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Module : CS596

HW : 1

Problem 1).

Box		Orange	Apple	Cherry	Total
	Red	1	4	5	10
	Blue	7	2	1	10
	Green	3	4	3	10
Total		11	10	9	30

Denote Red: R, Blue: B, Green: G, Orange: O,
Apple: A, Cherry: C

(a) $P(O|R) = \frac{1}{10}$; $P(O|G) = \frac{3}{10}$; $P(O|B) = \frac{7}{10}$

(b) $P(C, R) = P(C|R) \times P(R)$
 $= \frac{5}{10} \times 0.2 = 0.1$

$$P(C, G) = P(C|G) \times P(G)$$
$$= \frac{3}{10} \times 0.5 = 0.15$$

$$P(C, B) = P(C|B) \times P(B)$$
$$= 0.1 \times 0.3 = 0.03$$

Problem 2) $P(\text{defective}) = 30\% \Rightarrow P(\sim D) = 70\%$

Test 1 result

$$TP_1 = P(\text{positive} | D) = 80\% \Rightarrow FN_1 = P(\text{negative} | D) = 20\%$$

$$TN_1 = P(\text{negative} | \sim D) = 80\% \Rightarrow FP_1 = P(\text{positive} | \sim D) = 20\%$$

Test 2 result:

$$TP_2 = P(\text{positive} | D) = 90\% \Rightarrow FN_2 = P(\text{negative} | D) = 10\%$$

$$TN_2 = P(\text{negative} | \sim D) = 70\% \Rightarrow FP_2 = P(\text{positive} | \sim D) = 30\%$$

(a) Base on test 1 result:

$$P(D | \text{positive}) = \frac{P(\text{positive} | D) \times P(D)}{P(\text{positive})} = \frac{0.8 \times 0.3}{P(\text{positive})}$$

$$\begin{aligned} P(\text{positive}) &= P(\text{positive}, D) + P(\text{positive}, \sim D) \\ &= P(\text{positive} | D) P(D) + P(\text{positive} | \sim D) P(\sim D) \\ &= 0.8 \times 0.3 + 0.2 \times 0.7 = 0.38 \end{aligned}$$

$$\Rightarrow P(D | \text{positive}) = \frac{0.24}{0.38} = 0.605$$

Similarly, base on test 2 result:

$$\Rightarrow P(\text{positive}) = 0.9 \times 0.3 + 0.2 \times 0.7 = 0.41$$

$$\Rightarrow P(D | \text{positive}) = \frac{0.9 \times 0.3}{0.41} = 0.658$$

$$(b) P(D | \text{negative}) = \frac{P(\text{negative} | D) P(D)}{P(\text{negative} | D) P(D) + P(\text{negative} | \sim D) P(\sim D)}$$

Base on test result 1:

$$P(D | \text{negative}) = \frac{0.2 \times 0.3}{0.2 \times 0.3 + 0.8 \times 0.7} = \frac{0.06}{0.62} = 0.096$$

Base on test result 2:

$$P(D | \text{negative}) = \frac{0.1 \times 0.3}{0.1 \times 0.3 + 0.7 \times 0.7} = \frac{0.03}{0.52} = 0.057$$

(c) Assume total Walmart can sell X cameras

$$\text{Defective} = 0.3X, \text{ non Defective} = 0.7X.$$

The total cost of both "miss detection" & "false alarm":

$$\begin{aligned} \text{Total cost} &= \$12 \times \text{"miss detection" units} + \$6 \times \text{"false alarm" units} \\ &= [12 \times P(D|\text{negative}) \times P(\text{negative}) + 6 \times P(\text{positive}|\sim D) \times P(\sim D)] \times X \end{aligned}$$

* Base on test 1 result:

$$\frac{\text{Total cost}}{X} = 12 \times 0.096 \times 0.62 + 6 \times 0.2 \times 0.7 = 1.554$$

$$\Rightarrow \text{Total cost}_{(1)} = \$1.554X$$

* Base on test 2 result:

$$\frac{\text{Total cost}}{X} = 12 \times 0.057 \times 0.52 + 6 \times 0.3 \times 0.7 = 1.615$$

$$\Rightarrow \text{Total cost}_{(2)} = \$1.615X$$

Base on total cost calculated of both test 1 & test 2, we found test 2 actually costs more money than test 1. Hence, test 1 should be the preferred method.

Problem 3)

Name	HealtyBun	Burgerlet	YourBurg	FatBurger	GreasyJoint
Fat (X)	19	31	35	39	43
Calories (Y)	410	580	570	640	660

We need to find out the slope (w_1) & the intercept (w_0) in the linear equation $Y = w_0 + w_1 X$

Applying the formulas:

$$w_1 = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$= \frac{5(19 \times 410 + 31 \times 580 + 35 \times 570 + 39 \times 640 + 43 \times 660) - \sum x \sum y}{5(19^2 + 31^2 + 35^2 + 39^2 + 43^2) - (19 + 31 + 35 + 39 + 43)^2}$$

$$= \frac{495300 - (19 + 31 + 35 + 39 + 43) \times (410 + 580 + 570 + 640 + 660)}{29585 - 27889}$$

$$= \frac{17680}{1696} = 10.4245283$$

$$w_0 = \frac{\sum y - w_1 \sum x}{n} = \frac{1119.1037139}{5} = 223.8207$$

Hence, coef = 10.42.

intercept = 223.82.

Problem 4) Please refer to hw1_150201133.py