Exact Template Attacks with Spectral Computation

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Plan

- Introduction;

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- Formal proof;

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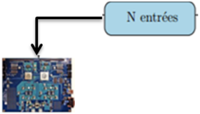
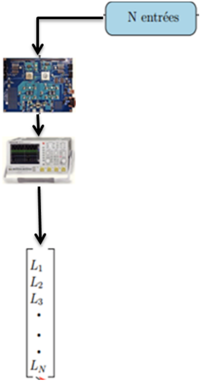
- Conclusion and perspectives.



Side-Channel attacks


N entrées
L_1
L_2
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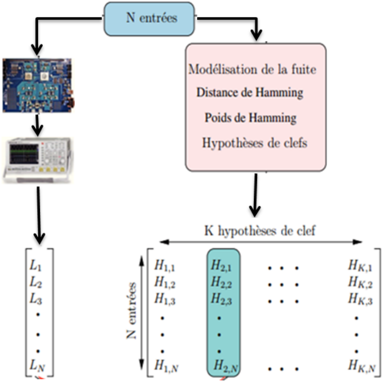
L_N



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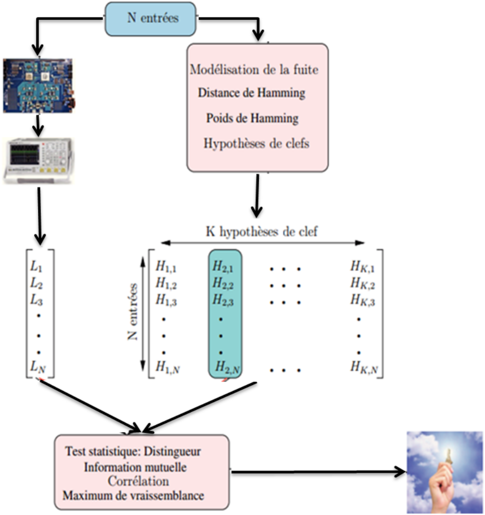

N entrées
Modélisation de la fuite
Distance de Hamming
Poids de Hamming
Hypothèses de clefs
K hypothèses de clef
N entrées
L_1 L_2 L_3 ... L_N
$H_{1,1}$ $H_{1,2}$ $H_{1,3}$ ... $H_{1,N}$
$H_{2,1}$ $H_{2,2}$ $H_{2,3}$ ... $H_{2,N}$
$H_{K,1}$ $H_{K,2}$ $H_{K,3}$ ... $H_{K,N}$
Test statistique: Distingueur
Information mutuelle
Corrélation
Maximum de vraissemblance

SCA

Introduction: Correlation Power Attack

The correlation between the real leakage M and the leakage model V applied to the key k is:

$$\rho_{M,V_k} = \frac{cov(M,V_k)}{\sigma_M.\sigma_{V_k}} = \frac{\langle M,V_k\rangle}{\|M\|\|V_k\|}$$

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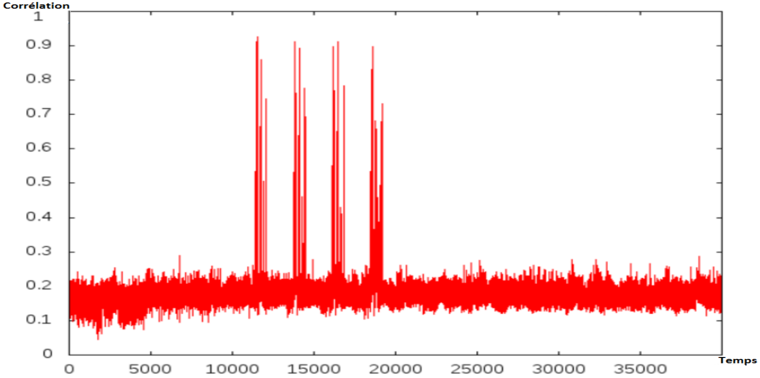

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$$T, k \to Z \to Y(Z) = Y \to X = Y + N.$$

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- Σ: the $D \times D$ covariance matrix of the noise.

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  - $N_q = X_q - Y_{q,k}$
  - $p_{N_q}(X_q - Y_{q,k}) = \frac{1}{\sqrt{(2\pi)^Q|\Sigma|}} e^{-\frac{1}{2}(X_q - Y_{q,k})^T \Sigma^{-1}(X_q - Y_{q,k})}$

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  - Since N is independent of Y_k:

$$p(X|Y_k) = p_N(X - Y_k) = \prod_q p_{N_q}(X_q - Y_{q,k})$$

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Theorem (Theorem 1 of [1])

Template attacks guess the key as:

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A scalability Problem!

Formal proof

the guessed key can be carried out by [5, 3]:

$$\hat{k} = \underset{k}{\operatorname{argmin}} \sum_{x=0}^{2^n-1} n_x(\tilde{L}_x - \tilde{M}_{x,k})^{\mathsf{T}} \Sigma^{-1} (\tilde{L}_x - \tilde{M}_{x,k}) \ . \tag{1}$$

Such that:

- n_x is the number of times the message x is involved,
- $\tilde{L}_x$ is the average trace over over all the traces corresponding to the same message x,
- $\tilde{M}_{x,k}$ is leakage model corresponding to the couple (x, k).
- $\Sigma = \frac{1}{N} L L^{\mathsf{T}} - \frac{1}{2^n} \tilde{M} \tilde{M}^{\mathsf{T}}$
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The attack (1) is more efficient in terms of computation, and memory space, than the theorem 1, as soon as the number of traces N is greater than the number of plaintexts involved in the leakage model (e.g. for AES it is $2^n = 256$).

Formal proof

To compute (1) without using the approximation by the LLN, contrary to the state of the art, one can consider.

Proposition (Exact Template Attack – Expression of the Maximum Likelihood Distinguisher)

$$\hat{k} = \underset{k}{\operatorname{argmin}} \sum_{x=0}^{2^n-1} n_x \tilde{M}_{x\oplus k}^{\mathsf{T}} \Sigma^{-1} \tilde{M}_{x\oplus k} - 2 \sum_{x=0}^{2^n-1} (n_x \tilde{L}_x^{\mathsf{T}})(\Sigma^{-1} \tilde{M}_{x\oplus k}) .$$

Spectral expression

Recalling that, for any pair of pseudo-Boolean functions f and g, we have:

$$\sum_{x=0}^{2^n-1} f(x).g(x \oplus k) = (f \otimes g)(k) = WHT(WHT(f) \bullet WHT(g))(k),$$

where

1. "$\bullet$" denotes the direct product between two pseudo-Boolean functions (that is, the term-to-term product),
2. "$\otimes$" denotes the convolution product between two pseudo-Boolean functions,
3. WHT denotes the Walsh-Hadamard Transform.

$$WHT(f)(u) = \sum_x (-1)^{u \cdot x} f(x).$$

Spectral expression

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\begin{aligned}
\hat{k} &= \underset{k}{\operatorname{argmin}} \sum_{x=0}^{2^n-1} n_x \tilde{M}_{x\oplus k}^{\mathsf{T}} \Sigma^{-1} \tilde{M}_{x\oplus k} - 2 \sum_{x=0}^{2^n-1} (n_x \tilde{L}_x^{\mathsf{T}})(\Sigma^{-1} \tilde{M}_{x\oplus k}) \\
&= \underset{k}{\operatorname{argmin}}\ n(.) \otimes \mathscr{M}(.)(k) - 2 \sum_{u=1}^{D} L_{cumul}[u] \otimes \tilde{\mathbb{M}}[u](k) \\
&= \underset{k}{\operatorname{argmin}}\ WHT\Big[WHT(n) \bullet WHT(\mathscr{M}) - 2 \sum_{u=1}^{D} WHT(L_{cumul}[u]) \bullet WHT(\tilde{\mathbb{M}}[u
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\end{aligned}
$$

So, we can carry out an exact template attack, by pre-processing $WHT(\mathscr{M})$, $WHT(\tilde{\mathbb{M}}[u])$ (for each u value), during the profiling phase, then guessing the key $\hat{k}$ accordingly.

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Results and experimental validation

- We employed raw traces from the SCA database (ASCAD) of the French National Agency for Information Systems Security (ANSSI) [2].
- The encryption algorithm target is a protected software implementation of AES running on an $\mathrm{ATMEGA\text{-}8515}$ μ-processor.

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- The target variable is $Z = \mathrm{SBox}(x[2] \oplus k[2])$.

Results and experimental validation: Success rate /#traces

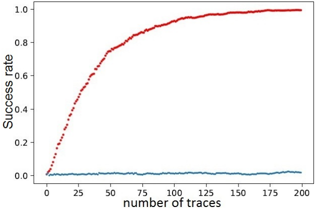

Figure: $D = 1$

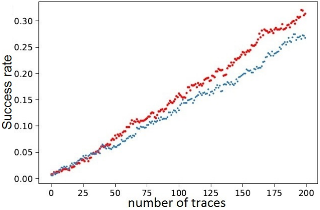

Figure: $D = 2$

Results and experimental validation: Success rate /#traces

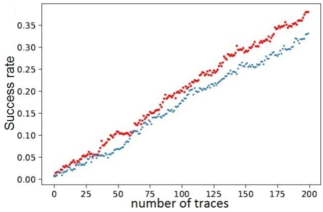

Figure: $D = 3$

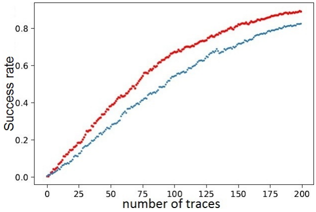

Figure: $D = 4$

Results and experimental validation: Success rate /#traces

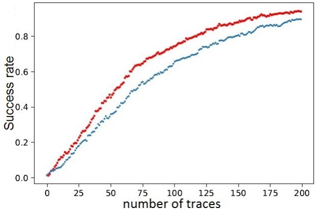

Figure: $D = 5$

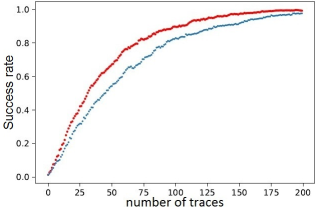

Figure: $D = 10$

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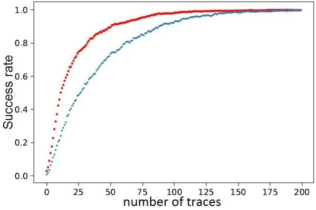


Figure: $D = 20$

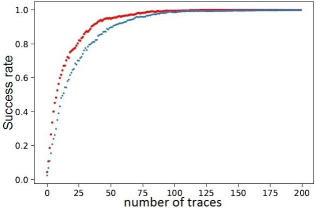
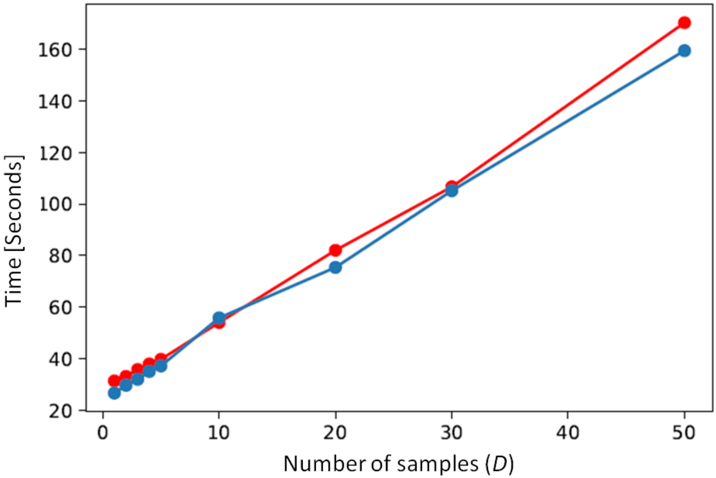


Figure: $D = 50$

Results and experimental validation: Computation time



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- Can be applied to any algorithms that involve SBox whose input is the *XOR*;
- A quasilinear instead of a quadratic time complexity (32*x* faster);
- Considerable gain in success rate comes at the expense of a marginal loss in computation time, which is explained in terms of complexity.

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- How these improvements behave with countermeasures,
- This approach should be extended to the Linear Regression Analysis (LRA) in [4].

Thank you for your attention

Nicolas Bruneau, Sylvain Guilley, Annelie Heuser, Damien Marion, and Olivier Rioul.
Optimal side-channel attacks for multivariate leakages and multiple models.
J. Cryptographic Engineering, 7(4):331–341, 2017.

Prouff Emmanuel, Strullu Remi, Benadjila Ryad, Cagli Eleonora, and Dumas Cecile.
Study of deep learning techniques for side-channel analysis and introduction to ascad database.
CoRR, pages 1–45, 2018.

Maamar Ouladj and Sylvain Guilley.
Side-Channel Analysis of Embedded Systems.
Springer, 2021.
ISBN: 978-3-030-77221-5.

Maamar Ouladj, Sylvain Guilley, and Emmanuel Prouff.
On the implementation efficiency of linear regression-based side-channel attacks.

In *Constructive Side-Channel Analysis and Secure Design - 11th International Workshop, COSADE 2020, Lugano, Switzerland, October 5-7, 2020, Proceedings (LNCS 12244)*, pages 147–172, 2020.

Maamar Ouladj, Nadia El Mrabet, Sylvain Guilley, Philippe Guillot, and Gilles Millérioux.
On the power of template attacks in highly multivariate context.
Journal of Cryptographic Engineering - JCEN, 2020.