Module MP(C)

Methods for predicting numerical variables

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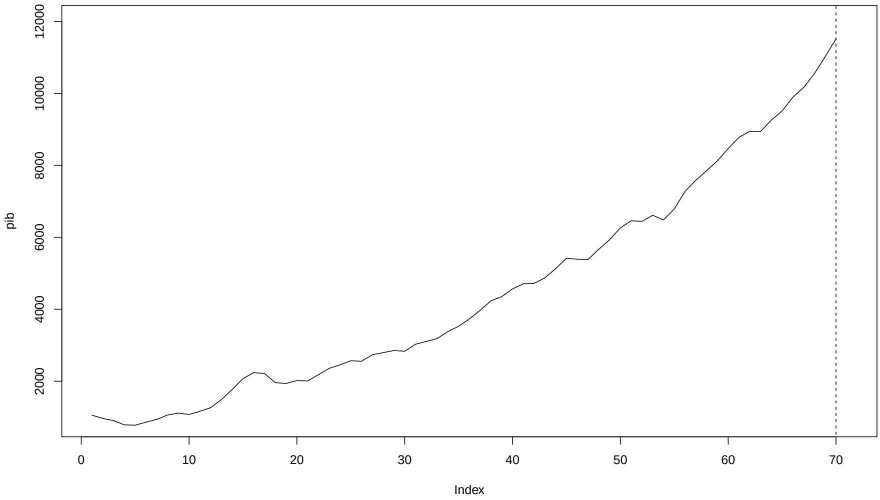
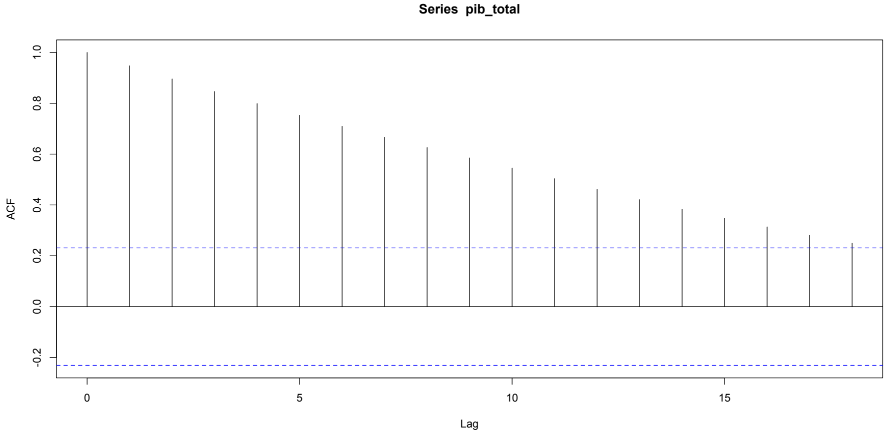
- Series with a trend but no seasonality
  - Context
  - Double Exponential Smoothing
  - Auto-regressive models
- Time series with a seasonal component
  - Context
  - Triple exponential smoothing
  - Auto-regressive models for seasonal time series

Series with a trend but no seasonality



Series with a trend but no seasonality



Series with a trend but no seasonality

Assumption : the series has a trend but no seasonality

$$x_t = f(t) + e_t,$$

with e_t the resisual component.

Problem : how to estimate $f(t)$?

The simplest case : $f(t) = a + b * t$. How to estimate a and b ?

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Double exponential smoothing (Holt's method)

At **every** time instant t, two parameters are estimated :

- a_t that represents an average level
- b_t that represents an average slope

As for the simple exponential smoothing, these parameters are estimated :

- taking into account all the past information
- giving more influence to recent values

Prediction :
if $x_1, \ldots, x_t$ is a time series,

$$\hat{x}_{t+h} = a_t + h \times b_t$$

Double exponential smoothing : update formulas

Update formulas for a_t and b_t:

$$\begin{cases} a_t & = \alpha\, x_t + (1-\alpha)(a_{t-1} + b_{t-1}) \\ b_t & = \beta(a_t - a_{t-1}) + (1-\beta) b_{t-1} \end{cases}$$

There are four parameters to initialize:

- α and $\beta \in [0, 1]$
- a_0 and b_0 initial level and slope

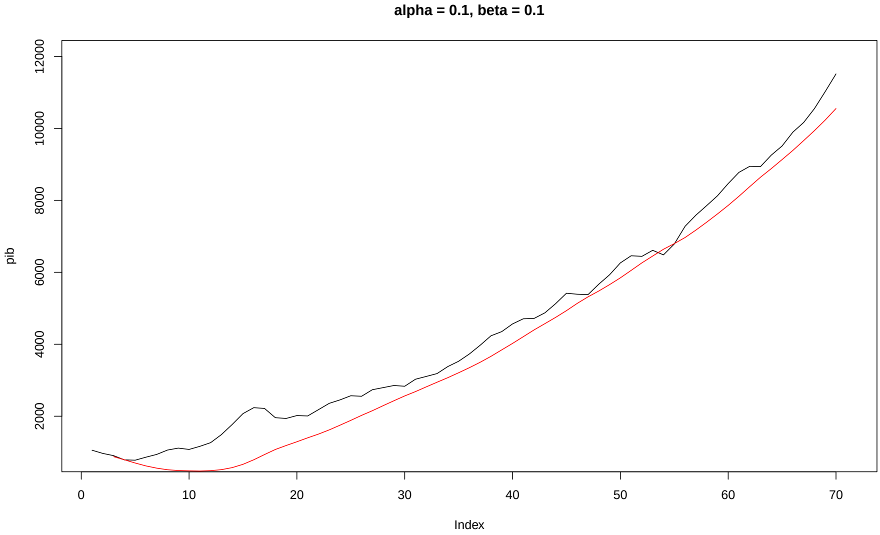
Double exponential smoothing: example

Impact of the parameters

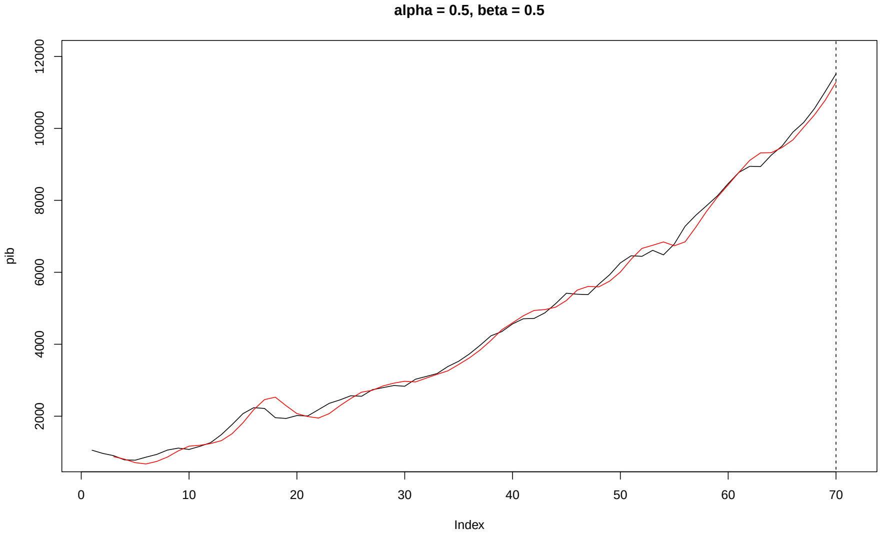
If α and β are small (close to 0), the predictions will be stable, but not very reactive to changes in the data.

Conversely, if α and β are big (close to 1), the predictions will be more fluctuating but also more reactive to changes

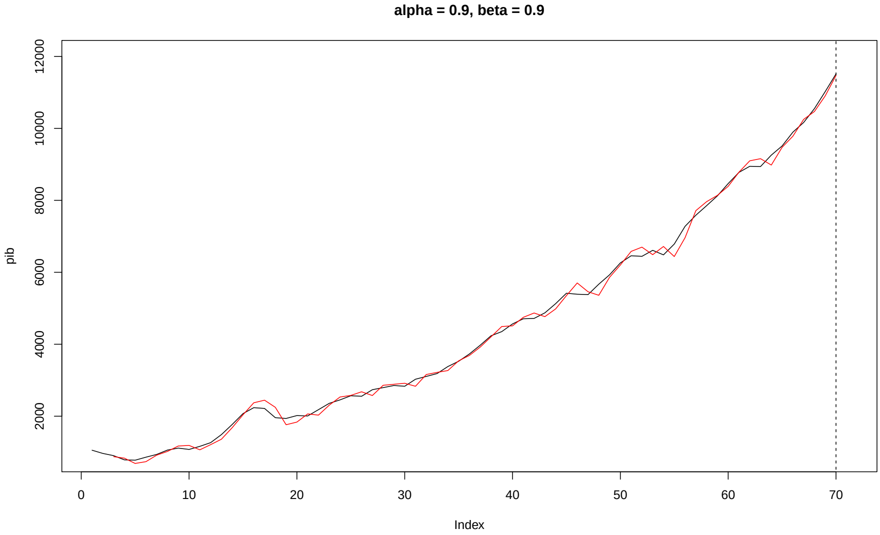
Double Exponential smoothing



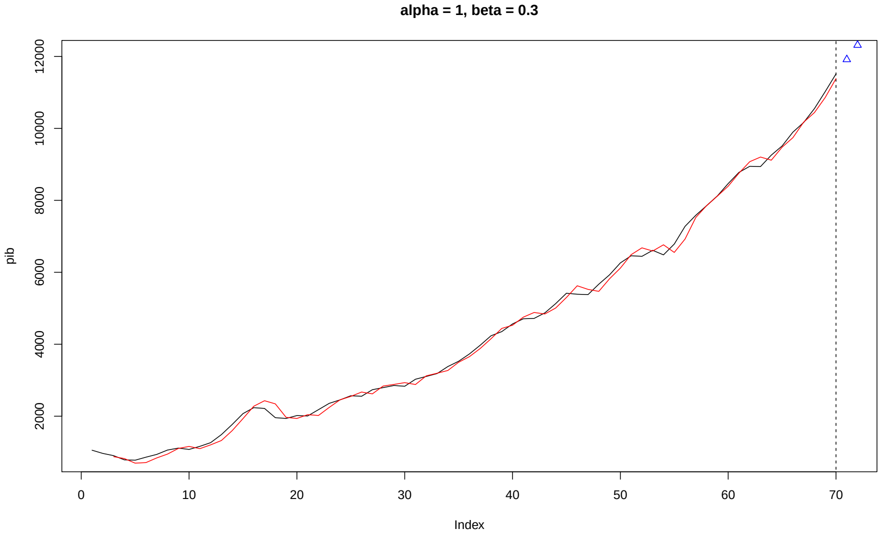
Double Exponential smoothing



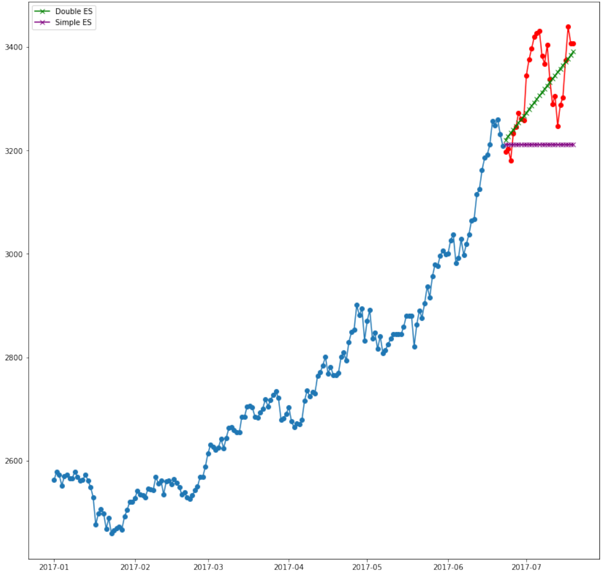
Double Exponential smoothing



Double Exponential smoothing



Double Exponential Smoothing



Choice of α and β

- Series with a trend but no seasonality
  - Context
  - Double Exponential Smoothing
  - Auto-regressive models
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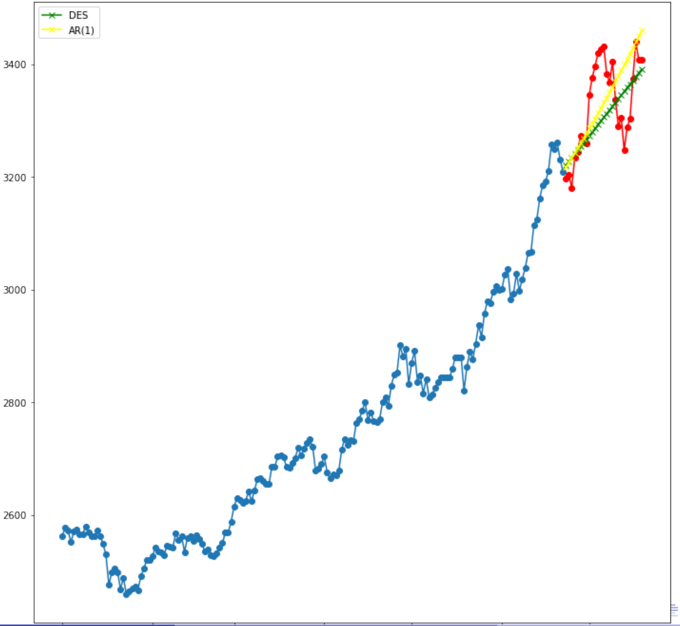
Auto-regressive models for series with trend

Classical AR models can be used for series with trend
Principle :

$$\hat{x}_n = \beta_0 + \sum_{k=1}^{p} \beta_k x_{n-k}$$

- p is the order of the model : AR(p)
- the coefficients β_j are estimated based on the original series

Auto-regressive models for series with trend

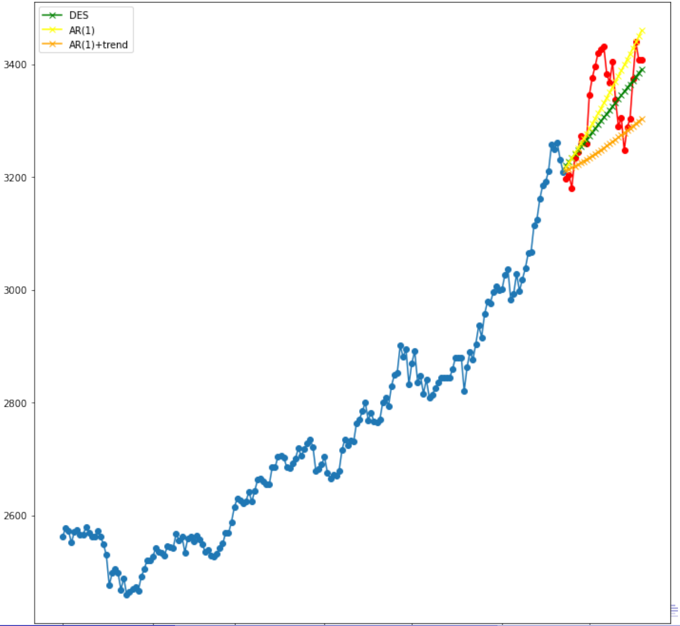


Auto-regressive models for series with trend

A new kind of auto-regressive model can be used for such a series :

$$\hat{x}_n = \beta_0 + b \times n + \sum_{k=1}^{p} \beta_k x_{n-k}$$

Auto-regressive models for series with trend



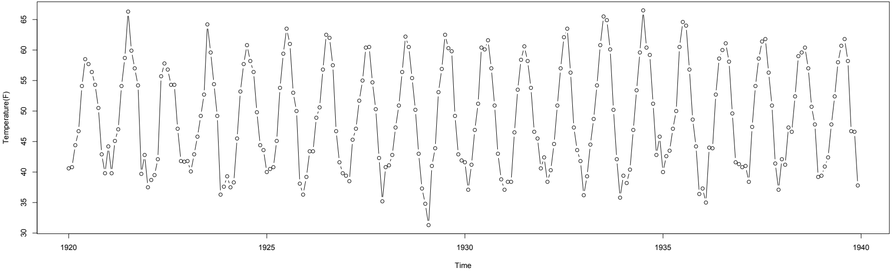
Summary

Methods for predicting a time series with a trend are :

- Double Exponential Smoothing
- Classical AR model
- AR model with a trend

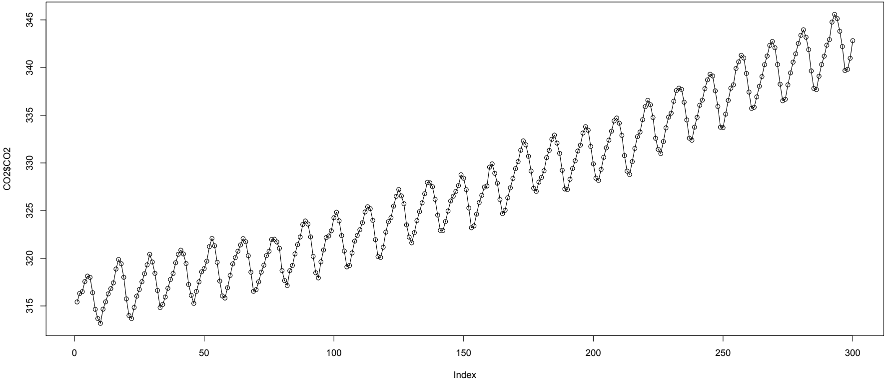
How to select the most adapted to a given time series ?

Example 1



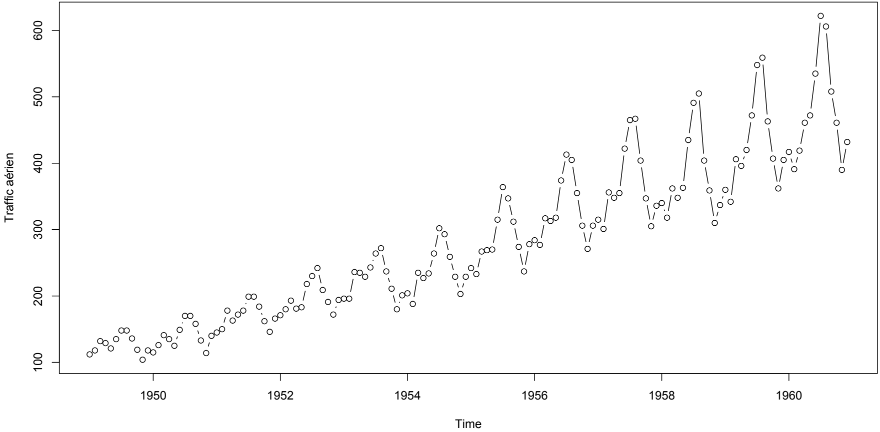
Temperature(F)
30
35
40
45
50
55
60
65
1920
1925
1930
1935
1940
Time

Example 2



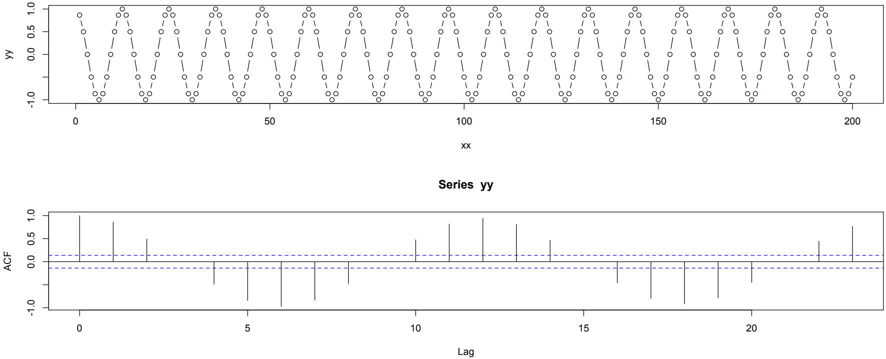
CO2$CO2
315
320
325
330
335
340
345
0
50
100
150
200
250
300
Index

Example 3

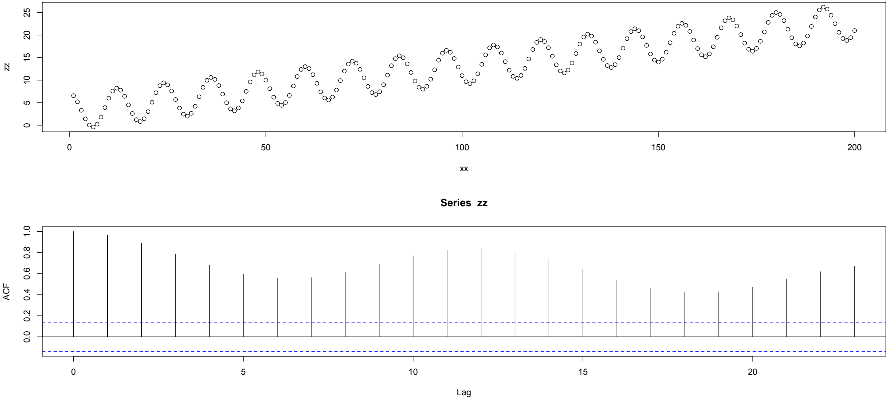


Traffic aérien
100
200
300
400
500
600
1950
1952
1954
1956
1958
1960
Time

Corresponding ACF



Corresponding ACF



Additive or multiplicative

Additive model

We assume that X_t can be written as

$$X_t = T_t + S_t + E_t$$

Multiplicative model

We assume that X_t can be written as

$$X_t = T_t * S_t * E_t$$

Time series with a seasonal component

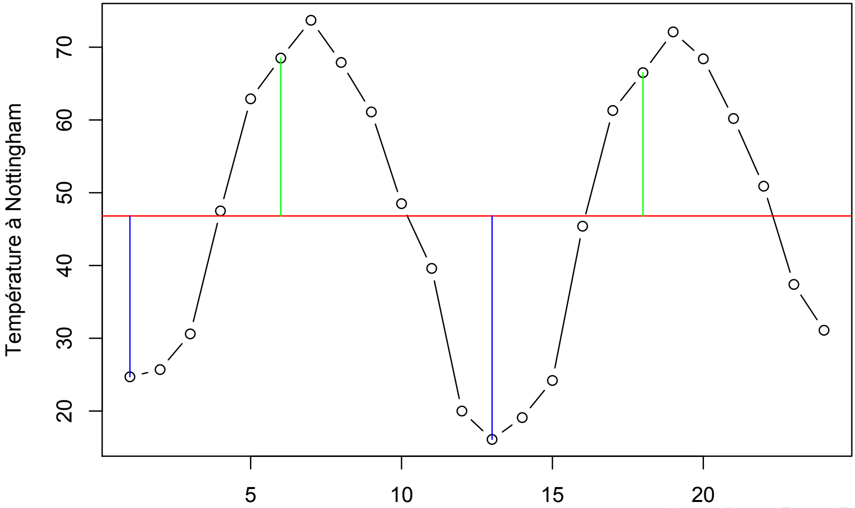
We assume here that the time series x_t has a seasonal component of period m (and possibly a trend)

$$x_t = f(t) + S_t + E_t$$

Objective : estimate $f(t)$ and S_t

"Shape" of S_t : m coefficients $s_1, \ldots, s_1$

Example



Time series with a seasonal component

Two main methods :

1. Triple Exponential Smoothing (additive or multiplicative)
2. Auto-regressive models

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Triple exponential smoothing

Principle : if $x_1, \dots, x_n$ is a time series with period m
At every time instant t, $m+2$ coefficients are learned :

- a a level coefficient
- b a slope coefficient
- $s_1, \dots, s_m$ the m seasonal coefficients

Prediction :

$$\hat{x}_{n+h} = a + h \times b + s_{1+(n+h-1)\,mod\,p}$$

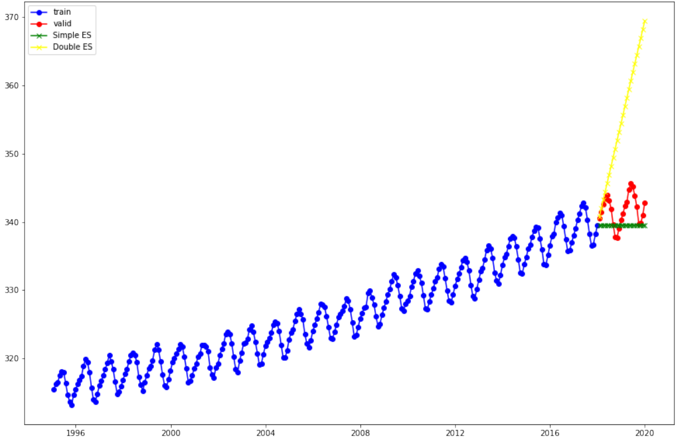
$$\hat{x}_{n+h} = (a + h \times b) \times (s_{1+(n+h-1)\,mod\,p})$$

The coefficients are learned iteratively, starting from initial values and updated at each time instant

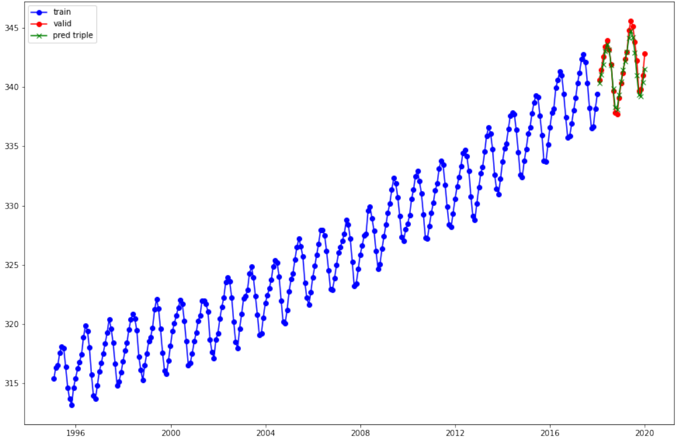
Triple exponential smoothing

Example :

Exponential smoothing



Exponential smoothing



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Auto-regressive models for seasonal time series

You can use the previous AR models or a new one:

- classical AR
- AR with a trend component
- seasonal AR (with trend or not)

AR or Triple ES for seasonal series in practice