**解 48.** 由于随机变量$Y$服从参数$\lambda = 1$的指数分布, 则有:

$$f(y) = \begin{cases} e^{-y} & y > 0 \\ 0 & y \le 0 \end{cases}$$

故由$F(y) = \int_{-\infty}^{y} f(t)dt$, 得:

$$F(y) = \begin{cases} 1 - e^{-y} & y > 0 \\ 0 & y \le 0 \end{cases}$$

则$P(Y \le a+1|Y > a) = \dfrac{P(a < Y \le a+1)}{P(Y > a)} = \dfrac{F(a+1) - F(a)}{1 - F(a)} = \dfrac{e^{-a} - e^{-a-1}}{e^{-a}} = 1 - e^{-1}$

**解 50.** 由$X$服从参数$\lambda = \dfrac{1}{5}$的指数分布, 得:

$$f(x) = \begin{cases} \dfrac{1}{5}e^{-\frac{1}{5}x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

故由$F(x) = \int_{-\infty}^{x} f(t)dt$, 得:

$$F(x) = \begin{cases} 1 - e^{-\frac{1}{5}x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

则单次接受服务的概率为$P(X \le 10) = F(10) = 1 - e^{-2}$.
设该客户在一个月内至少有一次未接受服务而离开为事件$A$.
则$P(A) = 1 - P(\bar{A}) = 1 - (P(X \le 10))^5 = 1 - (1 - e^{-2})^5$

**解 53.** 由于$X \sim N(1, 4)$, 故$\mu = 1, \sigma = 2$. 则有$\dfrac{X - \mu}{\sigma} \sim N(0, 1)$, 服从标准正态分布.
*(1).* $P(0 \le X \le 4) = P\left(\dfrac{0-1}{2} \le \dfrac{X-1}{2} \le \dfrac{4-1}{2}\right) = \Phi\left(\dfrac{3}{2}\right) - \Phi\left(-\dfrac{1}{2}\right) = \Phi\left(\dfrac{3}{2}\right) + \Phi\left(\dfrac{1}{2}\right) - 1$
查表得$P(0 \le X \le 4) = 0.9332 + 0.6915 - 1 = 0.6247$
*(2).* 由$P(X > c) = 1 - P(X \le c) = 2P(X \le c)$, 得$P(X \le c) = \dfrac{1}{3}$.
$P(X \le c) = P\left(\dfrac{X-1}{2} \le \dfrac{c-1}{2}\right) = \Phi\left(\dfrac{c-1}{2}\right) = \dfrac{1}{3}$
查表得: $\dfrac{c-1}{2} = -0.43$, 故$c = 0.14$

**解 56.** 设电源电压为$X$, 则$X \sim N(220, 225)$, 故$\mu = 220, \sigma = 15$. 则有$\dfrac{X - \mu}{\sigma} \sim N(0, 1)$, 服从标准正态分布.
$P(X \le 200) = P\left(\dfrac{X-220}{15} \le \dfrac{200-220}{15}\right) = \Phi\left(-\dfrac{-4}{3}\right) = 1 - \Phi\left(\dfrac{4}{3}\right) = 0.09177$

$$\begin{aligned} P(200 < X \le 240) &= P\left(\frac{200-220}{15} < \frac{X-220}{15} \le \frac{240-220}{15}\right) \\ &= \Phi\left(\frac{4}{3}\right) - \Phi\left(-\frac{4}{3}\right) \\ &= 2\Phi\left(\frac{4}{3}\right) - 1 \\ &= 0.81646 \end{aligned}$$

$P(X > 240) = 1 - P(X \le 200) - P(200 < X \le 240) = 0.09177$
设电子元件会损坏对应的事件为$A$, 则$P(A) = 0.1 \times P(X \le 200) + 0.3 \times P(X > 240) = 0.03671$

**解 2.**

$$\begin{aligned}P(Y=2) &= P(X\ge 2, Y=2)\\ &= P(X=2,Y=2)+P(X=3,Y=2)+P(X=4,Y=2)\\ &= P(X=2)P(Y=2)+P(X=3)P(Y=2)+P(X=4)P(Y=2)\\ &= \frac{1}{4}\times(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})=\frac{13}{48}\end{aligned}$$

**解 11.** *(1).* 对二维随机变量$(X,Y)$, 有$X<Y$.

$P(X=i,Y=j)=(1-p)^{i-1}p\cdot(1-p)^{j-i-1}p=p^2(1-p)^{j-2}$, 其中$i\ge 1, j\ge i+1$.

*(2).* $X$的边缘分布为:

$P(X=i)=\sum\limits_{j=2}^{+\infty}P(X=i,Y=j)=\sum\limits_{j=i+1}^{+\infty}\ p^2(1-p)^{j-2}=p(1-p)^{i-1}$

$Y$的边缘分布为:

$P(Y=j)=\sum\limits_{i=1}^{j-1}P(X=i,Y=j)=\sum\limits_{i=1}^{j-1}p^2(1-p)^{j-2}=(j-1)p^2(1-p)^{j-2}$

**解 19.** *(1).* 由分布函数定义可得:

$F(+\infty,+\infty)=1,\ F(+\infty,-\infty)=0,\ F(-\infty,+\infty)=0,\ F(-\infty,-\infty)=0.$ 即

$a(b+\frac{\pi}{2})(c+\frac{\pi}{2})=1,\ a(b+\frac{\pi}{2})(c-\frac{\pi}{2})=0,\ a(b-\frac{\pi}{2})(c+\frac{\pi}{2})=0,\ a(b+\frac{\pi}{2})(c+\frac{\pi}{2})=0$

解得: $a=\frac{1}{\pi^2},\ b=\frac{\pi}{2},\ c=\frac{\pi}{2}$

*(2).*

$$\begin{aligned}P(X>0,Y>0) &= P(X\le+\infty,Y\le+\infty)-P(X\le+\infty,Y\le 0)-P(X\le 0,Y\le+\infty)+P(X\le 0,Y\le 0)\\ &= F(+\infty,+\infty)-F(+\infty,0)-F(0,+\infty)+F(0,0)\\ &= 1-\frac{1}{\pi^2}\times\pi\times\frac{\pi}{2}-\frac{1}{\pi^2}\times\frac{\pi}{2}\times\pi+\frac{1}{\pi^2}\times\frac{\pi}{2}\times\frac{\pi}{2}\\ &= \frac{1}{4}\end{aligned}$$

*(3).* 设概率分布函数$F(x,y)$对应的概率密度函数为$f(x,y)$.

根据分布函数定义有: $F(x,y)=\int_{-\infty}^{y}dv\int_{-\infty}^{x}f(u,v)du$

其中$\int_{-\infty}^{x}f(u,v)du$是$v$的函数, 记为$f_1(v)$. 即得:

$F(x,y)=\int_{-\infty}^{y}f_1(v)dv$, 两边对$y$求导得: $\frac{\partial F(x,y)}{\partial y}=f_1(y)=\int_{-\infty}^{x}f(u,y)du$. 两边再对$x$求导得:

$\frac{\partial F(x,y)}{\partial x\partial y}=f(x,y)=\frac{1}{\pi^2(1+x^2)(1+y^2)}$

则$X$的边缘密度函数为:

$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy=\frac{1}{\pi(1+x^2)}$

$Y$的边缘密度函数为:

$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\frac{1}{\pi(1+y^2)}$

**解 21.** *(1).* 由分布函数$F(x,y)$的定义可得: $F(x,y)=\int_{-\infty}^{y}\int_{-\infty}^{x}f(u,v)dudv$, 算得

$$F(x,y)=\begin{cases}0 & x\le 0 \quad or \quad y\le 0\\ \sin x\sin y & 0<x<\dfrac{\pi}{2},0<y<\dfrac{\pi}{2}\\ \sin x & 0<x<\dfrac{\pi}{2},y\ge\dfrac{\pi}{2}\\ \sin y & x\ge\dfrac{\pi}{2},0<y<\dfrac{\pi}{2}\\ 1 & x\ge\dfrac{\pi}{2},y\ge\dfrac{\pi}{2}\end{cases}$$

*(2).* $P(0<X<\pi/4,\pi/4<Y<\pi/2)=\int_0^{\frac{\pi}{4}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos x\cos y dxdy=\dfrac{\sqrt{2}-1}{2}$

**解 27.** *(1).* 由题意知：$\iint_D f(x,y)dxdy=\iint_D c(R-\sqrt{x^2+y^2})dxdy=1$. 其中$D:x^2+y^2<R^2$
令$r^2=x^2+y^2$，$\theta=\arctan\dfrac{y}{x}$，则有：
$\int_0^{2\pi}d\theta\int_0^R cr(R-r)dr=\dfrac{c\pi R^3}{3}=1$，解得$c=\dfrac{3}{\pi R^3}$
*(2).* 由题意知：$P=\iint_{D'}f(x,y)dxdy=\iint_{D'}\dfrac{3}{\pi R^3}(R-\sqrt{x^2+y^2})dxdy$，其中$D':x^2+y^2\le r^2$.
令$\rho^2=x^2+y^2$，$\theta=\arctan\dfrac{y}{x}$，则有：
$\int_0^{2\pi}d\theta\int_0^r\dfrac{3}{\pi R^3}(R-\rho)\rho d\rho=\dfrac{3r^2R-2r^3}{R^3}$