

A decorative graphic on the left side of the slide, consisting of a mosaic of blue squares of various shades, arranged in a pattern that curves upwards and to the right, resembling a stylized wave or a digital data flow.

Intro. to Intelligent Systems

Sem. Ganjil 2023/2024

08–Adversarial Search

Lionov



**INFORMATIKA
UNPAR**

- Game Search
- Minimax & $\alpha - \beta$ Pruning
- Monte Carlo Tree Search
- Stochastic Game
- Partially Observable Games

Background

Adversarial

involving people opposing or disagreeing with each other

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Games are modeled as search problems and use heuristic evaluation functions.

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Consider a **zero-sum game** with an environment that:

- Deterministic
- Perfect information (fully observable)
- Sequential (taking turn)
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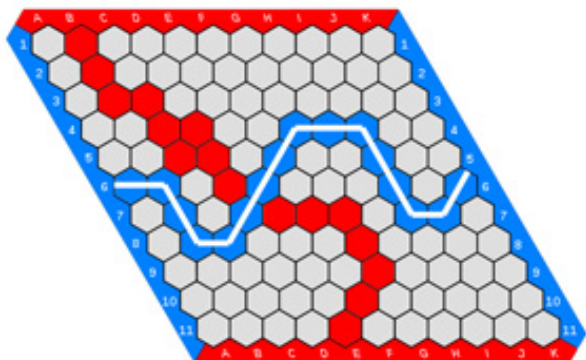
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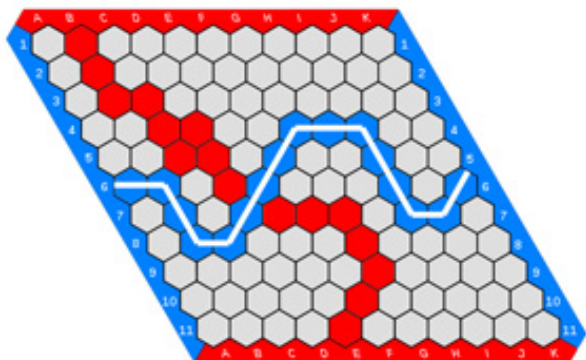
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Othello: human champions *refuse* to compete :D

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- **Utility**(s, p): A utility function (objective/ payoff function) that defines the final numeric value to players p when the game ends in terminal state s

State-space Graph in a Game

The initial state s_0 , ACTIONS, and RESULT define the **State-space Graph**

State-space Graph in a Game

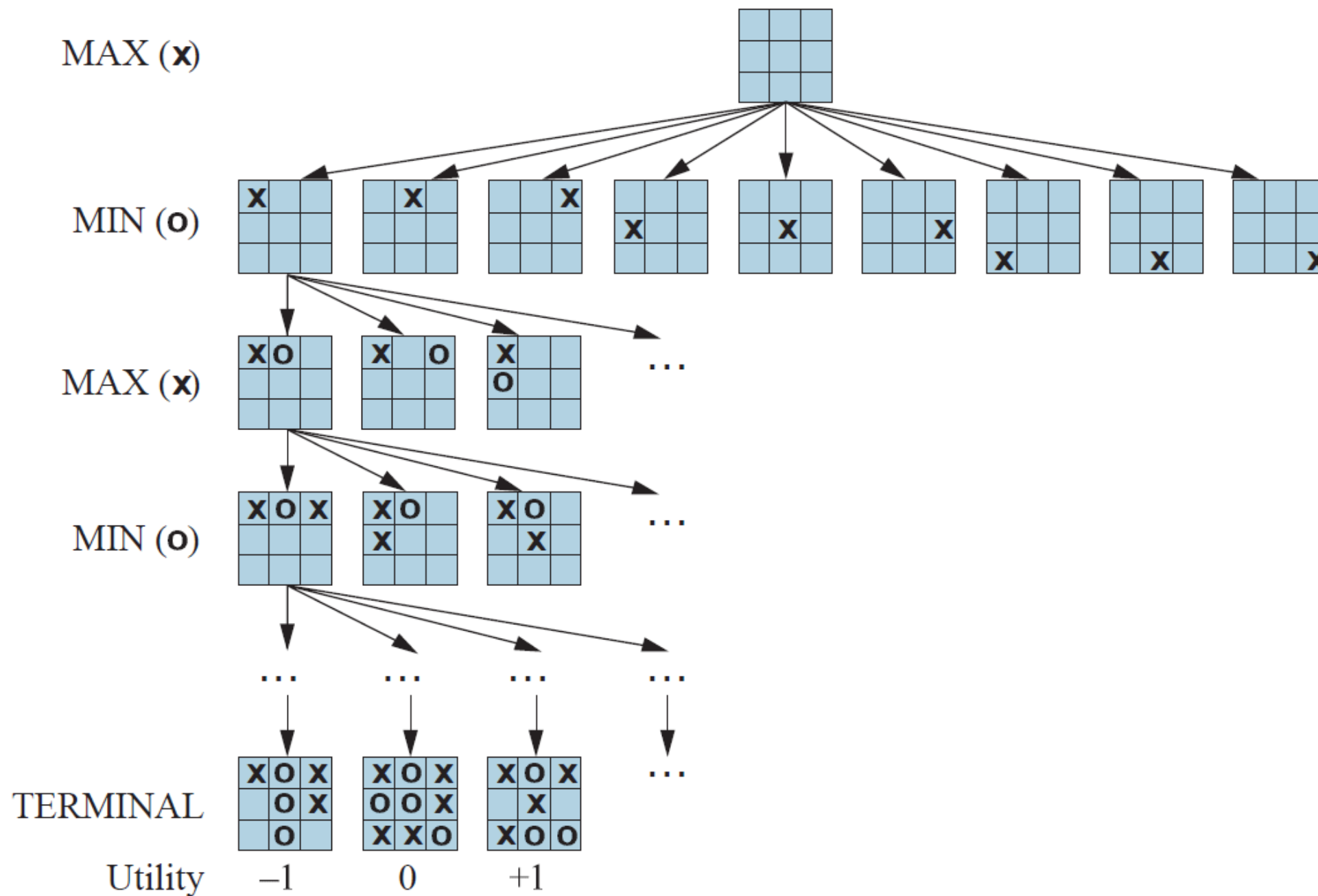
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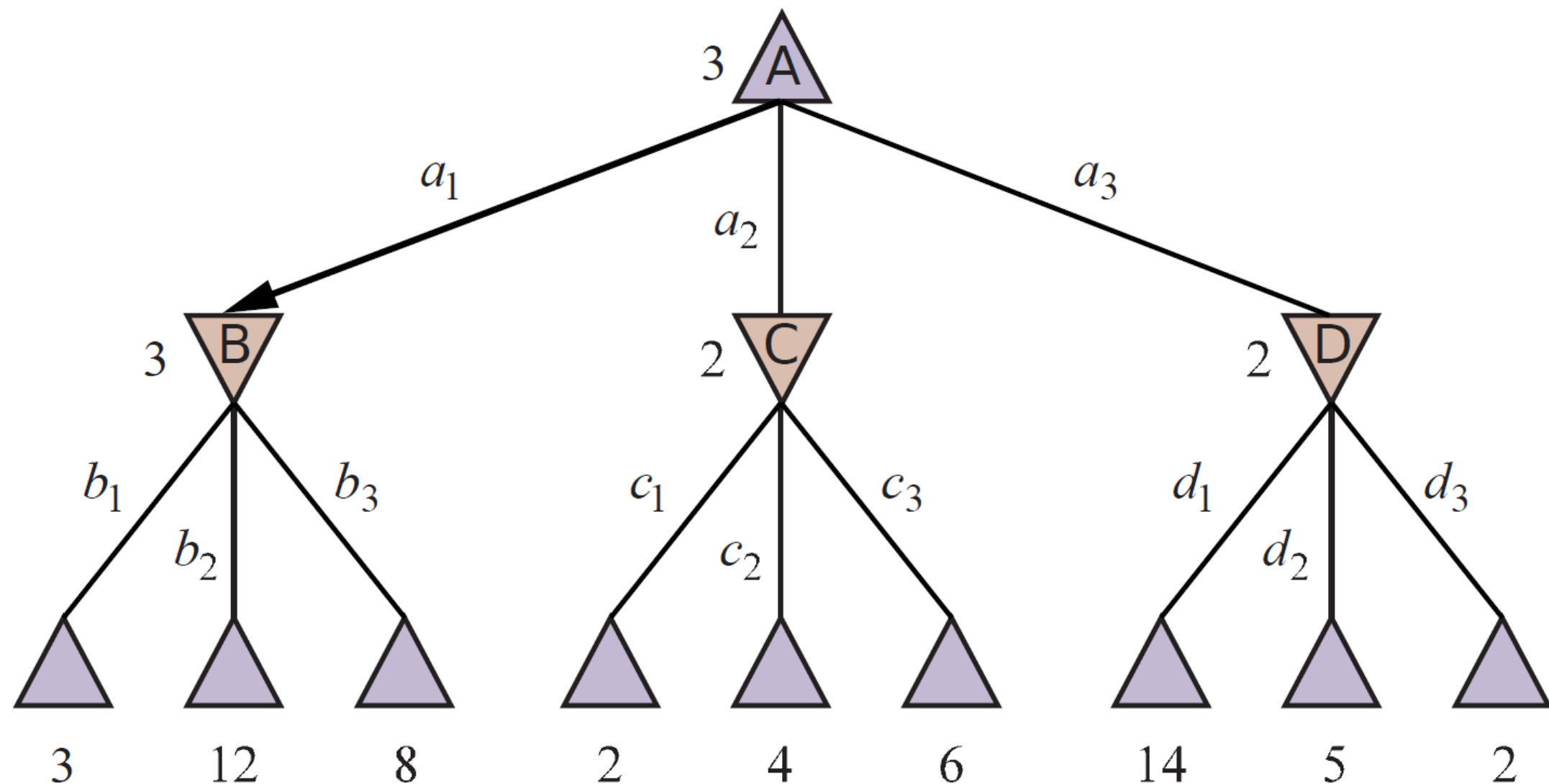
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$$\text{MINIMAX}(s) = \begin{cases} \text{UTILITY}(s, \text{MAX}) & \text{if IS-TERMINAL}(s) \\ \max_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{if TO-MOVE}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{if TO-MOVE}(s) = \text{MIN} \end{cases}$$

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We can identify the **minimax decision** at the root

Minimax Algorithm

function MINIMAX-SEARCH(*game, state*) **returns** *an action*
 player \leftarrow *game*.TO-MOVE(*state*)
 value, move \leftarrow MAX-VALUE(*game, state*)
 return *move*

function MAX-VALUE(*game, state*) **returns** a (*utility, move*) pair
 if *game*.IS-TERMINAL(*state*) **then return** *game*.UTILITY(*state, player*), *null*
 v $\leftarrow -\infty$
 for each *a* **in** *game*.ACTIONS(*state*) **do**
 v2, a2 \leftarrow MIN-VALUE(*game, game*.RESULT(*state, a*))
 if *v2* > *v* **then**
 v, move \leftarrow *v2, a*
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Time complexity is $O(b^m)$ and the space complexity is $O(bm)$

Where m is the maximum depth of the tree and there are b legal moves at each point

Minimax in Multiplayer Games

Returns a vector of n values (n = the number of players)

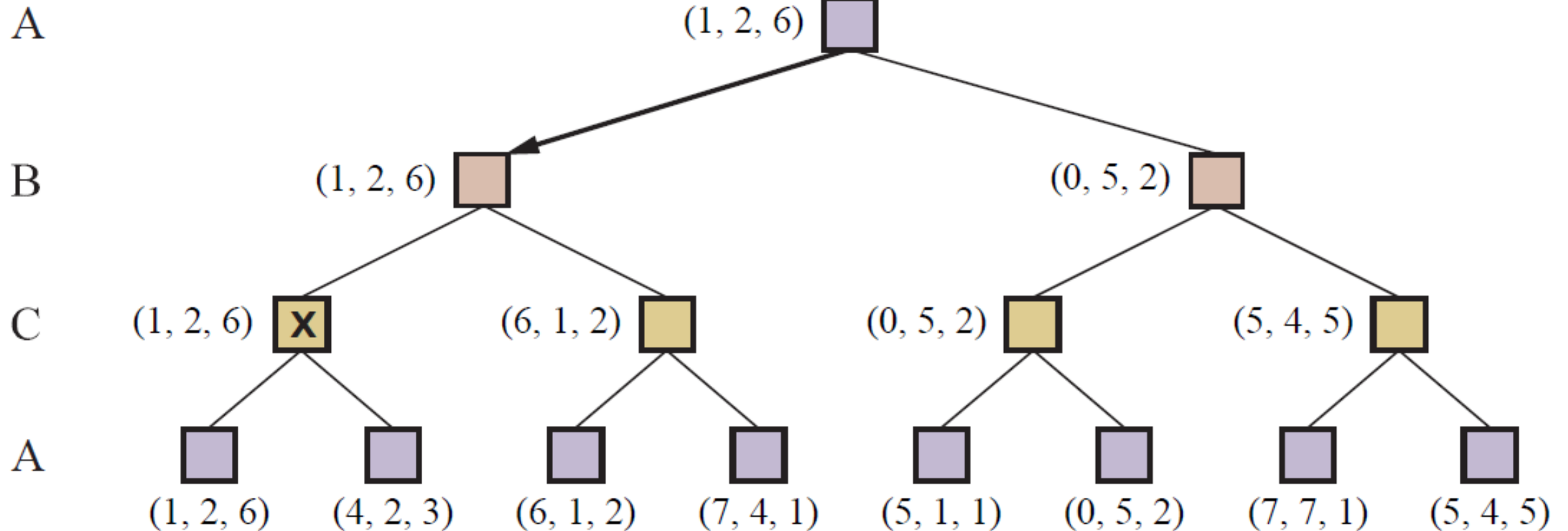
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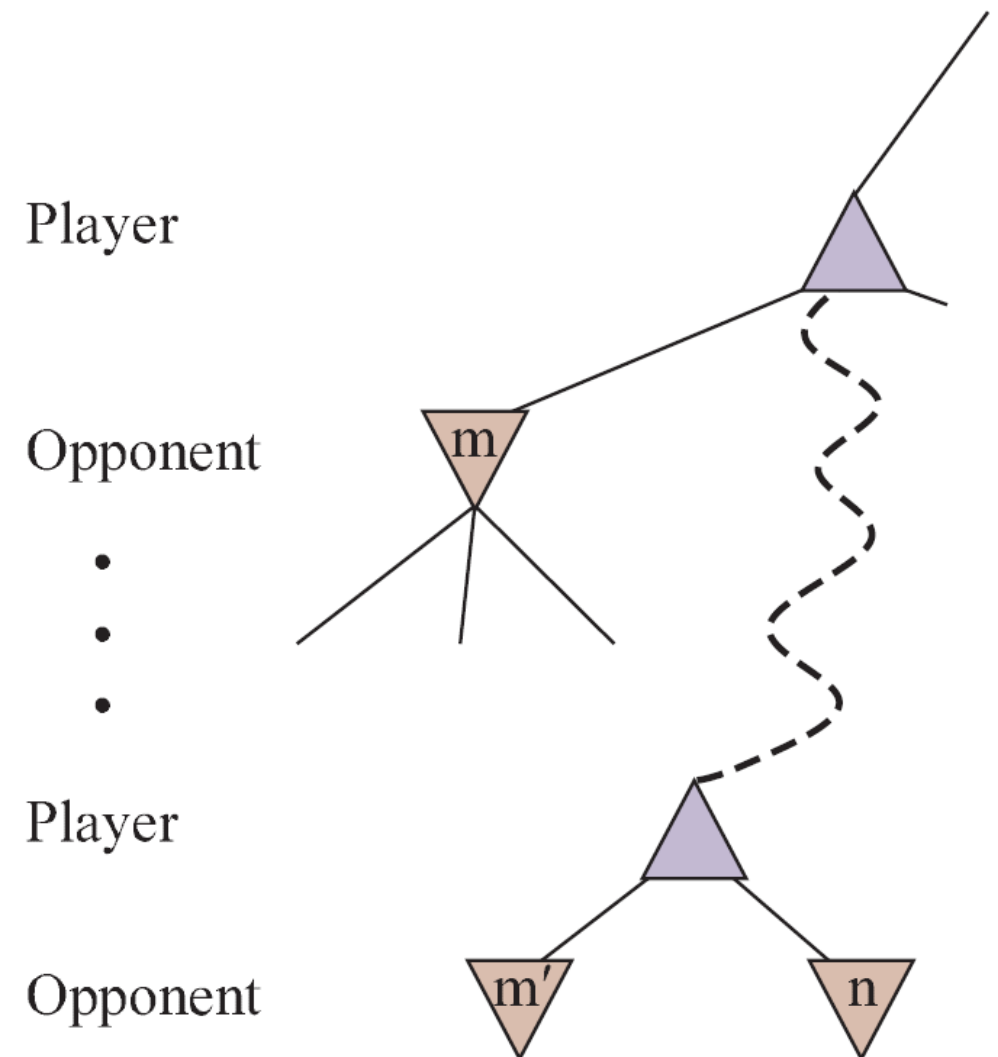
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The general principle:

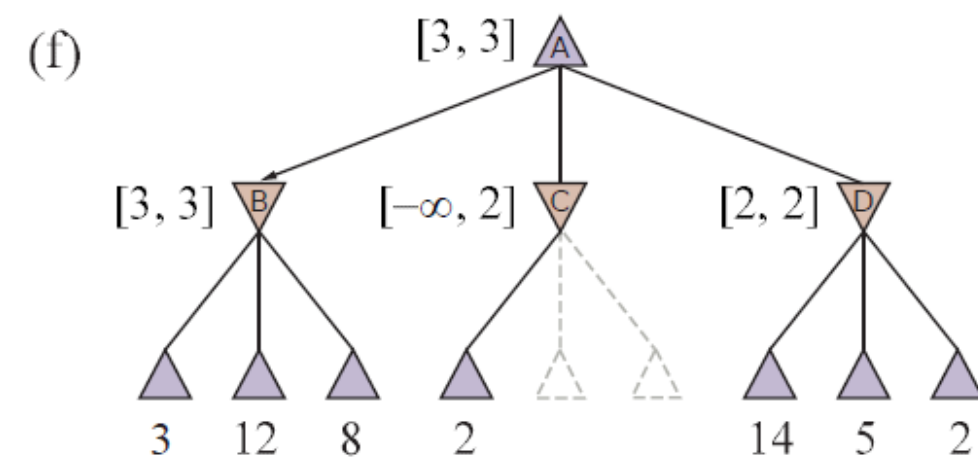
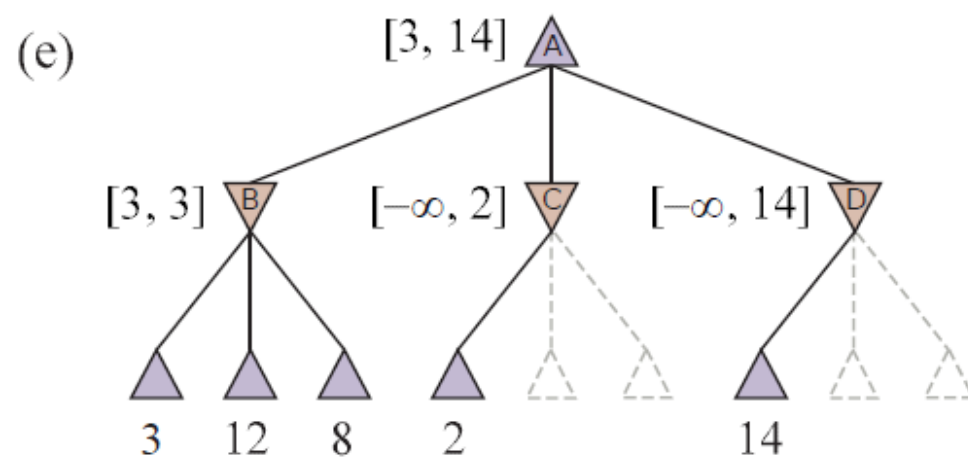
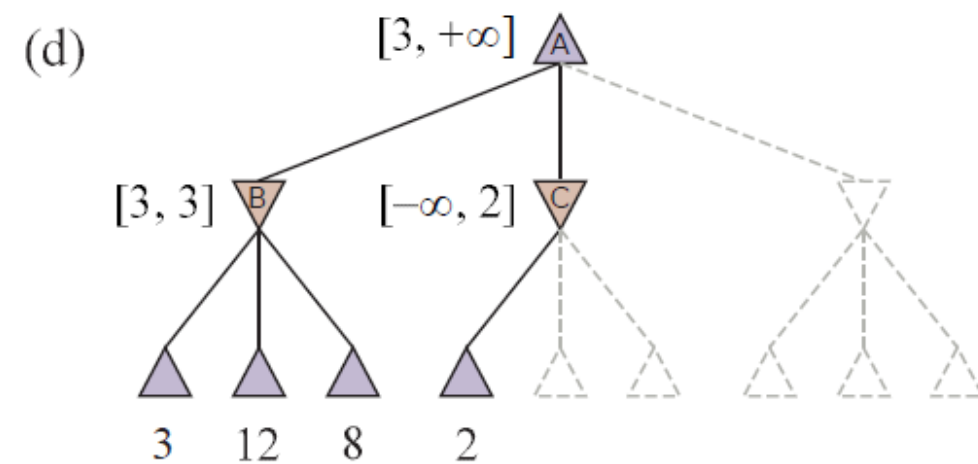
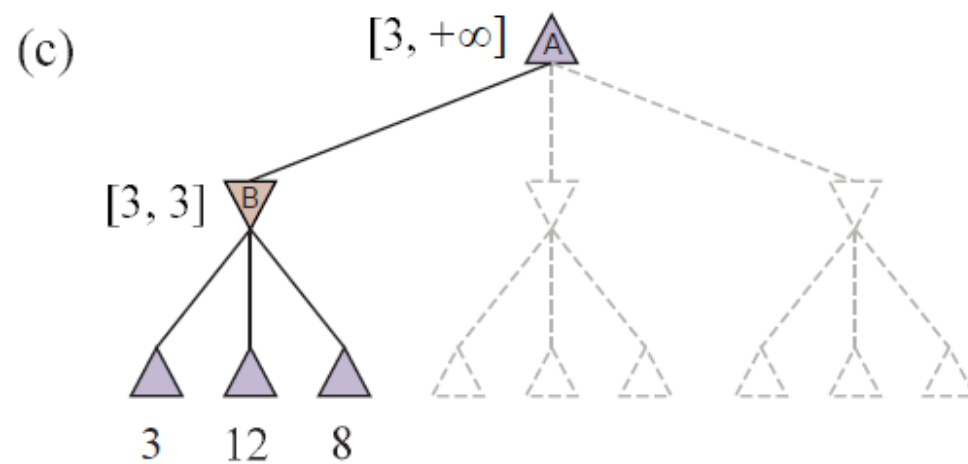
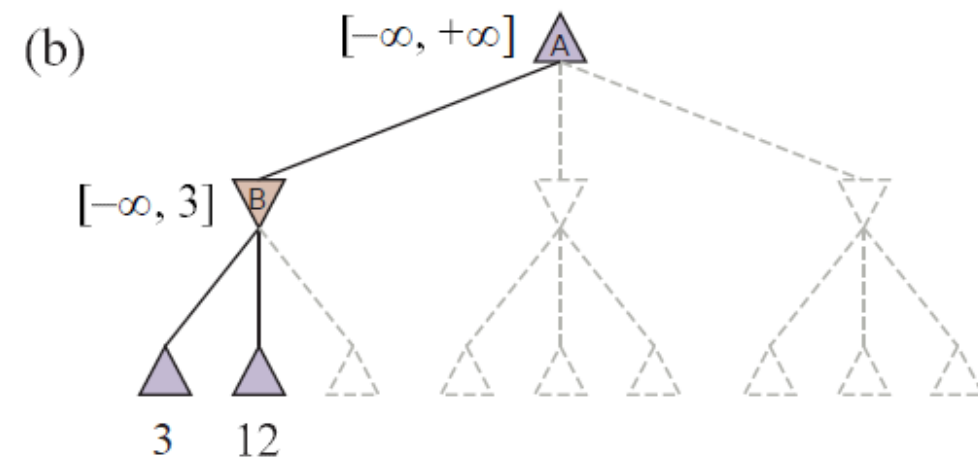
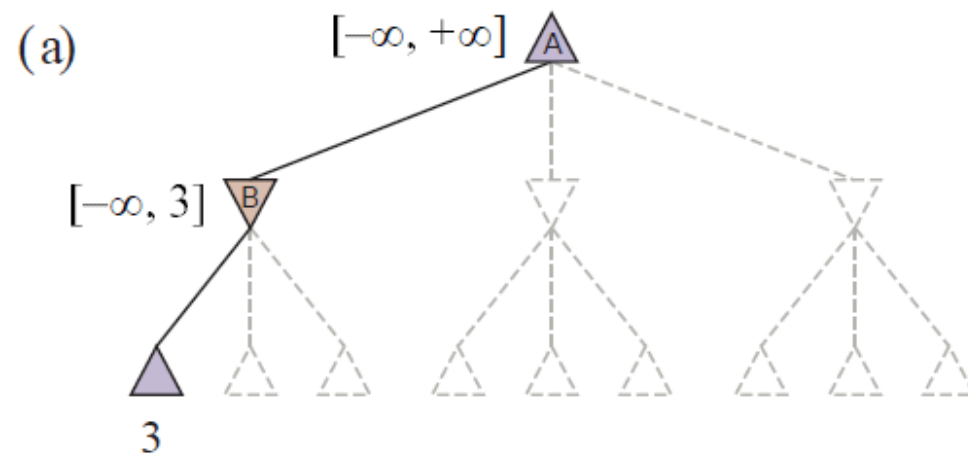
Consider a node n such that a player has a choice of moving to n .

If the player has a **better choice** either at the same level (m') or at any point higher up in the tree (m), then the player will never move to n .

Once we have found out enough about n (by examining some of its descendants) to reach this conclusion, we can prune it.



Alpha-Beta Pruning



$\alpha - \beta$ Pruning Algorithm

```
function ALPHA-BETA-SEARCH(game, state) returns an action  
  player  $\leftarrow$  game.TO-MOVE(state)  
  value, move  $\leftarrow$  MAX-VALUE(game, state,  $-\infty$ ,  $+\infty$ )  
  return move
```

```
function MAX-VALUE(game, state,  $\alpha$ ,  $\beta$ ) returns a (utility, move) pair  
  if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null  
  v  $\leftarrow -\infty$   
  for each a in game.ACTIONS(state) do  
    v2, a2  $\leftarrow$  MIN-VALUE(game, game.RESULT(state, a),  $\alpha$ ,  $\beta$ )  
    if v2 > v then  
      v, move  $\leftarrow$  v2, a  
       $\alpha \leftarrow$  MAX( $\alpha$ , v)  
    if v  $\geq \beta$  then return v, move  
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    v2, a2  $\leftarrow$  MAX-VALUE(game, game.RESULT(state, a),  $\alpha$ ,  $\beta$ )  
    if v2 < v then  
      v, move  $\leftarrow$  v2, a  
       $\beta \leftarrow$  MIN( $\beta$ , v)  
    if v  $\leq \alpha$  then return v, move  
  return v, move
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- **Type A strategy:** considers all possible moves to a certain depth in the search tree, and then uses a heuristic evaluation function to estimate the utility of states at that depth.
- **Type B strategy:** ignores moves that look bad, and follows promising lines “as far as possible”.

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Cut off the search early:

- Replace the terminal test by a **cutoff-test**: return TRUE for terminal states, otherwise its free to decide when to cut off the search, based on the search depth and any other properties of the state.

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Weighted combination of features of a game state:

$$\text{EVAL}(s) = \omega f_1(s) + \omega f_2(s) + \dots + \omega f_n(s) = \sum_i^n \omega f_i(s)$$

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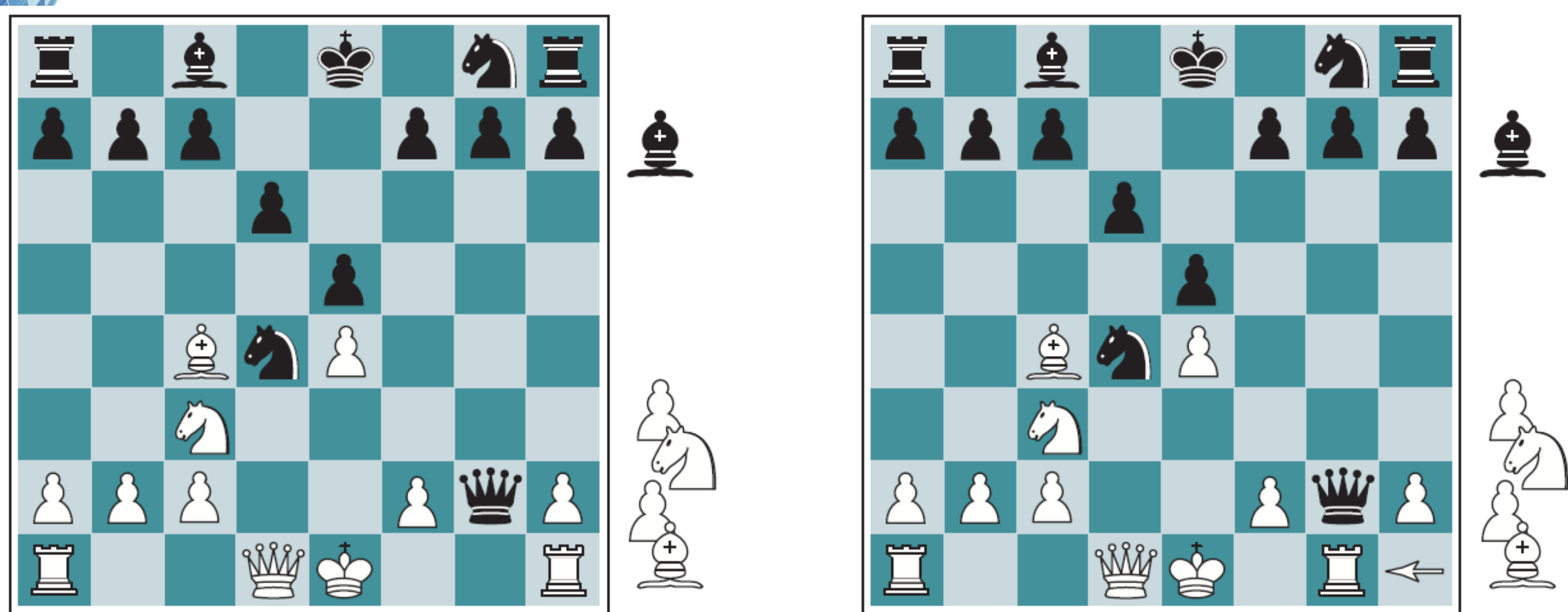
$$\text{EVAL}(s) = \omega f_1(s) + \omega f_2(s) + \dots + \omega f_n(s) = \sum_i^n \omega f_i(s)$$

The features and their weights are results of years of experience or estimated by machine learning techniques

Evaluation Function

The heuristic $\text{EVAL}(s, p)$ returns an *estimate* of the expected utility of state s

- For terminal states: $\text{EVAL}(s, p) = \text{UTILITY}(s, p)$.
- For nonterminal states: $\text{UTILITY}(\text{loss}, p) \leq \text{EVAL}(s, p) \leq \text{UTILITY}(\text{win}, p)$



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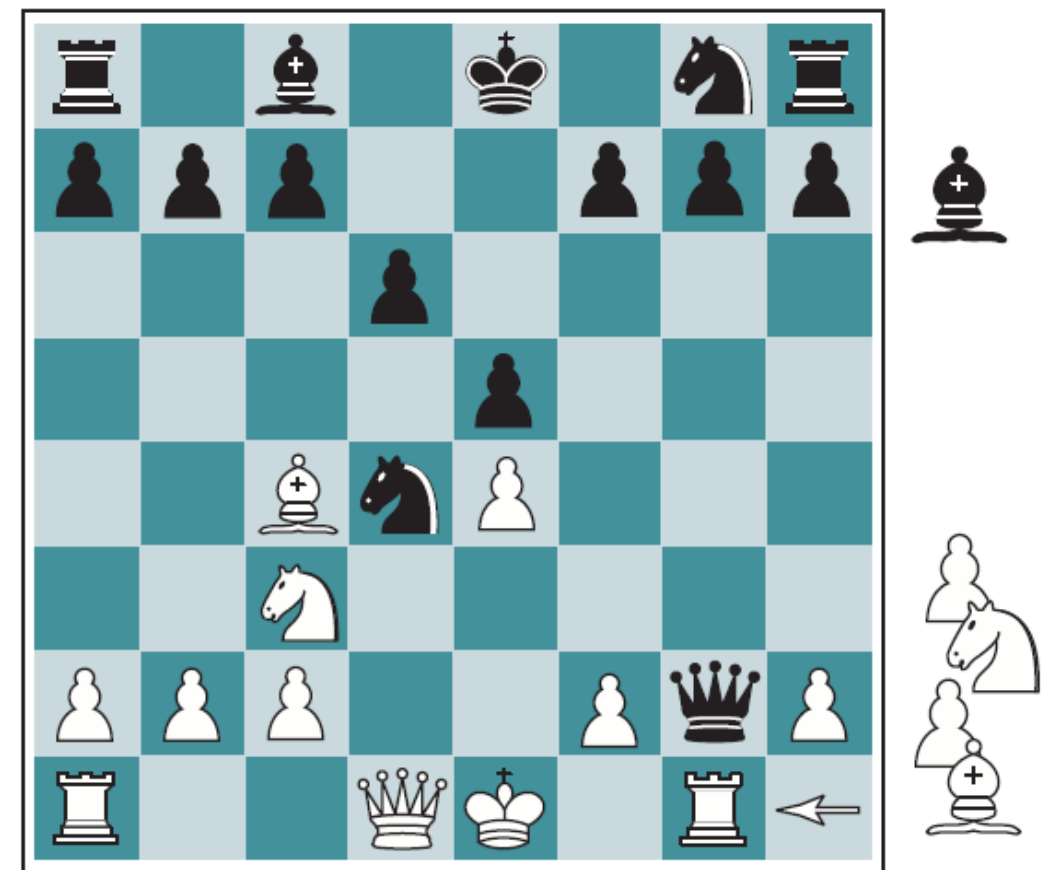
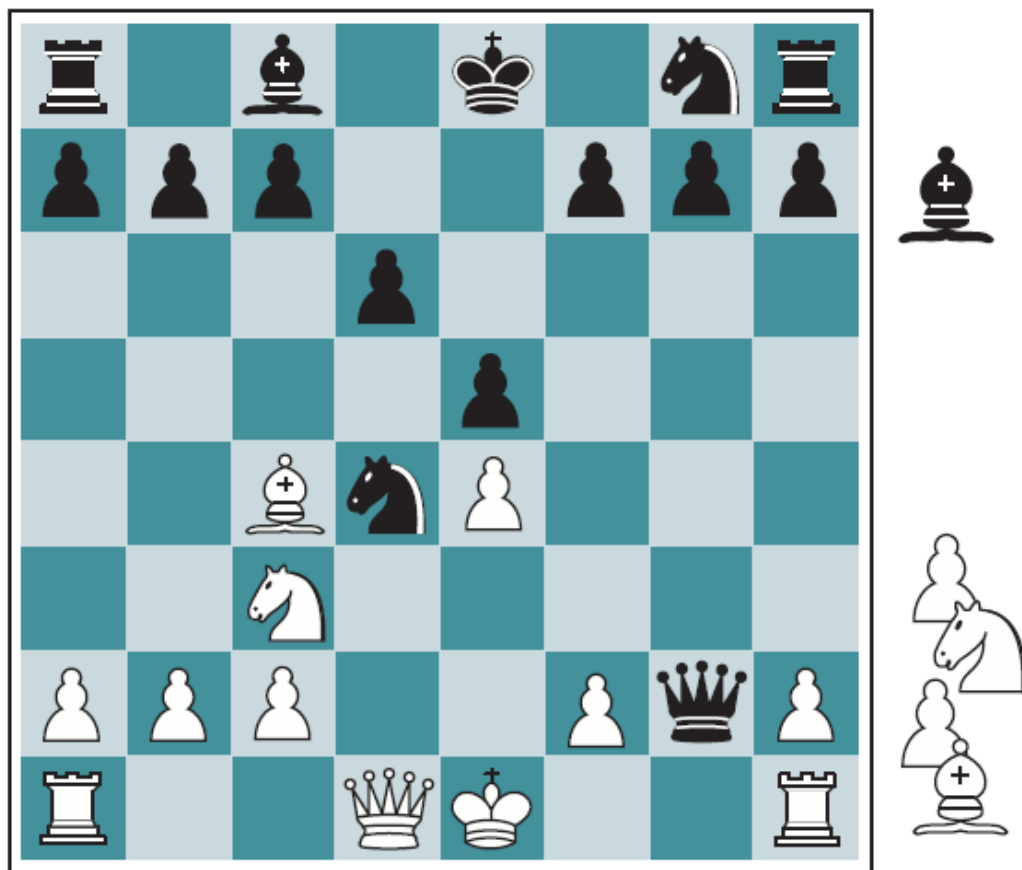
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This strategies may lead to error:



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Map every possible state to the best move in that state. The computer can play perfectly by looking up the right move in this table.

Monte Carlo Tree Search

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Pure Monte Carlo search: do **N** simulations starting from the current state and track which of the possible moves has the highest win percentage

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1. SELECTION
2. EXPANSION
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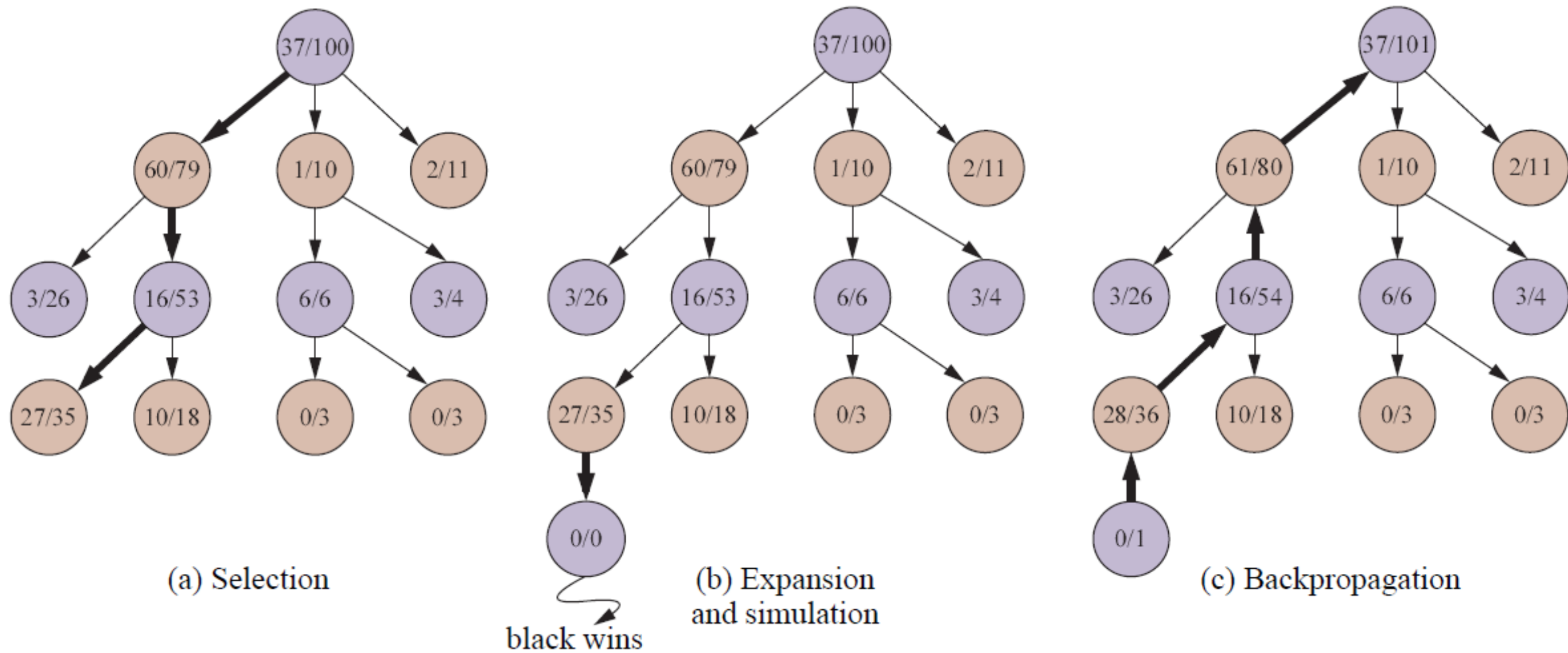
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```
graph TD; A((37/100)) --> B((60/79)); A --> C((1/10)); A --> D((2/11)); B --> E(( )); B --> F(( )); C --> G(( )); C --> H(( )); D --> I(( ))
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Exp. of a selection policy: “upper confidence bounds applied to trees” or **UCT**.
UCT ranks possible move based on an upper confidence bound formula **UCB1**

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function MONTE-CARLO-TREE-SEARCH($state$) **returns** *an action*

$tree \leftarrow \text{NODE}(state)$

while IS-TIME-REMAINING() **do**

$leaf \leftarrow \text{SELECT}(tree)$

$child \leftarrow \text{EXPAND}(leaf)$

$result \leftarrow \text{SIMULATE}(child)$

BACK-PROPAGATE($result, child$)

return the move in $\text{ACTIONS}(state)$ whose node has highest number of playouts

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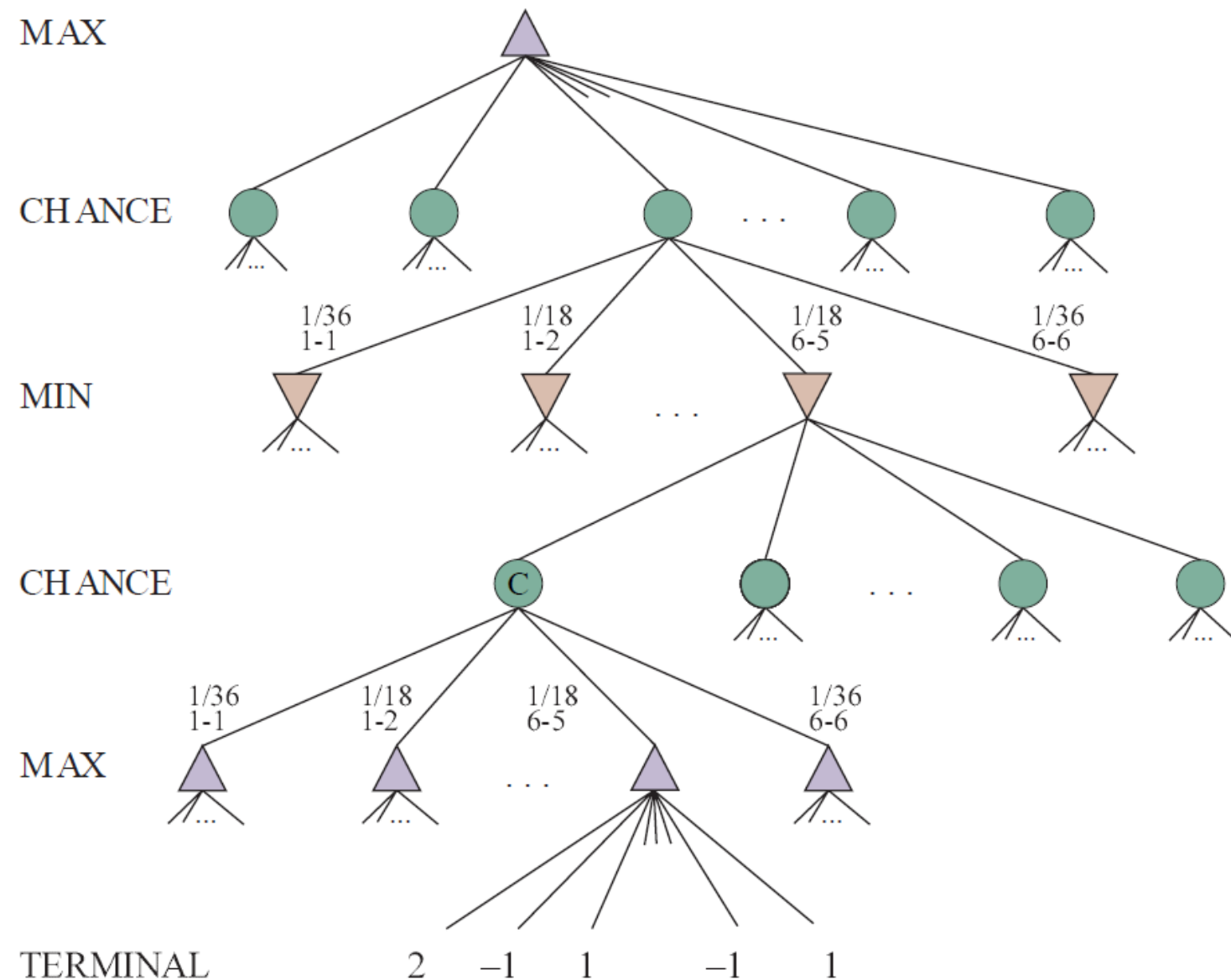
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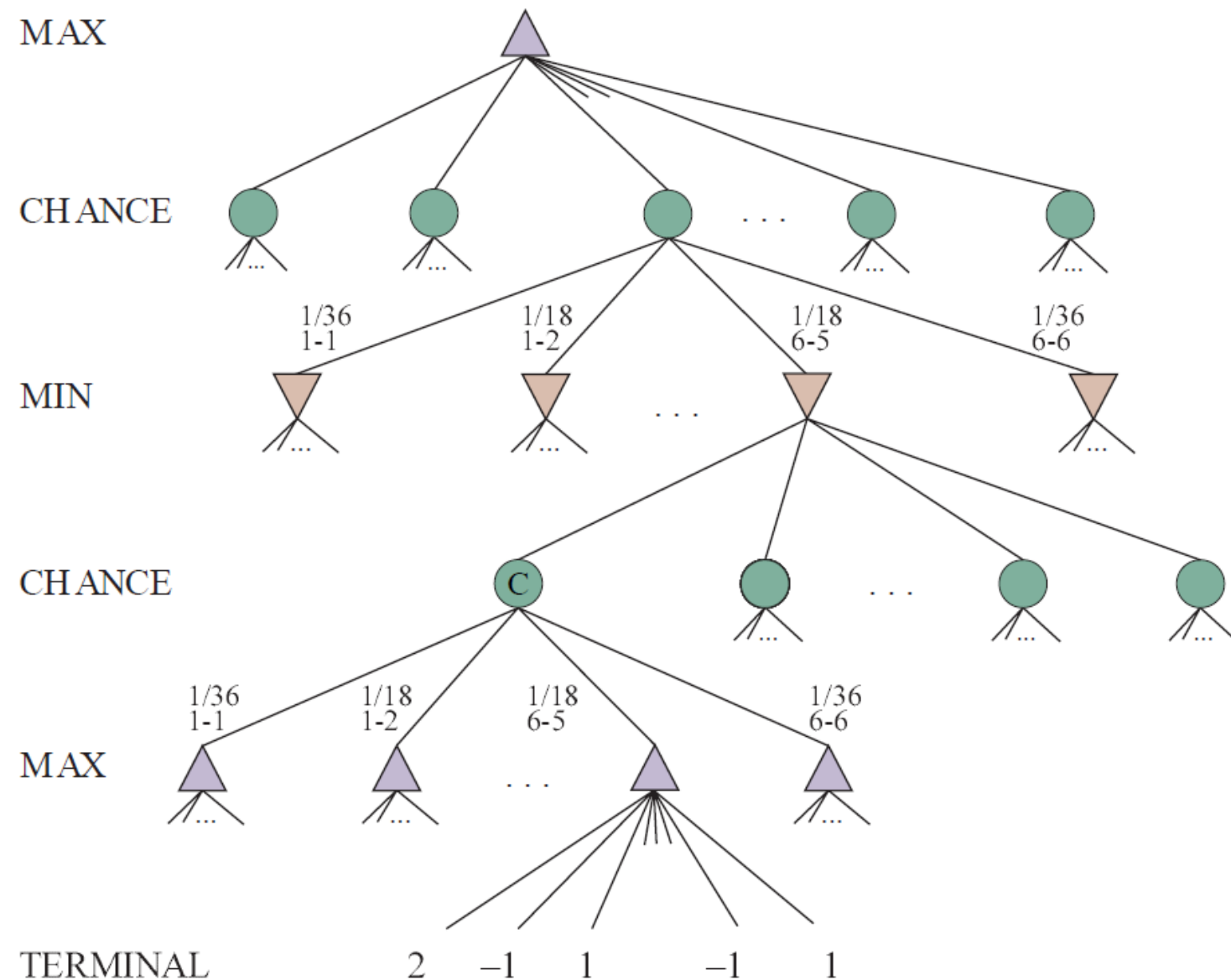
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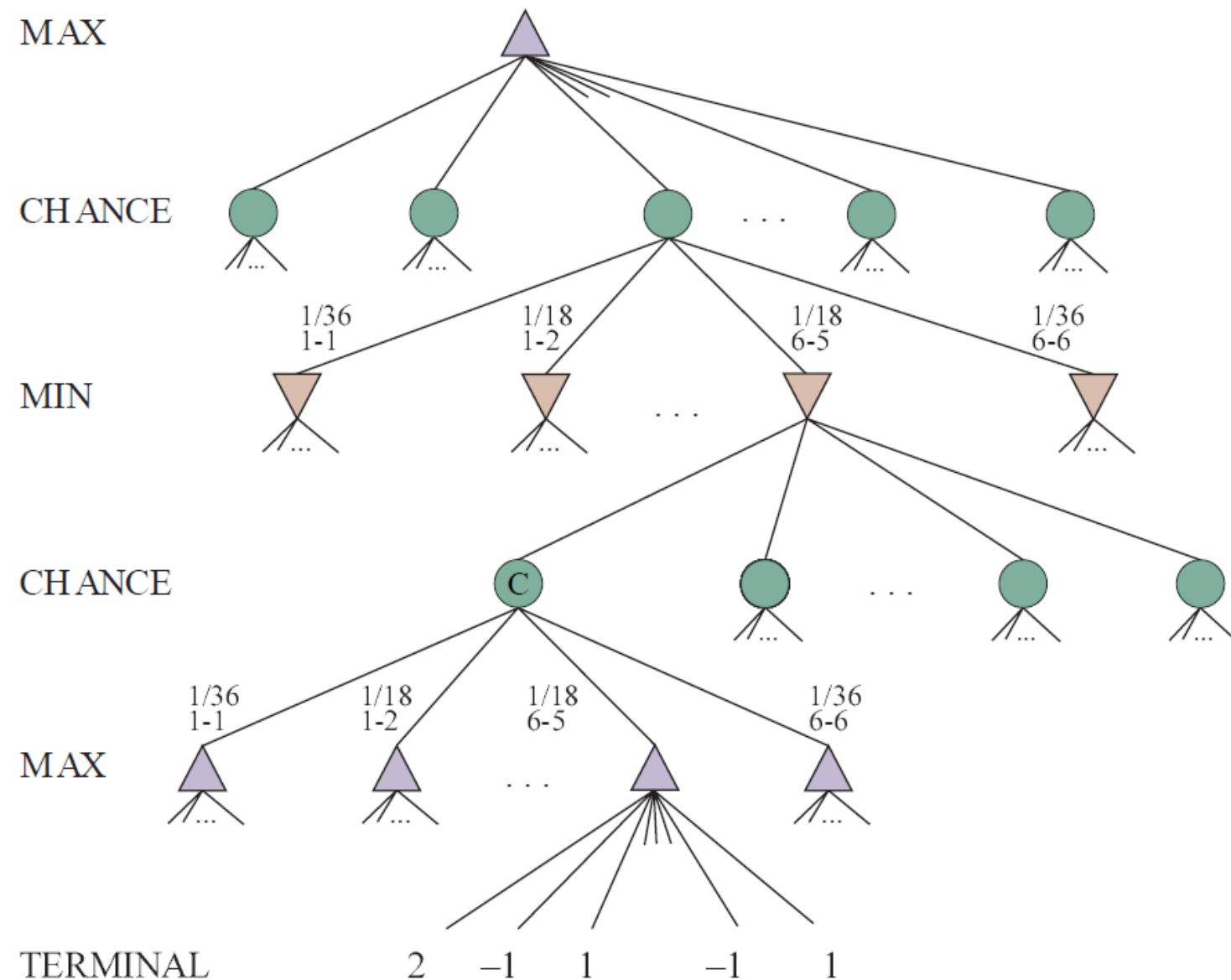
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Expectiminimax value for games with chance nodes, a generalization of the minimax value for deterministic games



Partially Observable Games

In games of imperfect information, such as Kriegspiel and poker, optimal play requires reasoning about the current and future belief states of each player. A simple approximation can be obtained by averaging the value of an action over each possible configuration of missing information.

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Calculating optimal decisions in complex games is intractable

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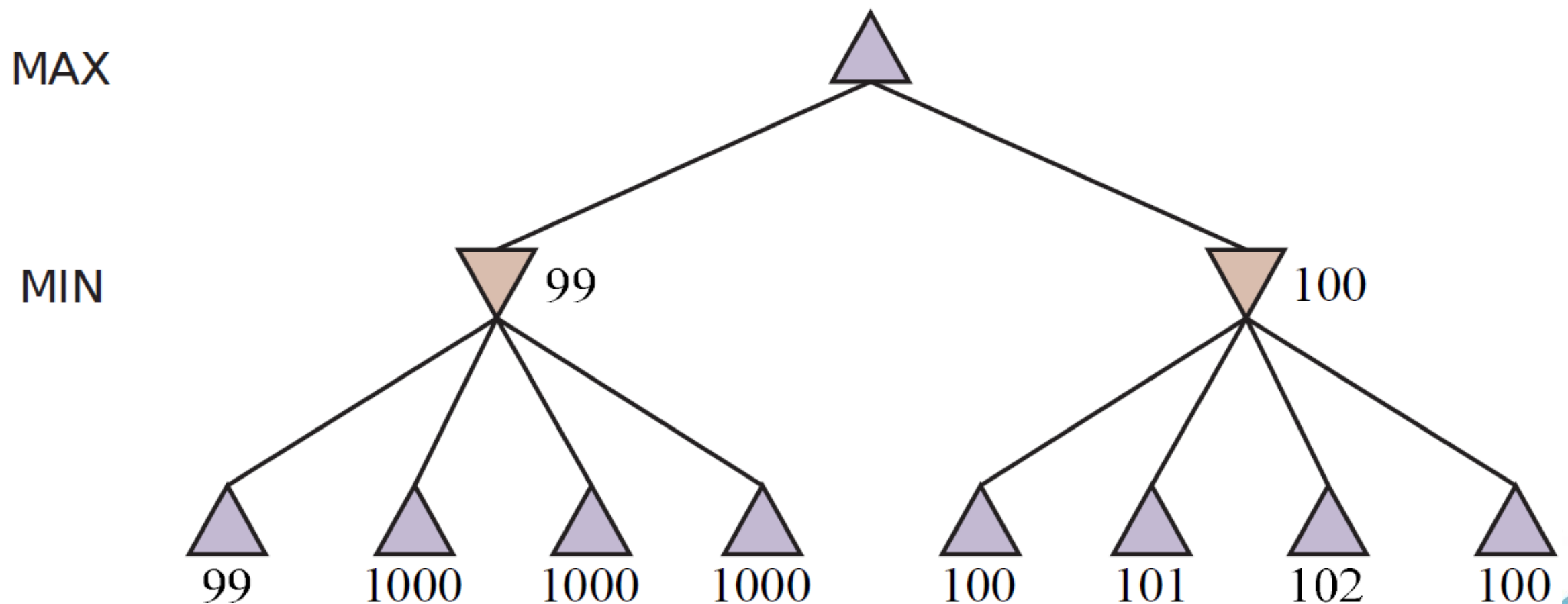
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- Reasoning are done at the level of individual moves. Humans play games differently: reasoning at the abstract level
- No ability to incorporate machine learning into the game search process



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