Question. **$T(n) = 1 + T(n-1)$**

$T(n) = 1 + (1+T(n-2))$

$T(n) = 2 + T(n-2)$

$T(n) = 3 + T(n-3)$

$T(n) = 4 + T(n-4)$

…..

$T(n) = k + T(n-k)$

For $k = (n-1)$

$T(n) = n – 1 + T(1)$

$T(n) = n$

i.e. $T(n) = \Theta(n)$

Q2: $T(n) = n + T(n-1)$

$T(n) = n + (n-1+T(n-2))$

$T(n) = 2n – 1 + T(n-2)$

$T(n) = 2n -1 + ((n-2) + T(n-3))$

$T(n) = 3n -3 + T(n-3)$

$T(n) = 3n -3 + n-3 + T(n-4))$

$T(n) = 4n – 6 + T(n-4)$

…..

$T(n) = kn – c + T(n-k)$

For $k = (n-1)$

$T(n) = n^2 -1 – c + T(1)$

i.e. **$T(n) = \Theta(n^2)$**

Q3: **$T(n) = 1 + T(n/2)$**

$T(n) = 1 + (1+T(n/4))$

$T(n) = 2 + T(n/4)$

$T(n) = 3 + T(n/8)$

$T(n) = 3 + T(n/2^3)$

…..

T(n) = k + T(n/2^k)

For k = logn => n = 2^k

T(n) = logn + T(1)

T(n) = 1 + log(n)

i.e. **T(n) = Theta(logn)**

Question4: T(n) = logn + T(n/2) // Assume T(1) = 1

T(n) = logn + (logn/2+T(n/4))

T(n) = logn + logn – log2 + T(n/4)

T(n) = 2logn – 1 + T(n/4)

T(n) = 2logn – 1 + (logn/4+T(n/8))

T(n) = 2logn – 1 + (logn – log 4 +T(n/8))

T(n) = 3logn – 3 + T(n/8)

T(n) = 3logn – 3 + (logn/8+T(n/16))

T(n) = 3logn – 3 + logn – log8+T(n/16)

T(n) = 4logn – 6 +T(n/16)

…..

T(n) = klogn – c + T(n/2^k)

Let n = 2^k => k = logn

T(n) = logn*logn – c + T(1)

T(n) = O($\log^2$n)

Question5: **T(n) = n + T(n/2)**

T(n) = n + (n/2+T(n/4))

T(n) = n + n/2 + T(n/4)

T(n) = n + n/2 + n/4 + T(n/8))

T(n) = n + n/2 + n/4 + T(n/8).

T(n) =n (1 + 1/2 + 1/4) + T(n/8)

…..

T(n) = n(1 + ½ + ¼ …….. + 1/k) + T(n/2k)

For k = n

T(n) = n(1+ ½ + ¼ + ….. + 1/n) + T(1/2)

// Assume T(1/2) = 1

T(n) = n.(1+1) + 1

T(n) = Theta(n)

1/2 + 1/4 + 1/8 + 1/16 + ⋯

From Wikipedia, the free encyclopedia

In mathematics, the infinite series **1/2 + 1/4 + 1/8 + 1/16 + · · ·** is an elementary example of a geometric series that converges absolutely.

There are many expressions that can be shown to be equivalent to the problem, such as the form: $2^{-1} + 2^{-2} + 2^{-3}$...

Its sum is

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1.$$

Or T(n) = 7/4 * n + T(n/8), T(n) = 15n/8 + T(n/16), T(n) = 15(k-1)n/2k + T(n/k)

Let k =n, T(n) =(n-1)*n/(2n) + T(1) => **T(n) = O(n)**

Question6: T(n) = n + 2T(n/2)

T(n) = n + 2(n/2+2T(n/4))

T(n) = n + 2n/2 + 4T(n/4)

T(n) = n + 2n/2 + 4(n/4 + 2T(n/8))

T(n) = n + n + n + 8T(n/8)

T(n) = 3n + 8(n/8 + 2T(n/16))

T(n) = 4n + 2T(n/2^4)

…..

T(n) = kn + 2T(n/2^4)

For k = logn => n = 2^k

T(n) = n.logn + T(1)

T(n) = O(nlogn)

Question7: T(n) = 2T(n-1)

T(n) = 2(2T(n-2))

T(n) = 4T(n-2)

T(n) = 4(2T(n-3))

T(n) = 8T(n-3)

T(n) = 8(2T(n-4))

T(n) = 16T(n-4)

T(n) = 2^4T(n-4)

……..

T(n) = 2^kT(n-k)

Let's k =n-1

T(n) = O(2^n)