Number System Conversion

There are many methods or techniques which can be used to convert numbers from one base to another. We'll demonstrate here the following −

- Decimal to Other Base System
- Other Base System to Decimal
- Other Base System to Non-Decimal
- Shortcut method − Binary to Octal
- Shortcut method − Octal to Binary
- Shortcut method − Binary to Hexadecimal
- Shortcut method − Hexadecimal to Binary

Decimal to Other Base System

Steps

- **Step 1** − Divide the decimal number to be converted by the value of the **new base**.
- **Step 2** − Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.
- **Step 3** − Divide the quotient of the previous divide by the new base.
- **Step 4** − Record the remainder from Step 3 as the next digit (to the left) of the new base number.

Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.

The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.

Example −

Decimal Number: 29_{10}

Calculating Binary Equivalent −

Step	Operation	Result	Remainder
Step 1	29 / 2	14	1
Step 2	14 / 2	7	0
Step 3	7 / 2	3	1
Step 4	3 / 2	1	1

Step 5	1 / 2	0	1

As mentioned in Steps 2 and 4, the remainders have to be arranged in the reverse order so that the first remainder becomes the Least Significant Digit (LSD) and the last remainder becomes the Most Significant Digit (MSD).

Decimal Number − 29_{10} = Binary Number − 11101_2.

Convert decimal fraction to binary number

A) Convert the integral part of decimal to binary equivalent

1. Divide the decimal number by 2 and store remainders in array.
2. Divide the quotient by 2.
3. Repeat step 2 until we get the quotient equal to zero.
4. Equivalent binary number would be reverse of all remainders of step 1.

B) Convert the fractional part of decimal to binary equivalent

1. Multiply the fractional decimal number by 2.
2. Integral part of resultant decimal number will be first digit of fraction binary number.
3. Repeat step 1 using only fractional part of decimal number and then step 2.

C) Combine both integral and fractional part of binary number.

Illustration:

Let's take an example for n = 4.47 k = 3

Step 1: Conversion of 4 to binary

1. 4/2: Remainder = 0: Quotient = 2
2. 2/2: Remainder = 0: Quotient = 1
3. 1/2: Remainder = 1: Quotient = 0

So equivalent binary of integral part of decimal is 100.

Step 2: Conversion of .47 to binary

1. 0.47 * 2 = 0.94, Integral part: 0
2. 0.94 * 2 = 1.88, Integral part: 1
3. 0.88 * 2 = 1.76, Integral part: 1

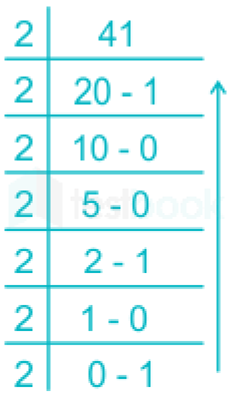
So equivalent binary of fractional part of decimal is .011

Step 3: Combined the result of step 1 and 2.

Final answer can be written as:
100 + .011 = 100.011

The given decimal number is $(41.6875)_{10}$

To convert the decimal number 41 into a binary number, we have to do the repeated division by 2, then we get



$(41)_{10} = (101001)_2$

Now, to convert the decimal number (0.6875) into a binary number, we have to do the repeated multiplication of 2 till we get zero.

$0.6875 \times 2 = \mathbf{1}.375$

$0.375 \times 2 = \mathbf{0}.75$

$0.75 \times 2 = \mathbf{1}.5$

$0.5 \times 2 = \mathbf{1}.0$

$(0.6875)_{10} = (1001)_2$

Therefore, the binary equivalent of $(41.6875)_{10}$ is $(101001.1001)_2$

Convert Decimal to Octal with Steps

Follow the steps given below to learn the decimal to octal conversion:

1. Write the given decimal number
2. If the given decimal number is less than 8 the octal number is the same.
3. If the decimal number is greater than 7 then divide the number by 8.
4. Note the remainder, we get after division
5. Repeat step 3 and 4 with the quotient till it is less than 8
6. Now, write the remainders in reverse order (bottom to top)
7. The resultant is the equivalent octal number to the given decimal number.

For example: Convert 1792 into an octal number.

Decimal Number	**Operation**	**Quotient**	**Remainder**	**Octal Number**
1792	÷ 8	224	0	0
224	÷ 8	28	0	00
28	÷ 8	3	4	400
3	÷ 8	0	3	3400

Example 1: Convert $(127)_{10}$ to Octal.

Solution: Divide 127 by 8

127 ÷ 8= 15(Quotient) and (7) Remainder

Divide 15 by 8 again.

15 ÷ 8 = 1(Quotient) and (7) Remainder

Divide 1 by 8, we get;

1 ÷ 8 = 0(Quotient) and (1) Remainder

Since the quotient is zero now, no more division can be done. So by taking the remainders in reverse order, we get the equivalent octal number.

Hence, $(127)_{10} = (177)_8$

Example 2: Convert 52_{10} to octal.

Solution: Divide 52 by 8

52 ÷ 8 = 6(Quotient) and (4) Remainder

Divide 6 by 8 again.

6 ÷ 8 = 0(Quotient) and (6) Remainder

Since the quotient is zero now, no more division can be done. So by taking the remainders in reverse order, we get the equivalent octal number.

Hence, $(52)_{10} = (64)_8$

Example 3: Convert 100_{10} to octal.

Solution: Divide 100 by 8

100 ÷ 8= 12(Quotient) and (4) Remainder

Divide 12 by 8 again.

$12 \div 8 = 1$(Quotient) and (4) Remainder

Divide 1 by 8, we get;

$1 \div 8 = 0$(Quotient) and (1) Remainder

Since the quotient is zero now, no more division can be done. So by taking the remainders in reverse order, we get the equivalent octal number.

Hence, $(100)_{10} = (144)_8$

To convert decimal number 41.6, we convert its integer and fraction part individually and then add them to get the equivalent octal number, as below:

To convert integer 41 to octal, follow these steps:

Divide 41 by 8 keeping notice of the quotient and the remainder. Continue dividing the quotient by 2 until you get a quotient of zero.

Then just write out the remainders in the reverse order to get the equivalent octal number.

41 / 8 = 5 with remainder 1

5 / 8 = 0 with remainder 5

Here is the answer to 41 decimals to octal number: **51**

For converting decimal fraction 0.6 to octal number, follow these steps:

Multiply 0.6 by 8 keeping notice of the resulting integer and fractional part. Continue multiplying by 8 until you get a resulting fractional part equal to zero (we calculate up to ten digits).

Then just write out the integer parts from the results of each multiplication to get equivalent octal number.

$0.6 \times 8 = 4 + 0.8$

$0.8 \times 8 = 6 + 0.4$

$0.4 \times 8 = 3 + 0.19999999999999$

$0.19999999999999 \times 8 = 1 + 0.59999999999991$

$0.59999999999991 \times 8 = 4 + 0.79999999999927$

$0.79999999999927 \times 8 = 6 + 0.39999999999418$

0.39999999999418 × 8 = 3 + 0.19999999995343

0.19999999995343 × 8 = 1 + 0.59999999962747

0.59999999962747 × 8 = 4 + 0.79999999701977

0.79999999701977 × 8 = 6 + 0.39999997615814

Here is the answer to 0.6 decimal to octal number:

0.4631463146

Therefore, decimal number 41.6 converted to octal is equal:

51.4631463146

Home work: Decimal to Hexadecimal

Conversion from Decimal to Hexadecimal number system

There are various direct or indirect methods to convert a decimal number into hexadecimal number. In an indirect method, you need to convert a decimal number into other number system (e.g., binary or octal), then you can convert into hexadecimal number by using grouping from binary number system and converting each octal digit into binary then grouping and convert these into hexadecimal number.

Example − Convert decimal number 105 into hexadecimal number.

First convert it into binary or octal number,

$= (100)_{10}$

$= (1x2^6+1x2^5+0x2^4+0x2^3+1x2^2+0x2^1+0x2^0)_{10}$ or $(1x8^2+4x8^1+4x8^0)_{10}$

Because base of binary and octal are 2 and 8 respectively.

$= (1100100)_2$ or $(144)_8$

Then convert each digit of octal number into 3 bit of binary number, then use grouping of 4 bit of binary number.

$= (1100100)_2$ or $(001\ 100\ 100)_2$

$= (110\ 0100)_2$

$= (0110\ 0100)_2$

$= (6\ 4)_{16}$

$= (64)_{16}$

However, there are two direct methods are available for converting a decimal number into Hexadecimal number − Converting with Remainders and Converting with Division. These are explained as following below.

(a) Converting with Remainders (For integer part)

This is a straightforward method which involve dividing the number to be converted. Let decimal number is N then divide this number from 16 because base of hexadecimal number system is 16. Note down the value of remainder, which will be: 0 to 15 (replace 10, 11, 12, 13, 14, 15 by A, B, C, D, E, F respectively). Again divide remaining decimal number till it became 0 and note every remainder of every step. Then write remainders from bottom to up (or in reverse order), which will be equivalent hexadecimal number of given decimal number. This is procedure for converting an **integer decimal** number, algorithm is given below.

- Take decimal number as dividend.
- Divide this number by 16 (16 is base of hexadecimal so divisor here).

- Store the remainder in an array (it will be: 0 to 15 because of divisor 16, replace 10, 11, 12, 13, 14, 15 by A, B, C, D, E, F respectively).
- Repeat the above two steps until the number is greater than zero.
- Print the array in reverse order (which will be equivalent hexadecimal number of given decimal number).

Note that dividend (here given decimal number) is the number being divided, the divisor (here base of hexadecimal, i.e., 16) in the number by which the dividend is divided, and quotient (remaining divided decimal number) is the result of the division.

Example − Convert decimal number 540 into hexadecimal number.

Since given number is decimal integer number, so by using above algorithm performing short division by 16 with remainder.

Division	**Remainder (R)**
540 / 16 = 33	12 = C
33 / 16 = 2	1
2 / 16 = 0	2
0 / 16 = 0	0

Now, write remainder from bottom to up (in reverse order), this will be 021C (or only 21C) which is equivalent hexadecimal number of decimal integer 540.

But above method cannot convert fraction part of a mixed (a number with integer and fraction part) hexadecimal number. For **decimal fractional** part, the method is explained as following below.

(b) Converting with Remainders (For fractional part)

Let decimal fractional part is M then multiply this number from 16 because base of hexadecimal number system is 16. Note down the value of integer part, which will be − 0 to 15 (replace 10, 11, 12, 13, 14, 15 by A, B, C, D, E, F respectively). Again multiply remaining decimal fractional number till it became 0 and note every integer part of result of every step. Then write noted results of integer part, which will be equivalent fraction hexadecimal number of given decimal number. This is procedure for converting a **fractional decimal** number, algorithm is given below.

- Take decimal number as multiplicand.
- Multiple this number by 16 (16 is base of hexadecimal so multiplier here).
- Store the value of integer part of result in an array (it will be: 0 to 15, because of multiplier 16, replace 10, 11, 12, 13, 14, 15 by A, B, C, D, E, F respectively).
- Repeat the above two steps until the number became zero.
- Print the array (which will be equivalent fractional hexadecimal number of given decimal fractional number).

Note that a multiplicand (here decimal fractional number) is that to be multiplied by multiplier (here base of hexadecimal, i.e., 16)

Example − Convert decimal fractional number 0.06640625 into hexadecimal number.

Since given number is decimal fractional number, so by using above algorithm performing short multiplication by 16 with integer part.

Multiplication	**Resultant integer part**
0.06640625 x 16=1.0625	1
0.0625 x 16 =1.0	1

Multiplication	Resultant integer part
0 x 16=0.0	0

Now, write these resultant integer part, this will be approximate 0.110 which is equivalent hexadecimal fractional number of decimal fractional 0.06640625.

Converting with Division

This method is guessing hexadecimal number of a decimal number. You need to draw a table of power of 16, For **integer part**, the algorithm is explained as following below.

- Start with any decimal number.
- List the powers of 16.
- Divide the decimal number by the largest power of 16.
- Find the remainder.
- Divide the remainder by the next power of 16.
- Repeat until you've found the full answer.

Example − Convert decimal number 380 into hexadecimal number.

According to above algorithm, table of power of 16,

Decimal	16^3=4096	16^2=256	16^1=16	16^0=1
Hexadecimal Digit	0	1	7	C

Divide the decimal number by the largest power of 16.

= 380 / 256 = 1.484375

So 1 will be first digit or most significant bit (MSB) of hexadecimal number.

Now, remainder will be,

= 380 - 1256 =124

Now, divide this

remainder by the next power of 16.

= 124 / 16 = 7.75

So 7 will be next digit or second most significant bit (MSB) of hexadecimal number.

Now, remainder will be,

= 124 - 716 = 12

Because remainder 12(= C) is less than base 16, so C (=12) will be as (least significant) bit of required hexadecimal number.

Therefore, 17C will be equivalent hexadecimal number of given decimal number

Other Base System to Decimal System

Steps

- **Step 1** − Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).
- **Step 2** − Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.
- **Step 3** − Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example

Binary Number − 11101_2

Calculating Decimal Equivalent −

Step	Binary Number	Decimal Number
Step 1	11101_2	$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	11101_2	**$(16 + 8 + 4 + 0 + 1)_{10}$**
Step 3	11101_2	29_{10}

Binary Number − 11101_2 = Decimal Number − 29_1

Hexadecimal Number System

The base of a hexadecimal system is 16. The 16 symbols involved in this system include 10 decimal digits and the first six letters of the English alphabet, i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. Here, the alphabets can be treated 10, 11, 12, 13, 14 and 15, respectively. Learn more about the hexadecimal system here.

Decimal Number System

A number system that uses digits from 0 to 9 to represent a number with base 10 is called the decimal number system. The number is expressed in base-10, where each value is denoted by 0 or the first nine positive integers. Each value in this number system has the place value of power 10. It means the digit at the tens place is ten times greater than the digit at the unit place.

Conversion from Hex to Decimal

As we know, number systems can be converted from one base to another. Thus, we can convert hexadecimal numbers to decimal easily. This number system conversion can be done as explained in the example given below:

Example:

Convert 7CF (hex) to decimal.

Solution:

Given hexadecimal number is 7CF.

In hexadecimal system,

7 = 7

C = 12

F = 15

To convert this into a decimal number system, multiply each digit with the powers of 16 starting from units place of the number.

$7CF = (7 \times 16^2) + (12 \times 16^1) + (15 \times 16^0)$

$= (7 \times 256) + (12 \times 16) + (15 \times 1)$

$= 1792 + 192 + 15$

$= 1999$

From this, the rule can be defined for the conversion from hex numbers to decimal numbers.

Suppose below is the hex number with n digits:

$d_{n-1} \dots d_3\ d_2\ d_1\ d_0$

Multiply each digit of the hex number with its corresponding powers of 16 and add them such as:

$d_{n-1} \times 16^{n-1} + \dots + d_3 \times 16^3 + d_2 \times 16^2 + d_1 \times 16^1 + d_0 \times 16^0$

Thus, the resultant number will be taken as base 10 or decimal number system.

$d_{n-1} \dots d_3\ d_2\ d_1\ d_0$ (hex) $= d_{n-1} \times 16^{n-1} + \dots + d_3 \times 16^3 + d_2 \times 16^2 + d_1 \times 16^1 + d_0 \times 16^0$ (decimal)

Hex to Decimal Converter

There is a tool available to convert the numbers from hexadecimal to decimal number system.

Hex to Decimal Table

The conversion table for the numbers from hexadecimal to decimal is given below:

Hexadecimal	Decimal (Equivalent Value)
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7

8	8
9	9
A	**10**
B	**11**
C	**12**
D	**13**
E	**14**
F	**15**

This table will help in representing the digits and letters individually in the large numbers in base 16 system as explained above.

Solved Examples

Let us have a look at the examples of conversion of numbers from hexadecimal (base 16) to the base 10 number system, with detailed explanations.

Example 1:

Convert $(1DA6)_{16}$ to decimal.

Solution:

$(1DA6)_{16}$

Here,

1 = 1

D = 13

A = 10

6 = 6

Thus,

$\mathbf{(1DA6)_{16}}$ = $(1 \times 16^3) + (13 \times 16^2) + (10 \times 16^1) + (6 \times 16^0)$

$= (1 \times 4096) + (13 \times 256) + (10 \times 16) + (6 \times 1)$

= 4096 + 3328 + 160 + 6

= 7590

Therefore, $(1DA6)_{16} = (7590)_{10}$

Example 2:

Convert $(E8B)_{16}$ to decimal system.

Solution:

$(E8B)_{16}$

Here,

E = 14

8 = 8

B = 11

Thus,

$(E8B)_{16} = (14 \times 16^2) + (8 \times 16^1) + (11 \times 16^0)$

$= (14 \times 256) + (8 \times 16) + (11 \times 1)$

$= 3584 + 128 + 11$

$= 3723$

Therefore, $(E8B)_{16} = (3723)_{10}$

Convert octal number $0.01_{(base\ 8)}$ into decimal form

The following table shows the places, octal number and the multipliers for the corresponding places.

place	ones	Decimal Point	tenths	hundredths
octal	0	.	0	1
multiplier	8^0		8^{-1}	8^{-2}

$= 0x8^0 + 0x8^{-1} + 0x8^{-2}$

$= 0 + 0 + 0.015625$

$= 0.015625$

So, the required decimal number is
$0.01_{(base\ 8)} = 0.015625_{(base\ 10)}$
Alternatively, $(0.01)_8 = (0.015625)_{10}$
Where, (base 10) means the number is in decimal number system and (base 8) means the number is in octal number system.

Convert octal number $7.12172_{(base\ 8)}$ into decimal form

The following table shows the places, octal number and the multipliers for the corresponding places.

place	ones	Decimal Point	tenths	hundredths	thousandths	ten thousandths	hundred thousandths
octal	7	.	1	2	1	7	2
multiplier	8^0		8^{-1}	8^{-2}	8^{-3}	8^{-4}	8^{-5}

$= 7x8^0 + 1x8^{-1} + 2x8^{-2} + 1x8^{-3} + 7x8^{-4} + 2x8^{-5}$

= 7 + 0.125 + 0.03125 + 0.001953125 + 0.001708984375 + 0.00006103515624

= 10.1599...

= 10.16 (approx. value)

So, the required decimal number is
$7.12172_{(base\ 8)} = 10.16_{(base\ 10)}$ (approx. value)
Alternatively, $(7.12172)_8 = (10.16)_{10}$ (approx. value)
Where, (base 10) means the number is in decimal number system and (base 8) means the number is in octal number system.

Other Base System to Non-Decimal System

Steps

- **Step 1** − Convert the original number to a decimal number (base 10).
- **Step 2** − Convert the decimal number so obtained to the new base number.

Example

Octal Number − 25_8

Calculating Binary Equivalent −

Step 1 − Convert to Decimal

Step	Octal Number	Decimal Number
Step 1	25_8	$((2 \times 8^1) + (5 \times 8^0))_{10}$
Step 2	25_8	$(16 + 5)_{10}$

Step 3	25_8	21_{10}

Octal Number − 25_8 = Decimal Number − 21_{10}

Step 2 − Convert Decimal to Binary

Step	Operation	Result	Remainder
Step 1	21 / 2	10	1
Step 2	10 / 2	5	0
Step 3	5 / 2	2	1
Step 4	2 / 2	1	0
Step 5	1 / 2	0	1

Decimal Number − 21_{10} = Binary Number − 10101_2

Octal Number − 25_8 = Binary Number − 10101_2

Shortcut method - Binary to Octal

Steps

- **Step 1** − Divide the binary digits into groups of three (starting from the right).
- **Step 2** − Convert each group of three binary digits to one octal digit.

Example

Binary Number − 10101_2

Calculating Octal Equivalent −

Step	Binary Number	Octal Number
Step 1	10101_2	010 101
Step 2	10101_2	2_8 5_8
Step 3	10101_2	25_8

Binary Number − 10101_2 = Octal Number − 25_8

Shortcut method - Octal to Binary

Steps

- **Step 1** − Convert each octal digit to a 3-digit binary number (the octal digits may be treated as decimal for this conversion).
- **Step 2** − Combine all the resulting binary groups (of 3 digits each) into a single binary number.

Example

Octal Number − 25_8

Calculating Binary Equivalent −

Step	Octal Number	Binary Number
Step 1	25_8	2_{10} 5_{10}
Step 2	25_8	010_2 101_2
Step 3	25_8	**010101_2**

Octal Number − 25_8 = Binary Number − 10101_2

Shortcut method - Binary to Hexadecimal

Steps

- **Step 1** − Divide the binary digits into groups of four (starting from the right).
- **Step 2** − Convert each group of four binary digits to one hexadecimal symbol.

Example

Binary Number − 10101_2

Calculating hexadecimal Equivalent −

Step	Binary Number	Hexadecimal Number
Step 1	10101_2	**0001 0101**
Step 2	10101_2	1_{10} 5_{10}
Step 3	10101_2	15_{16}

Binary Number − 10101_2 = Hexadecimal Number − 15_{16}

Shortcut method - Hexadecimal to Binary

Steps

- **Step 1** − Convert each hexadecimal digit to a 4-digit binary number (the hexadecimal digits may be treated as decimal for this conversion).
- **Step 2** − Combine all the resulting binary groups (of 4 digits each) into a single binary number.

Example

Hexadecimal Number − 15_{16}

Calculating Binary Equivalent −

Step	Hexadecimal Number	Binary Number
Step 1	15_{16}	1_{10} 5_{10}
Step 2	15_{16}	0001_2 0101_2
Step 3	15_{16}	00010101_2

Hexadecimal Number − 15_{16} = Binary Number − 10101_2

Complements

Given a Binary Number as a string, print its 1's and 2's complements.

1's complement of a binary number is another binary number obtained by toggling all bits in it, i.e., transforming the 0 bit to 1 and the 1 bit to 0. In the 1's complement format , the positive numbers remain unchanged . The negative numbers are obtained by taking the 1's complement of positive counterparts. for example +9 will be represented as 00001001 in eight-bit notation and -9 will be represented as 11110110, which is the 1's complement of 00001001.

Examples:

1's complement of "0111" is "1000"

1's complement of "1100" is "0011"

2's complement of a binary number is 1, added to the 1's complement of the binary number. In the 2's complement representation of binary numbers, the MSB represents the sign with a '0' used for plus sign and a '1' used for a minus sign. the remaining bits are used for representing magnitude. positive magnitudes are represented in the same way as in the case of sign-bit or 1's complement representation. Negative magnitudes are represented by the 2's complement of their positive counterparts.

Examples:

2's complement of "0111" is "1001"

2's complement of "1100" is "0100"

Another trick to finding two's complement:

Step 1: Start from the Least Significant Bit and traverse left until you find a 1. Until you find 1, the bits stay the same

Step 2: Once you have found 1, let the 1 as it is, and now

Step 3: Flip all the bits left into the 1.

Illustration

Suppose we need to find 2s Complement of 100100

Step 1: Traverse and let the bit stay the same until you find 1. Here x is not known yet. Answer = xxxx00 –

Step 2: You found 1. Let it stay the same. Answer = xxx100

Step 3: Flip all the bits left into the 1. Answer = 011100.

Hence, the 2s complement of 100100 is 011100.