Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Q1_Solution: Option B

The solution for the above question is attached in the .ipynb format as mentioned by the assignment team.
The file Assignment 2_set2Q1.ipynb contains the solution to the above question.

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.
    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Q2_Solution:
A. False:

because as mentioned that the data is normally distributed the mean is 38 and standard deviation is 6 then it's at most age will be 38+6=44years and exceeding 44 means the data is considered as outlier and its almost negligible when the data is symmetrical.
So, the age of the greater number of employees is less than 44years.

B.True:
Because mean=38, z-score for age 30 is z = (30 - 38) / 6 = -1.33 then by seeing the probability of z-score from z-table I.e., 0.0918.
Now multiply with total 0 .0918 * 400=36.72 employees so almost 36 employees.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Given that x1 and x2 are identically distributed normal random variables with mean μ and variance σ^2 then,

$2X_1$: as the x1 is normal random variable following normal distribution, then mean=2μ and variance=4σ2

$X_1 + X_2$: as both x1 and x2 are normal random variable following normal distribution, then mean=2μ and variance=2σ2

The difference will be in the variance of the data, mean il be the same

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

Q4_Solution: Option D

The solution for the above question is attached in the .ipynb format as mentioned by the assignment team.
The file Assignment 2_set2Q4.ipynb contains the solution to the above question.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5^{th} percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss in a given year?

Q5(A)_Solution:

Total Profit=Division1+Division2

mean=5+7=12, stdev= sqrt $(3^2 + 4^2)$=sqrt (9+16) = 5.

Z_score at 95% is –1.96

The range in dollars is $(\mu - z\sigma, \mu + z\sigma) = (12 - 1.96 * 5, 12 + 1.96 * 5) = (2.2, 21.8)$.

The range in rupees is (2.2 * 45, 21.8 * 45) = (99, 981).

Q5(B)_Solution:

Z-score value for 5^{th} percentile is –1.645, mean=5+7=12, stdev= sqrt $(3^2 + 4^2)$=sqrt (9+16) = 5. Therefore, the corresponding value in dollars is $\mu + z\sigma = 12 + (-1.645) * 5 \approx 4.775$. In Rupees, this value is $4.775 * 45 \approx 215$.

Q5(C)_Solution:

As the mean profit of division 1 is lower than division 2, hence division 1 has a larger probability of making a loss in a given year