Vector : $n \times 1$ matrix.

example:

$$y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \leftarrow \text{3 dimensional vector, } \underline{\mathbb{R}^4}$$

$y_2 = 2$

1 indexed v/s 0 indexed:

$$\underbrace{y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\text{Default}} \qquad y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

Addition & Scalar Multiplication.

* Addition only works on same dimension matrixes.
* Scalar

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}$$

Matrix Vector Multiplication.

Matrix Vector

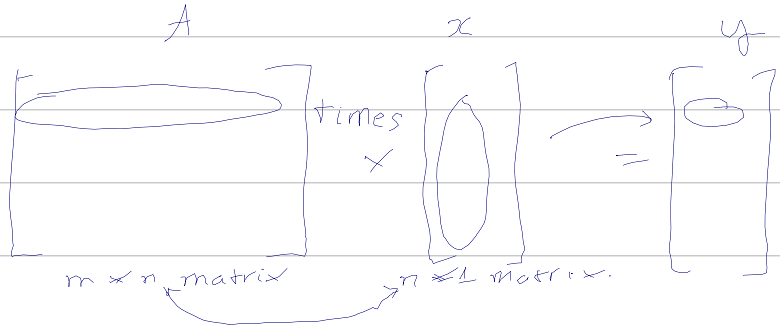
$$\underset{3 \times 2}{\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}} \underset{2 \times 1}{\begin{bmatrix} 1 \\ 5 \end{bmatrix}} = \underset{3 \times 1 \text{ matrix}}{\begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}}$$

$1 \times 1 + 3 \times 5 = 16$

$4 \times 1 + 0 \times 5 = 4$

$2 \times 1 + 1 \times 5 = 7$

Generalize



Implementation:

House sizes:

- $\to$ 2104
- $\to$ 1416
- $\to$ 1534
- $\to$ 852

$h_\theta(x) = -40 + 0.25x$.

the hypothesis to predict the house prices

Construct a matrix, vector.

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} h_\theta(2014) \\ h_\theta(1416) \\ \\ \\ \end{bmatrix}$$

$\Rightarrow$ $\underbrace{\text{prediction}}_{4\times 1}$ = Data Matrix $\times$ Parameters

Matrix Multiplication:

Example:

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

$\Rightarrow$ Outcome $= \begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}$

Details:

$$\underset{m \times n}{\overset{A}{[\quad]}} \times \underset{n \times o}{\overset{B}{[\quad]}} = \underset{m \times o}{\overset{C}{[\quad]}}$$

times

- take matrix A & multiply with each column of B to become column of C.

Implementation.

House sizes

$$\begin{cases} 2014 \\ 1416 \\ 1534 \\ 852 \end{cases}$$

And 3 competing hypotheses for predicting prices

1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$
3. $h_\theta(x) = -150 + 0.4x$

$\Rightarrow$ Matrix

$$\begin{bmatrix} 1 & 2014 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

prediction of 1st hypotheses — 2nd — third

Matrix Multiplication Properties.

A & B are matrices.

1. $A \times B \neq B \times A$ (not commutative)
2. $A \times B \times C \longrightarrow$ a) $(A \times B) \times C$ (Associative)
   b) $A \times (B \times C)$
3. **Identity matrix**: denoted I (or $I_{n \times n}$)

example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, informally $\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

$$\underset{m \times n}{A} \times \underset{n \times n}{I} = \underset{m \times m}{I} \times \underset{m \times n}{A} = A$$

Note:
$AB \neq BA$ unless $B = I$
$AI = IA$

Inverse:

If A is $m \times m$ (square) matrix, and if it has an inverse,

$$A(A^{-1}) = A^{-1}A = I$$

$A \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ has NO inverse

Eg: $$\underset{\substack{2 \times 2 \\ A}}{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}} \underset{A^{-1}}{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2 \times 2}$$

Matrix that don't have inverse are "singular" or "degenerate"

Matrix Transpose

Example:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$$

Def: if $B = A^T$

$$B_{ij} = A_{ji}$$