1

(a).

In this problem $A = (1, 1, 1)$, $\nabla f = (e^{x_1}, 2e^{2x_2}, 2e^{2x_3})^T$ and $\nabla^2 f = diag\{e^{x_1}, 4e^{2x_2}, 4e^{2x_3}\}$ So the KKT system is

$$\begin{pmatrix} e^{x_1} & 0 & 0 & 1 \\ 0 & 4e^{2x_2} & 0 & 1 \\ 0 & 0 & 4e^{2x_3} & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ \lambda \end{pmatrix} = \begin{pmatrix} -e^{x_1} \\ -2e^{2x_2} \\ -2e^{2x_3} \\ 0 \end{pmatrix}$$

By solving the KKT system we get the Newton direction at $\boldsymbol{x} = (x_1, x_2, x_3)$ is

$$\boldsymbol{d} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} \frac{4e^{-x_1} - e^{-2x_2} - e^{-2x_3}}{4e^{-x_1} + e^{-2x_2} + e^{-2x_3}} \\ \frac{-4e^{-x_1} + 3e^{-2x_2} - e^{-2x_3}}{2(4e^{-x_1} + e^{-2x_2} + e^{-2x_3})} \\ \frac{-4e^{-x_1} - e^{-2x_2} + 3e^{-2x_3}}{2(4e^{-x_1} + e^{-2x_2} + e^{-2x_3})} \end{pmatrix}$$

(b).

I will directly paste the outputs which show the process of iteration below.
iteration 0: [0. 1. 0.]
iteration 1: [0.55783402 0.55270748 -0.1105415]
iteration 2: [0.74171111 0.22388047 0.03440841]
iteration 3: [0.83735858 0.09139269 0.07124873]
iteration 4: [0.8464719 0.07685719 0.07667091]
iteration 5: [0.84657358 0.07671322 0.0767132]
iteration 6: [0.84657359 0.0767132 0.0767132]
optimal value: 4.663287963194248
This result is the same as what we have shown in the previous homework.

2

(a).

By using the Log Barrier function, the approximating equality constrained problem is

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \boldsymbol{c}^T\boldsymbol{x} - \frac{1}{t}\sum_{i=1}^{n} log(x_i)$$

$$s.t. \qquad \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$$

(b).

$$\nabla f = \boldsymbol{c} - \frac{1}{t}\begin{pmatrix} \frac{1}{x_1} \\ \frac{1}{x_2} \\ \frac{1}{x_3} \\ \cdots \\ \frac{1}{x_n} \end{pmatrix}, \nabla^2 f = \frac{1}{t} diag\{\frac{1}{x_1^2}, \frac{1}{x_2^2}, \frac{1}{x_3^2}, \dots, \frac{1}{x_n^2}\}$$

(c).

The implementation can be found in file LP.py

(d).

By introducing two variables we get the standard form of this LP.

$$\min_{(x_1,x_2,s_1,s_2)\in\mathbb{R}^4} -x_1 - 3x_2$$

$$s.t. \quad x_1 + x_2 + s_1 = 6$$

$$-x_1 + 2x_2 + s_2 = 8$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Then we solve it. The output is
iteration 0: [2 1 3 8]
iteration 1: [1.63310136 3.90397091 0.46292773 1.82515955]
iteration 2: [1.34599628 4.59596597 0.05803775 0.15406434]
iteration 3: [1.33436048 4.65965606 0.00598345 0.01504836]

iteration 4: [1.33343377e+00 4.66596530e+00 6.00928006e-04 1.50318063e-03]
iteration 5: [1.33334340e+00 4.66659622e+00 6.03805159e-05 1.50958601e-04]
iteration 6: [1.33333435e+00 4.66665953e+00 6.11807327e-06 1.52952233e-05]
iteration 7: [1.33333344e+00 4.66666592e+00 6.35922351e-07 1.58980583e-06]
iteration 8: [1.33333335e+00 4.66666658e+00 7.10430528e-08 1.77607681e-07]
iteration 9: [1.33333333e+00 4.66666666e+00 1.84101098e-09 4.60247546e-09]
optimal value: -15.333333327196652

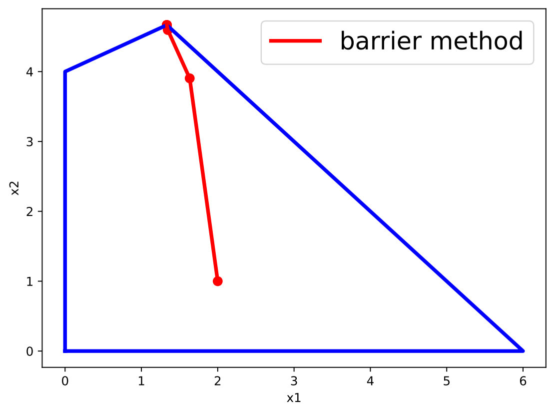

Figure 1: trajectories

We can see the optimal variable is $(x_1^*, x_2^*, s_1^*, s_2^*) = (\frac{4}{3}, \frac{14}{3}, 0, 0)$, optimal value is -15.3333.

3

(a).

$$\begin{aligned} \max_{\boldsymbol{\mu}\in\mathbb{R}^4} \quad & -6\mu_1 - 8\mu_2 \\ s.t. - \mu_1 + \mu_2 + \mu_3 &= -1 \\ -\mu_1 - 2\mu_2 + \mu_4 &= -3 \\ \mu_1, \mu_2, \mu_3, \mu_4 &\geq 0 \end{aligned}$$

(b).

$$\begin{aligned} \max_{\boldsymbol{\mu}\in\mathbb{R}^2} \quad & -6\mu_1 - 8\mu_2 \\ s.t. - \mu_1 + \mu_2 &\leq -1 \\ -\mu_1 - 2\mu_2 &\leq -3 \\ \mu_1, \mu_2 &\geq 0 \end{aligned}$$

(c).

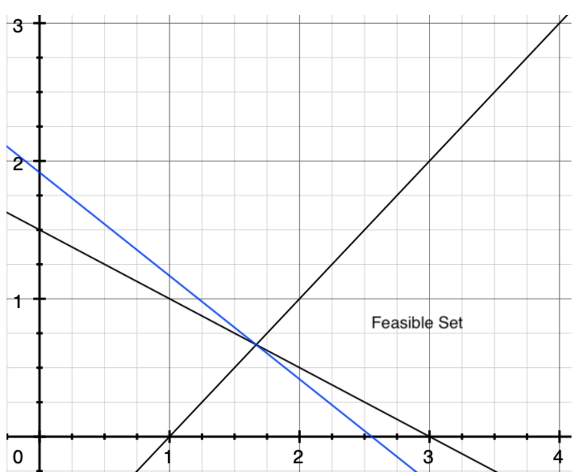

Figure 2: graphical solution

In the figure above the optimal solution is the intersection of the blue line with the feasible set. The dual optimal solution is $(\mu_1^*, \mu_2^*) = (\frac{5}{3}, \frac{2}{3})$, the dual optimal value and the primal optimal value are both $-\frac{46}{3}$ or approximately -15.33.

(d).

iteration 0: [4. 1. 2. 3.]
iteration 1: [2.1603025 0.54793137 0.61237113 0.25616523]
iteration 2: [1.72345017 0.6491511 0.07429907 0.02175236]
iteration 3: [1.67238171 0.66488279 0.00749892 0.0021473]
iteration 4: [1.66723914e+00 6.66487789e-01 7.51348892e-04 2.14716385e-04]
iteration 5: [1.66672417e+00 6.66648696e-01 7.54772486e-05 2.15653129e-05]
iteration 6: [1.66667249e+00 6.66664846e-01 7.64760038e-06 2.18503079e-06]
iteration 7: [1.66666727e+00 6.66666477e-01 7.94902929e-07 2.27115120e-07]
iteration 8: [1.66666673e+00 6.66666646e-01 8.88038268e-08 2.53725245e-08]
iteration 9: [1.66666667e+00 6.66666666e-01 2.30125234e-09 6.57497929e-10]
dual optimal value: -15.333333339470014
We can see the dual optimal solution is $\boldsymbol{\mu}^* = (\frac{5}{3}, \frac{2}{3}, 0, 0)$ with dual optimal value -15.3333