Artificial Intelligence

Sem. Ganjil 2023/2024

10. Reasoning with Uncertainty

Lionov

Overview

- Conditional Probability
- Bayes' Rule
- Bayesian Network

Uncertainty

Uncertainty: “a typical feature” of real world problems for the agent
  cause: partial observability, non-determinism, and adversaries

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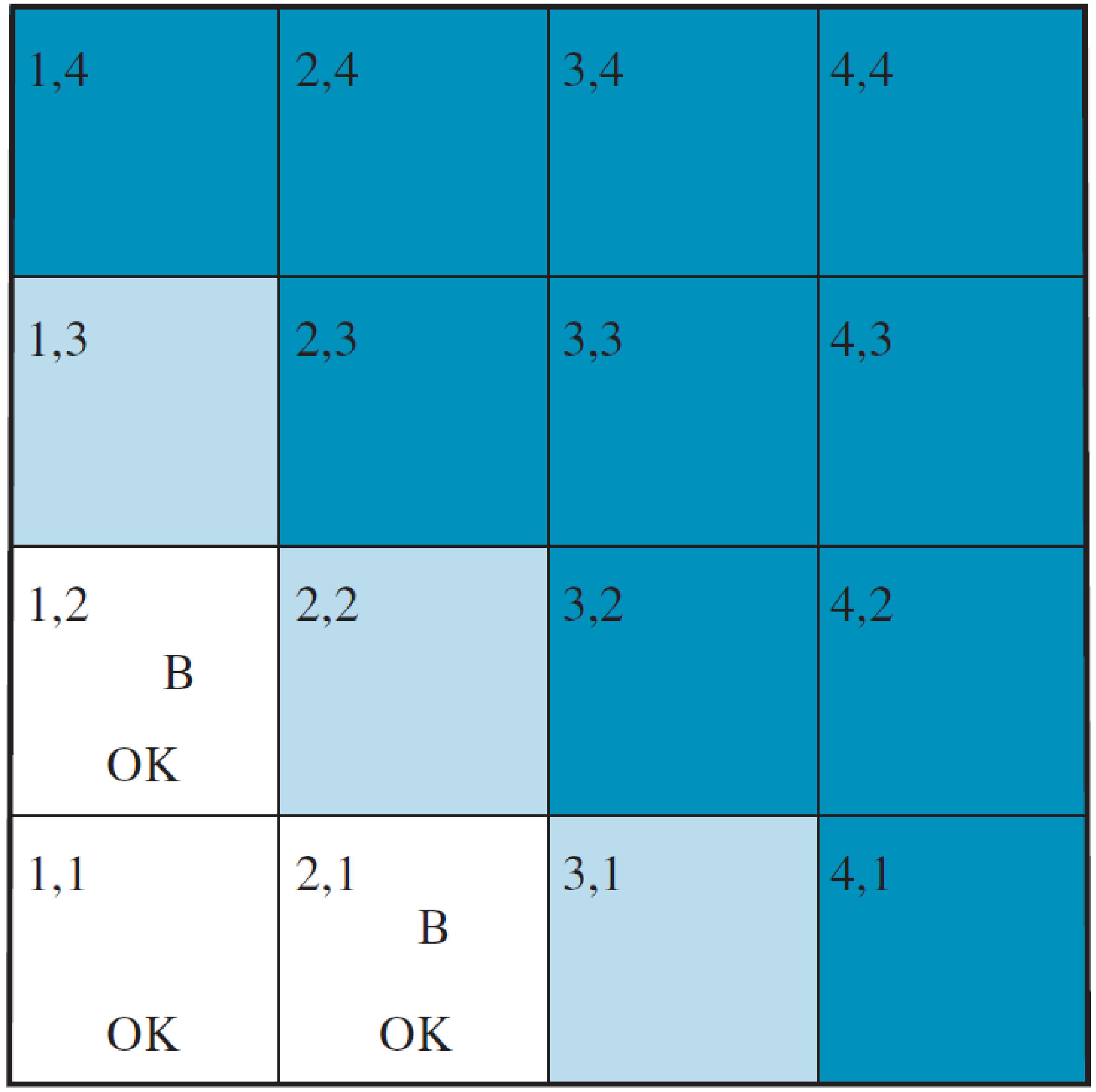

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Decision Theory = Probability Theory + Utility Theory

Decision-Theory Agent

```
function DT-AGENT(percept) returns an action
   persistent: belief_state, probabilistic beliefs about the current state of the world
               action, the agent's action

   update belief_state based on action and percept
   calculate outcome probabilities for actions,
      given action descriptions and current belief_state
   select action with highest expected utility
      given probabilities of outcomes and utility information
   return action
```

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Product Rule: $P(a \land b) = P(a \mid b)P(b)$

$$P(a \mid b) = \frac{P(a \land b)}{P(b)}$$, holds whenever $P(b) > 0$

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$P(Cavity = true) = P(cavity) = 0.4$, $P(Cavity = false) = P(\neg cavity) = 0.6$
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Probability Distribution: for a random variable X, $P(X)$ gives the values of $P(X = x_i)$ for each possible i. E.g.: $\mathcal{P}(Weather) = \langle 0.6, 0.1, 0.28, 0.01 \rangle$ for sun, rain, cloud, snow.

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Conditional Distribution: $\mathcal{P}(X|Y)$ gives the values of $P(X = x_i|Y = y_i)$ for each possible i, j pair

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Full Joint Distribution

Needs notation for distributions on multiple random variables

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E.g. given $Toothache$, $Cavity$, and $Catch$, the full joint distribution is a $2 \times 2 \times 2$ table

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

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For a set of random variables $X_1, X_2, \ldots X_n$, $\mathcal{P}(X_1, X_2, \ldots X_n)$ must obey:

- $\mathcal{P}(X_1, X_2, \ldots X_n) > 0$
- $\sum_{(X_1, X_2, \ldots X_n)} \mathcal{P}(X_1, X_2, \ldots X_n) = 1$

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Probabilistic Inference

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Computation of posterior probabilities given observed evidence

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Marginalization: summing out, combined by adding and remove other variables

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E.g. $\mathcal{P}(cavity) = \langle 0.2, 0.8 \rangle$

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Using the product rule, we obtain a rule called **conditioning**:

$$\mathcal{P}(Y) = \sum_z \mathcal{P}(Y|z)P(z)$$

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We mostly want conditional probabilities of some variables, given evidence about others

E.g.: compute the probability of a cavity, given evidence of a toothache
$P(cavity \mid toothache) = 0.6$ and $P(\neg cavity \mid toothache) = 0.4$

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Normalization: ensuring the distribution of $\mathcal{P}$ adds up to 1 (using α)

$$\mathcal{P}(Cavity \mid toothache) = \alpha\mathcal{P}(Cavity, toothache)$$
$$= \alpha\langle 0.12, 0.08\rangle = \langle 0.6, 0.4\rangle$$

Note that $P(toothache)$ is not even needed!

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General inference procedure:

- query involves a single variable X (e.g. Cavity)
- **E** is the list of evidence variables (e.g. Toothache)
- **e** is the list of observed values for **E**
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where summation is over all possible combinations of values of **Y**

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- **Y** the remaining unobserved variables (e.g. Catch)

$$P(X \mid e) = \alpha P(X, e) = \alpha \sum_y P(X, e, y)$$

where summation is over all possible combinations of values of **Y**

Problems: needs $O(2^n)$ space and $O(2^n)$ time to process

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Unfortunately, clean separation of entire sets of variables is quite rare because it based on knowledge of the domain. We need more subtle methods!!!

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The general form of Bayes' Rule with normalization is

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Diagnostic knowledge is often more fragile than causal knowledge!

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The decomposition of probabilistic domains through conditional independence is one of the most important developments in AI

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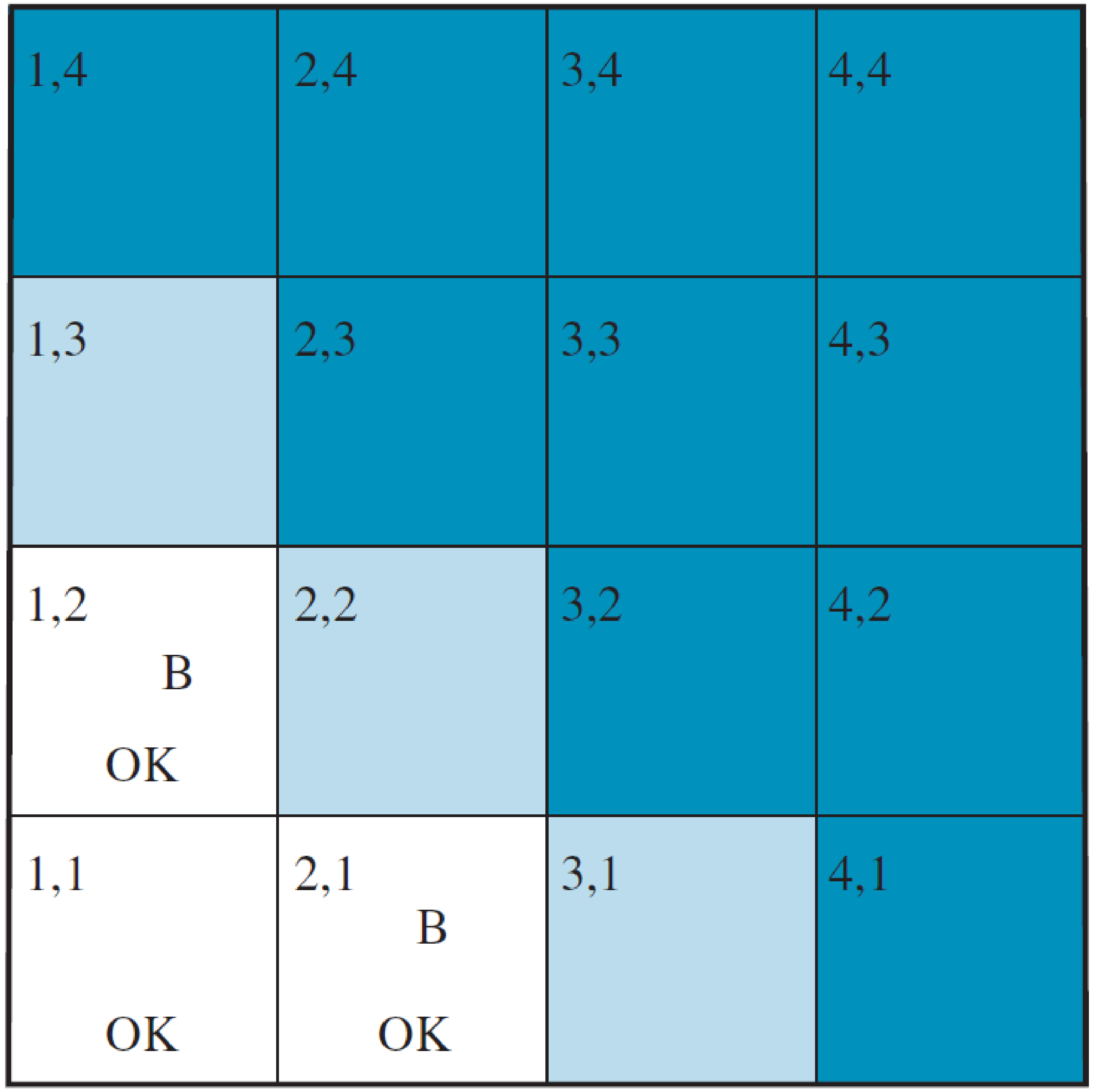
- Categorize a new document: check which keywords appear in the document and then apply above equation to obtain the posterior probability distribution

Probabilistic Reasoning in Wumpus World

The sensor of an agent gets “Breeze” in [1,2] and [2,1]



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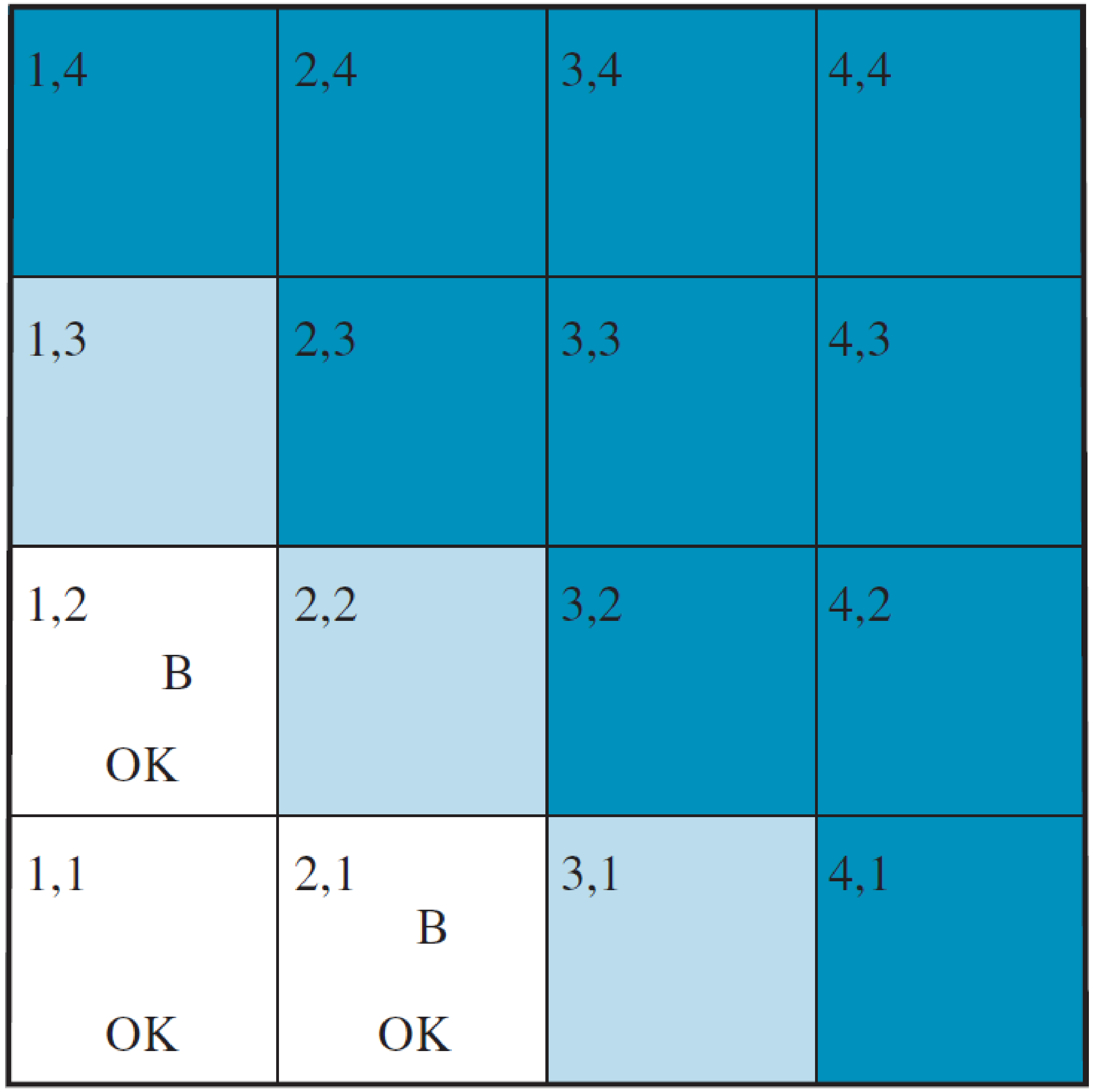

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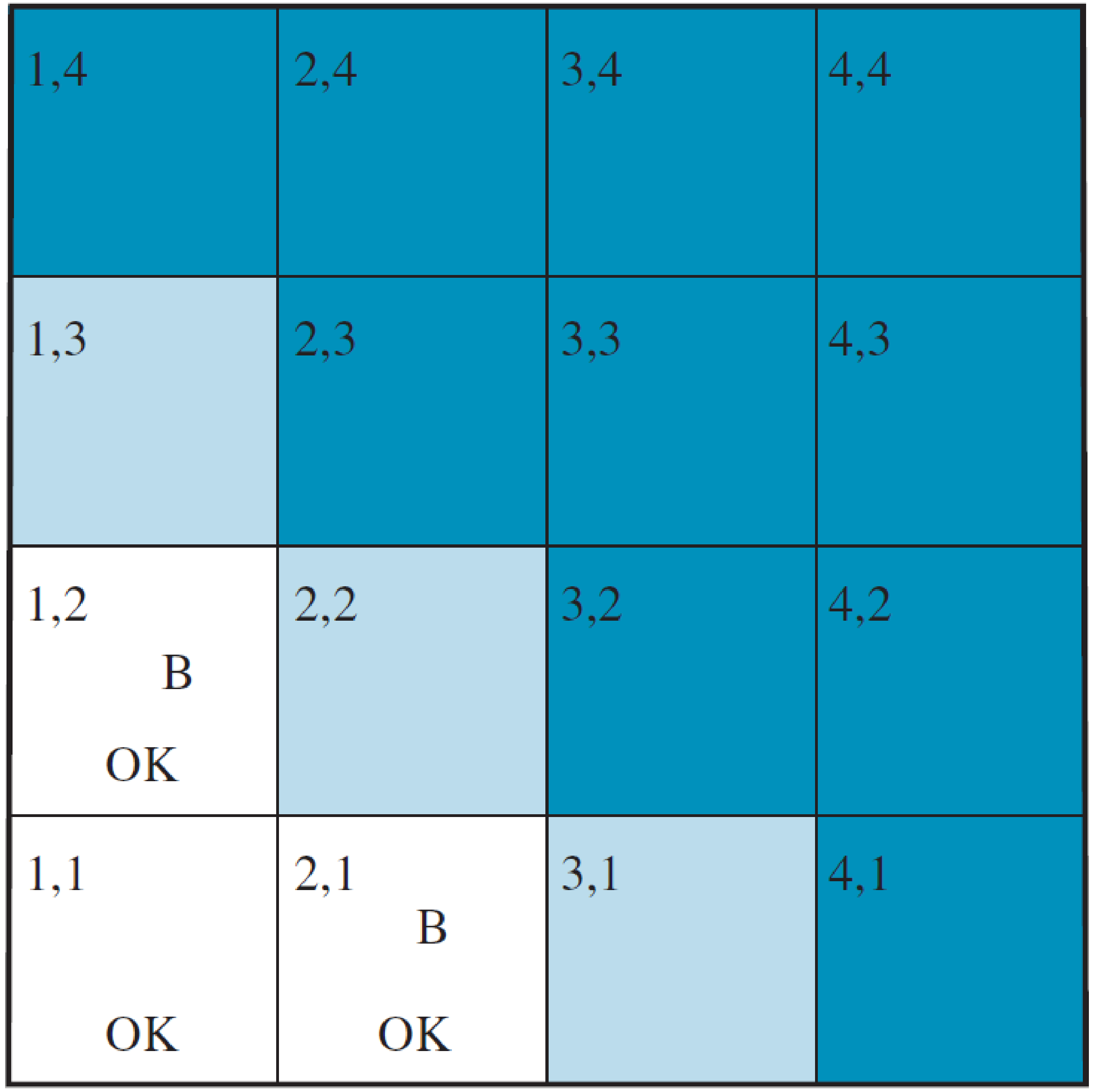

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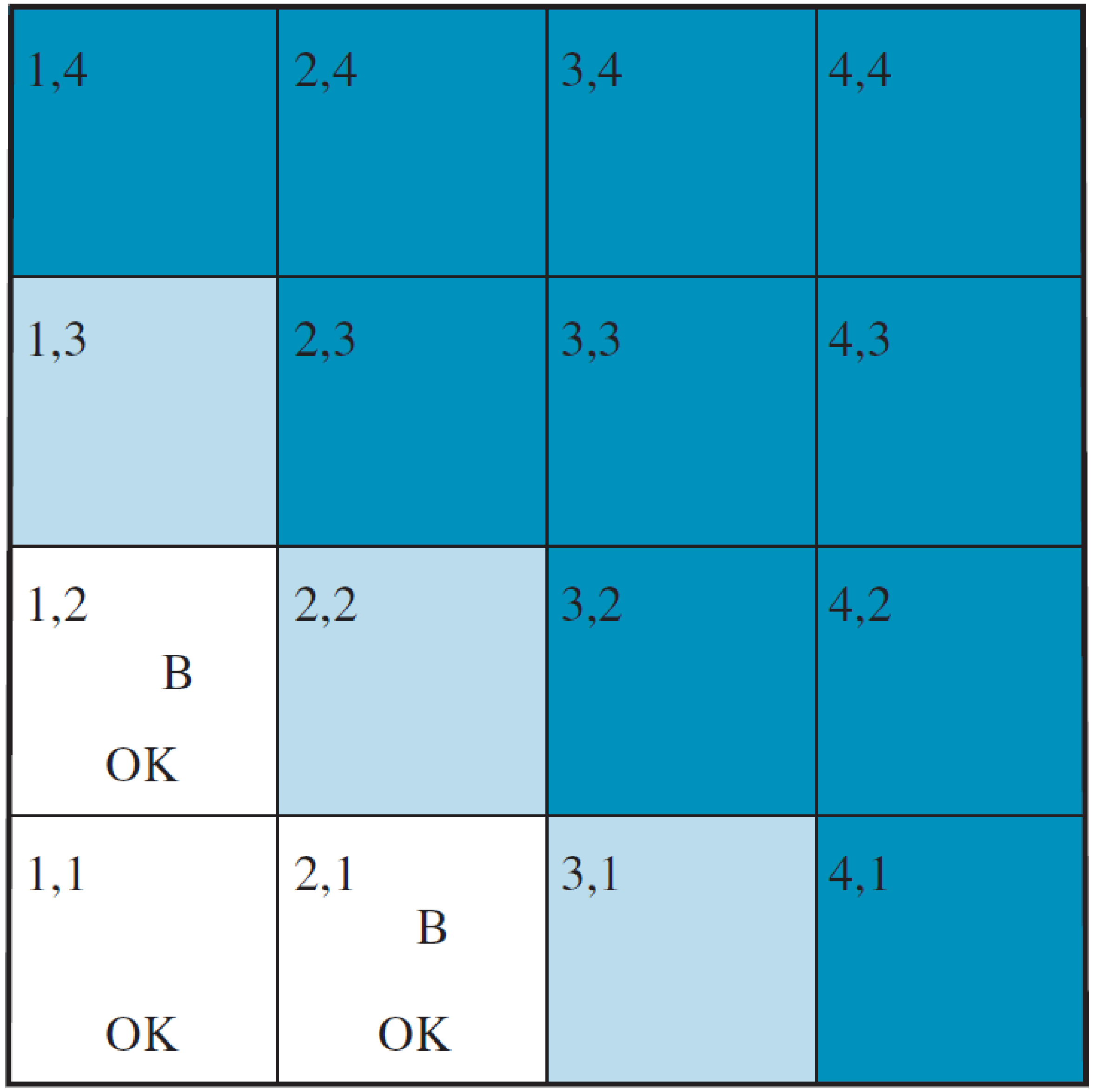

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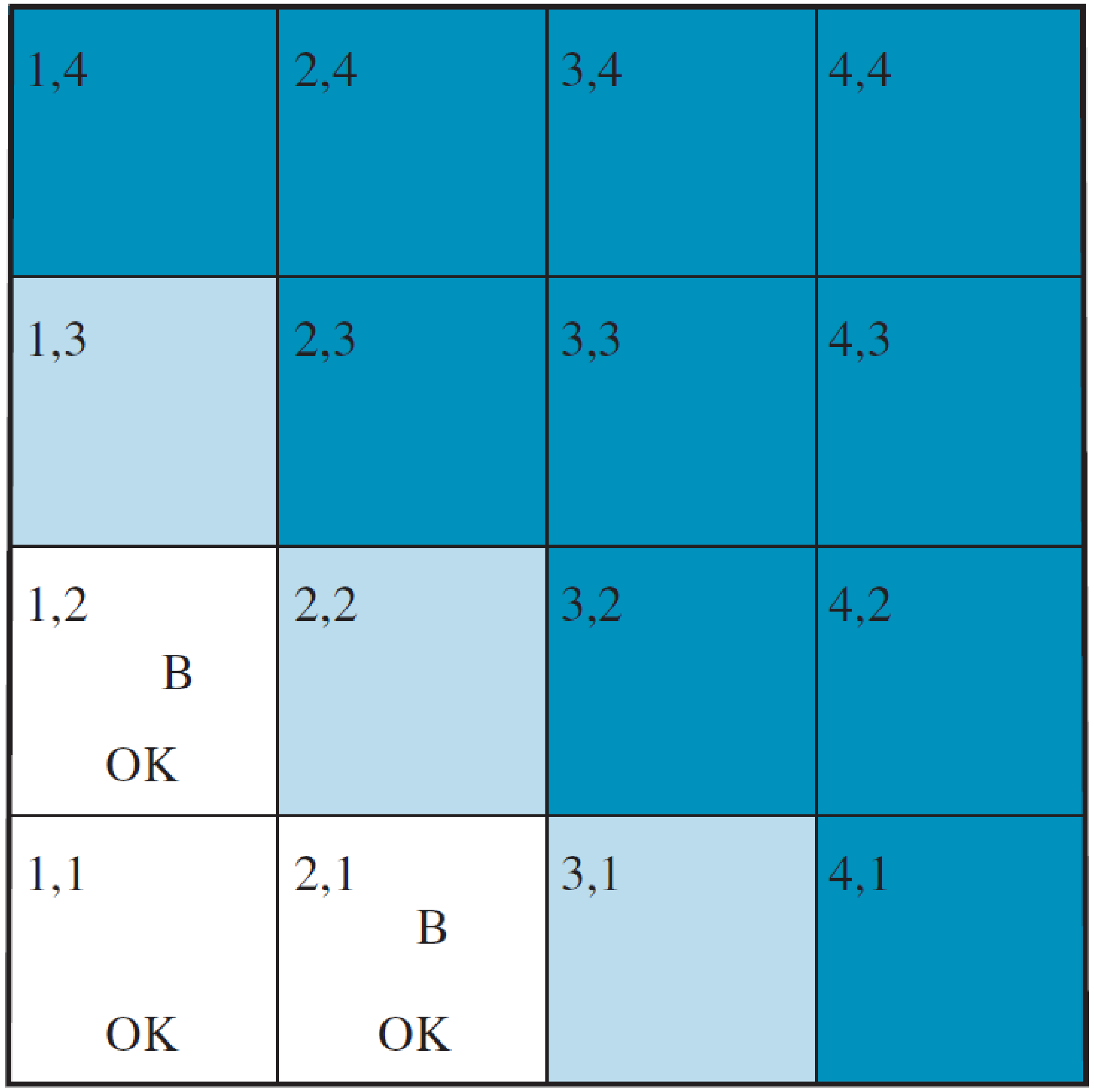

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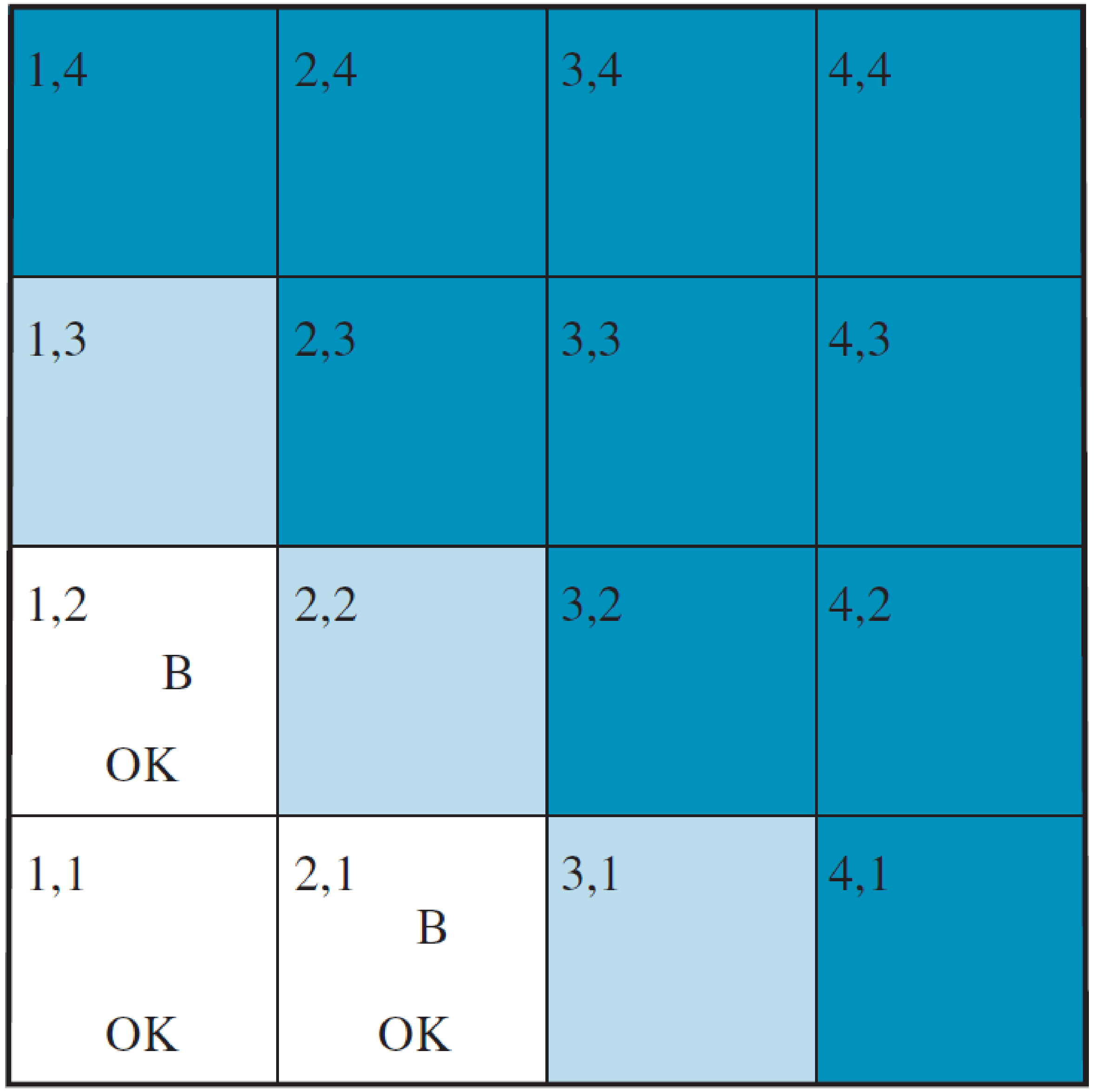

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- For each square, $P[i, j]$ is TRUE iff room [i,j] contains a pit
- For each square, $B[i, j]$ is TRUE iff room [i,j] is breezy and we know the value of B_{11}, B_{12}, and B_{21},

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$\mathcal{P}(P_{11}, \ldots, P_{44}) = \prod_{i,j=1,1}^{4,4} \mathcal{P}(P_{ij})$. For world with n pits, it is $0.2^n \times 0.8^{16-n}$.

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Probabilistic Reasoning in Wumpus World

Specify the full joint distribution and apply the product rule:

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There are 12 unknown room, hence the summation contains 2^{12} terms

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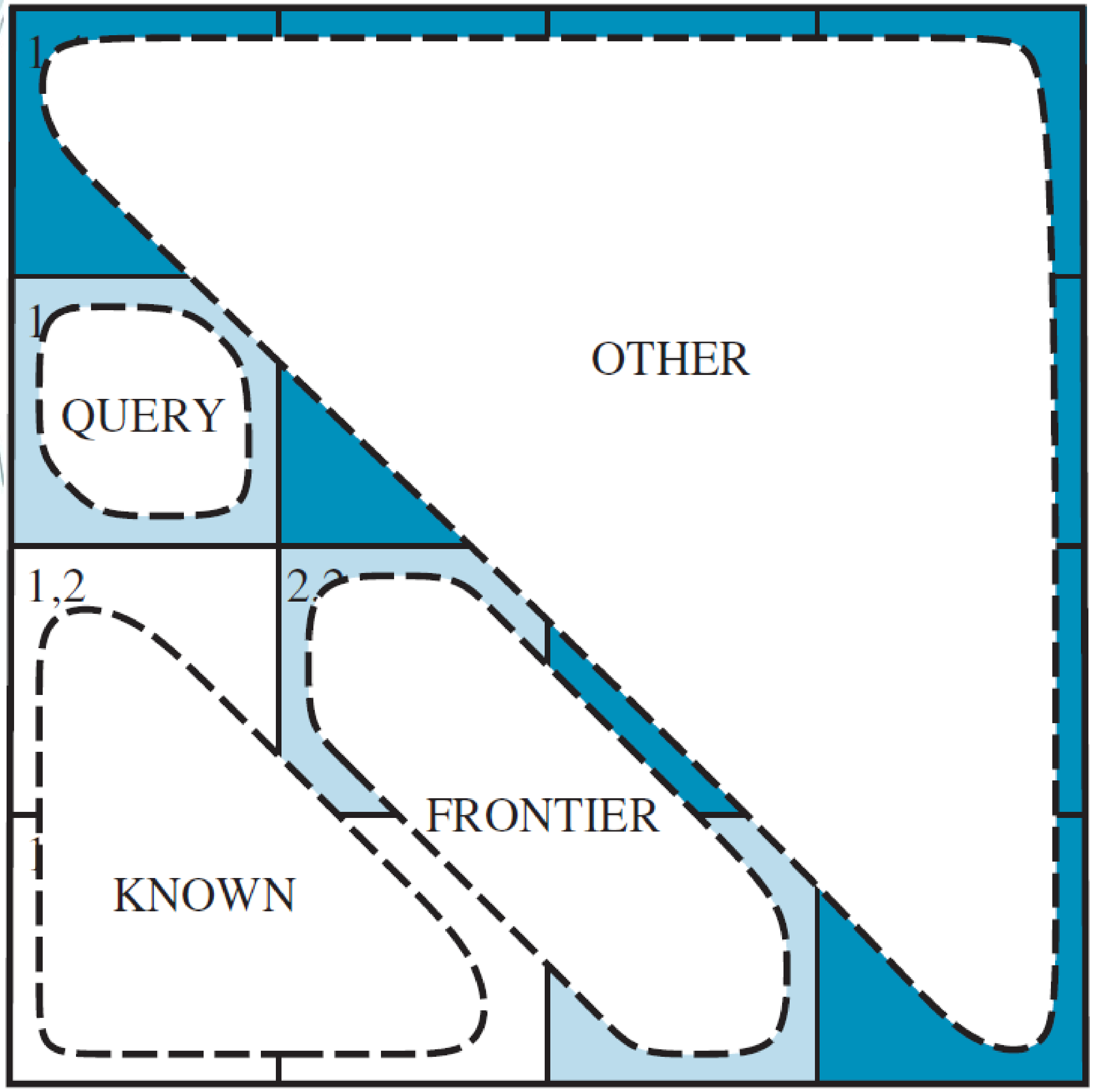

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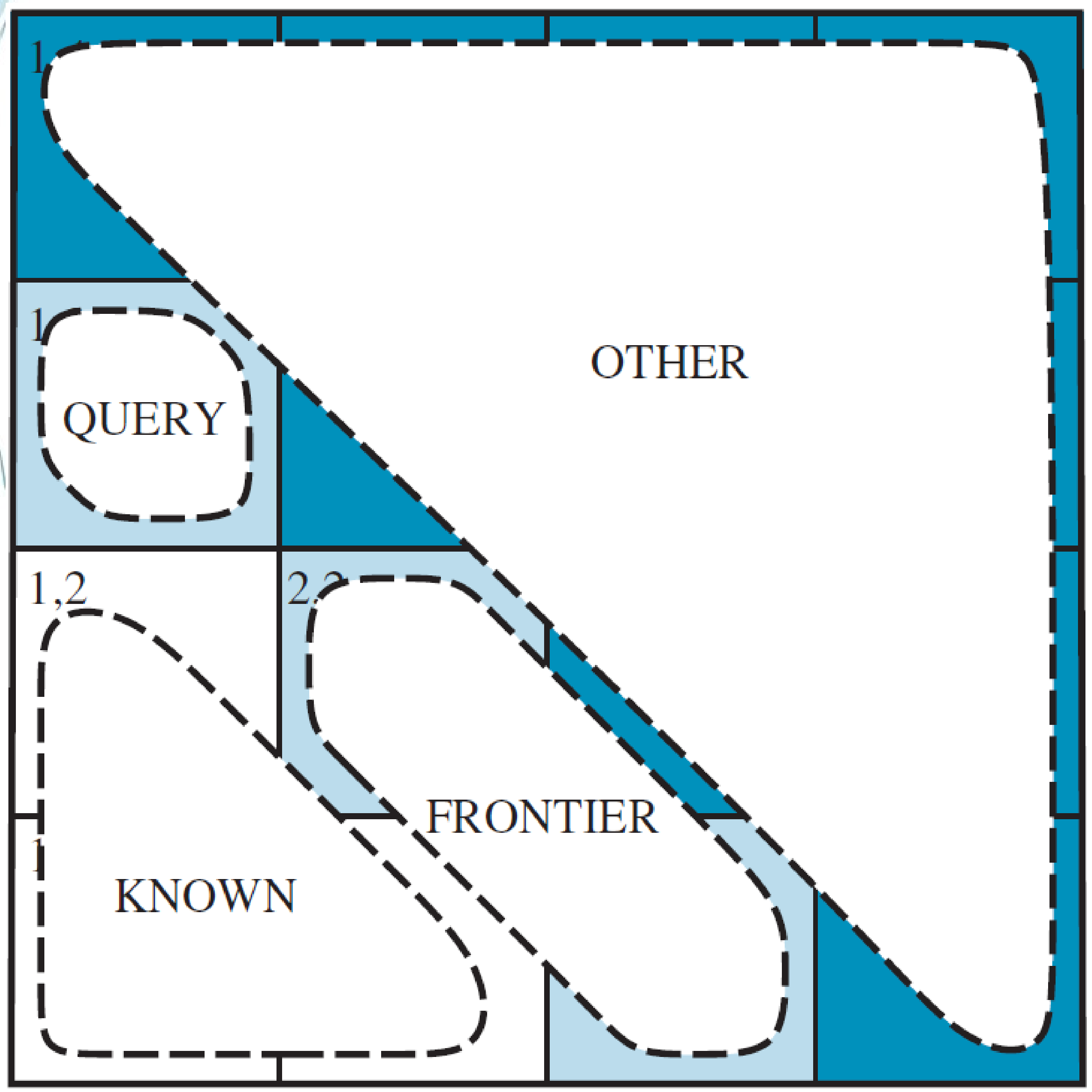

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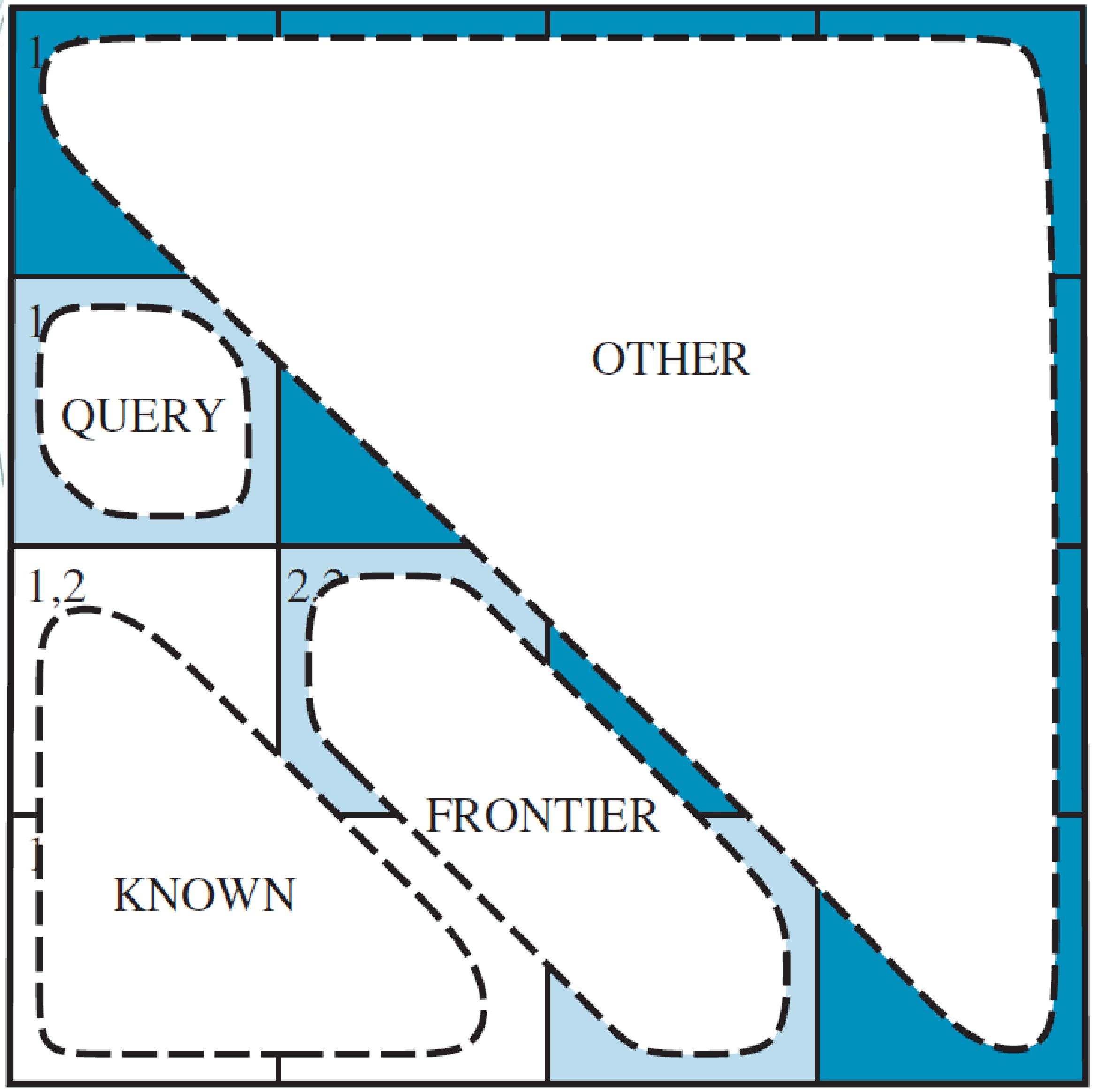

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Insight: the breezes are conditionally independent of the other variables, given the frontier and query variables

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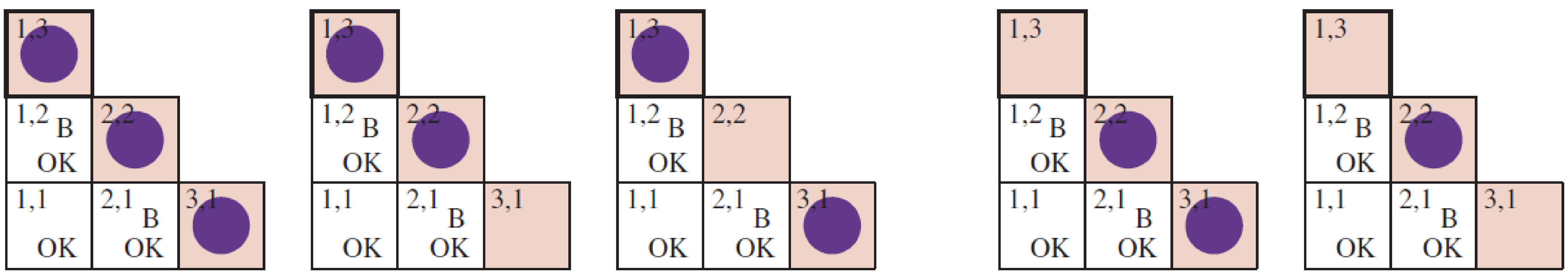

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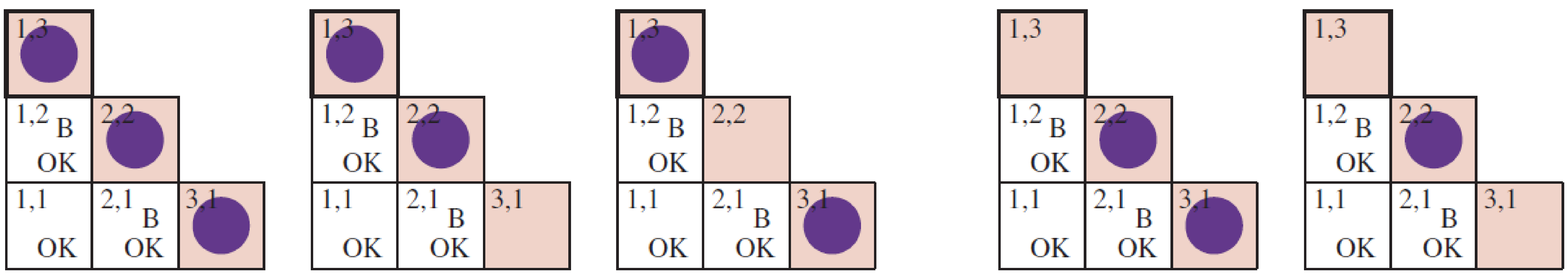

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$$P_{13} = P_{31} = \langle 0.31, 0.69 \rangle \qquad P_{22} = \langle 0.86, 0.14 \rangle$$

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