



# Artificial Intelligent

Sem. Ganjil 2023/2024

## 09. Logical Agent

Lionov



**INFORMATIKA**  
**UNPAR**

- Knowledge-Based Agent
- The Wumpus World
- Propositional Logic
- Method Checking
- Theorem Proving
- Intro. to First Order Logic

# Preliminaries



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- A sentence is an assertion about the world.

## Logical AI

*“The idea is that an agent can represent knowledge of its world, its goals and the current situation by sentences in logic and decide what to do by inferring that a certain action or course of action is appropriate to achieve its goals.”*

John McCarthy



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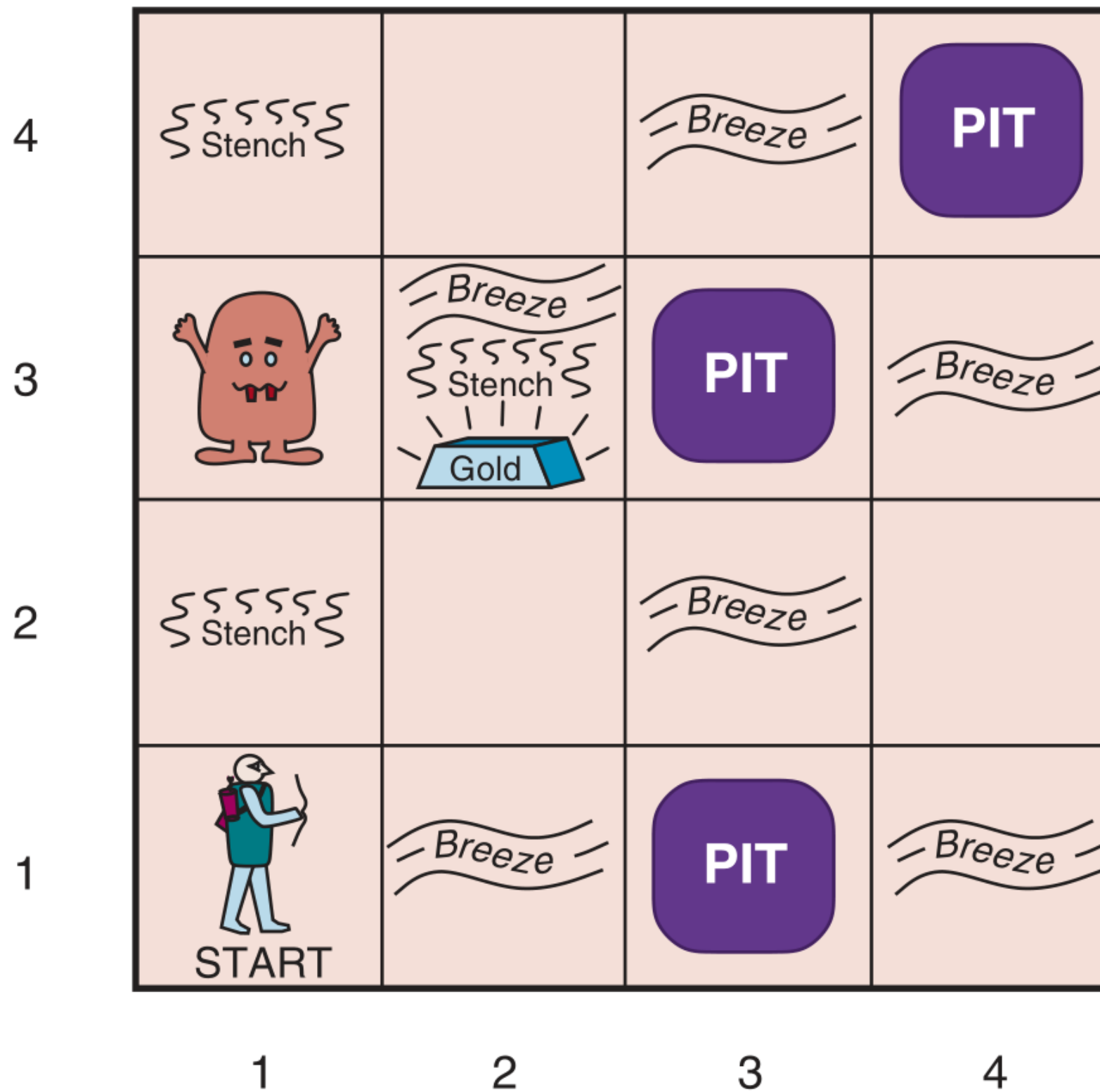
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    - query what is known (ASK)
- function** KB-AGENT(*percept*) **returns** an *action*  
**persistent:** *KB*, a knowledge base  
*t*, a counter, initially 0, indicating time

```
TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))  
action  $\leftarrow$  ASK(KB, MAKE-ACTION-QUERY(t))  
TELL(KB, MAKE-ACTION-SENTENCE(action, t))  
t  $\leftarrow$  t + 1  
return action
```



# The Wumpus World



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**Environment:** 4x4 grid rooms (walls surrounding the grid), Start in [1,1], facing east. The locations of the gold and the Wumpus are chosen randomly (not at [1,1]) with a uniform distribution. Each square (not at [1,1]) can be a pit with probability 0.2.



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**Sensors:** *Stench* and *Breeze*: adjacent to the wumpus and a pit, respectively. *Glitter*: square with gold. *Bump*: walks into a wall. *Scream*: the wumpus is killed. Given to the agent as a list of five symbols, ex.: [Stench, Breeze, None, None, None].

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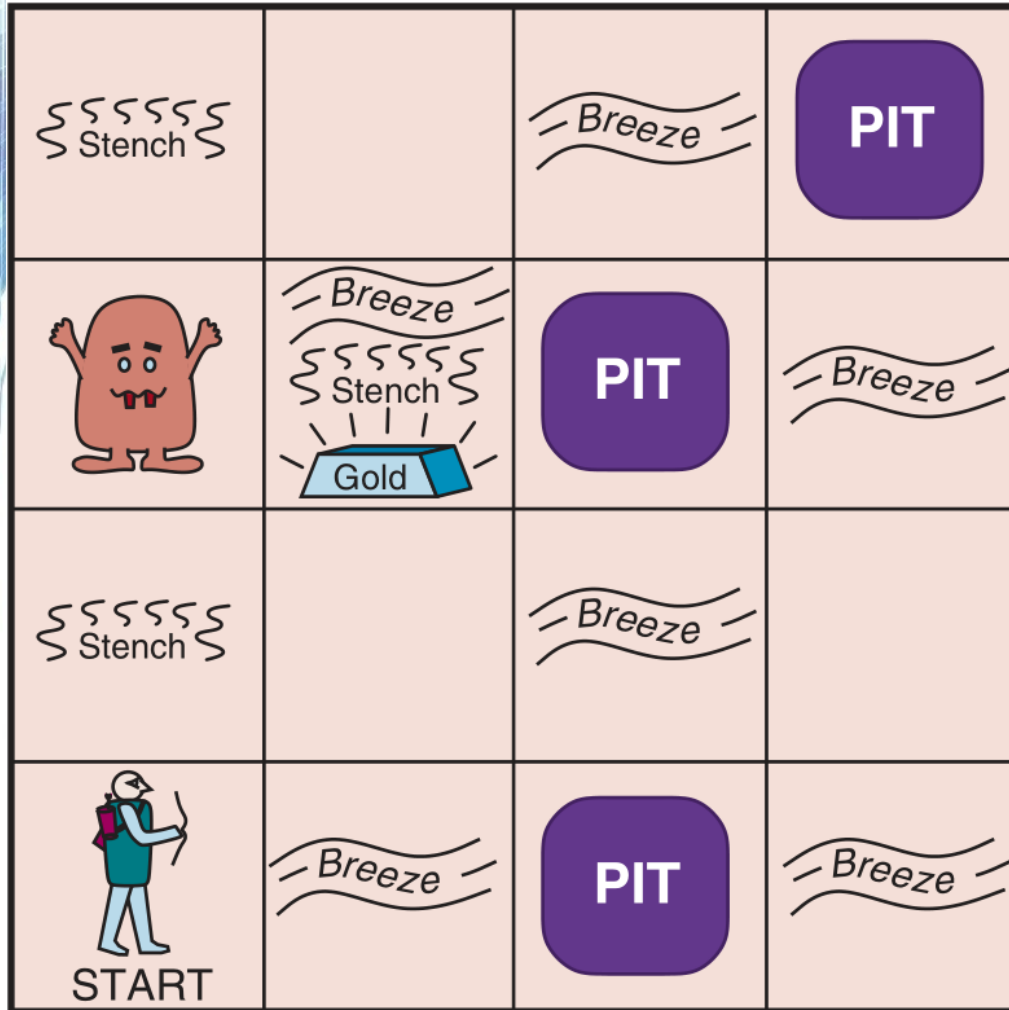
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**Properties:** Partially Observable, Static, Discrete, Single-agent, Deterministic, Sequential



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1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2	3,2	4,2
1,1 A OK	2,1 OK	3,1	4,1

**A** = Agent  
**B** = Breeze  
**G** = Glitter, Gold  
**OK** = Safe square  
**P** = Pit  
**S** = Stench  
**V** = Visited  
**W** = Wumpus

The initial situation: after percept [None, None, None, None, None]

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After moving to [2,1] and perceiving [None, Breeze, None, None, None]

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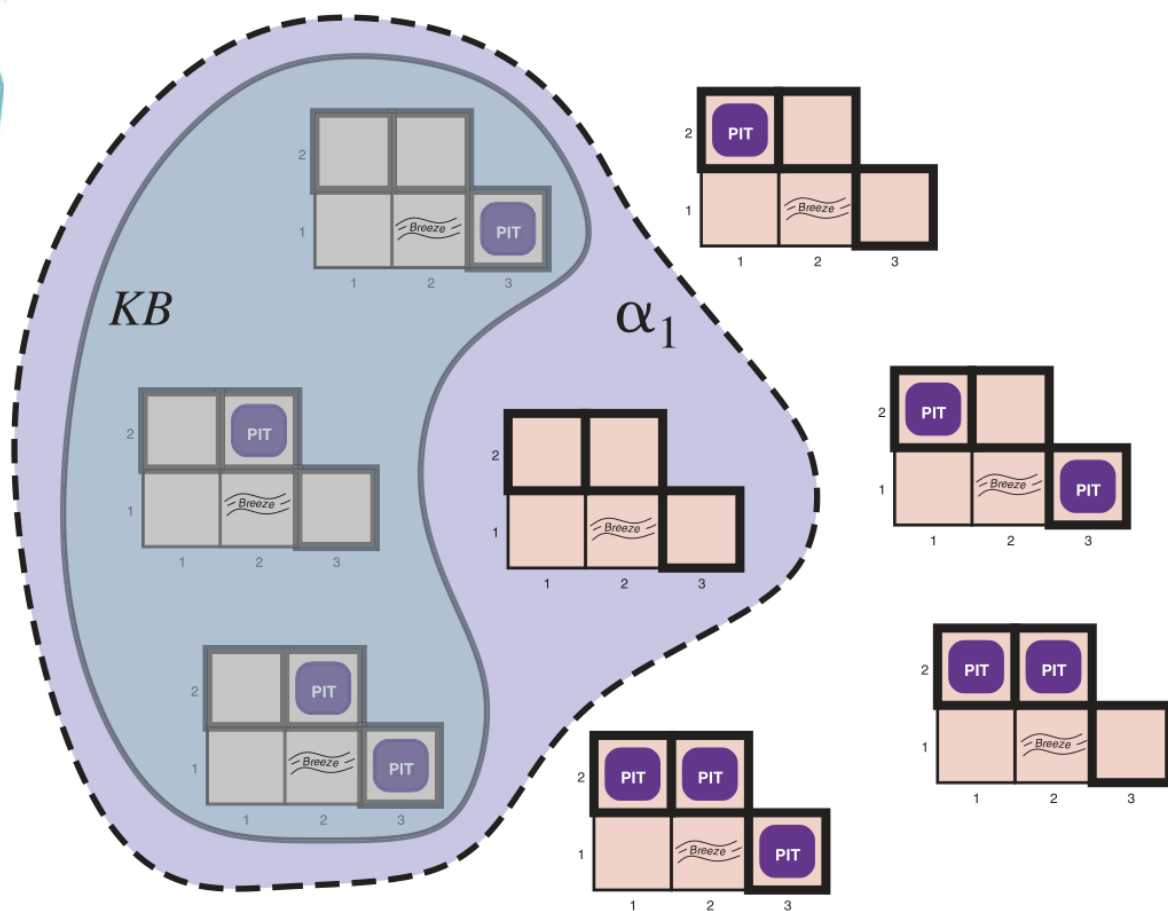
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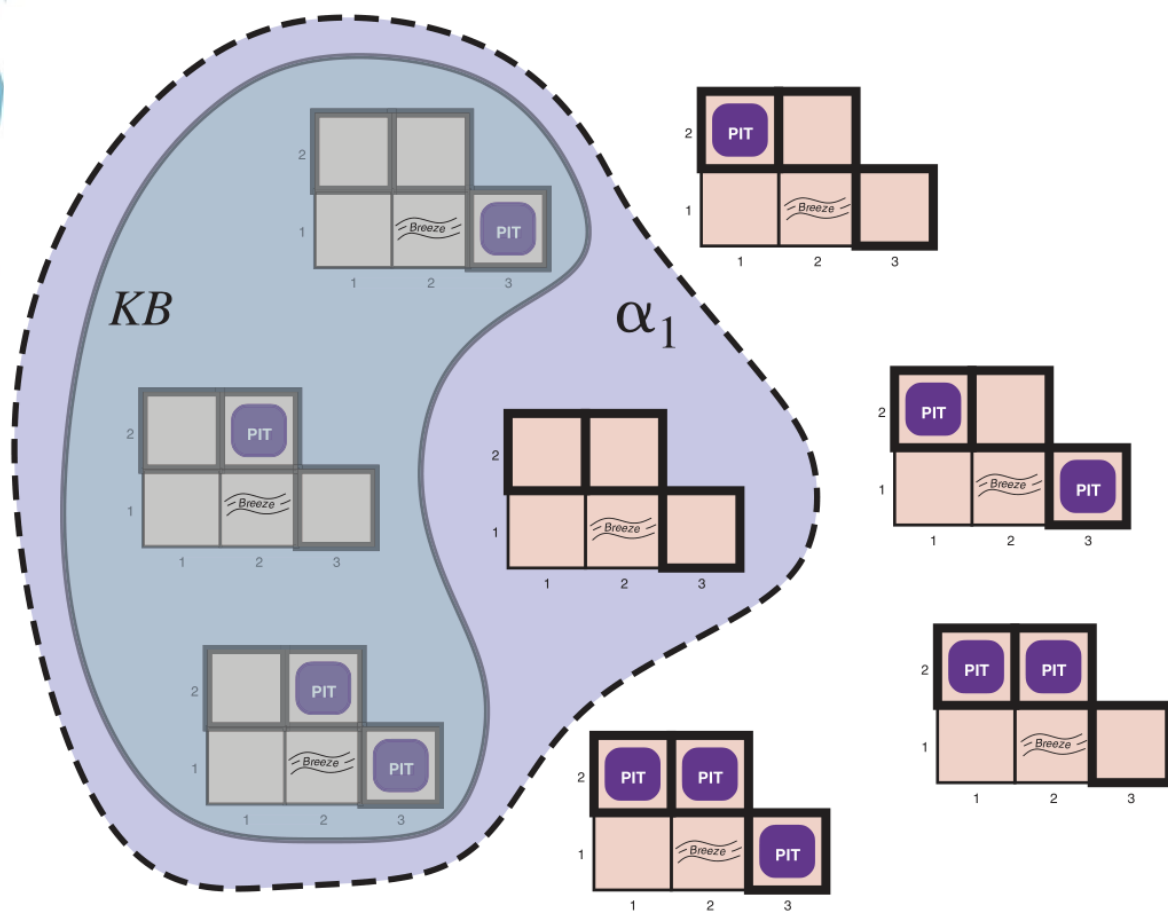
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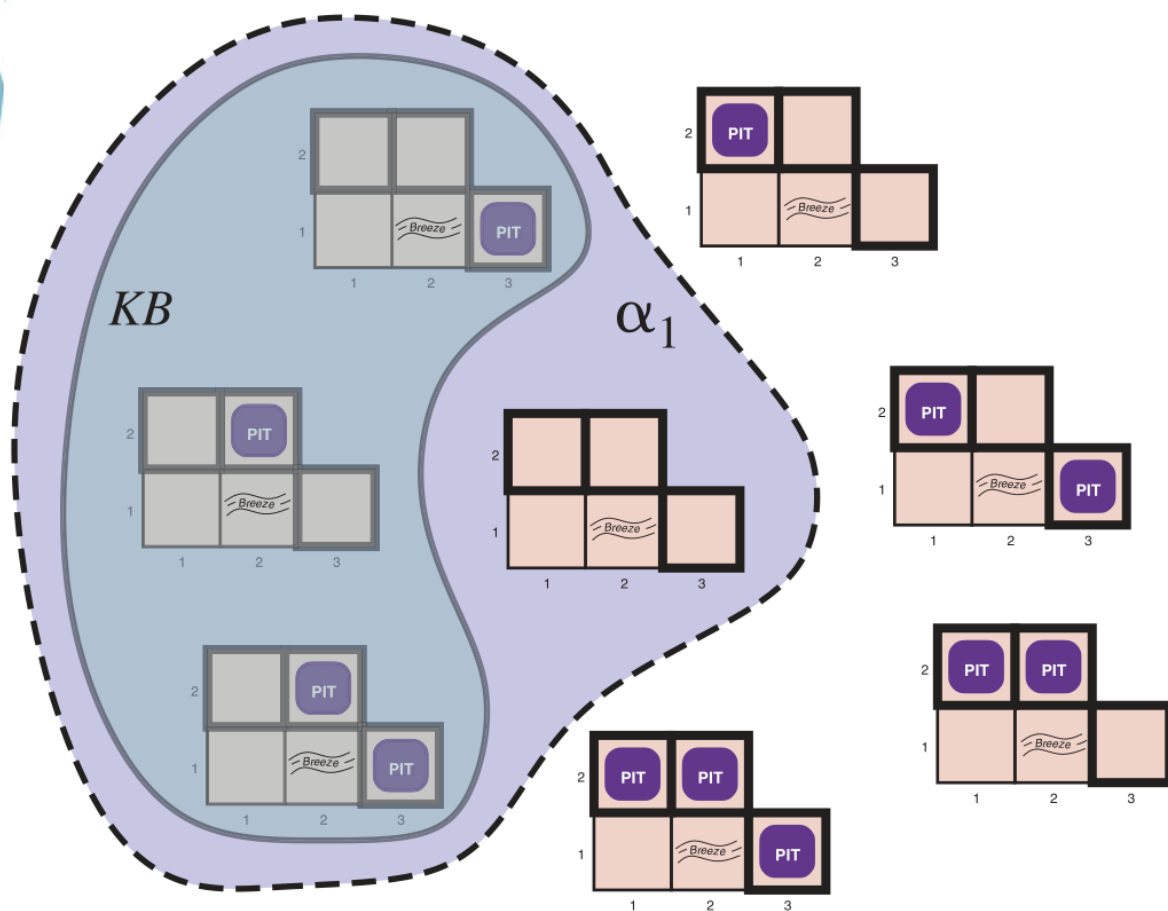


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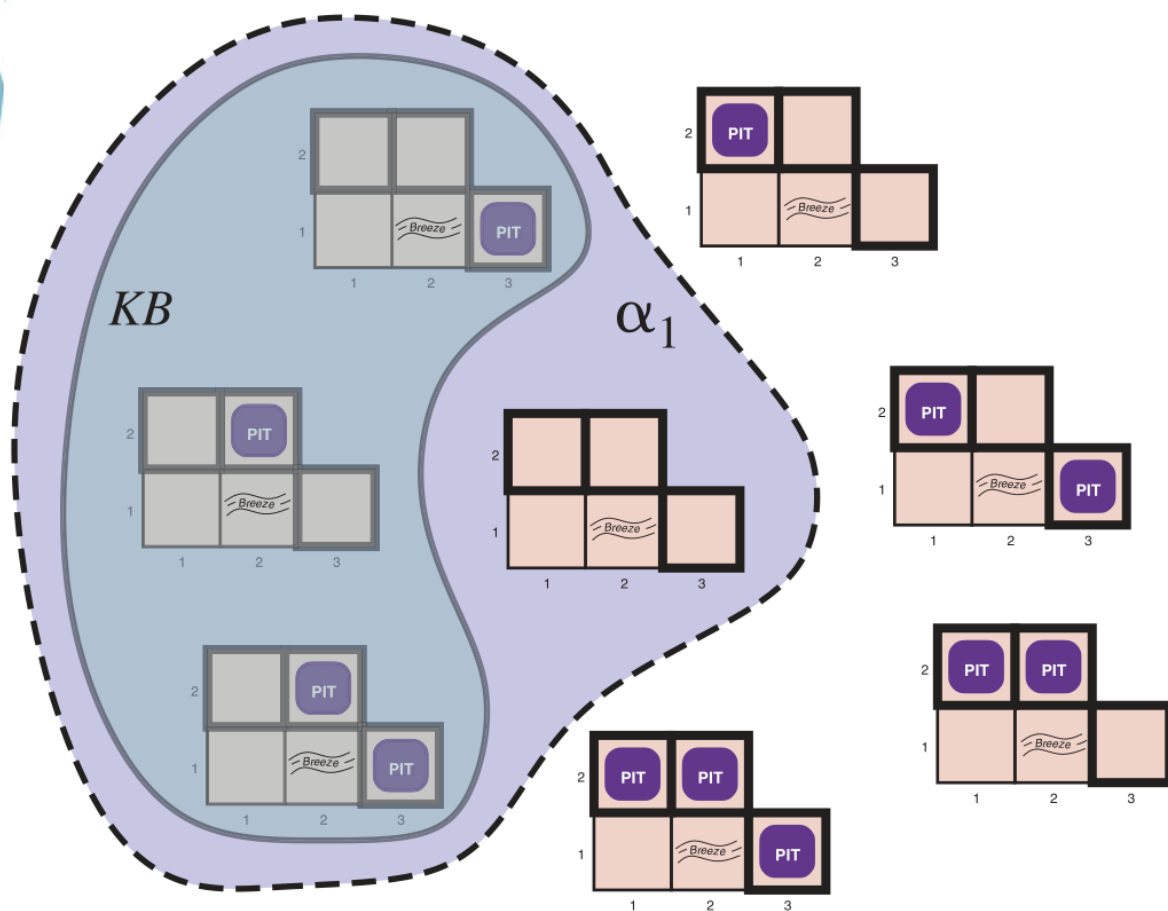


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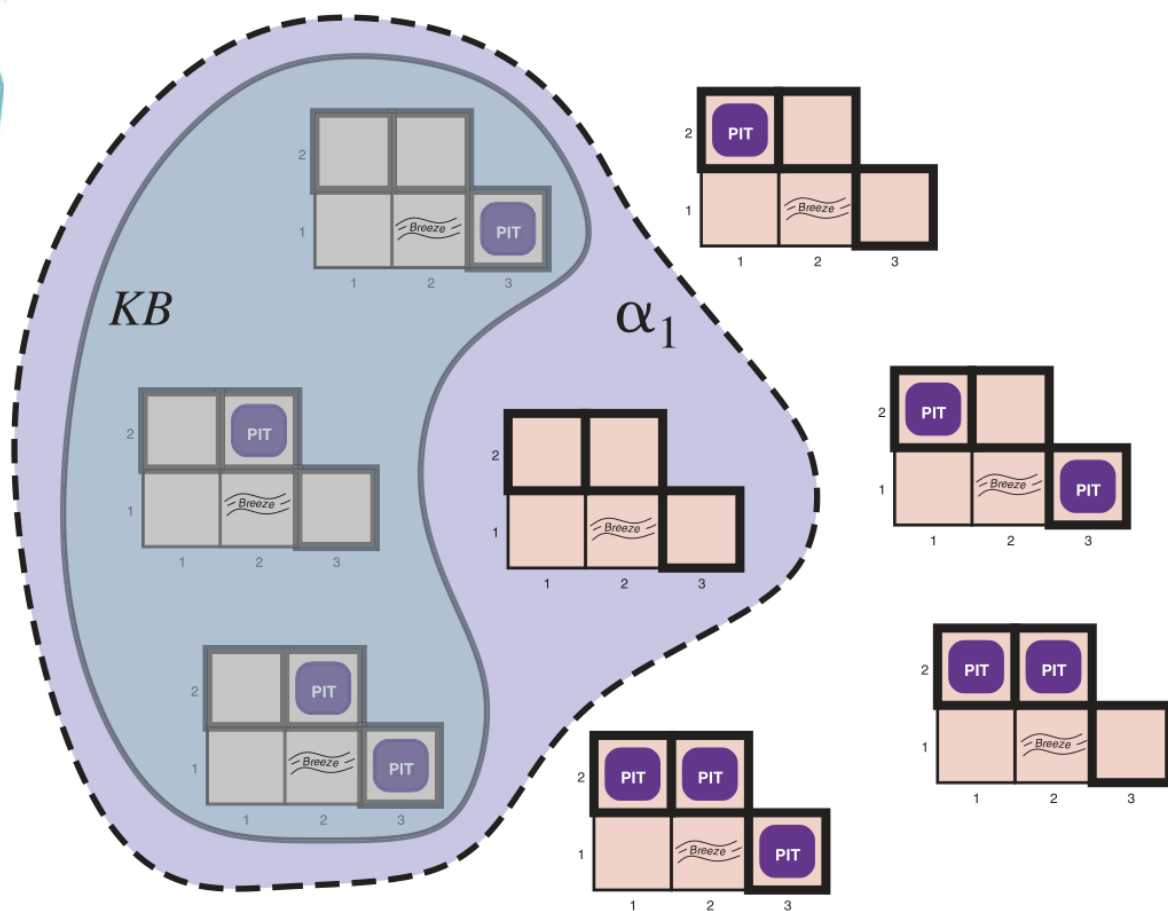
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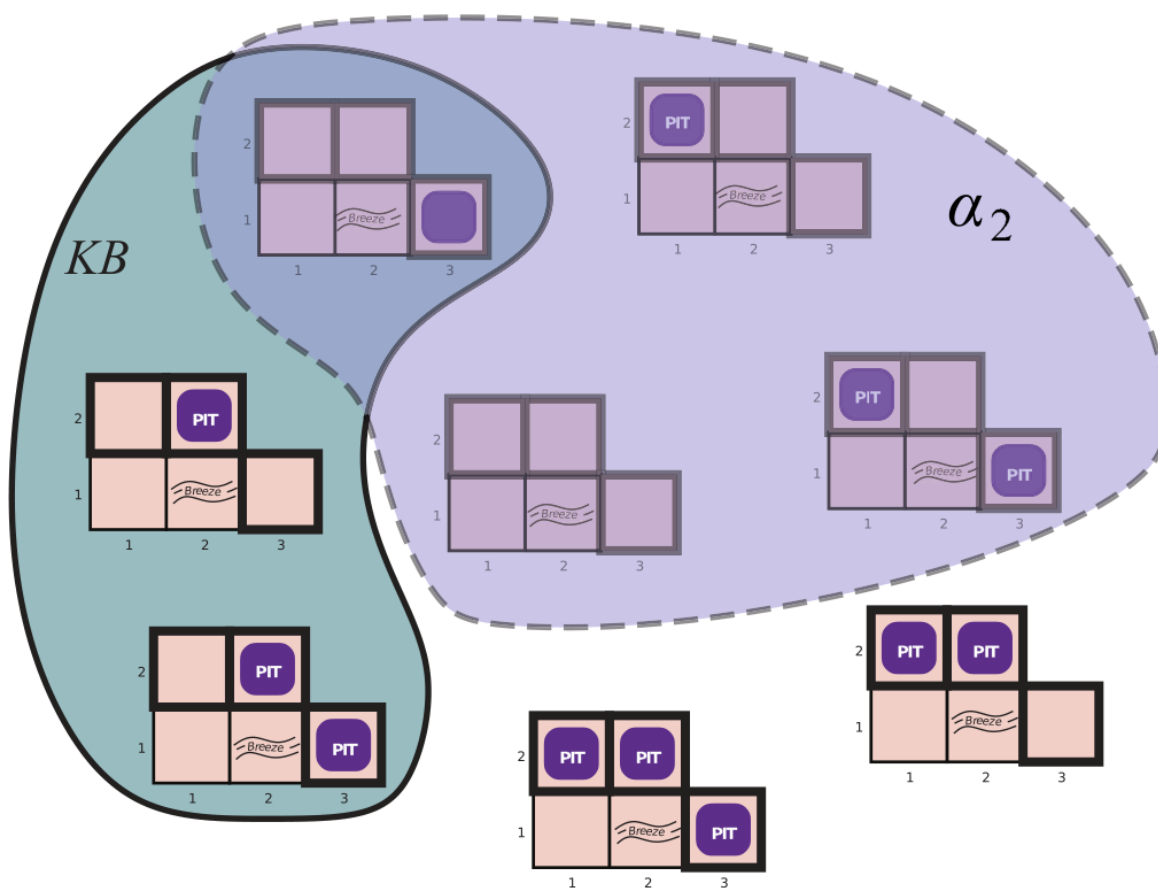
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- in every model in which  $\mathcal{KB}$  is true, so does  $\alpha_1$
- hence,  $\mathcal{KB} \models \alpha_1$

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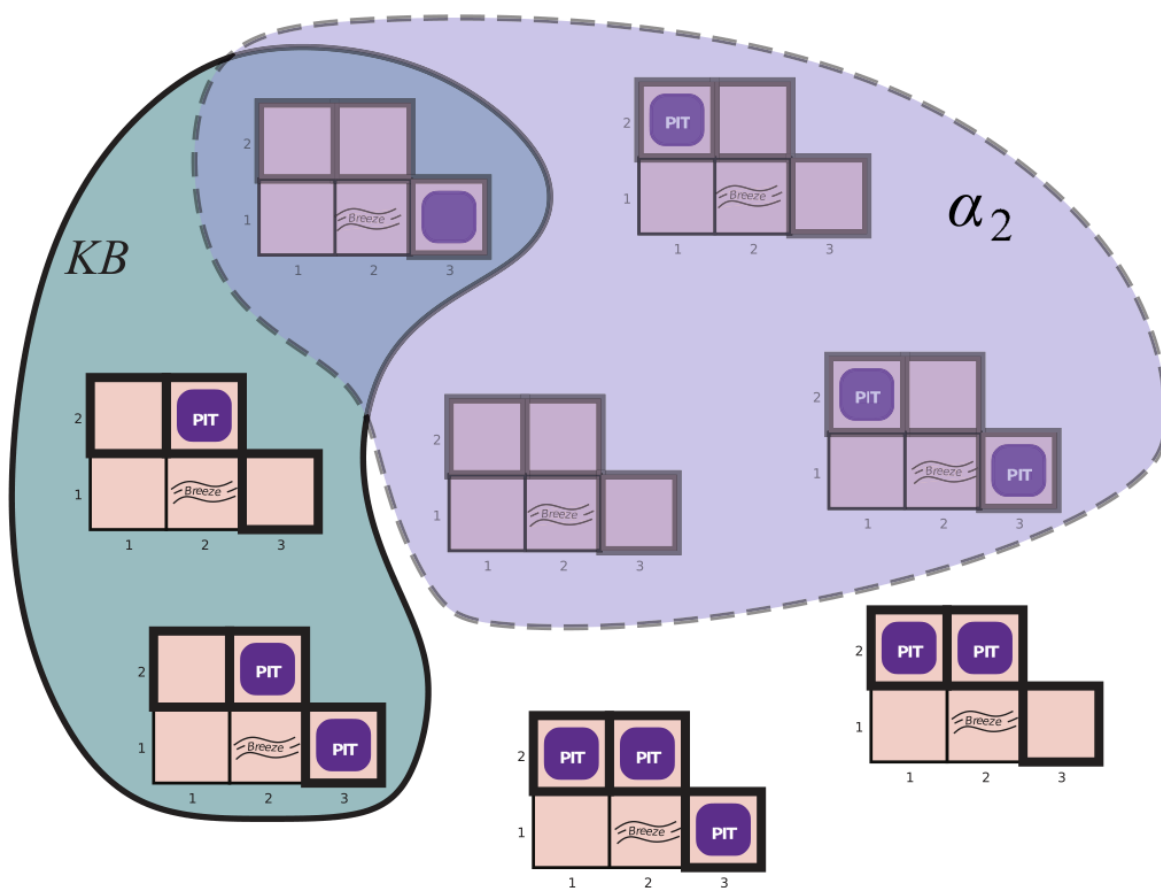
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- $\alpha_2$  = “There is no pit in [2,2]”





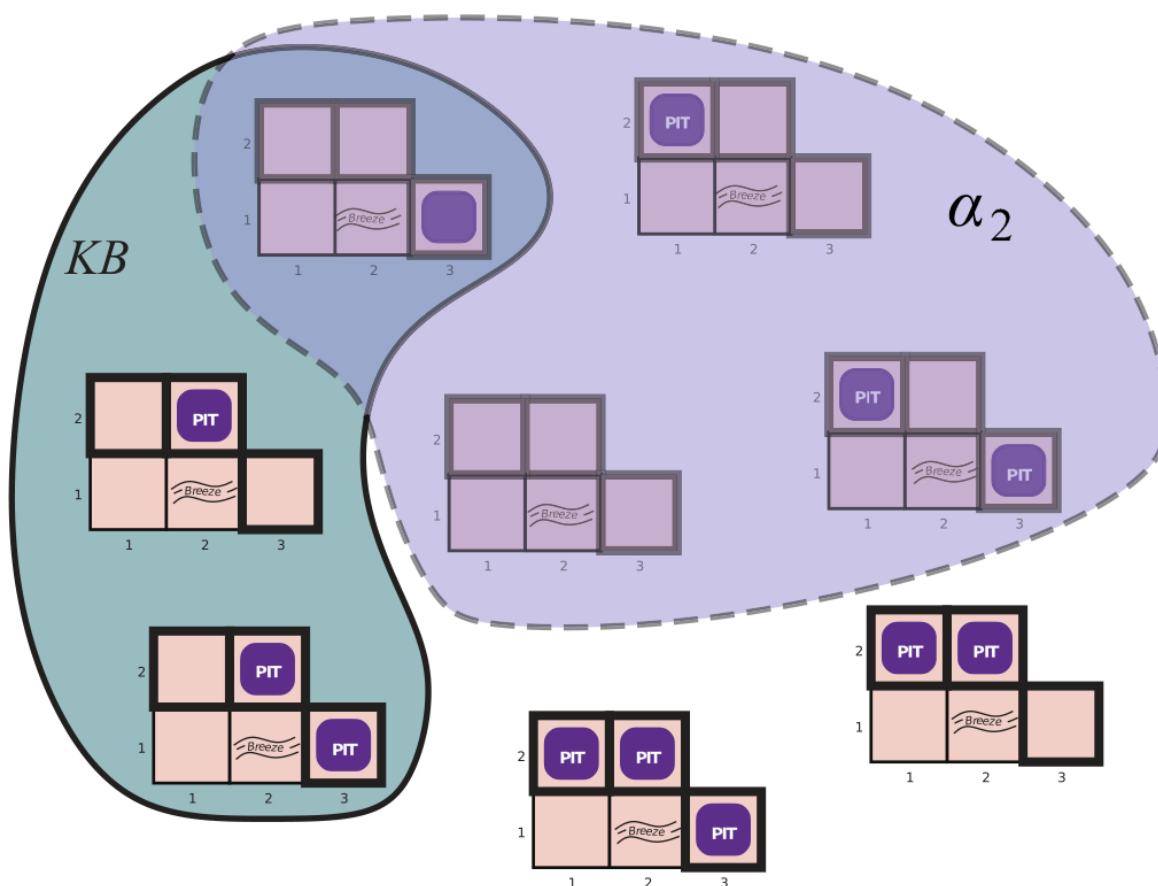
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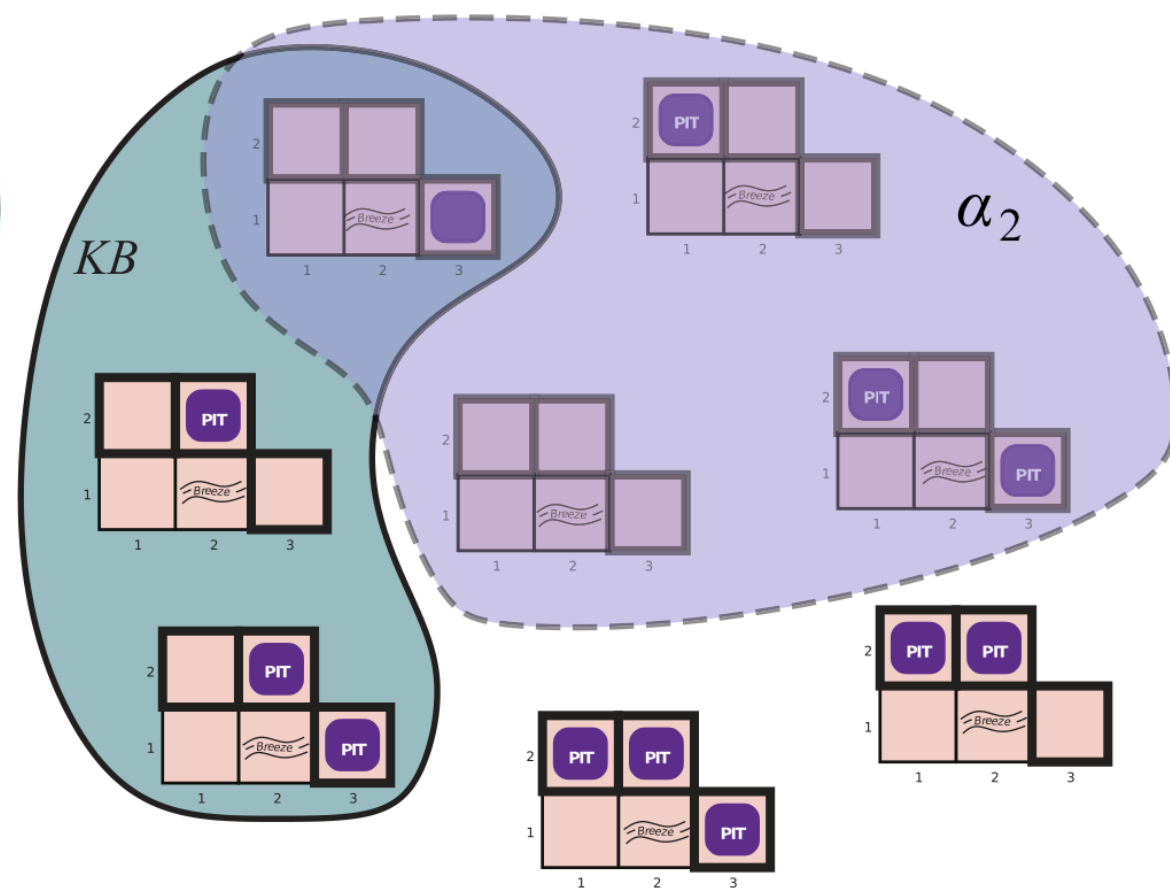


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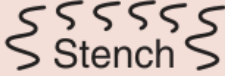

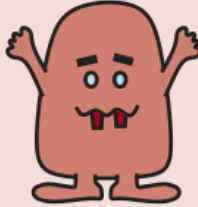
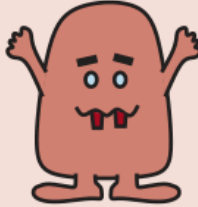
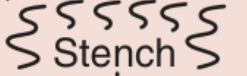
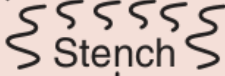

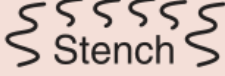




A **BNF (Backus-Naur Form)** grammar of sentences in propositional logic:

$$\begin{aligned} \text{Sentence} &\rightarrow \text{AtomicSentence} \mid \text{ComplexSentence} \\ \text{AtomicSentence} &\rightarrow \text{True} \mid \text{False} \mid P \mid Q \mid R \mid \dots \\ \text{ComplexSentence} &\rightarrow ( \text{Sentence} ) \\ &\mid \neg \text{Sentence} \\ &\mid \text{Sentence} \wedge \text{Sentence} \\ &\mid \text{Sentence} \vee \text{Sentence} \\ &\mid \text{Sentence} \Rightarrow \text{Sentence} \\ &\mid \text{Sentence} \Leftrightarrow \text{Sentence} \end{aligned}$$

OPERATOR PRECEDENCE :  $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

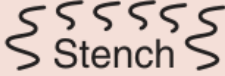


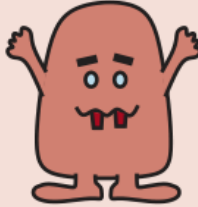

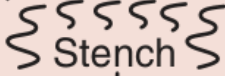



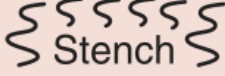







# The Wumpus world $\mathcal{KB}$

 Stench		 Breeze	<b>PIT</b>
	 Breeze  Stench  Gold	<b>PIT</b>	 Breeze
 Stench		 Breeze	
 START	 Breeze	<b>PIT</b>	 Breeze

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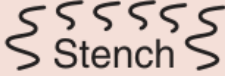


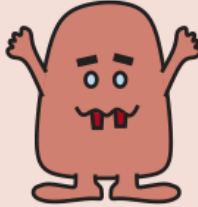

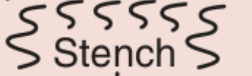









Symbols for each  $[x, y]$ :

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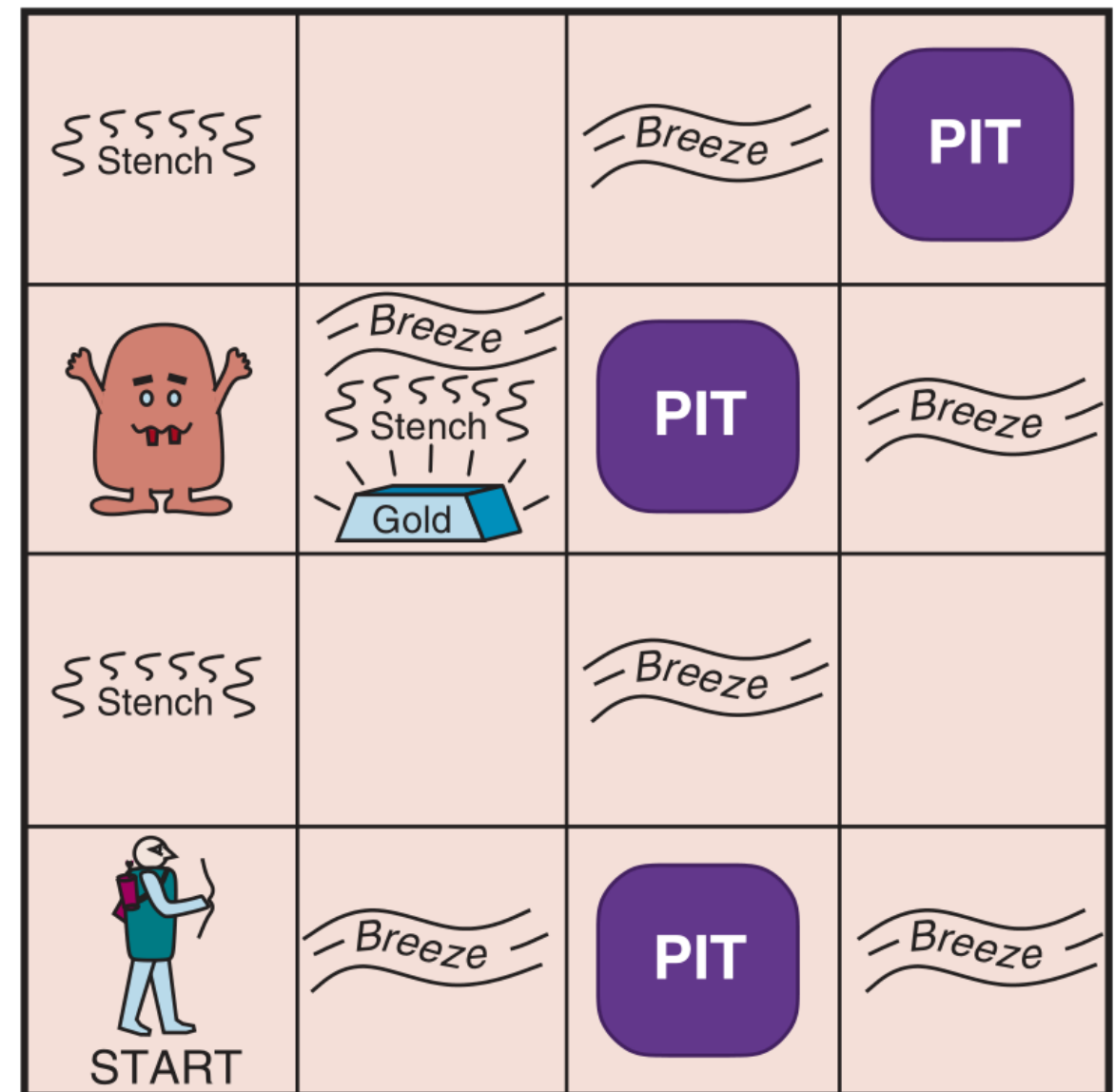
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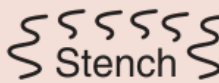


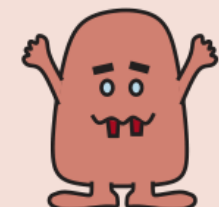

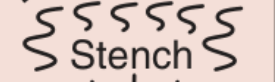



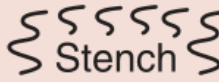









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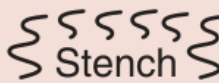


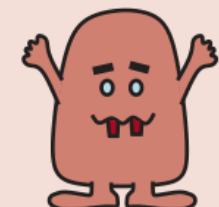

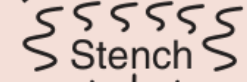



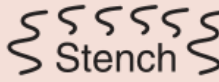





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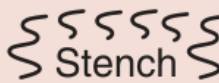


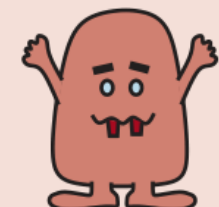
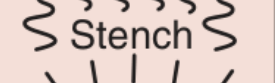
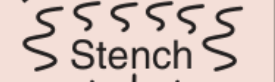



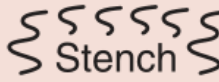





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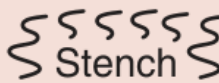


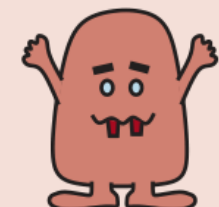

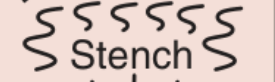



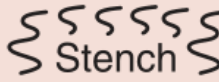





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 Stench		 Breeze	 PIT
	 Breeze  Stench  Gold	 PIT	 Breeze
 Stench		 Breeze	
 START	 Breeze	 PIT	 Breeze



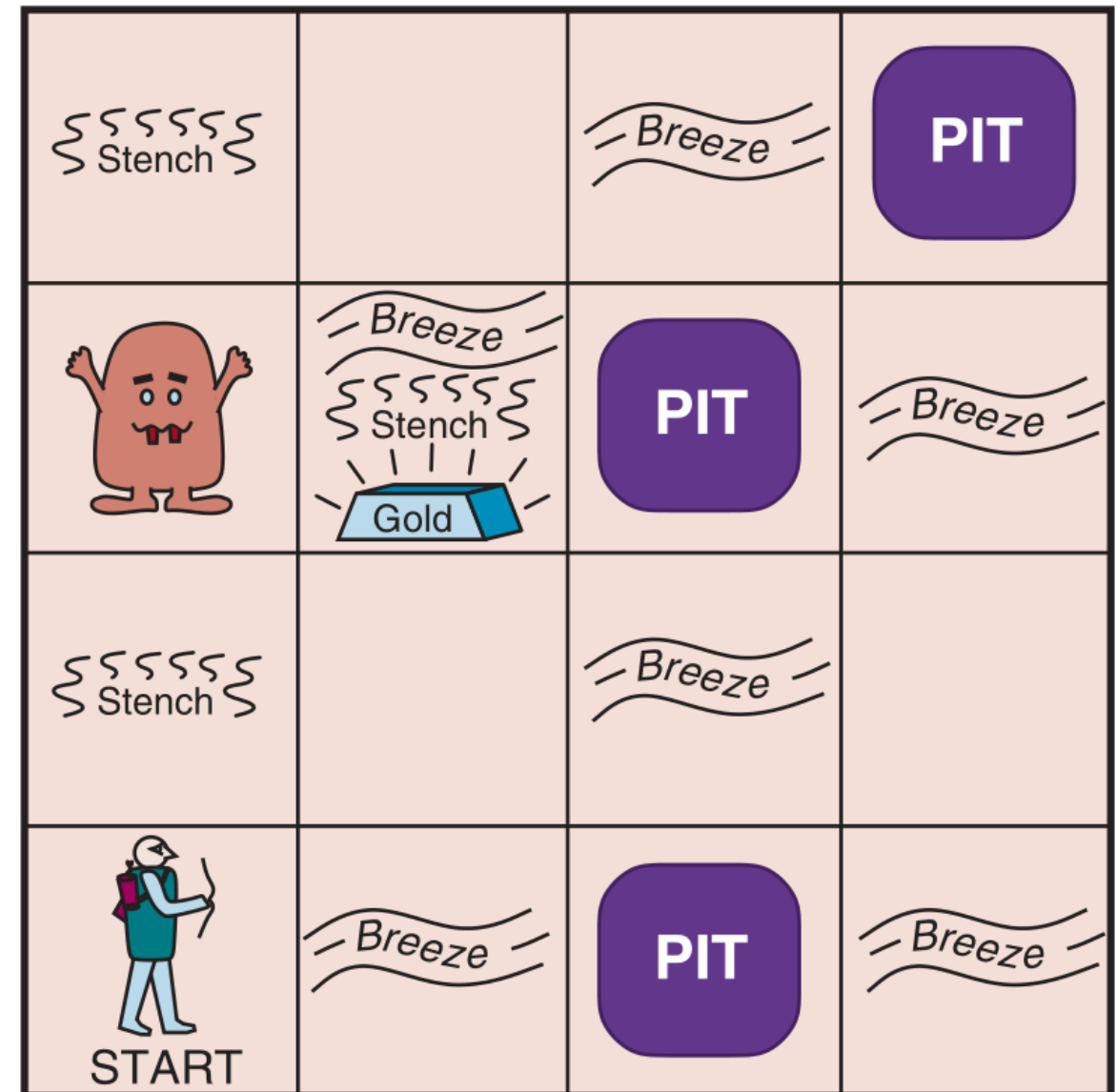
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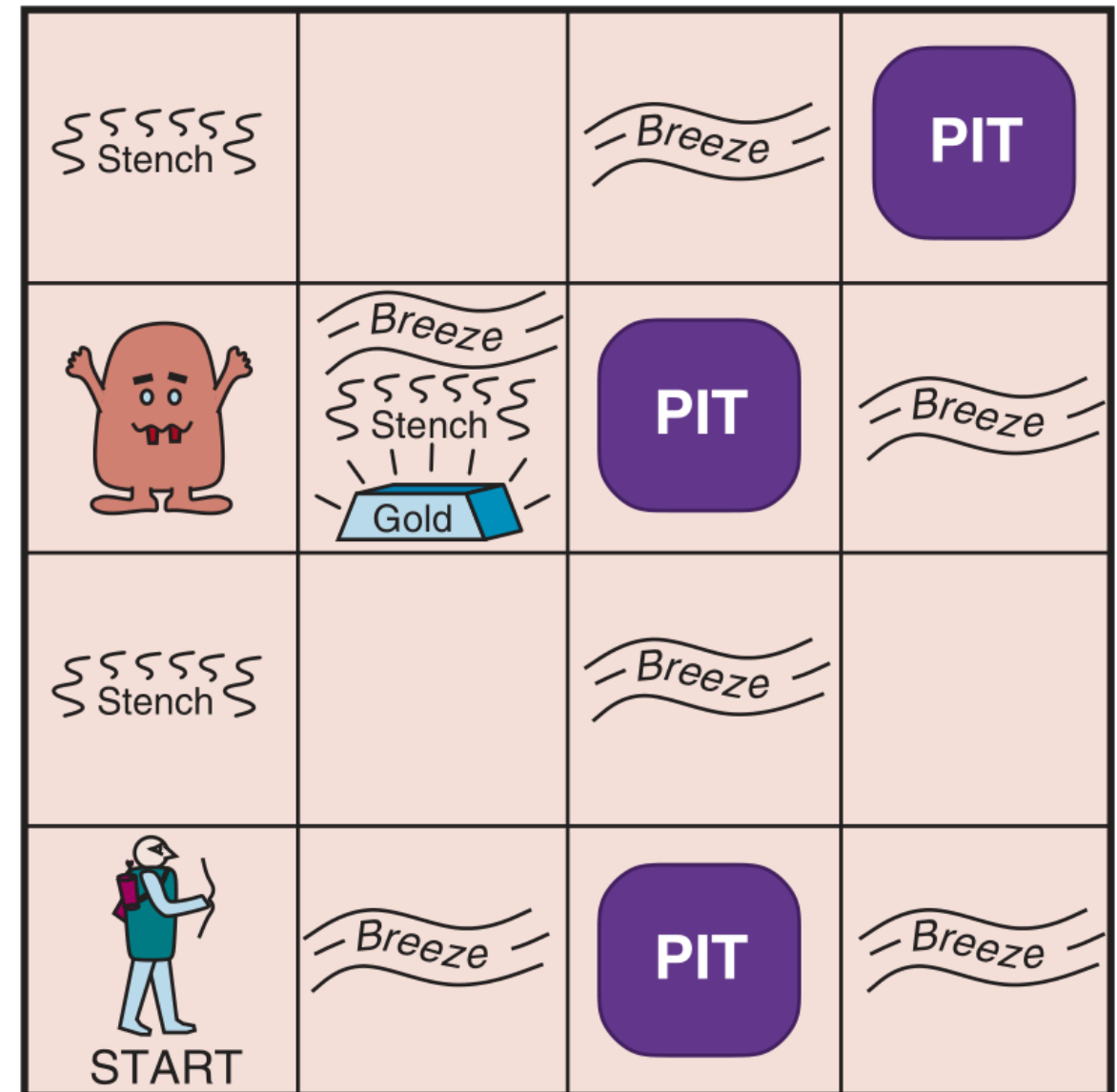
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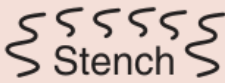


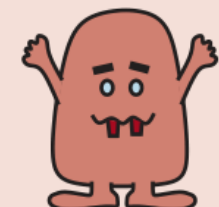
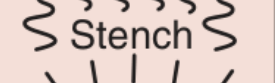
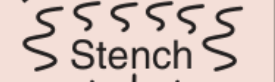



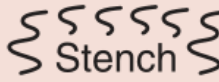





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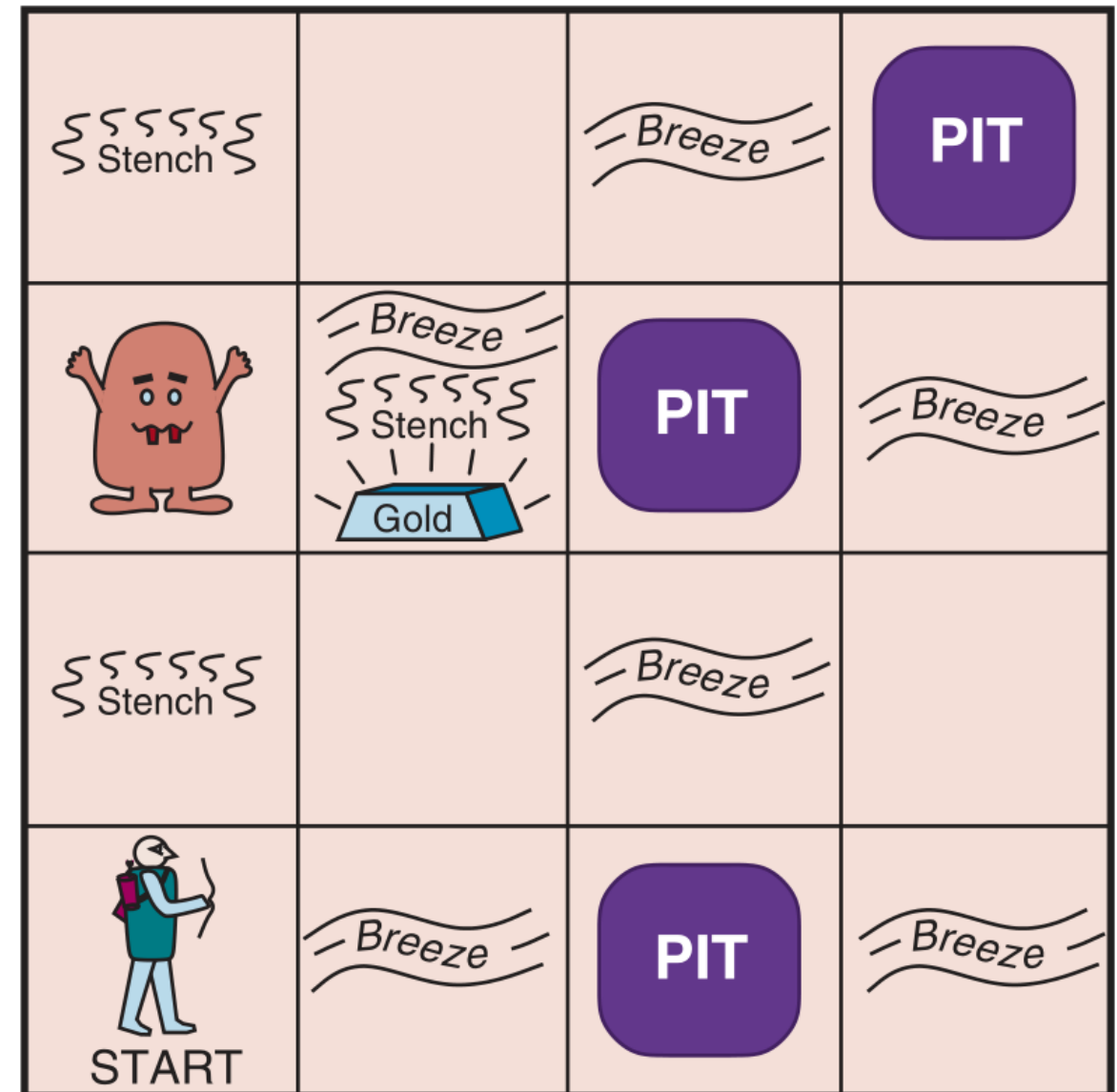
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Questions (based on above  $\mathcal{KB}$ ):

- $\mathcal{KB} \models P_{1,2}$
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# Model Checking

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$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$KB$
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	false	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	true	true	true	true	true	true	<u>true</u>
false	true	false	false	true	false	false	true	false	false	true	true	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	true	true	true	true	true	true	false	true	true	false	true	false



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$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$KB$
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	false	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	true	true	true	true	true	true	<u>true</u>
false	true	false	false	true	false	false	true	false	false	true	true	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	true	true	true	true	true	true	false	true	true	false	true	false

If  $\mathcal{KB}$  and  $\alpha$  contain  $n$  symbols, then there are  $2^n$  models and the complexity of the enumeration (algorithm) is  $O(2^n)$

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Ways to ensure **completeness**:

- Proof by *resolution*: use powerful inference rules (resolution rules)
- *Forward or Backward chaining*: use of modus ponens on a restricted form of propositions (Horn clauses)

Resolution: a single inference rule

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Unit resolution rule:

$$\frac{l_1 \vee \dots \vee l_i \vee \dots \vee l_k, \quad m}{l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k}$$

where  $l_i$  and  $m$  are complementary literals

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Resolution inference rule

$$\frac{l_1 \vee \dots \vee l_i \vee \dots \vee l_k, \quad m_1 \vee \dots \vee m_j \vee \dots \vee m_n}{l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

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$$\frac{l_1 \vee \dots \vee l_i \vee \dots \vee l_k, \quad m_1 \vee \dots \vee m_j \vee \dots \vee m_n}{l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

where  $l_i$  and  $m_j$  are complementary literals

The  $\mathcal{KB}$  must be transformed to CNF (Conjunctive Normal Form): Conjunction of disjunction of literals. Example:  $(A \vee B \vee \neg C) \wedge (C \vee \neg D)$

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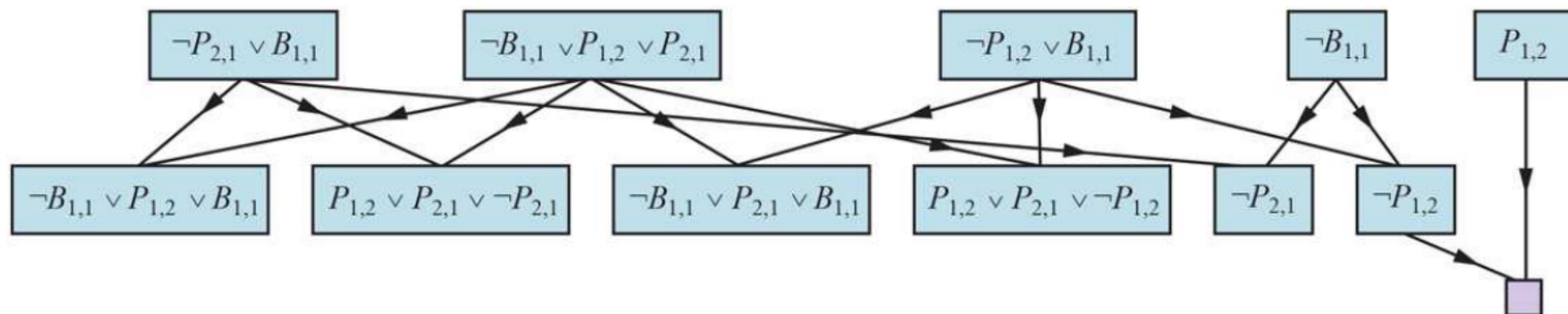
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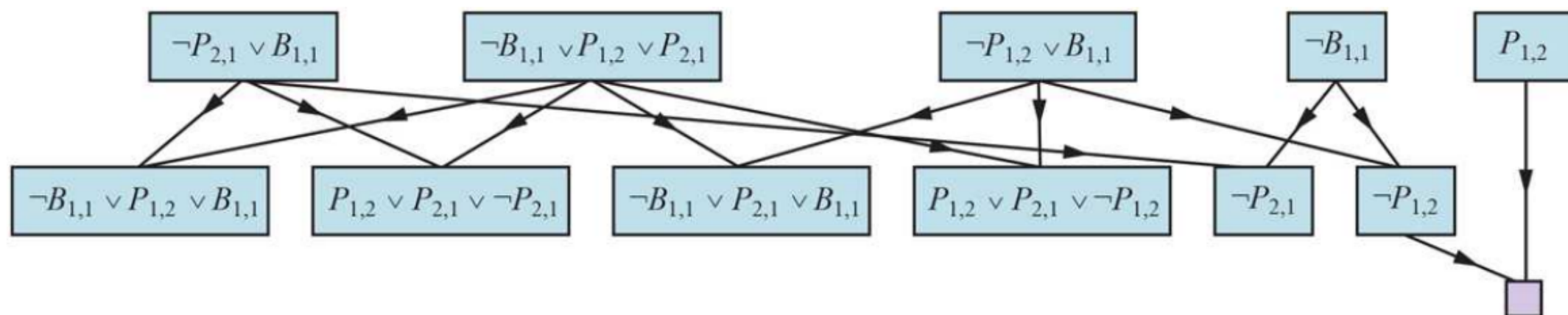
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- An empty clause is yielded, meaning the query is proven

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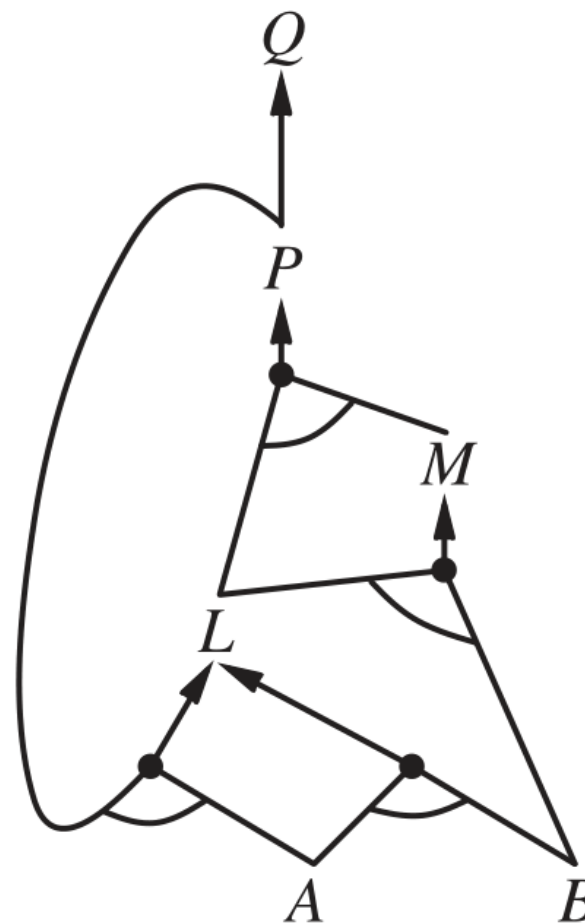
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- All birds except dodo fly:  $\forall x \text{ bird}(x) \wedge \neg \text{dodo}(x) \implies \text{fly}(x)$
- Some wombat like ice-cream:  $\exists x \text{ wombat}(x) \wedge \text{likes}(x, \text{ice} - \text{cream})$

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Note on natural language:

Conversion between natural language and logical expressions is possible due to the expressiveness of FOL. This is very valuable in many areas, such as development of virtual assistants like Alexa, Cortana, Siri, and many more.



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While models are encoded explicitly, there are some limitations:

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- rule-based and do not use data like machine learning
- do not handle uncertainty like probability, although fuzzy logic allows for degree of truth



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