# CS 61A Summer 2025

# Trees, Python Lists

Discussion 4: July 8, 2025

## Getting Started

In this discussion, don't use a Python interpreter to run code until you are confident your solution is correct. Figure things out and check your work by *thinking* about what your code will do. Not sure? Draw an environment diagram!

## Lists

Some of you already know list operations that we haven't covered yet, such as `append`. Don't use those today. All you need are list literals (e.g., `[1, 2, 3]`), item selection (e.g., `s[0]`), list addition (e.g., `[1] + [2, 3]`), `len` (e.g., `len(s)`), and slicing (e.g., `s[1:]`). Use those! There will be plenty of time for other list operations when we introduce them later this week.

The most important thing to remember about lists is that a non-empty list `s` can be split into its first element `s[0]` and the rest of the list `s[1:]`.

```
>>> s = [2, 3, 6, 4]
>>> s[0]
2
>>> s[1:]
[3, 6, 4]
```

A list comprehension describes the elements in a list and evaluates to a new list containing those elements.

There are two forms:

```
[<expression> for <element> in <sequence>]
[<expression> for <element> in <sequence> if <conditional>]
```

Here's an example that starts with `[1, 2, 3, 4]`, picks out the even elements `2` and `4` using `if i % 2 == 0`, then squares each of these using `i*i`. The purpose of `for i` is to give a name to each element in `[1, 2, 3, 4]`.

```
>>> [i*i for i in [1, 2, 3, 4] if i % 2 == 0]
[4, 16]
```

This list comprehension evaluates to a list of:

- The value of `i*i`
- For each element `i` in the sequence `[1, 2, 3, 4]`
- For which `i % 2 == 0`

In other words, this list comprehension will create a new list that contains the square of every even element of the original list `[1, 2, 3, 4]`.

We can also rewrite a list comprehension as an equivalent `for` statement, such as for the example above:

```
>>> result = []
>>> for i in [1, 2, 3, 4]:
...     if i % 2 == 0:
...         result = result + [i*i]
>>> result
[4, 16]
```

### Q1: Even weighted

Write a function that takes a list `s` and returns a new list that keeps only the even-indexed elements of `s` and multiplies them by their corresponding index. First approach this problem with a normal `for` loop (without list comprehension).

```
def even_weighted_loop(s):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> even_weighted_loop(x)
    [0, 6, 20]
    """
    "*** YOUR CODE HERE ***"
```

Now that you've done it with a `for` loop, try it with a list comprehension!

```
def even_weighted_comprehension(s):
    """
    >>> x = [1, 2, 3, 4, 5, 6]
    >>> even_weighted_comprehension(x)
    [0, 6, 20]
    """
    return [_____________________________________________]
```

# Dictionaries

A dictionary is a Python data structure that maps *keys* to *values*. For a dictionary `d`:

- Each key maps to exactly one value, which you can access with `d[<key>]`.
- If you try `d[<key>]` but `<key>` is not a valid key in `d`, you'll get an error.
- You can use `<key> in d` to test (`True` or `False`) whether a key is in `d`.
- `d.get(<key>, <default>)` will return the value that `<key>` maps to in `d`, or `<default>` if `<key>` is not a key in `d`.
- The sequence of keys or values or key-value pairs can be accessed using `d.keys()` or `d.values()` or `d.items()`, respectively.
- Keys can be of any immutable type (like strings, numbers, and tuples) but not mutable types (like lists).
- You can use a `d` in a `for` loop (such as `for k in d`). Doing so will iterate through the keys in `d`.

### Q2: Happy Givers

In a certain discussion section, some people exchange gifts for the holiday season. We call two people **happy givers** if they give gifts to each other. Implement a function `happy_givers`, which takes in a `gifts` dictionary that maps people in the section to the person they gave a gift to. `happy_givers` outputs a list of all the happy givers in the section. The order of the list does not matter.

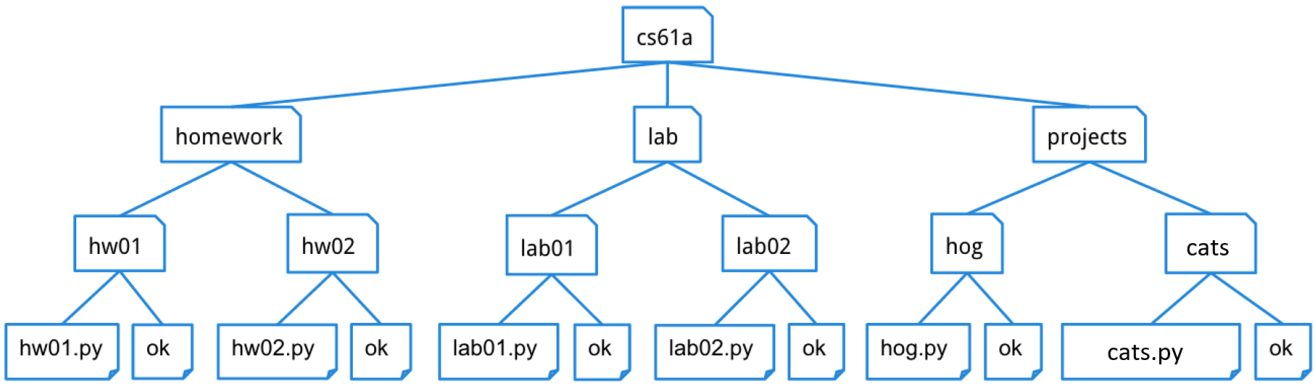
Note that if someone received but did not give a gift, they will not appear in the `gifts` dictionary as a key. (They'll appear only as a value.) You can assume that no one gives themself a gift.

Once you've found a solution, as a challenge, attempt to implement your solution in one line using a list comprehension.

```
def happy_givers(gifts):
    """
    >>> gift_recipients = {
    ...     "Alice": "Eve", # Alice gave a gift to Eve
    ...     "Bob": "Finn",
    ...     "Christina": "Alice",
    ...     "David": "Gina", # Gina is not a key because she didn't give anyone a gift
    ...     "Eve": "Alice",
    ...     "Finn": "Bob",
    ... }
    >>> list(sorted(happy_givers(gift_recipients))) # Order does not matter
    ['Alice', 'Bob', 'Eve', 'Finn']
    """
    "*** YOUR CODE HERE ***"
```

# Trees

A `tree` is a data structure that represents a hierarchy of information. A file system is a good example of a tree structure. For example, within your `cs61a` folder, you have folders separating your `projects`, `lab` assignments, and `homework`. The next level is folders that separate different assignments, `hw01`, `lab01`, `hog`, etc., and inside those are the files themselves, including the starter files and `ok`. Below is an incomplete diagram of what your `cs61a` directory might look like.



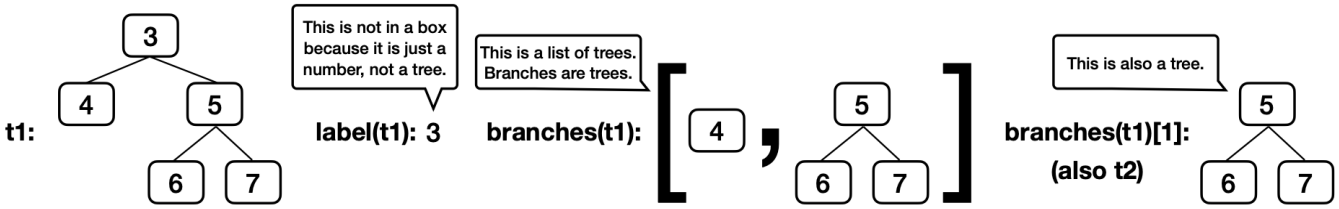

**cs61a_tree**

As you can see, unlike trees in nature, the tree abstract data type is drawn with the root at the top and the leaves at the bottom.

For a tree `t`: - Its root label can be any value, and `label(t)` returns it. - Its branches are trees, and `branches(t)` returns a list of branches. - An identical tree can be constructed with `tree(label(t), branches(t))`. - You can call functions that take trees as arguments, such as `is_leaf(t)`. - That's how you work with trees. No `t == x` or `t[0]` or `x in t` or `list(t)`, etc. - There's no way to change a tree (that doesn't violate an abstraction barrier).

Here's an example tree `t1`, for which its branch `branches(t1)[1]` is `t2`.

```
t2 = tree(5, [tree(6), tree(7)])
t1 = tree(3, [tree(4), t2])
```




**Example Tree**

A **path** is a sequence of trees in which each is the parent of the next.

You don't need to know how `tree`, `label`, and `branches` are implemented in order to use them correctly, but here is the implementation from lecture.

*Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.*

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch), 'branches must be trees'
    return [label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_leaf(tree):
    return not branches(tree)

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

### Q3: Maximum Path Sum

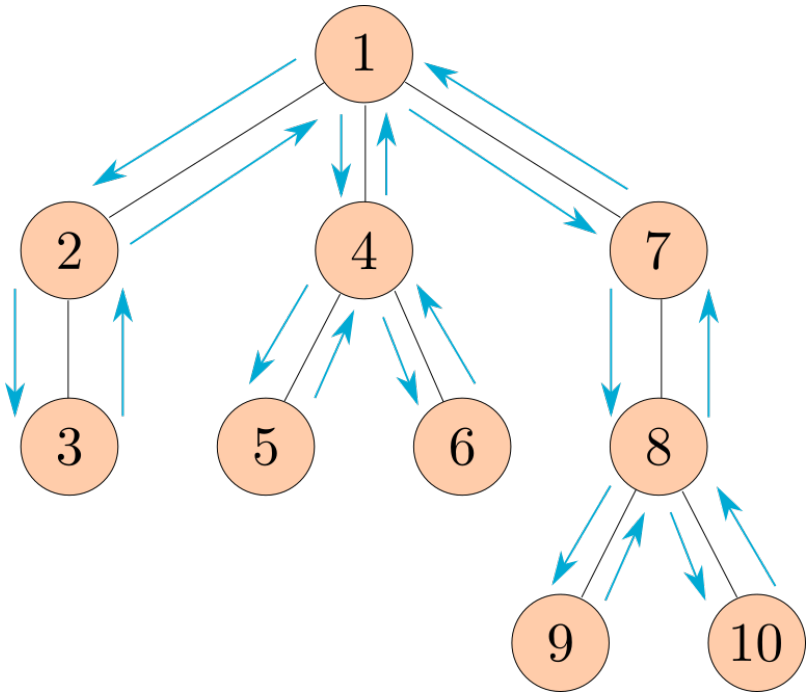
Write a function that takes in a tree and returns the maximum sum of the values along any path in the tree. Recall that a path is from the tree's root to any leaf.

```
def max_path_sum(t):
    """Return the maximum path sum of the tree.
    >>> t = tree(1, [tree(5, [tree(1), tree(3)]), tree(10)])
    >>> max_path_sum(t)
    11
    """
    "*** YOUR CODE HERE ***"
```

### Q4: Preorder

Define the function `preorder`, which takes in a tree as an argument and returns a list of all the entries in the tree in the order that `print_tree` would print them.

The following diagram shows the order that the nodes would get printed, with the arrows representing function calls.




**Preorder Traversal**

Note: This ordering of the nodes in a tree is called a preorder traversal.

```
def preorder(t):
    """Return a list of the entries in this tree in the order that they
    would be visited by a preorder traversal (see problem description).

    >>> numbers = tree(1, [tree(2), tree(3, [tree(4), tree(5)]), tree(6, [tree(7)])])
    >>> preorder(numbers)
    [1, 2, 3, 4, 5, 6, 7]
    >>> preorder(tree(2, [tree(4, [tree(6)])]))
    [2, 4, 6]
    """
    "*** YOUR CODE HERE ***"
```

### Q5: Find Path

Implement `find_path`, which takes a tree `t` with unique labels and a value `x`. It returns a list containing the labels of the nodes along a path from the root of `t` to a node labeled `x`.

If `x` is not a label in `t`, return `None`. Assume that the labels of `t` are unique.

```
def find_path(t, x):
    """
    >>> t2 = tree(5, [tree(6), tree(7)])
    >>> t1 = tree(3, [tree(4), t2])
    >>> find_path(t1, 5)
    [3, 5]
    >>> find_path(t1, 4)
    [3, 4]
    >>> find_path(t1, 6)
    [3, 5, 6]
    >>> find_path(t2, 6)
    [5, 6]
    >>> print(find_path(t1, 2))
    None
    """
    if ________________________________________:
        return ________________________________________
    ________________________________________:
        path = ________________________________________
        if path:
            return ________________________________________
    return None
```

You're done! Excellent work this week. Please be sure to fill out your TA's attendance form to get credit for this discussion!