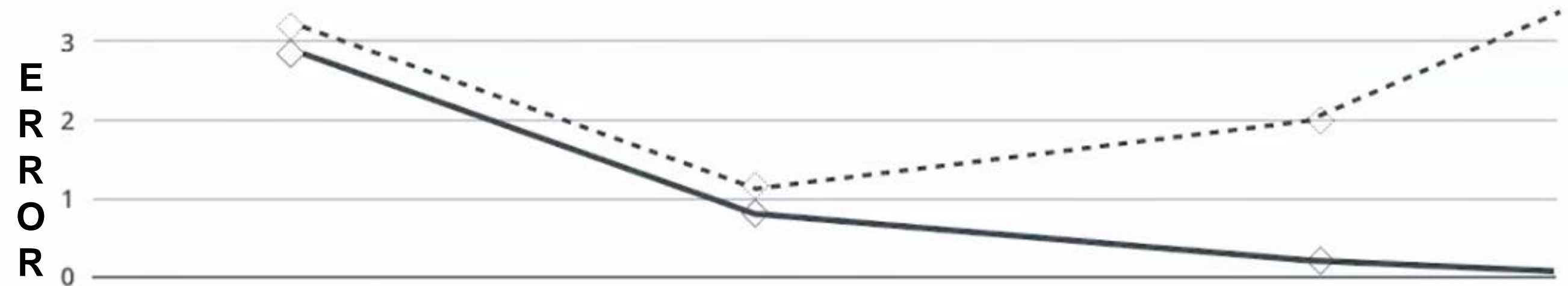
QUADRATIC MODEL

Degree = 2

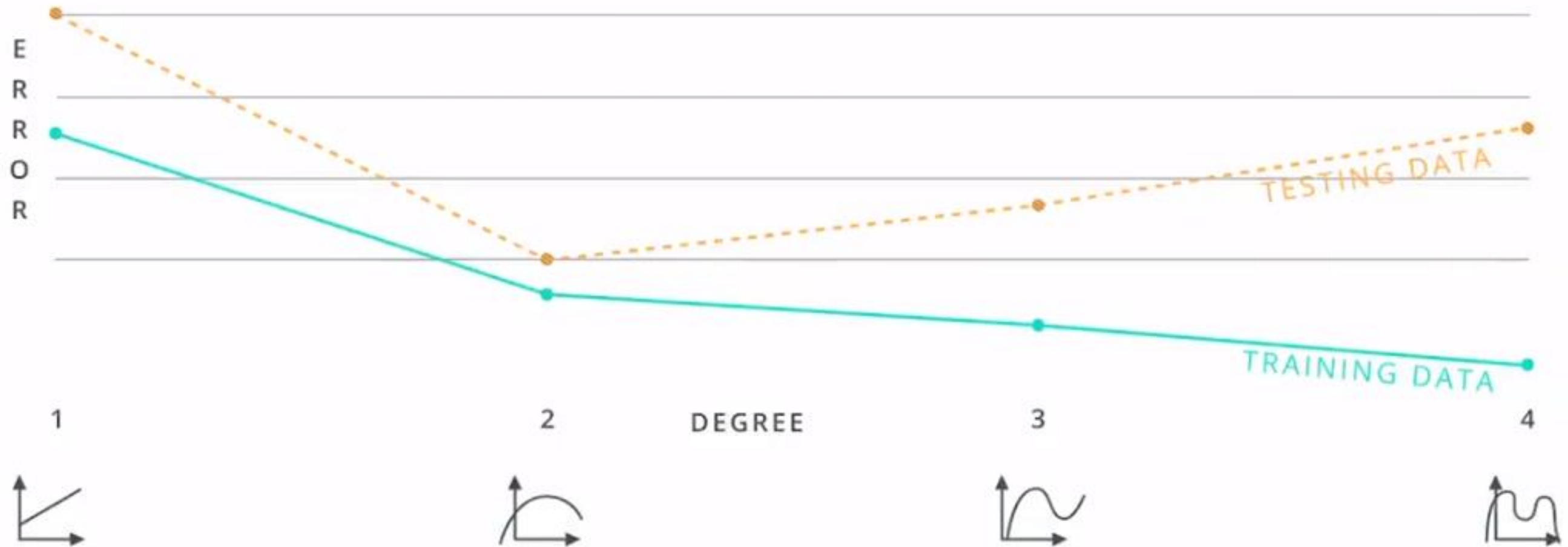


POLYNOMIAL MODEL

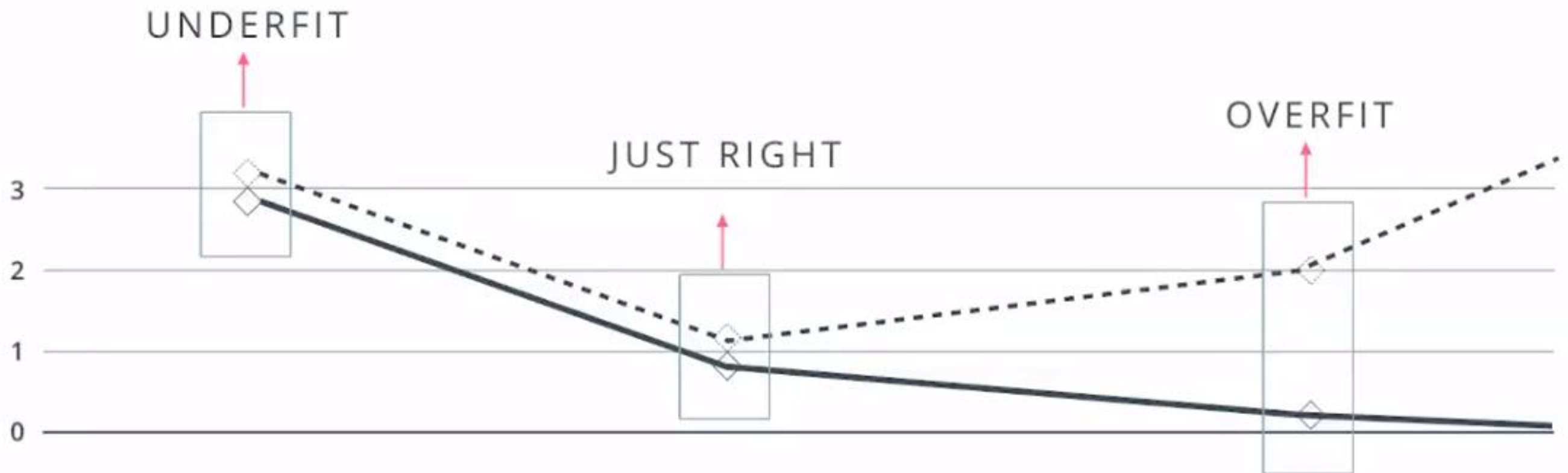
Degree = 6



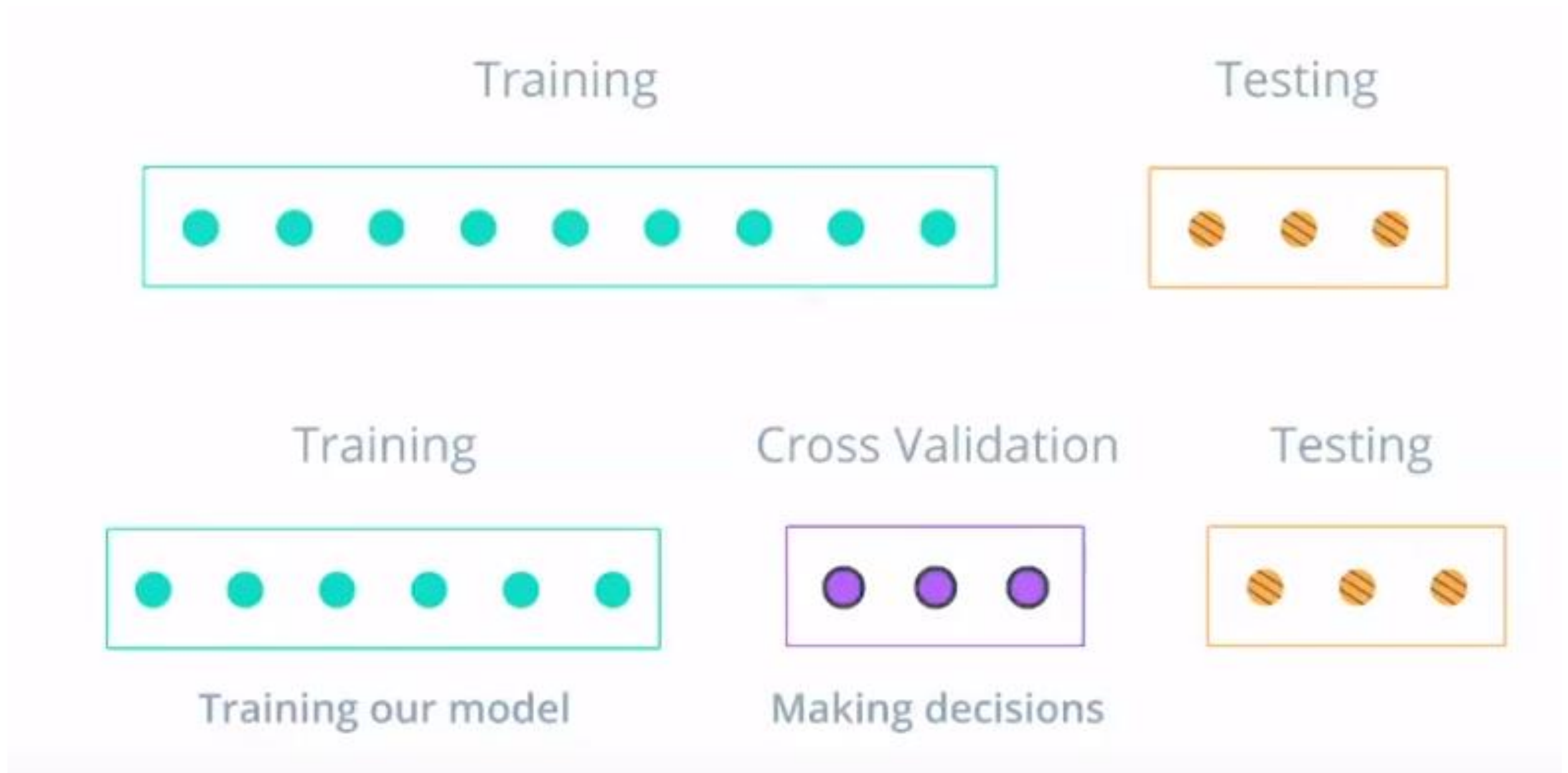
Model Complexity



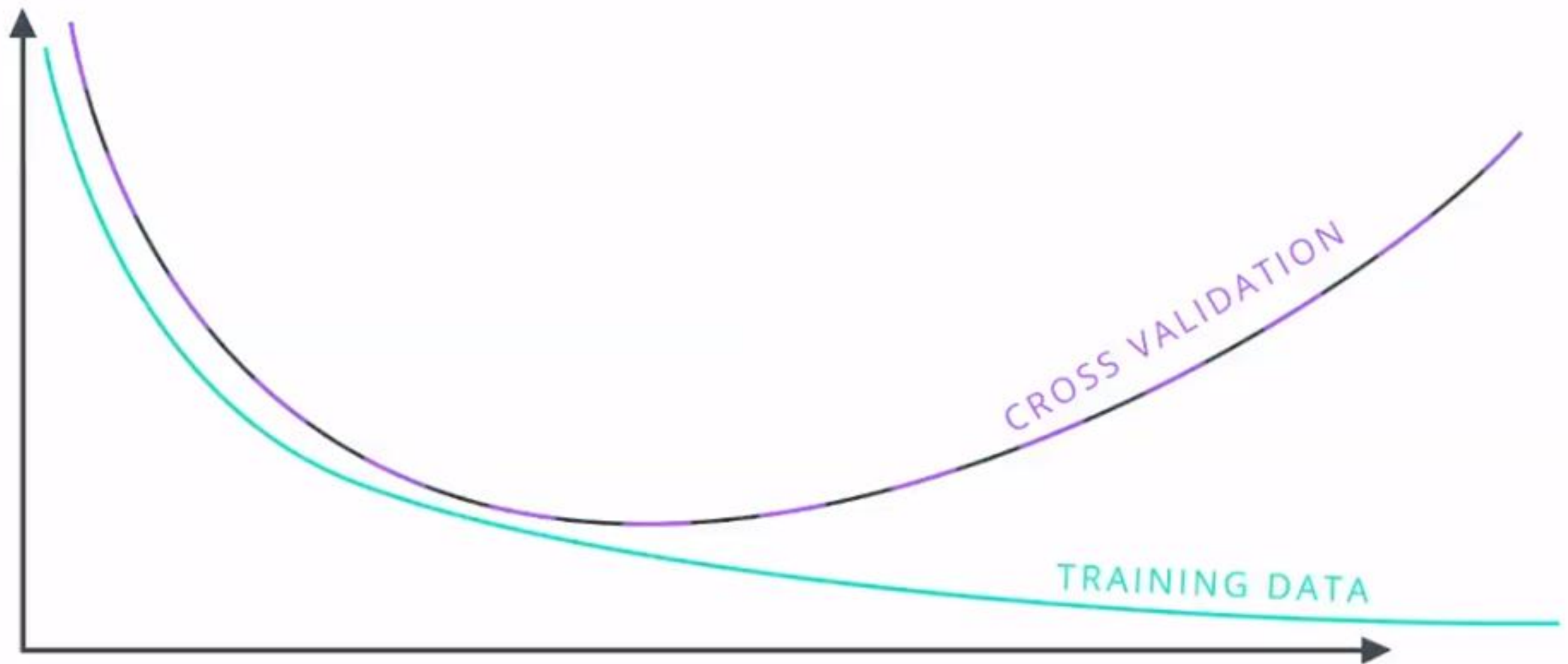
Model Complexity



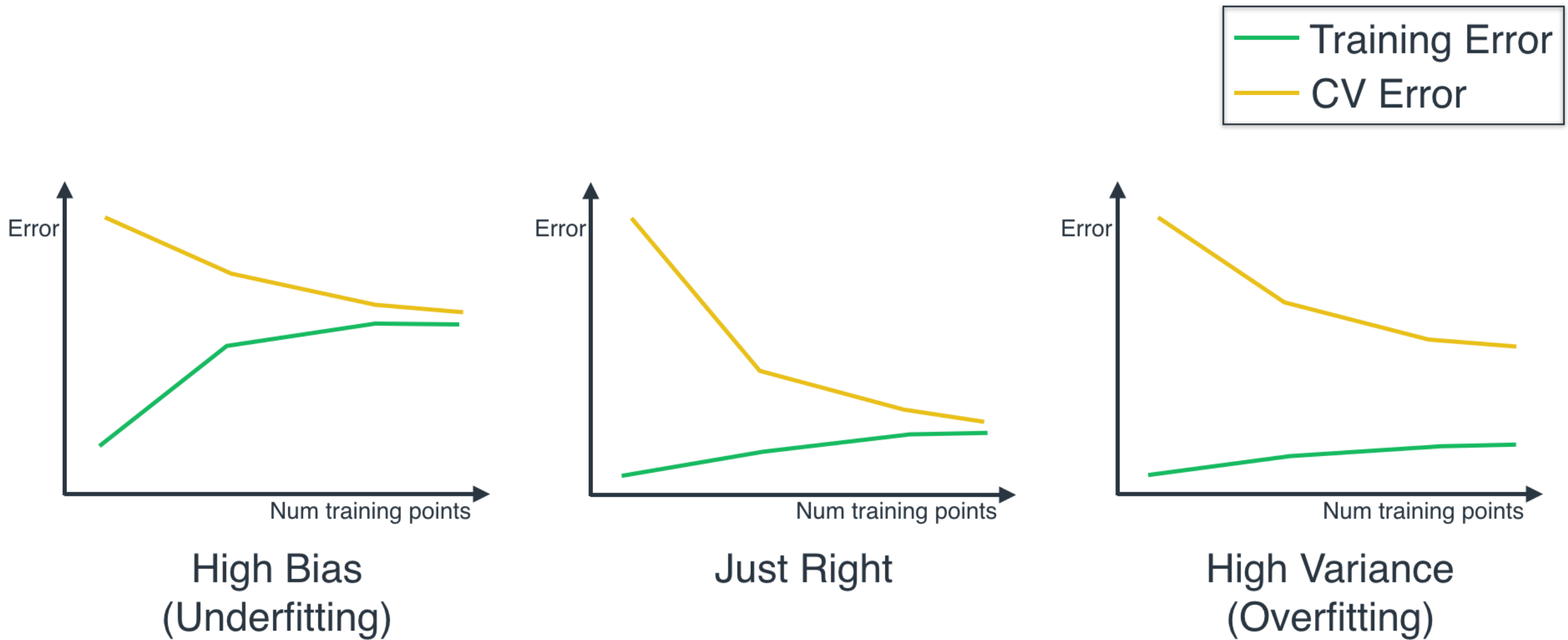
Model Complexity



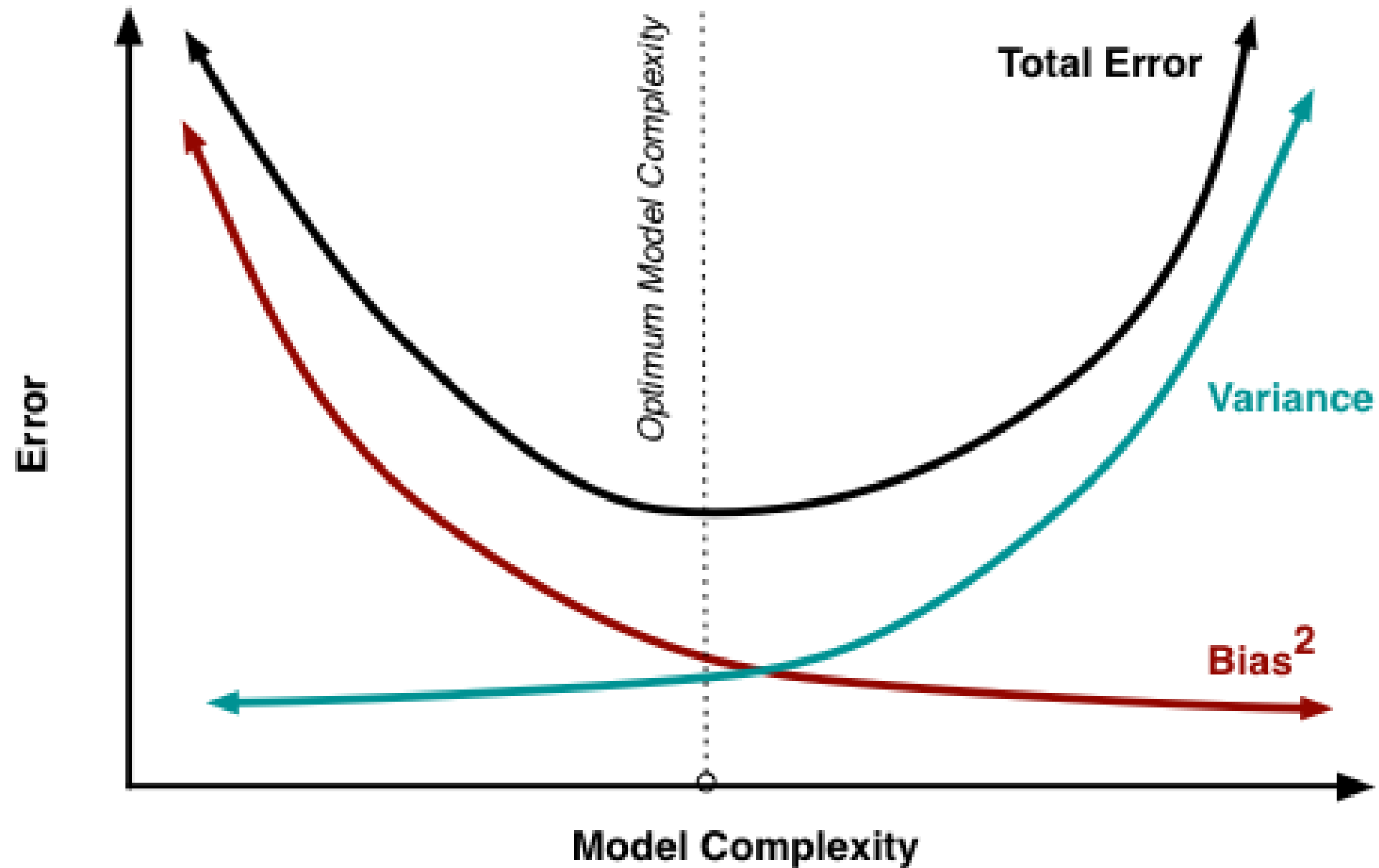
Model Complexity



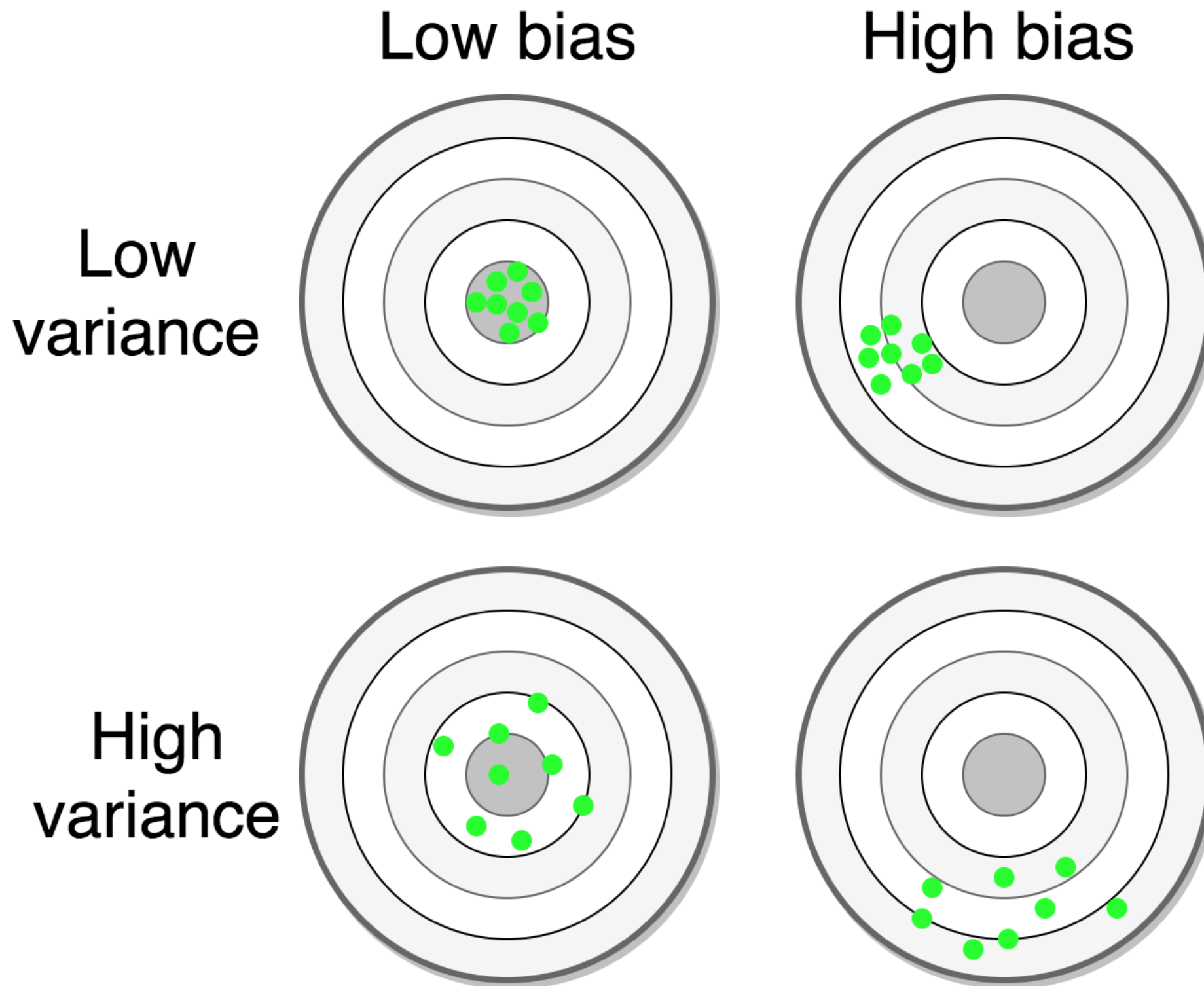
Impact of training points



Bias Variance Trade-off



Bias Variance Trade-off



<https://elitedatascience.com/bias-variance-tradeoff>

Regularization / Shrinkage

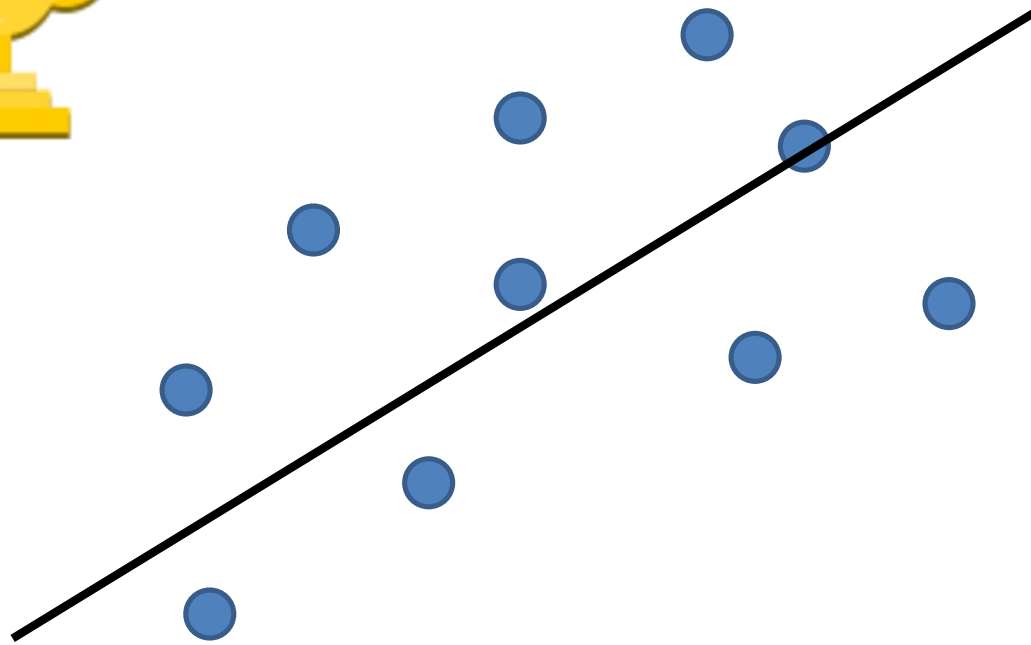
Ridge / L2

$$\sum_{i=1}^n \left(y_i - \beta_0 - \sum_{j=1}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^p \beta_j^2 = \text{RSS} + \lambda \sum_{j=1}^p \beta_j^2$$

Lasso / L1

$$\sum_{i=1}^n \left(y_i - \beta_0 - \sum_{j=1}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^p |\beta_j| = \text{RSS} + \lambda \sum_{j=1}^p |\beta_j|$$

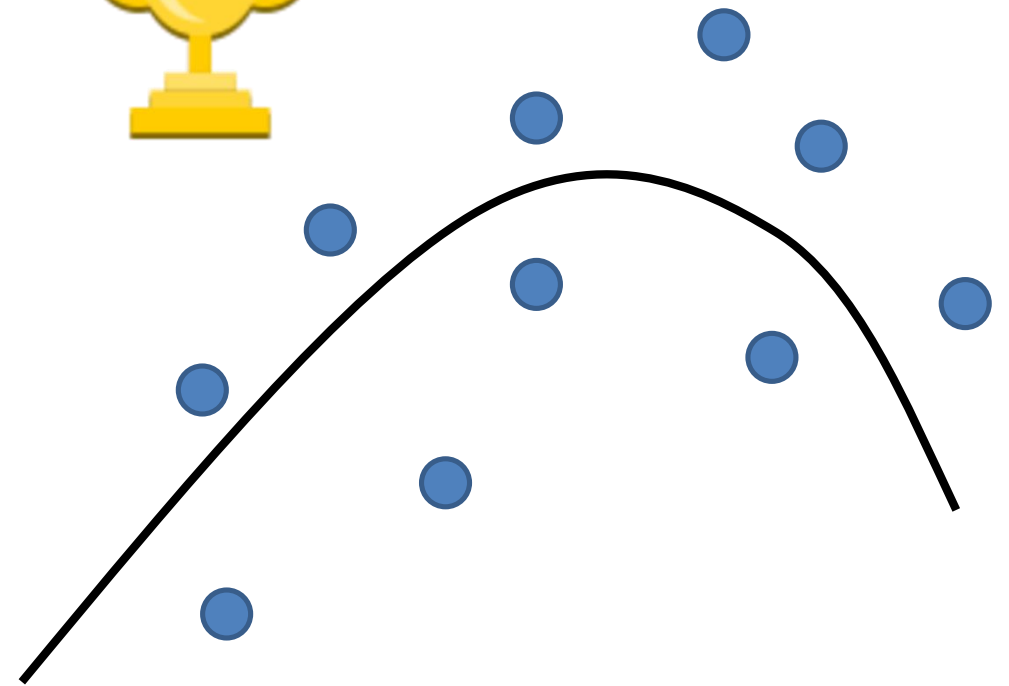
SIMPLE MODEL



ERROR: 

$$3x_1 + 4x_2 + 5$$

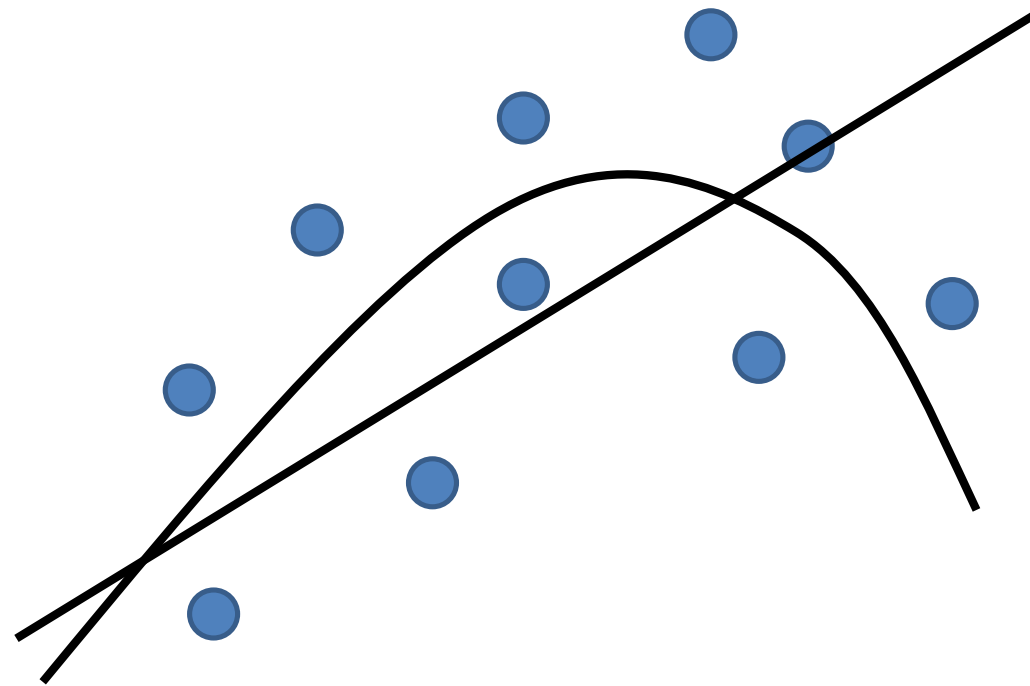
COMPLEX MODEL



ERROR: 

$$2x_1^3 - 2x_1^2x_2 - 4x_2^3 + 3x_1^2 + 6x_1x_2 + 4x_2^2 + 5$$

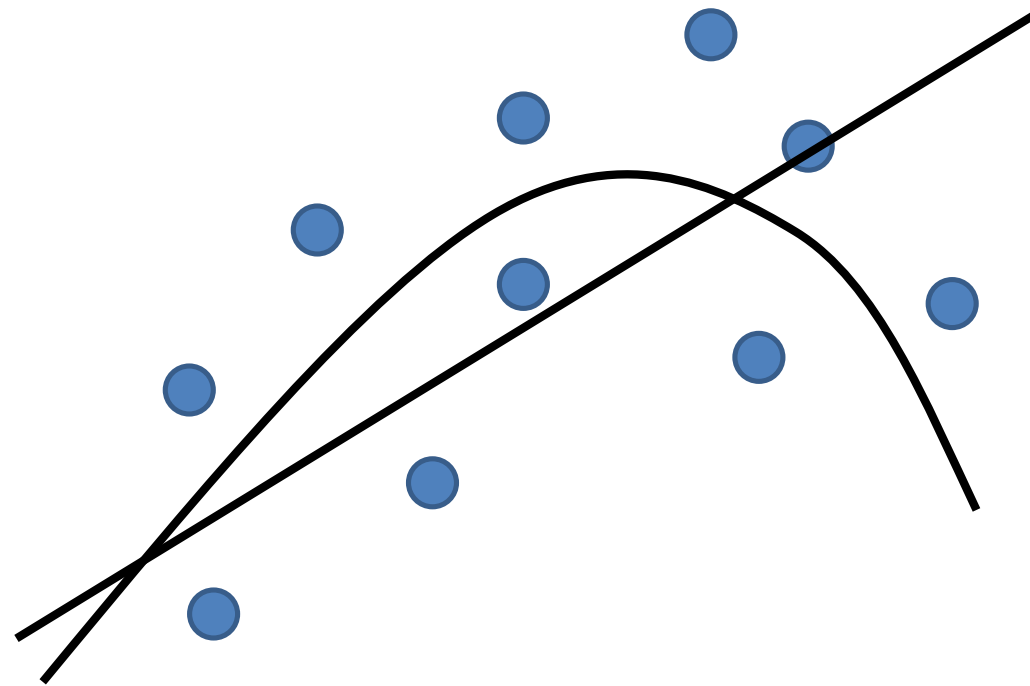
L1 (Lasso) Regularization



$$2x_1^3 - 2x_1^2x_2 - 3x_1x_2^2 + 4x_2^3 + 5x_1^2 + 6x_1x_2 + 4x_2^2 + 5$$

$$|2| + |-2| + |-3| + |4| + |5| + |6| + |4| + |5| = 21$$

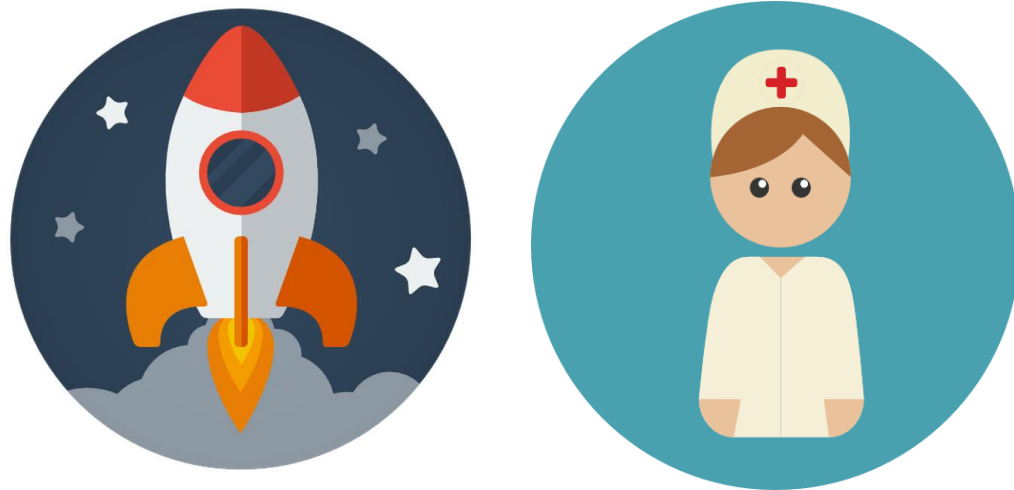
L2 (Ridge) Regularization



$$2x_1^3 - 2x_1^2x_2 - 3x_1x_2^2 + 4x_2^3 + 5x_1^2 + 6x_1x_2 + 4x_2^2 + 5$$

$$2^2 + (-2)^2 + 3^2 + 4^2 + 5^2 + 6^2 + 4^2 = 85$$

Simple vs Complex Models



**Requires LOW ERROR
OK if it's COMPLEX**

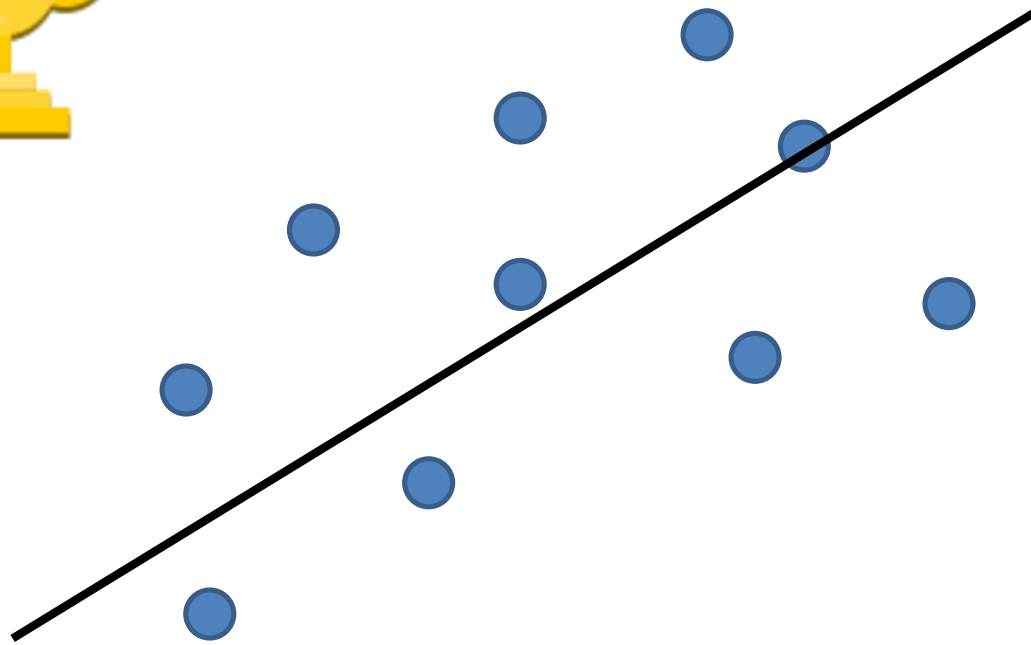
**PUNISHMENT on
COMPLEXITY should be
SMALL**



**Requires SIMPLICITY
OK with ERRORS**

**PUNISHMENT on
COMPLEXITY should be
BIG**

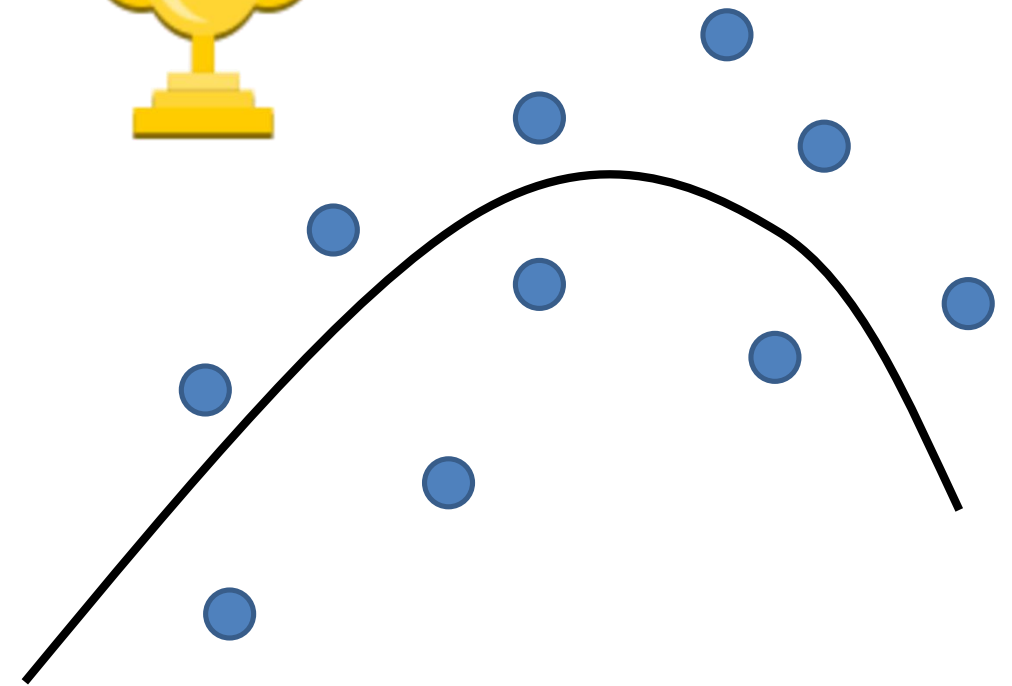
The λ Parameter



λ

ERROR:

$$3x_1 + 4x_2 + 5$$



λ

ERROR:

$$2x_1^3 - 2x_1^2x_2 - 4x_2^3 + 3x_1^2 + 6x_1x_2 + 4x_2^2 + 5$$

SAVE λ