DEPARTMENT OF LEARNING STUFF

LEVEL 4 CLASS TEST

Statistics 101

7/11/2024

DURATION: 2 hours

Paper Instructions

There are **THREE** sections. Candidates should answer **ALL THREE** sections.

Each section is worth the same amount of marks in total, and a mark in square brackets is assigned to each part of a question.

Student is permitted to remove question paper from examination venue.

Do not turnover until instructed to do so.

SECTION 1

1. A random sample of size $n = 48$ is taken from a large population with known standard deviation $\sigma = 26$. If the sample mean is 74, calculate to 2 decimal places the following confidence intervals for the population mean μ:

   (a) The 90% confidence interval. **[3]**

   (b) The 95% confidence interval. **[3]**

   (c) The 99% confidence interval. **[3]**

2. A random sample of size $n = 25$ is taken from a large population. The sample mean is $\bar{x} = 84$, and the sample standard deviation is $s = 32$.

   (a) Using this information, construct a 95% confidence interval for the population mean μ. Enter the interval using square brackets, separating the two values with commas $[\ldots, \ldots]$: **[2]**

   (b) Suppose you are told that 32 is also the value of the population standard deviation σ. Construct a new 95% confidence interval and compare this with your previous calculation. **[3]**

3. A pair of dice is tossed 105 times and the sum is seven 20 times.

   (a) If the dice were fair, what would be the probability of the sum being seven? Round to 3 decimal places if required. **[3]**

   (b) We can reject the hypothesis that the dice are fair using a two tailed test at 5% significance. True or False? **[3]**

   (c) We can reject the hypothesis that the dice are fair using a one tailed test at 5% significance. True or False? Explain which type of test you think is more appropriate in this case. **[3]**

4. You are told that a random variable X has a population standard deviation of 14.

   (a) How large a sample n would you need in order to be able to estimate the population mean to within ± 3.3 at the 90% confidence level? Round your answer up to an integer. **[2]**

SECTION 2

1. A sample of 100 UK households have the following distribution:

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Household size	Frequency f_i
1	27
2	33
3	18
4	14
5	7
6	1

(a) Calculate the mean household size $\bar{x}$. Round your answer to two decimal places if needed.   **[3]**

(b) Calculate the median household size $\tilde{x}$.   **[2]**

(c) Calculate the mode.   **[2]**

(d) Calculate the sample standard deviation s, rounded to 2 decimal places.   **[3]**

2. You are told $P(A) = 0.34$, $P(B) = 0.71$ and $P(A \text{ or } B) = 0.84$. Are the following statements true or false? Provide an explanation in each case.

(a) A and B are independent.   **[3]**

(b) A and B are mutually exclusive.   **[3]**

(c) If two events that occur with non-zero probabilities are mutually exclusive, then they cannot be independent.   **[4]**

3. A ball is drawn from a box containing 11 red, 27 white, 21 blue and 17 orange balls. Find the probability (rounded to 2 decimal places if required) that it is:

(a) Orange or red.   **[1]**

(b) Not red and not blue.   **[1]**

(c) Not blue.   **[1]**

(d) White.   **[1]**

SECTION 3

1. Suppose 22% of customers at Aldi pay by cash. In a random sample of 13 customers, what is the probability (rounded to 4 decimal places) that:

(a) Two of the customers pay by cash?   **[2]**

(b) At least two of the customers pay by cash?   **[2]**

2. What is the probability, rounded to 4 decimal places:

   (a) Of getting exactly 7 heads and 1 tails in 8 tosses of a fair coin?   **[3]**

   (b) Of getting 2 sixes in 8 rolls of a fair dice?   **[3]**

3. If $Z \sim N(0, 1)$, find the critical value z_0, rounded to 2 decimal places, such that:

   (a) $P(Z \geq z_0) = 0.09$ **[2]**

   (b) $P(Z \leq z_0) = 0.25$ **[2]**

   (c) $P(-z_0 \leq Z \leq z_0) = 0.53$ **[2]**

4. Using the table of areas for the standard Normal distribution, calculate the areas under $N(0, 1)$, rounded to 3 decimal places:

   (a) Between -1.645 and 1.645   **[2]**

   (b) For values greater than 2.575   **[2]**

   (c) Between -1.25 and 1.25   **[2]**

   (d) Between 0.5 and 1.96.   **[2]**

End