Homework 1

1 Written Assignment

a

The shape of the image will still be a circular disk, with different radius.

Explanations: We use the same setting as our lecture notes, i.e. $\boldsymbol{r_o} = (x_o, y_o, z_o)$ denotes the coordinate of actual object pixel and $\boldsymbol{r_i} = (x_i, y_i, f)$ denotes its image. We set the pinhole $(x, y, z) = (0, 0, 0)$ and let the optical axis be the z-axis. Now we prove our argument by deriving the equation of the image of the disk.

The equation of the circular disk in original space can be described as $(x_o - a)^2 + (y_o - b)^2 = c^2$ where (a, b) is its center and c is its radius. Then by applying the rule of similar triangles we get $(x_i, y_i) = (\frac{fx_o}{z_o}, \frac{fy_o}{z_o})$, then by substituting the coordinates we get the equation of image disk as $(\frac{z_o x_i}{f} - a)^2 + (\frac{z_o y_i}{f} - b)^2 = c^2$, or equivalently

$$(x_i - \frac{af}{z_o})^2 + (y_i - \frac{bf}{z_o}) = (\frac{cf}{z_o})^2$$

which is still a circular disk in image plane.

b

Case 1: plane $y = 0$

The direction vectors on this plane can be expressed generally as $(x_i, 0, z_i)$. Then the vanishing points of the family of lines represented by these family of vectors can be expressed as $(f\frac{x_i}{z_i}, 0)$ by our formula from lecture slides. Then we know the vanishing points $(f\frac{x_i}{z_i})$ actual lie on line $y = 0$ in the image plane.

Case 2: plane $x = 0$

Now the direction vectors are $(0, y_i, z_i)$ and the vanishing points are $(0, f\frac{y_i}{z_i})$. This time the vanishing points lie on the line $x = 0$ in the image plane.

c

Generally, the normal vector of plane $Ax + By + Cz + D = 0$ is (A, B, C), thus any direction vector (x_i, y_i, z_i) that lies in this plane must satisfy that $Ax_i + By_i + Cz_i = 0$. Then we know that the vanishing point of line (x_i, y_i, z_i) is $(f\frac{x_i}{z_i}, f\frac{y_i}{z_i})$. (Note that z_i cannot be zero, otherwise there will not be a vanishing point for these lines parallel to the image plane)

These vanishing points $(f\frac{x_i}{z_i}, f\frac{y_i}{z_i})$ actually lie on line

$$Ax + By + Cf = 0$$

2 Programming Assignment Documentation and Results