5. Overall Percentage of relevant images

```r
# percentage of images supplied
p_s1 <- 0.15
p_s2 <- 0.2
p_s3 <- 0.25
p_s4 <- 0.40

# percentage of relevant images
p_ri1 <- 0.5
p_ri2 <- 0.6
p_ri3 <- 0.8
p_ri4 <- 0.85

# total percentage
p_t <- (p_s1*p_ri1) + (p_s2*p_ri2) + (p_s3*p_ri3) + (p_s4*p_ri4)
cat("The total percentage of relevant images is", round(p_t*100, 2), "%.")
```

```
## The total percentage of relevant images is 73.5 %.
```

6. A fair coin is tossed twice.

Let E1 be the event that both tosses have the same outcome, that is, E1=(HH, TT). Let E2 be the event that the first toss is ahead, that is, E2=(HH, HT). Let E3 be the event that the second toss is ahead, that is, E3=(TH, HH). Show that E1, E2, and E3 are pairwise independent but not mutually independent.

Let all events be S = (HH, TT, HT, TH).

E1 = (HH, TT), has a probability of 1/2 given all the events above and since the coin is fair.

Same with both E2 = (HH, HT) and E3 = (TH, HH) which both have a probability of 1/2, meaning all events have the same probability.

To show pairwise independence: $P(E1 \cap E2) = P(E1)P(E2)$, $P(E2 \cap E3) = P(E2)P(E3)$, and $P(E1 \cap E3) = P(E1)P(E3)$

and since $P(E1 \cap E2) = P(HH) = 1/4$, as shown with S = (HH, TT, HT, TH), $P(E2 \cap E3) = P(HH) = 1/4$, $P(E1 \cap E3) = P(HH) = 1/4$,

and $P(E1)P(E2) = 1/2 * 1/2 = 1/4$, $P(E2)P(E3) = 1/2 * 1/2 = 1/4$, $P(E1)P(E3) = 1/2 * 1/2 = 1/4$,

we can see that, indeed, $P(E1 \cap E2) = P(E1)P(E2) = 1/4$, $P(E2 \cap E3) = P(E2)P(E3) = 1/4$, $P(E1 \cap E3) = P(E1)P(E3) = 1/4$.

All pairwise events are equal to 1/4 and so they are proven to be pairwise independent.

As for their mutual independence, we must show that: $P(E1 \cap E2 \cap E3) = P(E1)P(E2)P(E3)$.

$P(E1 \cap E2 \cap E3)$ is P(HH) which is 1/4, so

$1/4 = 1/2 * 1/2 * 1/2$

and $1/4 \neq 1/8$.

While the events may be pairwise independent, since $P(E1 \cap E2 \cap E3)$ is not equal to $P(E1)P(E2)P(E3)$, they are not mutually independent.

Github Link: https://github.com/SylTana/APM1110-QUIJANO-JULIAN_PHILIP/tree/main/FA4