Transformation In 2-Dimension

Course Title: Computer Graphics

Course Code: CSE – 413

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Introduction of Transformations

Computer Graphics provide the facility of viewing object from different angles. The architect can study building from different angles i.e.

1.Front Evaluation
2.Side elevation
3.Top plan

A Cartographer can change the size of charts and topographical maps. So if graphics images are coded as numbers, the numbers can be stored in memory. These numbers are modified by mathematical operations called as Transformation.

The purpose of using computers for drawing is to provide facility to user to view the object from different angles, enlarging or reducing the scale or shape of object called as Transformation.

Two essential aspects of transformation are given below:

1.Each transformation is a single entity. It can be denoted by a unique name or symbol.

2.It is possible to combine two transformations, after connecting a single transformation is obtained, e.g., A is a transformation for translation. The B transformation performs scaling. The combination of two is C=AB. So C is obtained by concatenation property.

There are two complementary points of view for describing object transformation.

1.Geometric Transformation: The object itself is transformed relative to the coordinate system or background. The mathematical statement of this viewpoint is defined by geometric transformations applied to each point of the object.

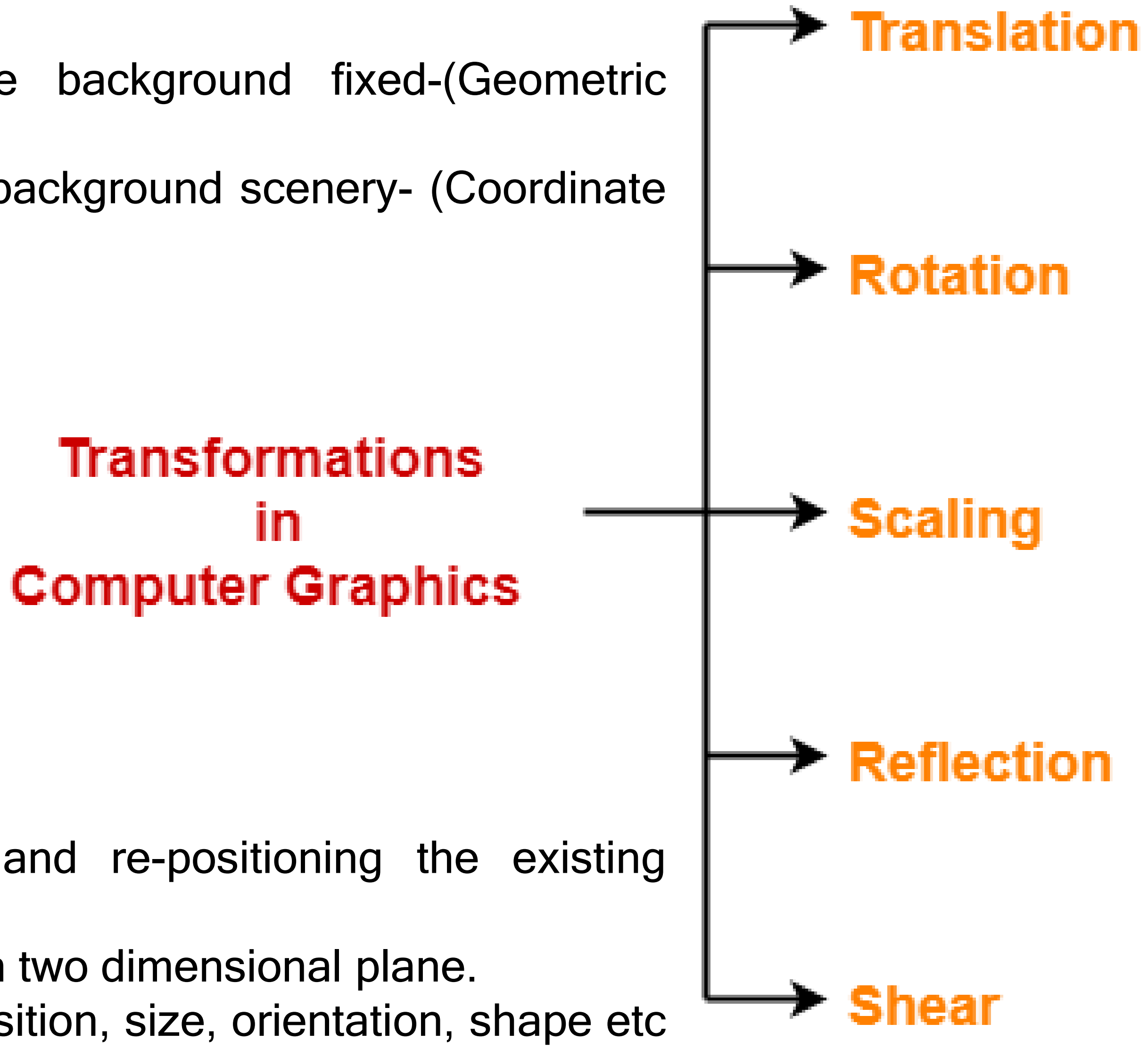
2.Coordinate Transformation: The object is held stationary while the coordinate system is transformed relative to the object. This effect is attained through the application of coordinate transformations.

An example that helps to distinguish these two viewpoints:

•The movement of an automobile against a scenic background we can simulate this by
•Moving the automobile while keeping the background fixed-(Geometric Transformation)
•We can keep the car fixed while moving the background scenery- (Coordinate Transformation)

Types of Transformations:

1. Translation
2. Scaling
3. Rotating
4. Reflection
5. Shearing



2D Transformation in Computer Graphics-

Definition-1: In Computer graphics,
Transformation is a process of modifying and re-positioning the existing graphics.

Definition-2: 2D Transformations take place in a two dimensional plane.
Transformations are helpful in changing the position, size, orientation, shape etc of the object.

2D Translation in Computer Graphics-

In Computer graphics,

2D Translation is a process of moving an object from one position to another in a two dimensional plane. The movement of objects without deforming the shape of the object is Translation.

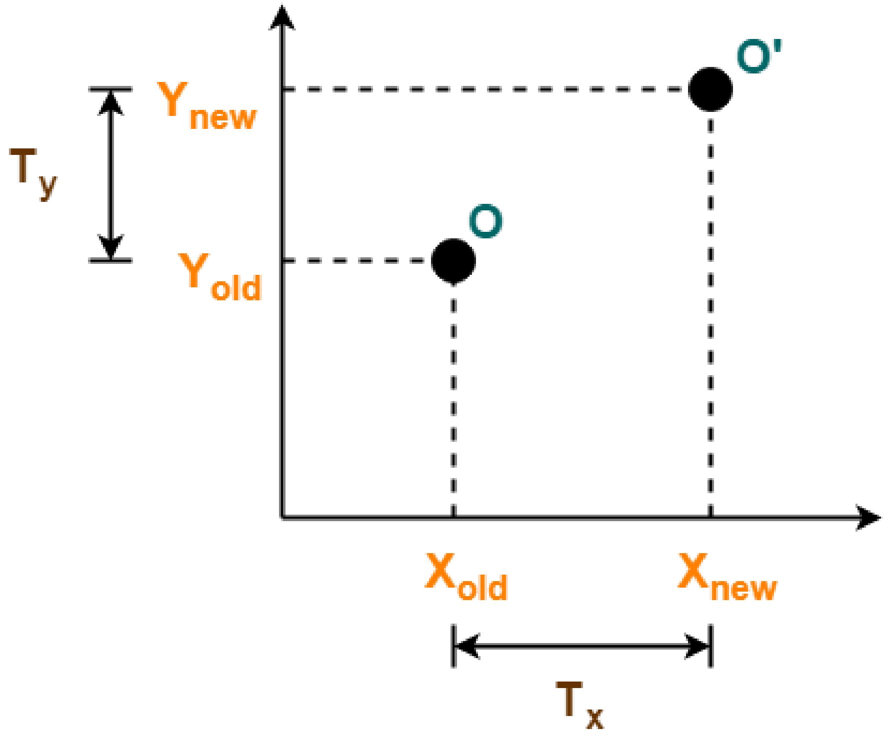
Consider a point object O has to be moved from one position to another in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the object O after translation = (X_{new}, Y_{new})
- Translation vector or Shift vector = (T_x, T_y)

Given a Translation vector (T_x, T_y)-

- T_x defines the distance the X_{old} coordinate has to be moved.
- T_y defines the distance the Y_{old} coordinate has to be moved.



2D Translation in Computer Graphics

This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- $X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Translation Matrix

In Matrix form, the above translation equations may be represented as-

- The homogeneous coordinates representation of (X, Y) is (X, Y, 1).
- Through this representation, all the transformations can be performed using matrix / vector multiplications.

The above translation matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Translation Matrix

(Homogeneous Coordinates Representation)

PRACTICE PROBLEMS BASED ON 2D TRANSLATION IN COMPUTER GRAPHICS-

Problem-01:

Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Solution-

Old center coordinates of C = (X_{old}, Y_{old}) = (1, 4)

- Translation vector = (T_x, T_y) = (5, 1)

Let the new center coordinates of C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 1 + 5 = 6$
- $Y_{new} = Y_{old} + T_y = 4 + 1 = 5$

Thus, New center coordinates of C = (6, 5).

Alternatively,

In matrix form, the new center coordinates of C after translation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Thus, New center coordinates of C = (6, 5).

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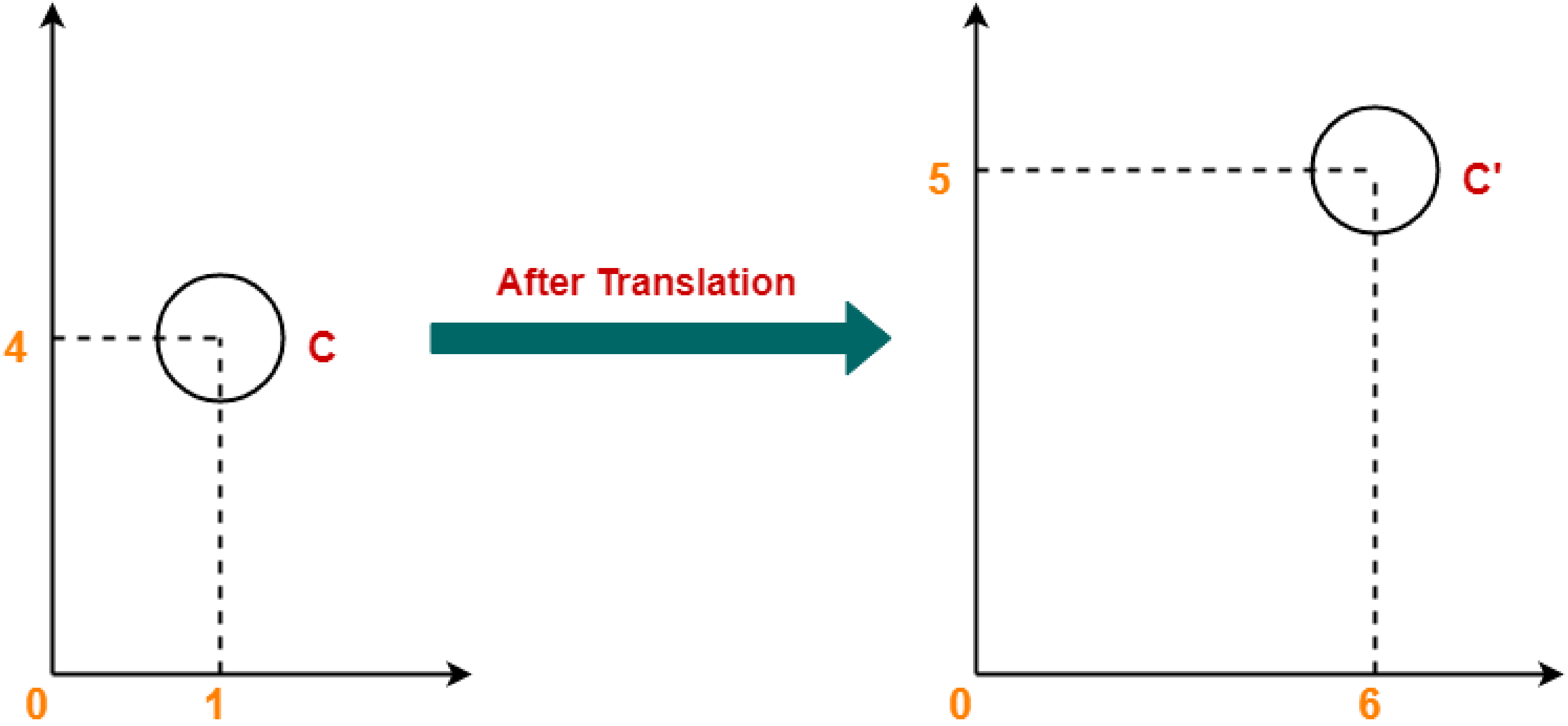


Problem-02:

Given a square with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.

Solution-

Old coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)

- Translation vector = $(T_x, T_y) = (1, 1)$

<u>For Coordinates A(0, 3)</u>

Let the new coordinates of corner A = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner A = (1, 4).

<u>For Coordinates B(3, 3)</u>

Let the new coordinates of corner B = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner B = (4, 4).

For Coordinates C(3, 0)

Let the new coordinates of corner C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner C = (4, 1).

For Coordinates D(0, 0)

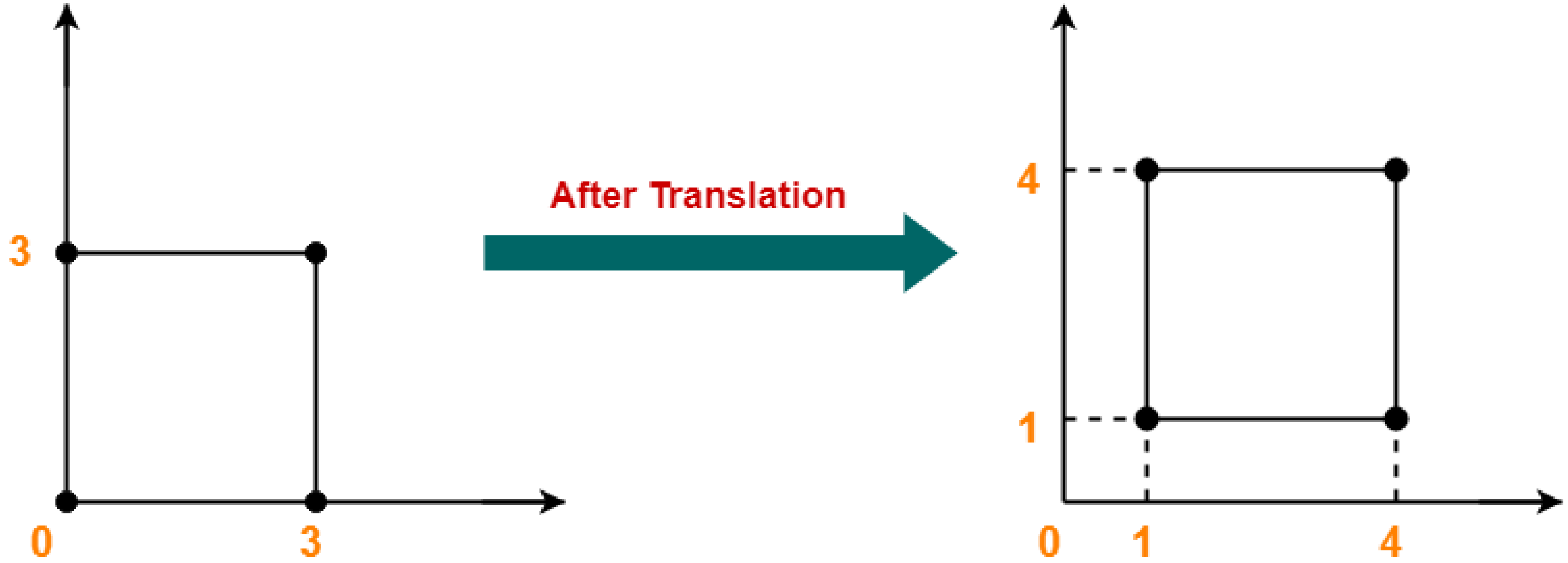
Let the new coordinates of corner D = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner D = (1, 1).

Thus, New coordinates of the square = A (1, 4), B(4, 4), C(4, 1), D(1, 1).



2D Rotation in Computer Graphics-

In Computer graphics,

- ✓ 2D Rotation is a process of rotating an object with respect to an angle in a two dimensional plane.
- ✓ Rotation is a type of transformation that is very often used in computer graphics and image processing.
- ✓ Rotation is a process of rotating an object with respect to an angle in a two-dimensional plane.
- ✓ It is a process of changing the angle of the object which can be clockwise or anticlockwise, while we have to specify the angle of rotation and rotation point. A rotation point is also called a pivot point.

There are two types of rotations according to the direction of the movement of the object. These are:

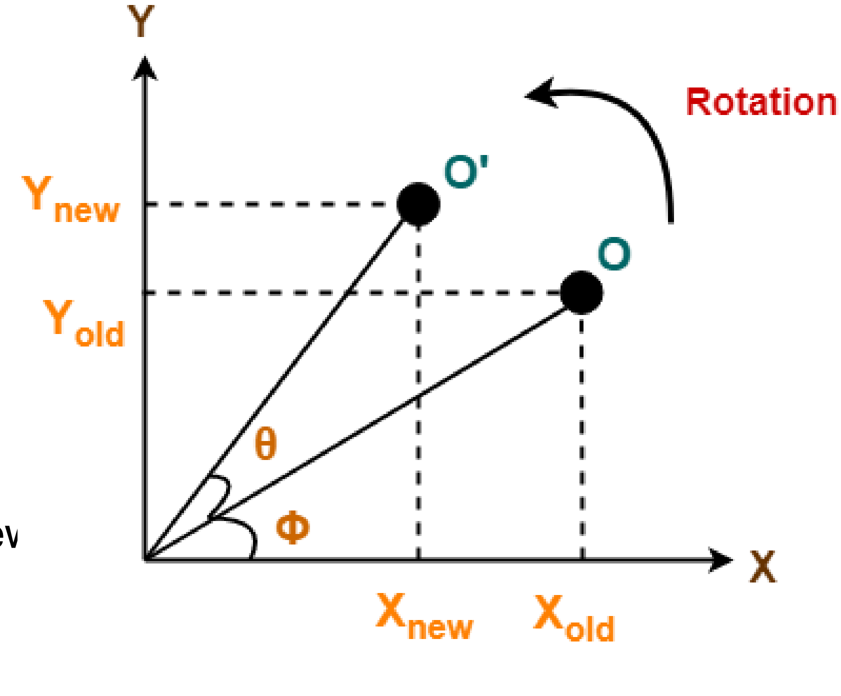
- **Anti-clockwise rotation**
- **Clockwise rotation**

The positive value of the rotation angle rotates an object in an anti-clockwise direction while the negative value of the rotation angle rotates an object in a clockwise direction. When we rotate any object, then every point of that object is rotated by the same angle. For example, a straight line is rotated by the endpoints with the same angle and the line is re-drawn between the new endpoints. Also, the polygon is rotated by shifting every vertex with the help of the same rotational angle. Same for circle also, it can be obtained by center position by the specified angle.

Consider a point object O has to be rotated from one angle to another in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = $(X_{new}, Y_{new}$



2D Rotation in Computer Graphics

This rotation is achieved by using the following rotation equations-

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Rotation Matrix

For homogeneous coordinates, the above rotation matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Rotation Matrix

(Homogeneous Coordinates Representation)

Problem-01:

Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Solution:- We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

- Old ending coordinates of the line = $(X_{old}, Y_{old}) = (4, 4)$
- Rotation angle = $\theta = 30^\circ$

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$\mathbf{X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta}$

$= 4 \times \cos 30^\circ - 4 \times \sin 30^\circ$

$= 4 \times (\sqrt{3} / 2) - 4 \times (1 / 2)$

$= 2\sqrt{3} - 2$

$= 2(\sqrt{3} - 1)$

$= 2(1.73 - 1)$

$= 1.46$

$\mathbf{Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta}$

$= 4 \times \sin 30^\circ + 4 \times \cos 30^\circ$

$= 4 \times (1 / 2) + 4 \times (\sqrt{3} / 2)$

$= 2 + 2\sqrt{3}$

$= 2(1 + \sqrt{3}) = 2(1 + 1.73) = 5.46$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

Alternatively,

In matrix form, the new ending coordinates of the line after rotation may be obtained as-

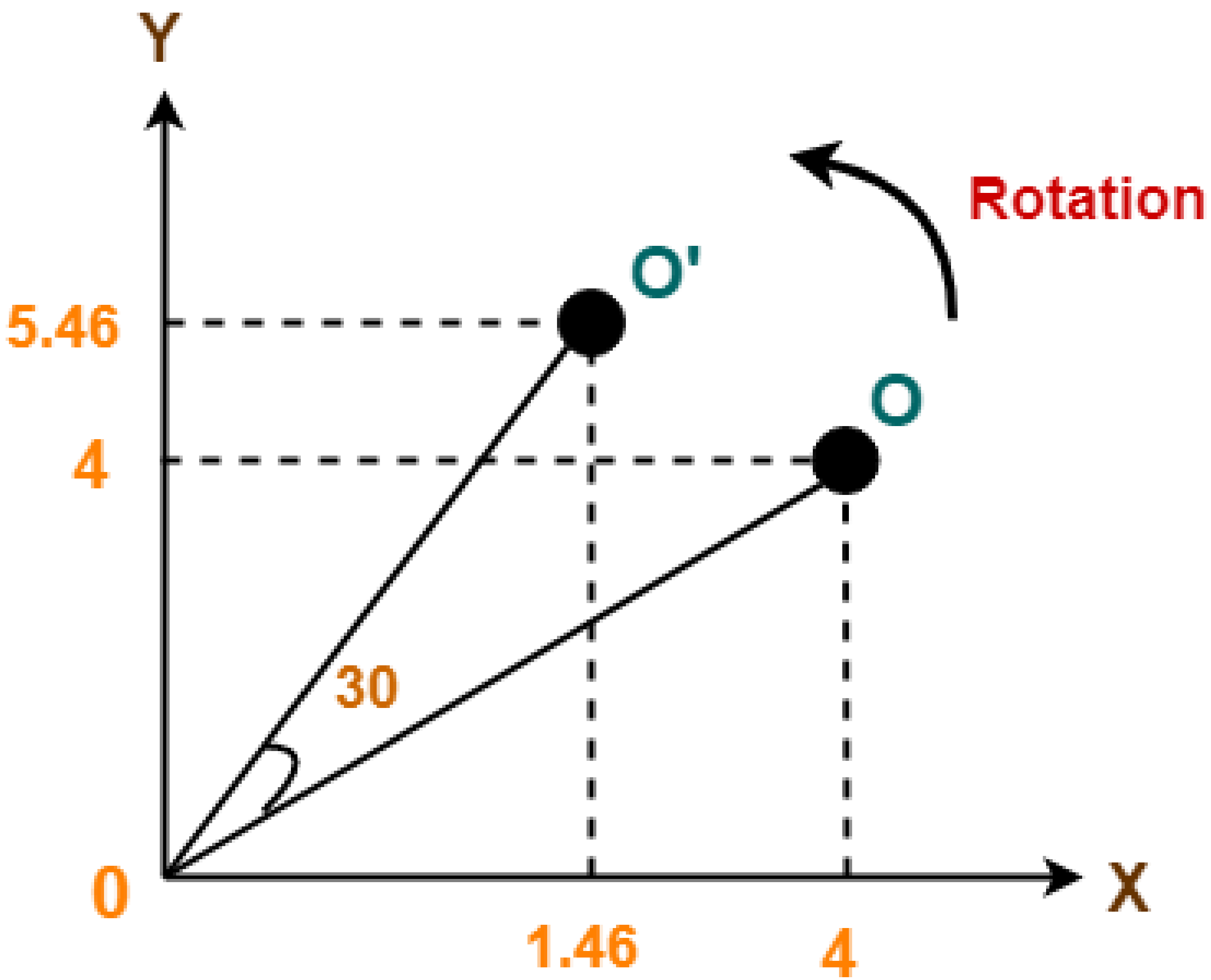
$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$



Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

Problem-02:

Given a line segment with starting point as O (0, 0) and ending point as A (5, 5). Apply 30-degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Solution-

We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

Given-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (5, 5)
- Rotation angle = θ = 30°

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta = 5 \times \cos30^o - 5 \times \sin30^o$

$= 5 \times (\sqrt{3} / 2) - 5 \times (1 / 2)$

$= 4.33 - 2.5$

$= 1.83$

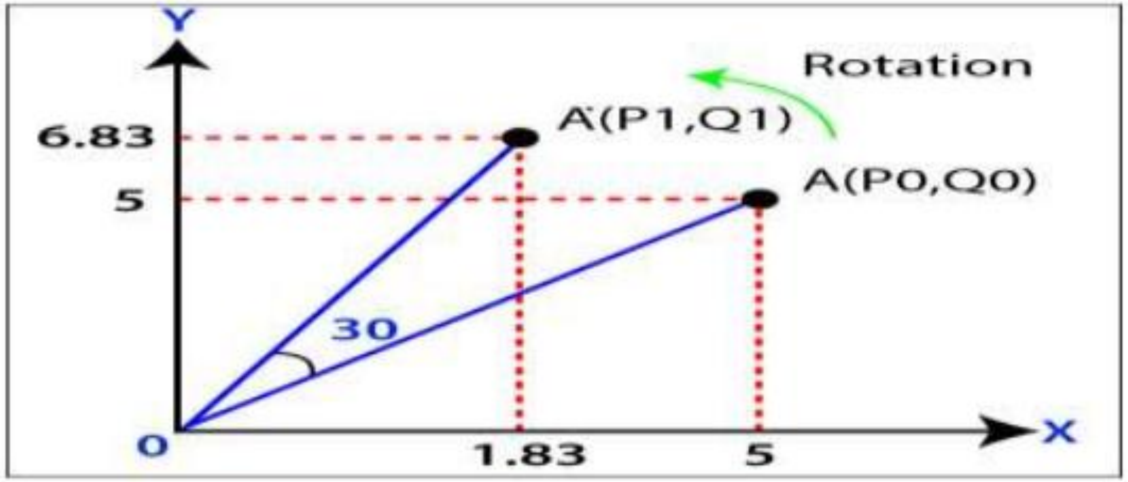
$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta = 5 \times \sin30^o + 5 \times \cos30^o$

$= 5 \times (1 / 2) + 5 \times (\sqrt{3} / 2)$

$= 2.5 + 4.33$

$= 6.83$

Thus, New ending coordinates of the line after rotation = A' (1.83, 6.83).



Problem-03:

Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

Solution:- We rotate a polygon by rotating each vertex of it with the same rotation angle.

Old corner coordinates of the triangle = A (0, 0), B(1, 0), C(1, 1)

Rotation angle = θ = 90º

For Coordinates A(0, 0)

Let the new coordinates of corner A after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

X_{new}

$= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$= 0 \times \cos 90^\circ - 0 \times \sin 90^\circ$

$= 0$

Y_{new}

$= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$= 0 \times \sin 90^\circ + 0 \times \cos 90^\circ$

$= 0$

Thus, New coordinates of corner A after rotation = (0, 0).

For Coordinates B(1, 0)

The new coordinates of corner B after rotation = (X_{new}, Y_{new}).

X_{new}

$= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$= 1 \times \cos 90^\circ - 0 \times \sin 90^\circ$

$= 0$

Y_{new}

$= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$= 1 \times \sin 90^\circ + 0 \times \cos 90^\circ$

$= 1 + 0$

$= 1$

Thus, New coordinates of corner B after rotation = (0, 1).

For Coordinates C(1, 1)

The new coordinates of corner C after rotation = (X_{new}, Y_{new}).

X_{new}

$= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$= 1 \times \cos 90^\circ - 1 \times \sin 90^\circ$

$= 0 - 1$

$= -1$

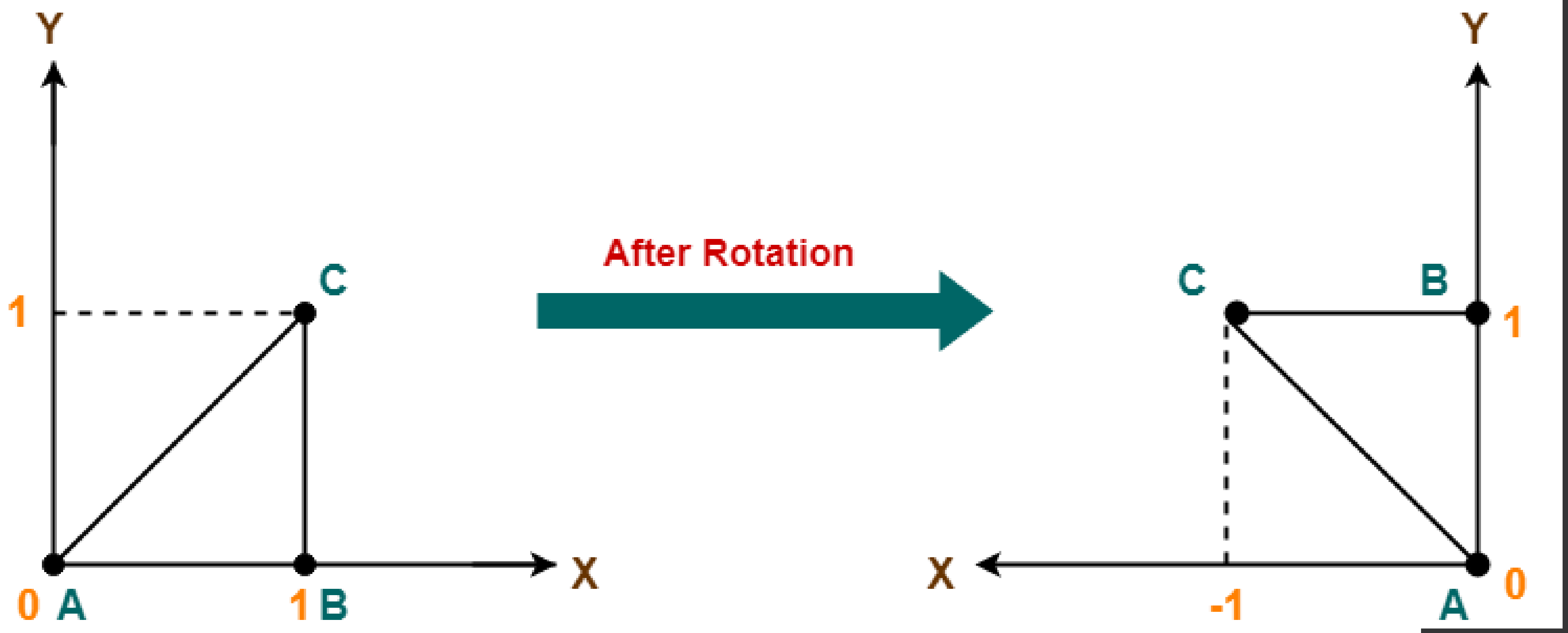
Y_{new}

$= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$= 1 \times \sin 90^\circ + 1 \times \cos 90^\circ$

$= 1 + 0$

$= 1$



Thus, New coordinates of corner C after rotation = (-1, 1).

Thus, New coordinates of the triangle after rotation = A (0, 0), B(0, 1), C(-1, 1).

Problem-04:

Rotate a line AB whose endpoints are C (3, 4) and D (12, 15) about origin through a 45° anticlockwise direction.

Solution-

- Given- Rotation angle = θ = 45°

Rotation of the point A (3, 4)

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (3, 4)

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta = 3 \times \cos 45^\circ - 4 \times \sin 45^\circ = 3 \times (1 / \sqrt{2}) - 4 \times (1 / \sqrt{2})$

$= 3 \times 0.707 - 4 \times 0.707$

$= -0.707$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta = 3 \times \sin 45^\circ + 4 \times \cos 45^\circ$

$= 3 \times (1 / \sqrt{2}) + 4 \times (1 / \sqrt{2}) = 3 \times 0.707 + 4 \times 0.707$

$= 4.949$

Thus, New ending coordinates of the line after rotation = C (-0.707, 4.949)

Rotation of the point B (12, 15)

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (12, 15)

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta = 12 \times \cos 45^o - 15 \times \sin 45^o = 12 \times (1 / \sqrt{2}) - 15 \times (1 / \sqrt{2})$

$= 12 \times 0.707 - 15 \times 0.707$

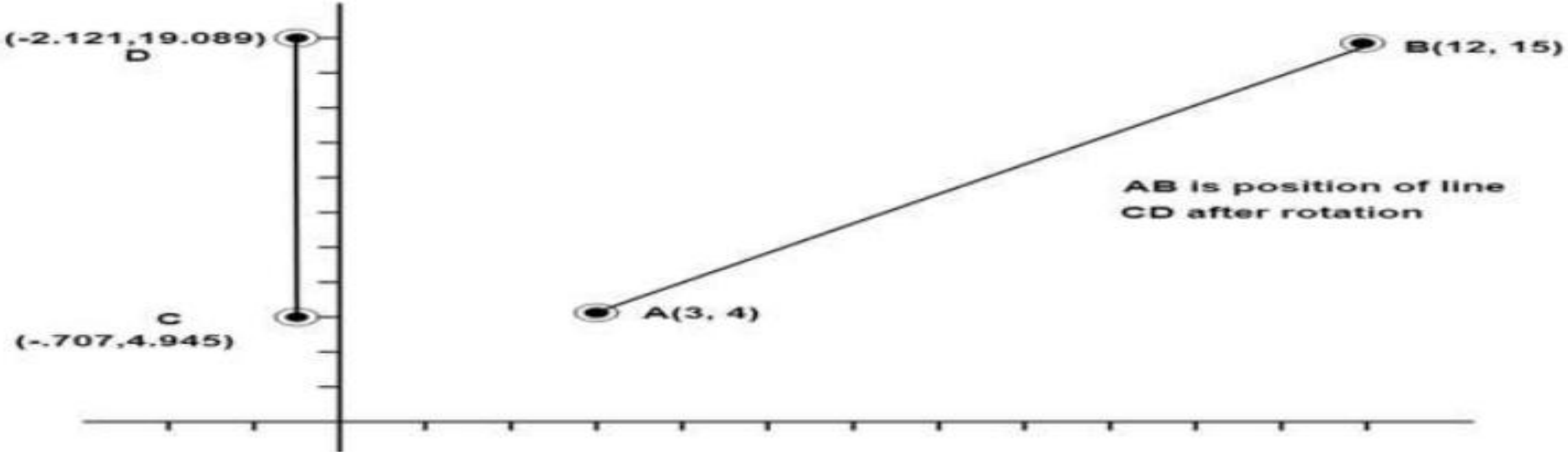
$= -2.121$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta = 12 \times \sin 45^o + 15 \times \cos 45^o$

$= 12 \times (1 / \sqrt{2}) + 15 \times (1 / \sqrt{2}) = 12 \times 0.707 + 15 \times 0.707$

$= 19.089$

Thus, New ending coordinates of the line after rotation = D (-2.121, 19.089)



Alternatively,

In matrix form, the new ending coordinates of the line after rotation of the point A (3, 4) may be obtained as-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (3, 4)

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

$$\begin{bmatrix} Xnew \\ Ynew \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ x } \begin{bmatrix} Xold \\ Yold \end{bmatrix}$$

$$= \begin{bmatrix} \cos45^0 & -\sin45^0 \\ \sin45^0 & \cos45^0 \end{bmatrix} \text{ x } \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \text{ x } \cos45^0 - 4 \text{ x } \sin45^0 \\ 3 \text{ x } \sin45^0 + 4 \text{ x } \cos45^0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \text{ x } 0.707 - 4 \text{ x } 0.707 \\ 3 \text{ x } 0.707 + 4 \text{ x } 0.707 \end{bmatrix}$$

$$= \begin{bmatrix} -0.707 \\ 4.949 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = C (-0.707, 4.949)

In matrix form, the new ending coordinates of the line after rotation of the point B (12, 15) may be obtained as-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (12, 15)

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

$$\begin{bmatrix} Xnew \\ Ynew \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ x } \begin{bmatrix} Xold \\ Yold \end{bmatrix}$$

$$= \begin{bmatrix} \cos45^0 & -\sin45^0 \\ \sin45^0 & \cos45^0 \end{bmatrix} \text{ x } \begin{bmatrix} 12 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \text{ x } \cos45^0 - 15 \text{ x } \sin45^0 \\ 12 \text{ x } \sin45^0 + 15 \text{ x } \cos45^0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \text{ x } 0.707 - 15 \text{ x } 0.707 \\ 12 \text{ x } 0.707 + 15 \text{ x } 0.707 \end{bmatrix}$$

$$= \begin{bmatrix} -2.121 \\ 19.089 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = D (-2.121, 19.089)

Problem-05:

Rotate a line AB whose endpoints are A (2, 5) and B (6, 12) about origin through a 30° clockwise direction.

Solution-

- Given- Rotation angle = θ = 30° but it is clockwise direction, so θ = - θ

Rotation of the point A (2, 5)

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (2, 5)

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta = X_{old} \times \cos(-\theta) - Y_{old} \times \sin(-\theta) = X_{old} \times \cos\theta + Y_{old} \times \sin\theta$

$= 2 \times \cos 30° + 5 \times \sin 30°$

$= 2 \times (\sqrt{3} / 2) + 5 \times (1 / 2)$

$= 2 \times 0.866 + 5 \times 0.5$

$= 4.232$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta = X_{old} \times \sin(-\theta) + Y_{old} \times \cos(-\theta) = -X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$= - 2 \times \sin 30° + 5 \times \cos 30°$

$= -2 \times (1 / 2) + 5 \times (\sqrt{3} / 2)$

$= -2 \times 0.5 + 5 \times 0.866$

$= 3.33$

Thus, New ending coordinates of the line after rotation = A' (4.232, 3.33)

Rotation of the point B (6, 12)

- Old ending coordinates of the line = $(X_{old}, Y_{old}) = (6, 12)$

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta = X_{old} \times \cos(-\theta) - Y_{old} \times \sin(-\theta) = X_{old} \times \cos\theta + Y_{old} \times \sin\theta$

$= 6 \times \cos 30^o + 12 \times \sin 30^o$

$= 6 \times (\sqrt{3} / 2) + 12 \times (1 / 2)$

$= 6 \times 0.866 + 12 \times 0.5 = 11.196$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta = X_{old} \times \sin(-\theta) + Y_{old} \times \cos(-\theta) = -X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

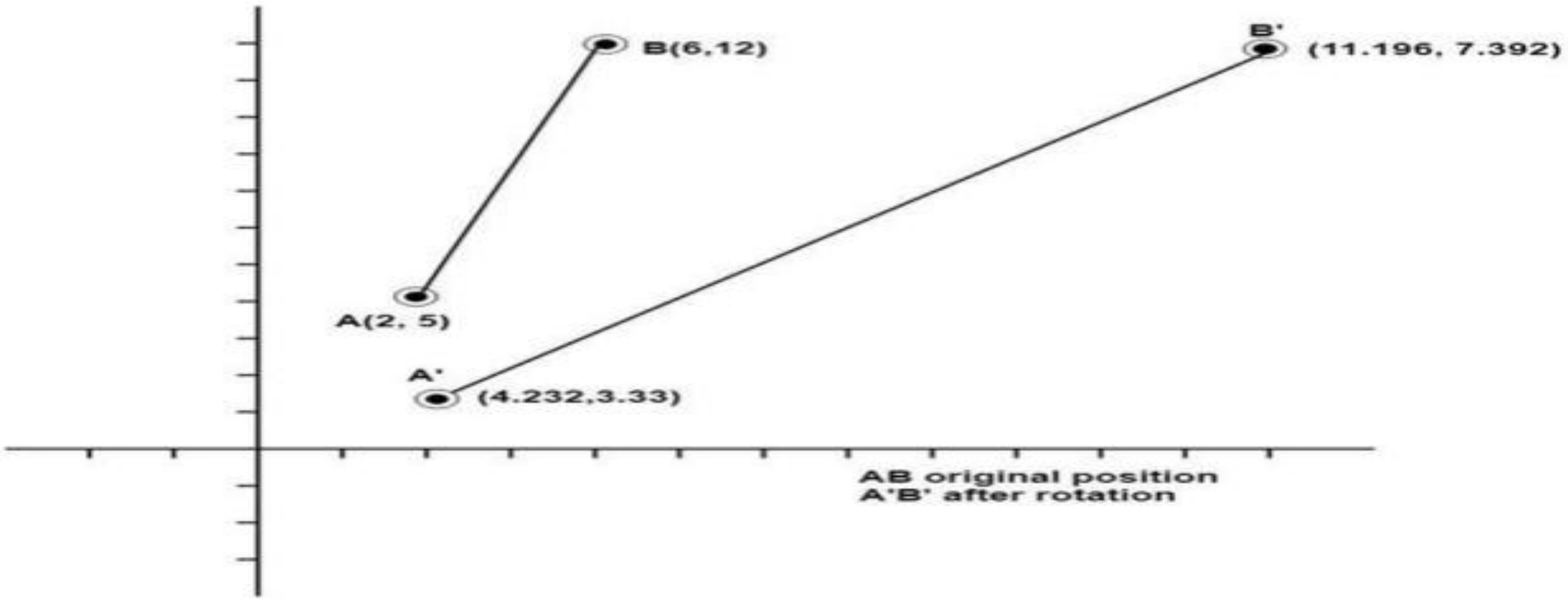
$= - 6 \times \sin 30^o + 12 \times \cos 30^o$

$= -6 \times (1 / 2) + 12 \times (\sqrt{3} / 2)$

$= -6 \times 0.5 + 12 \times 0.866$

$= 7.392$

Thus, New ending coordinates of the line after rotation = B' (11.196, 7.392)



Alternatively,

Matrix Form

- Given- Rotation angle = θ = 30° but it is clockwise direction, so θ = - θ

We know,

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

In matrix form, the new ending coordinates of the line after rotation of the point A (2, 5) may be obtained as-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (2, 5)

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

$$\begin{bmatrix} Xnew \\ Ynew \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} x \begin{bmatrix} Xold \\ Yold \end{bmatrix}$$

$$= \begin{bmatrix} \cos30^0 & \sin30^0 \\ -\sin30^0 & \cos30^0 \end{bmatrix} x \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \text{ x } \cos30^0 + 5 \text{ x } \sin30^0 \\ 2 \text{ x } \sin30^0 + 5 \text{ x } \cos30^0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \text{ x } (\sqrt{3} / 2) + 5 \text{ x } (1 / 2) \\ -2 \text{ x } (1 / 2) + 5 \text{ x } (\sqrt{3} / 2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \text{ x } 0.866 + 5 \text{ x } 0.5 \\ -2 \text{ x } 0.5 + 5 \text{ x } 0.866 \end{bmatrix}$$

$$= \begin{bmatrix} 4.232 \\ 3.33 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = A' (4.232, 3.33)

In matrix form, the new ending coordinates of the line after rotation of the point B (6, 12) may be obtained as-may be obtained as-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (6, 12)

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

$$\begin{bmatrix} Xnew \\ Ynew \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} x \begin{bmatrix} Xold \\ Yold \end{bmatrix}$$

$$= \begin{bmatrix} \cos 30^0 & \sin 30^0 \\ -\sin 30^0 & \cos 30^0 \end{bmatrix} x \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \text{ x } \cos 30^0 \ + \ 12 \text{ x } \sin 30^0 \\ 6 \text{ x } \sin 30^0 \ + \ 12 \text{ x } \cos 30^0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \text{ x } (\sqrt{3} / 2) \ + \ 12 \text{ x } (1 / 2) \\ -6 \text{ x } (1 / 2) \ + \ 12 \text{ x } (\sqrt{3} / 2) \end{bmatrix}$$

$$= \begin{bmatrix} 6 \text{ x } 0.866 \ + \ 12 \text{ x } 0.5 \\ -6 \text{ x } 0.5 \ + \ 12 \text{ x } 0.866 \end{bmatrix}$$

$$= \begin{bmatrix} 11.196 \\ 7.392 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = B' (11.196, 7.392)

Problem-06:

(a) Find the matrix that represents rotation of an object by 30° about the origin.

(b) What are the new coordinates of the point P (2, −4) after the rotation?

Solution-

We know,

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(a)

$$R_{30^\circ} = \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(*b*) So the new coordinates can be found by multiplying:

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} \sqrt{3}+2 \\ 1-2\sqrt{3} \end{pmatrix}$$

Problem-07

Prove that 2D rotations about the origin are commutative i.e. $R_1 R_2 = R_2 R_1$.

Proof- R_1 and R_2 are rotation matrices.

$$R_1 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 \\ sin\theta_1 & cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and } R_2 = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 \\ sin\theta_2 & cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 * R_2 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 \\ sin\theta_1 & cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 \\ sin\theta_2 & cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2 + 0 & -cos\theta_1 sin\theta_2 - sin\theta_1 cos\theta_2 + 0 & 0+0+0 \\ sin\theta_1 cos\theta_2 + cos\theta_1 sin\theta_2 + 0 & -sin\theta_1 sin\theta_2 + cos\theta_1 cos\theta_2 + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2 & -cos\theta_1 sin\theta_2 - sin\theta_1 cos\theta_2 & 0 \\ sin\theta_1 cos\theta_2 + cos\theta_1 sin\theta_2 & -sin\theta_1 sin\theta_2 + cos\theta_1 cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{......................(1)}$$

$$R_2 * R_1 = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 \\ sin\theta_2 & cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 \\ sin\theta_1 & cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta_2 cos\theta_1 - sin\theta_2 sin\theta_1 + 0 & -cos\theta_2 sin\theta_1 - sin\theta_2 cos\theta_1 + 0 & 0+0+0 \\ sin\theta_2 cos\theta_1 + cos\theta_2 sin\theta_1 + 0 & -sin\theta_2 sin\theta_1 + cos\theta_2 cos\theta_1 + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2 & -cos\theta_1 sin\theta_2 - sin\theta_1 cos\theta_2 & 0 \\ sin\theta_1 cos\theta_2 + cos\theta_1 sin\theta_2 & -sin\theta_1 sin\theta_2 + cos\theta_1 cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{......................(2)}$$

From equation (1) and (2), we get

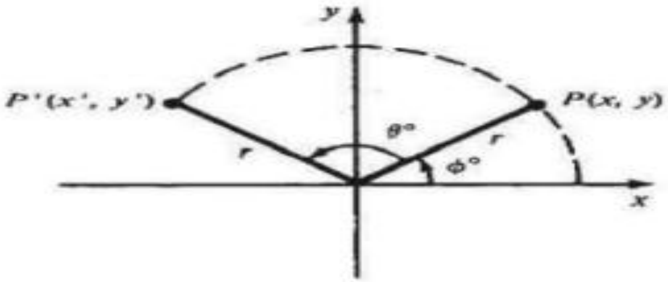
$R_1 R_2 = R_2 R_1$

(Hence Proved)

1. **Derive the transformation that rotates an object point θ° about the origin. Write the matrix representation for this rotation.**

Solution-

Refer to following figure,



Definition of the trigonometric functions sin and cos yields

$$x' = r\cos(\theta + \phi) \qquad y' = r\sin(\theta + \phi)$$

and

$$x = r\cos\phi \qquad y = r\sin\phi$$

Using trigonometric identities, we obtain

$$r\cos(\theta + \phi) = r(\cos\theta\cos\phi - \sin\theta\sin\phi) = x\cos\theta - y\sin\theta$$

and

$$r\sin(\theta + \phi) = r(\sin\theta\cos\phi + \cos\theta\sin\phi) = x\sin\theta - y\cos\theta$$

or

$$x' = x\cos\theta - y\sin\theta \qquad y' = x\sin\theta + y\cos\theta$$

Writing $P' = \begin{pmatrix} x' \\ y' \end{pmatrix}$, $P = \begin{pmatrix} x \\ y \end{pmatrix}$, and

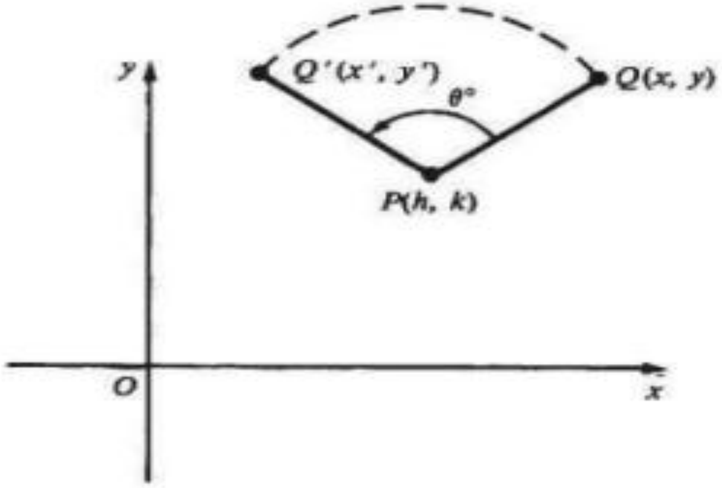
$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

we can now write $P' = R_\theta \cdot P$.

2. Describe the transformation that rotates an object point, Q(x, y), θ° about a fixed center of rotation P(h, k)

Solution-

Refer to following figure,



We determine the transformation $R_{\theta, P}$ in three steps:

(1) translate so that the center of rotation P is at the origin,

(2) perform a rotation of θ degrees about the origin, and

(3) translate P back to (h, k).

Using v = h**I** + k**J** as the translation vector, we build $R_{\theta, P}$ by composition of transformations

$$R_{\theta, O'} = T_{\mathbf{v}} \cdot R_{\theta} \cdot T_{-\mathbf{v}}$$

2D Scaling in Computer Graphics-

In computer graphics, scaling is a process of modifying or altering the size of objects.
Scaling may be used to increase or reduce the size of object.
Scaling subjects the coordinate points of the original object to change.
Scaling factor determines whether the object size is to be increased or reduced.
If scaling factor > 1, then the object size is increased.
If scaling factor < 1, then the object size is reduced.

Consider a point object O has to be scaled in a 2D plane. Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Scaling factor for X-axis = S_x
- Scaling factor for Y-axis = S_y
- New coordinates of the object O after scaling = (X_{new}, Y_{new})

This scaling is achieved by using the following scaling equations-

- $X_{new} = X_{old} \text{ x } S_x$
- $Y_{new} = Y_{old} \text{ x } S_y$

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

For homogeneous coordinates, the above scaling matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Scaling Matrix

(Homogeneous Coordinates Representation)

Practice Problems Based On 2D Scaling

Problem-01:

Find the transformation that scales (with respect to the origin) by (a) a units in the X direction, (b) b units in the Y direction, and (c) simultaneously a units in the X direction and b units in the Y direction.

SOLUTION

(a) The scaling transformation applied to a point $P(x, y)$ produces the point (ax, y). We can write this in matrix form as $S_{a,1} \cdot P$, or

$$\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ y \end{pmatrix}$$

(b) As in part (a), the required transformation can be written in matrix form as $S_{1,b} \cdot P$. So

$$\begin{pmatrix} 1 & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ by \end{pmatrix}$$

(c) Scaling in both directions is described by the transformation $x' = ax$ and $y' = by$. Writing this in matrix form as $S_{a,b} \cdot P$, we have

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ by \end{pmatrix}$$

Problem-02:

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.
Solution-
Given-
Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
Scaling factor along X axis = 2
Scaling factor along Y axis = 3

<u>For Coordinates A(0, 3)</u>

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9

<u>For Coordinates B(3, 3)</u>

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates D(0, 0)

Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

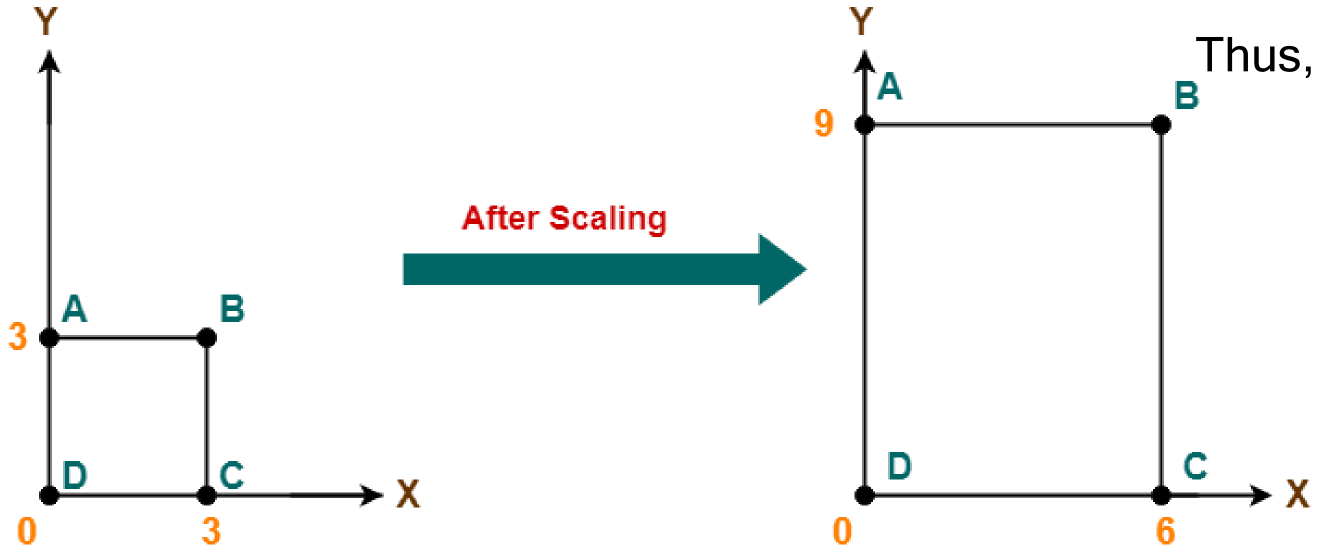
For Coordinates C(3, 0)

Let the new coordinates of corner C after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).



Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).

Problem-03:

Prove that 2D Scaling transformations are commutative i.e, $S_1 S_2 = S_2 S_1$.

Solution: S_1 and S_2 are scaling matrices

$$S_1 = \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_1 * S_2 = \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Sx_1 Sx_2 & 0 & 0 \\ 0 & Sy_1 Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{..........................equation 1}$$

$$S_2 * S_1 = \begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Sx_2 Sx_1 & 0 & 0 \\ 0 & Sy_2 Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{..............................equation 2}$$

From equation 1 and 2

$$S_1 S_2 = S_2 S_1. \text{ Hence Proved}$$

2D Reflection

- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of mirror.
- The size of reflected object is same as the size of original object.

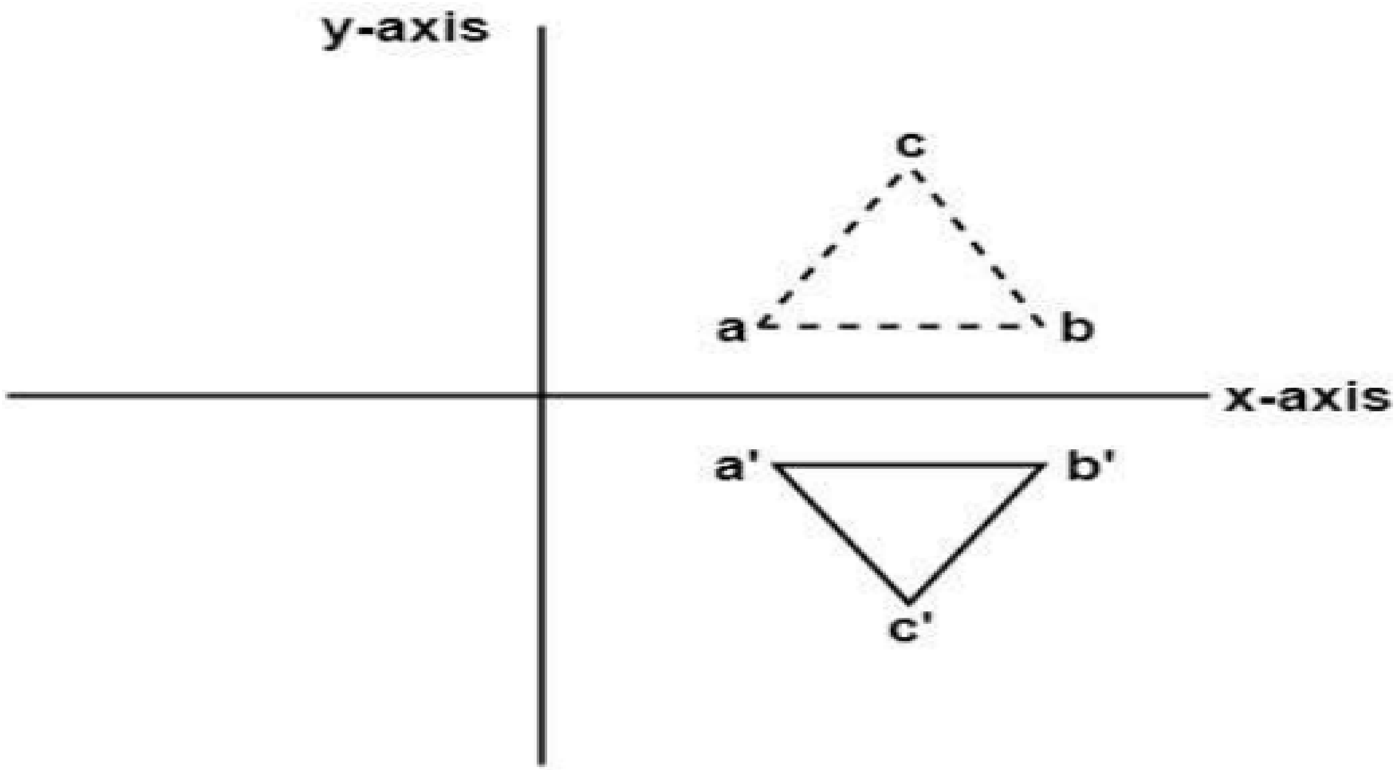
Consider a point object O that has to be reflected in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the reflected object O after reflection = (X_{new}, Y_{new})

Reflection on X-Axis:

In this transformation value of x will remain same whereas the value of y will become negative. Following figures shows the reflection of the object axis. The object will lie another side of the x-axis.



This reflection is achieved by using the following reflection equations-

- $X_{new} = X_{old}$
- $Y_{new} = -Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

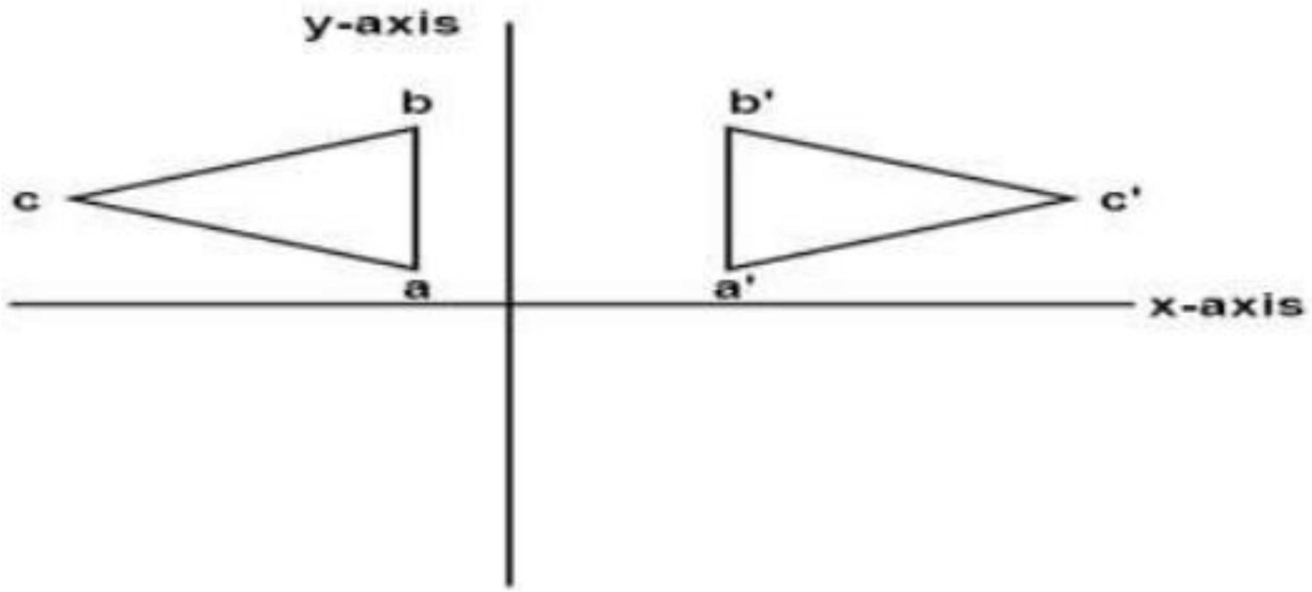
(Reflection Along X Axis)

(Homogeneous Coordinates Representation)

Reflection on Y-Axis:

Here the values of x will be reversed, whereas the value of y will remain the same. The object will lie another side of the y-axis.

The following figure shows the reflection about the y-axis



This reflection is achieved by using the following reflection equations-

- $X_{new} = -X_{old}$
- $Y_{new} = Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

(Homogeneous Coordinates Representation)

Practice Problems Based On 2D Reflection

Problem-01:

Given a triangle with coordinate points A (3, 4), B (6, 4), C (5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Solution-

Given-

Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)

Reflection has to be taken on the X axis

For Coordinates A (3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 3$
- $Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner A after reflection = (3, -4).

For Coordinates B (6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 6$
- $Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner B after reflection = (6, -4).

For Coordinates C (5, 6)

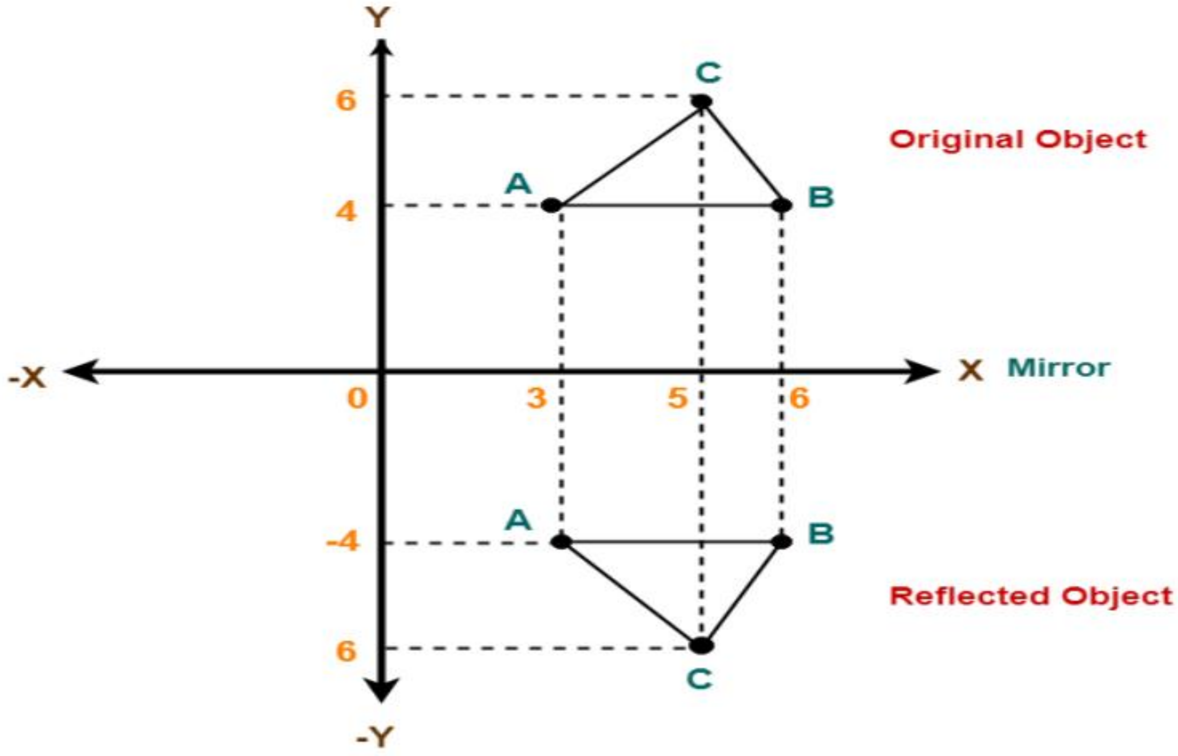
Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 5$
- $Y_{new} = -Y_{old} = -6$

Thus, New coordinates of corner C after reflection = (5, -6).

Thus, New coordinates of the triangle after reflection = A (3, -4), B (6, -4), and C (5, -6).



Problem-02:

Given a triangle with coordinate points A (3, 4), B (6, 4), C (5, 6). Apply the reflection on the Y axis and obtain the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the triangle = A (3, 4), B (6, 4), C (5, 6)
- Reflection has to be taken on the Y axis

For Coordinates A (3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -3$
- $Y_{new} = Y_{old} = 4$

Thus, New coordinates of corner A after reflection = (-3, 4).

For Coordinates B (6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -6$
- $Y_{new} = Y_{old} = 4$

Thus, New coordinates of corner B after reflection = (-6, 4).

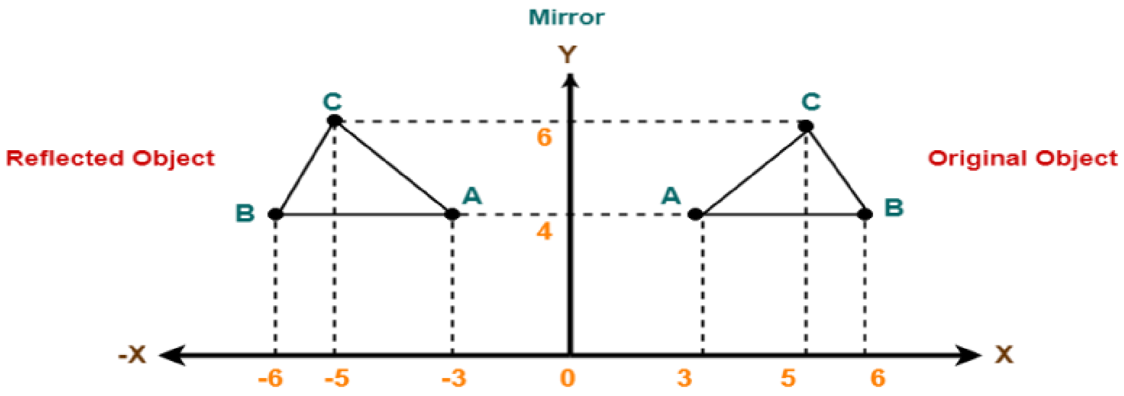
For Coordinates C (5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -5$
- $Y_{new} = Y_{old} = 6$

Thus, New coordinates of corner C after reflection = (-5, 6).



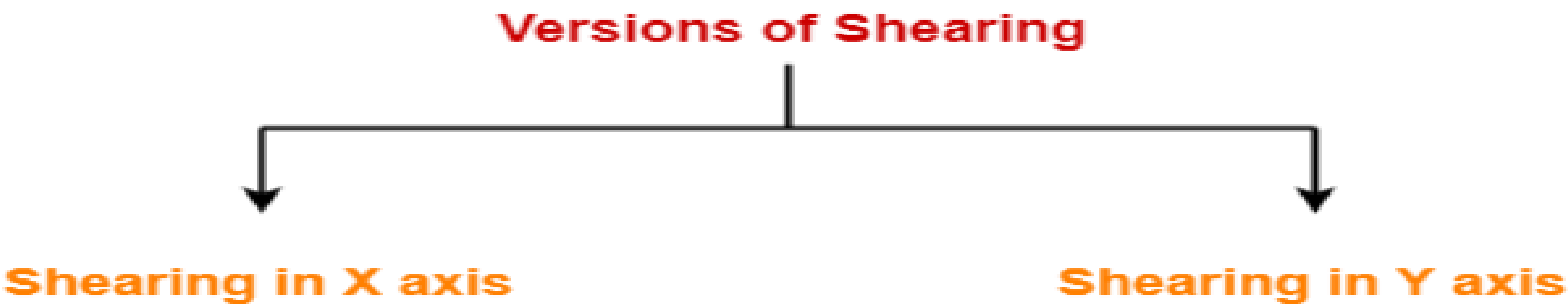
Thus, New coordinates of the triangle after reflection = A (-3, 4), B (-6, 4), C (-5, 6).

2D Shearing

In Computer graphics, 2D Shearing is an ideal technique to change the shape of an existing object in a two-dimensional plane.

In a two-dimensional plane, the object size can be changed along X direction as well as Y direction.

So, there are two versions of shearing-



Consider a point object O that has to be sheared in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Shearing parameter towards X direction = Sh_x
- Shearing parameter towards Y direction = Sh_y
- New coordinates of the object O after shearing = (X_{new}, Y_{new})

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

- $X_{new} = X_{old} + Sh_x \times Y_{old}$
- $Y_{new} = Y_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In X axis)

(Homogeneous Coordinates Representation)

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} + Sh_y \times X_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In Y axis)

(Homogeneous Coordinates Representation)

Practice Problems Based On 2D Shearing

Problem-01:

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the triangle = A (1, 1), B (0, 0), C (1, 0)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2

Shearing in X Axis-

For Coordinates A (1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$
- $Y_{new} = Y_{old} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

For Coordinates B (0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C (1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 0 = 1$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner C after shearing = (1, 0).

Thus, New coordinates of the triangle after shearing in X axis = A (3, 1), B (0, 0), C (1, 0).

Shearing in Y Axis-

For Coordinates A (1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

Shearing in Y Axis-

For Coordinates A (1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B (0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 0 = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C (1, 0)

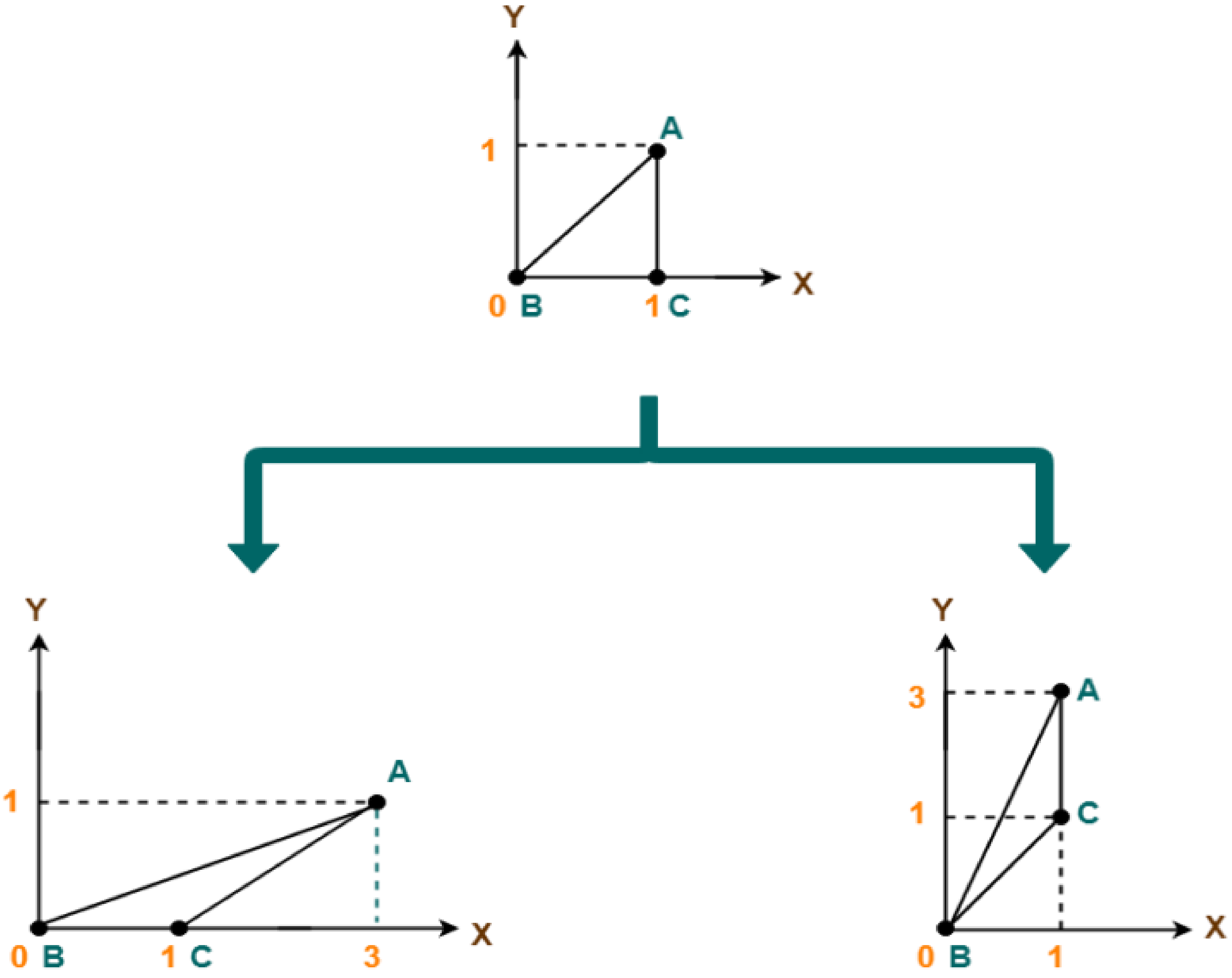
Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 1 = 2$

Thus, New coordinates of corner C after shearing = (1, 2).

Thus, New coordinates of the triangle after shearing in Y axis = A (1, 3), B (0, 0), C (1, 2).



Shearing in X Axis

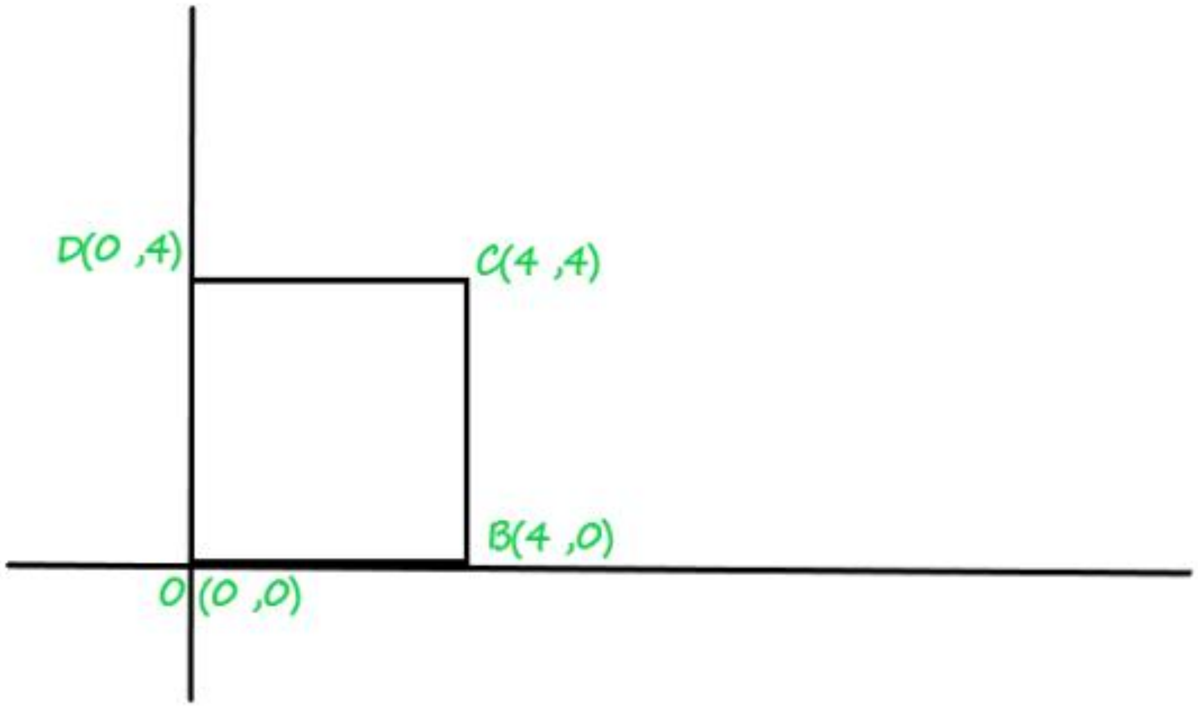
Shearing in Y Axis

More Problems 2D Transformation:

Problem-01:

Consider we have a square O (0, 0), B (4, 0), C (4, 4), D (0, 4) on which we first apply scaling transformation where given scaling factor is $S_x=S_y=0.5$ and then we apply rotation transformation in clockwise direction it by 90^*(angle), in last we perform reflection transformation about origin.

The square O, A, C, D looks like:



Square given (Fig.1)

First, we perform scaling transformation over a 2-D object:

Representation of scaling condition:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sx \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

For coordinate O (0, 0):

$$O \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2-D object after scaling:

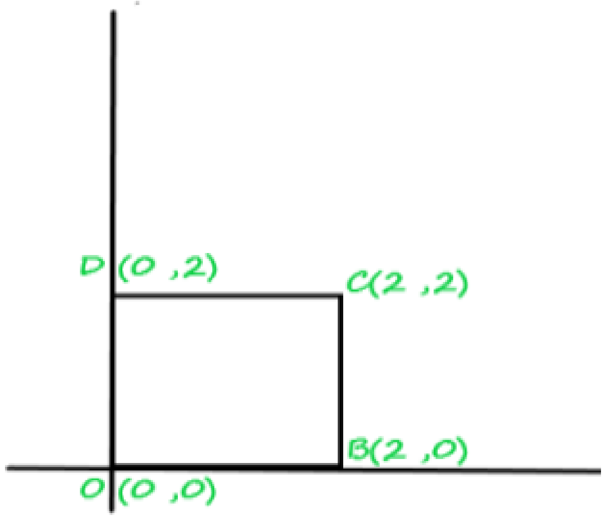


Fig.2

For coordinate B (4, 0):

$$B \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

For coordinate C (4, 4):

$$C \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

For coordinate D (0, 4):

$$D \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

*Now, we'll perform rotation transformation in clockwise-direction on Fig.2 by 90^{θ}:
The condition of rotation transformation of 2-D object about origin is:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos & sin \\ -sin & cos \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\text{Cos}90 = 0$$
$$\sin 90 = 1$$

For coordinate O (0, 0):

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For coordinate B (2, 0):

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

For coordinate C (2, 2):

$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$C \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

For coordinate D (0, 2):

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$D \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

2-D object after rotating about origin by 90^{*} angle:

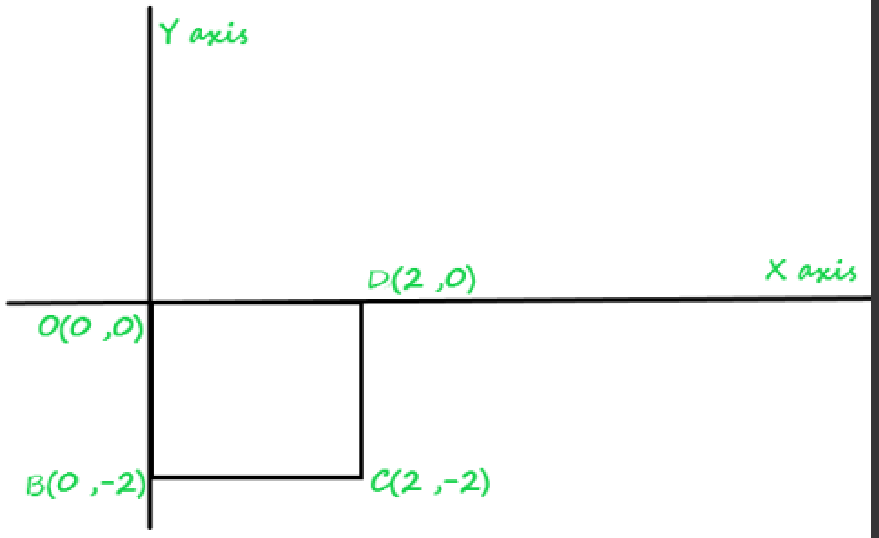


Fig.3

*Now, we'll perform third last operation on Fig.3, by reflecting it about origin:
The condition of reflecting an object about origin is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

For coordinate O (0, 0):

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$O \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For coordinate B' (0, 0):

$$B' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$B' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

Note: The above finale result of Fig.4, that we get after applying all transformation one after one in a serial manner.

For coordinate C' (0, 0):

$$C' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$C' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

For coordinate D' (0, 0):

$$D' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$D' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

The final 2-D object after reflecting about origin, we get:

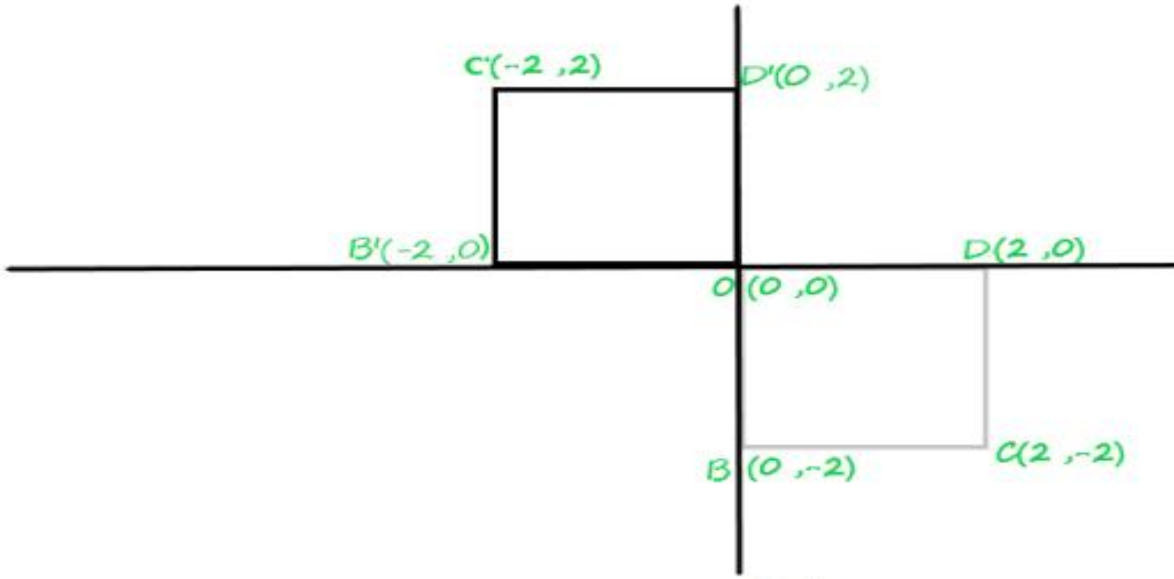


Fig.4