解 1.(1). 由于$P(X=k)=p(1-p)^{k-1}$, 则随机变量X的期望为:

$EX=\sum_{k=1}^{+\infty}kP(X=k)=\sum_{k=1}^{+\infty}kp(1-p)^{k-1}=p\sum_{k=1}^{+\infty}k(1-p)^{k-1}$.

记$S=\sum_{k=1}^{+\infty}k(1-p)^{k-1}$. 则有:

$(1-p)S=\sum_{k=1}^{+\infty}k(1-p)^{k}$

两式相减即得$S-(1-p)S=pS=\sum_{k=1}^{+\infty}(1-p)^{k-1}=\dfrac{1}{p}$. 故$EX=pS=\dfrac{1}{p}$

解 6. 由于随机变量X分布函数为$F(x)$, 则密度函数为$f(x)=(F(x))'=\dfrac{1}{2}\varphi(x)+\dfrac{1}{4}\varphi(\dfrac{x-4}{2})$

则随机变量X的数学期望为:

$$\begin{aligned}EX=\int_{-\infty}^{+\infty}xf(x)=\frac{1}{2}\int_{-\infty}^{+\infty}x\varPhi(x)dx+\frac{1}{4}\int_{-\infty}^{+\infty}x\varPhi\left(\frac{x-4}{2}dx\right)\\&=\frac{1}{4\sqrt{2\pi}}\left(2\int_{-\infty}^{+\infty}xe^{-x^2/2}dx+\int_{-\infty}^{+\infty}xe^{-(x-4)^2/8}dx\right)\\&=\frac{1}{4\sqrt{2\pi}}\left(0+8\sqrt{2\pi}\right)\\&=2\end{aligned}$$

解 7. 由概率密度函数的性质有$\int_{-\infty}^{+\infty}f(x)dx=1$. 即:

$$\int_{-\infty}^{+\infty}f(x)dx=\int_{-\infty}^{+\infty}ce^{-x^2+x}dx=c\int_{-\infty}^{+\infty}e^{-\left(x-\frac{1}{2}\right)^2}e^{\frac{1}{4}}d\left(x-\frac{1}{2}\right)=ce^{\frac{1}{4}}\sqrt{\pi}=1$$

解得: $c=\dfrac{1}{\sqrt{\pi}}e^{-\frac{1}{4}}$

随机变量X的期望为$EX=\int_{-\infty}^{+\infty}xf(x)$, 则:

$$\begin{aligned}EX&=\int_{-\infty}^{+\infty}xf(x)dx=c\int_{-\infty}^{+\infty}xe^{-x^2+x}dx\\&=ce^{\frac{1}{4}}\int_{-\infty}^{+\infty}xe^{-\left(x-\frac{1}{2}\right)^2}dx\xlongequal{t=x-\frac{1}{2}}ce^{\frac{1}{4}}\int_{-\infty}^{+\infty}\left(t+\frac{1}{2}\right)e^{-t^2}dt\\&=ce^{\frac{1}{4}}\frac{\sqrt{\pi}}{2}=\frac{1}{2}\end{aligned}$$

解 8.

$$EX=\int_{-\infty}^{+\infty}xf(x)dx=\int_{0}^{+\infty}xf(x)dx=\int_{0}^{+\infty}\frac{x^{n+1}e^{-x}}{n!}dx$$

记$I(n)=\int_{0}^{+\infty}\dfrac{x^{n+1}e^{-x}}{n!}dx$. 则:

$$I(n)=\int_{0}^{+\infty}\frac{x^{n+1}e^{-x}}{n!}dx=-\int_{0}^{+\infty}\frac{x^{n+1}}{n!}d(e^{-x})=-\frac{x^{n+1}e^{-x}}{n!}\Big|_0^{+\infty}+\frac{n+1}{n}\int_{0}^{+\infty}\frac{x^{n}e^{-x}}{(n-1)!}dx=\frac{n+1}{n}I(n-1)$$

故$EX=I(n)=\dfrac{n+1}{n}\dfrac{n}{n-1}\cdots\dfrac{2}{1}I(0)=(n+1)\int_{0}^{+\infty}xe^{-x}dx=n+1$

解 9.(1). 由于$X\sim f(x)$, $f(x)=\dfrac{x}{\sigma^2}\exp\left\{-\dfrac{x^2}{2\sigma^2}\right\}$, 则:

$$\begin{aligned}EX&=\int_{-\infty}^{+\infty}xf(x)dx=\int_{0}^{+\infty}\frac{x^2}{\sigma^2}\exp\left\{-\frac{x^2}{2\sigma^2}\right\}dx=-\int_{0}^{+\infty}xd\left(\exp\left\{-\frac{x^2}{2\sigma^2}\right\}\right)\\&=-x\exp\left\{-\frac{x^2}{2\sigma^2}\right\}\Big|_0^{+\infty}+\int_{0}^{+\infty}\exp\left\{-\frac{x^2}{2\sigma^2}\right\}dx=\sqrt{\frac{\pi}{2}}\sigma\end{aligned}$$

解 9.(3).

$EX = \int_{-\infty}^{+\infty} xf(x)dx = \int_0^{+\infty} k\left(\frac{x}{\lambda}\right)^k \exp\left\{-\left(\frac{x}{\lambda}\right)^k\right\} dx$

令$t = \frac{x}{\lambda}$, 则$dx = \lambda dt$, 有:

$EX = \int_0^{+\infty} k\left(\frac{x}{\lambda}\right)^k \exp\left\{-\left(\frac{x}{\lambda}\right)^k\right\} dx = k\lambda \int_0^{+\infty} t^k e^{-t^k} dt$

再令$\xi = t^k$, 则$dt = \frac{1}{k}\xi^{\frac{1-k}{k}} d\xi$, 有:

$EX = k\lambda \int_0^{+\infty} t^k e^{-t^k} dt = \lambda \int_0^{+\infty} \xi^{\frac{1}{k}} e^{-\xi} d\xi = \lambda \Gamma\left(1 + \frac{1}{k}\right) = \frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)$