CS5020: Nonlinear Optimisation: Theory and Algorithms
Coding Exercise - 2 (5 Marks)

(1) Let $f(x) = a|x - b| + c$.

(a) Apply gradient descent with diminishing step size to minimise $f(x)$.

(b) Demonstrate the problem for 3 different cases by choosing different a, b, c settings, i.e., case 1 : $a_1 > 0, b_1 > 0, c_1 > 0$, case 2 : $a_2 > 0, b_2 < 0, c_2 > 0$, case 3 : $a_3 > 0, b_3 > 0, c_3 < 0$.

(c) Plot the error $e_t = |x_t - x_*|$ versus iteration t. Plot $f(x_t)$ versus t.

Note: This question has total 3 demonstrations, and total of 6 plots (2 plots per demonstration).

(2) Let $f(x) = ax^2 + bx + c$.

(a) Apply gradient descent with constant size to minimise $f(x)$.

(b) Demonstrate the problem for 3 different cases by choosing different a, b, c settings, i.e., case 1 : $a_1 > 0, b_1 > 0, c_1 > 0$, case 2 : $a_2 > 0, b_2 < 0, c_2 > 0$, case 3 : $a_3 > 0, b_3 > 0, c_3 < 0$.

(c) In each of the 3 cases, choose step size α such that algorithm (i) converges from one side of the solution, (ii) oscillates on both sides of the solution, and (iii) diverges.

(d) Plot the error $e_t = |x_t - x_*|$ versus iteration t. Plot $f(x_t)$ versus t.

Note: This question has total $3 \times 3 = 9$ demonstrations, and total of 18 plots (2 plots per demonstration).

(3) Let $f(x) = x^4 + 0.5x^3 - 4x^2$.

(a) Apply gradient descent with constant size and heavy ball momentum to minimise $f(x)$.

(b) For a fixed step size α, choose 3 different momentum coefficients β such that algorithm (i) converges to local minima (ii) escapes local minima and converges to global minima and (iii) diverges.

(c) Plot the error $e_t = |x_t - x_*|$ versus iteration t. Plot $f(x_t)$ versus t.

Note: This question has total 3 demonstrations, and total of 6 plots (2 plots per demonstration).

(4) Let $f(x) = x^4 + 0.5x^3 - 4x^2$.

(a) Apply Newton method to minimise $f(x)$.

(b) Choose 2 different initial points such that algorithm (i) converges to local minima (ii) converges to global minima.

(c) Plot the error $e_t = |x_t - x_*|$ versus iteration t. Plot $f(x_t)$ versus t. Verify rate of convergence by plotting $\frac{e_t}{e_{t+1}^2}$ if it is quadratic rate (or $\frac{e_t}{e_{t+1}}$ if it is linear rate).

Note: This question has total 2 demonstrations, and total of 6 plots (3 plots per demonstration).