Strategy Backtest

Reading Notes of Efficiently Inefficient #Ch3

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[1] Lasse Heje Pedersen, 2015. "Efficiently Inefficient: How Smart Money Invests and Market Prices Are Determined," Economics Books, Princeton University Press, edition 1, number 10441.

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Profit Sources

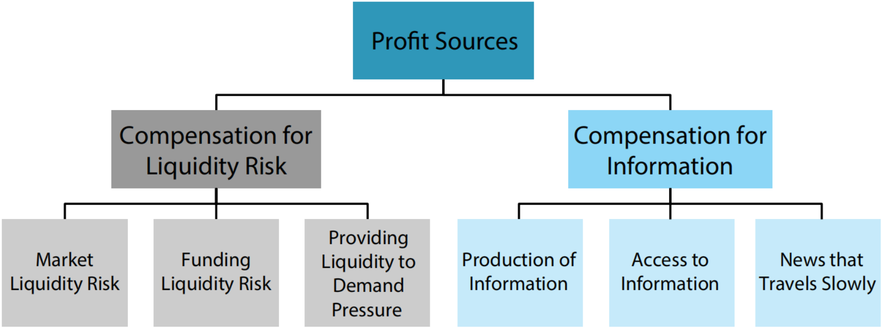

Figure 3.1. The main sources of profit for hedge fund strategies.

Compensation for Information

The information contained in market prices must be efficiently inefficient, reflecting enough information to make it difficult to make money, but not so efficient that no one wants to collect information and trade on it (Grossman and Stiglitz 1980).

Why do these effects arise?

1. Some investors (“noise traders”) suffer from **behavioral biases** and make common mistakes that push prices away from fundamentals.
    - They are partly corrected, but only partly due to the limits of arbitrage (Shleifer and Vishny 1997, Shleifer 2000).
2. In the real world all “arbitrage” trades are risky, so **arbitrage** occurs only to a **limited extent**.
    - Limited by its costs and risks, arbitrage trading does not completely eliminate any mispricing; an efficiently inefficient level of mispricing persists.

Real-world Arbitrage Limits

1. Arbitrage is often subject to **fundamental risk**.
   - If a hedge fund buys a cheap security, say, an undervalued oil company, there is still a risk that the security will underperform due to a random event (the CEO dies in a car accident or the oil rig blows up).
2. Arbitrage is subject to **noise trader risk**.
   - If a hedge fund buys a cheap security, it might become even cheaper before the price approaches the fundamental value (De Long, Shleifer, Summers, and Waldmann 1993). This situation leads to short-term losses for the hedge fund (even if the trade was "right" ex ante), which can lead to capital redemptions, and the fund may not live to see the upside.
3. Hedge funds may try to **ride a bubble** rather than trading against it.
   - As George Soros has done during the Internet bubble—especially when the hedge fund thinks that other smart investors will delay trading against the mispricing and "pop" the bubble (Abreu and Brunnermeier 2003 and Brunnermeier and Nagel 2004). Because of these risks and the liquidity risks discussed next, hedge funds limit the size of the positions that they take and therefore competing arbitrageurs may not completely eliminate mispricings.

New Trading Ideas Hunting

When you are looking for new cool trading ideas, think about whether there is:

- **information** that most investors overlook
- new ways to **combine** various sources of information
- a smart way to get the information **fast**
- what type of information is **not fully reflected** in the price because of limited arbitrage.

Liquidity Risk

Liquidity risk consists of

- the risk of rising transaction costs (market liquidity risk);
- the risk of running out of cash, especially for a leveraged hedge fund (funding liquidity risk);
- the risk of accommodating demand pressure.

Liquidity risk is not just a limit of arbitrage, however; it affects market prices directly as it creates a liquidity risk premium.
Said differently, the pricing of liquidity risk is a natural component of an efficiently inefficient market (whereas other limits of arbitrage only matter if some noise traders push the prices out of line in the first place).

Market Liquidity Risk

The risk that you cannot get out or that you will have to pay large transaction costs is called market liquidity risk.

- Investors want to be compensated for taking market liquidity risk, and therefore illiquid securities are cheap and earn higher average gross returns.

Liquidity-adjusted CAPM

$$\mathbb{E}(R^i - \mathrm{TC}^i) = R^f + \beta^i \lambda \tag{1}$$

- R^i is a security i's return.
- TC^i is a security i's transaction cost.
- is the risk premium.
- β^i is the security's covariance with the net return of the overall market M:

$$\beta^i = \frac{\mathrm{cov}(R^i - \mathrm{TC}^i, R^M - \mathrm{TC}^M)}{\mathrm{var}(R^M - \mathrm{TC}^M)}$$

Liquidity-adjusted CAPM (continued)

Gross returns are determined by

$$\mathbb{E}(R^i) = R^f + \mathbb{E}(\mathrm{TC}^i) + (\beta^{R^i,R^M} + \beta^{\mathrm{TC}^i,\mathrm{TC}^M} - \beta^{\mathrm{TC}^i,R^M} - \beta^{R^i,\mathrm{TC}^M})\lambda \quad (2)$$

- $(\beta^{R^i,R^M}$ is the standard market beta, which depends on the covariance between the security's own return and the market return.
- $\beta^{\mathrm{TC}^i,\mathrm{TC}^M}$ means that investors require a higher return for a security with more commonality in liquidity. In other words, traders do not like to hold a security whose liquidity dries up when one cannot trade many other things.
- $\beta^{\mathrm{TC}^i,R^M}$ implies that investors should be compensated for holding a security that becomes illiquid when the market falls, because this is often when you really need the money.
- $\beta^{R^i,\mathrm{TC}^M}$ implies that investors want to be compensated for holding securities that drop in value during a liquidity crisis.

Funding Liquidity Risk

The risk that a hedge fund cannot fund the position throughout the life of the trade.

Said differently, it is the risk of being forced to unwind positions as the fund hits a margin constraint or gets uncomfortably close.

CAPM Adjustment

$$\mathbb{E}(R^i) = R^f + \beta^i \lambda + m^i \psi \tag{3}$$

- m^i is the margin requirement of security i.
- ψ is the compensation for tying up capital.

Another implication for funding constraints and leverage constraints is that many investors prefer to buy risky securities over applying leverage to safer securities. This implication can help explain why riskier securities tend to offer lower risk-adjusted returns than safer ones within each asset class.

Combination of Market and Funding Liquidity Risk

$$
\begin{aligned}
\mathbb{E}(R^i) = & R^f + \beta^i \lambda + \text{market liquidity risk compensation} \\
& + \text{funding liquidity risk compensation}
\end{aligned} \tag{4}
$$

Market and funding liquidity risks are interconnected: Securities with high transaction costs and market liquidity risk tend to also be difficult to finance, and vice versa.

Providing Liquidity to Demand Pressure

Alpha source: The tendency of hedge funds to profit by providing liquidity to demand pressure.
For example, if a security faces unusual buying pressure, its price is elevated, which means that its future expected return is abnormally low. Other securities may be abandoned, leading to low prices and high expected returns. In such situations, a contrarian trading strategy becomes profitable, effectively providing liquidity by buying low and selling high.

Demand Pressure Sources

1. **Corporate Events**: such as a merger.
2. **Hedging Demand**: such as hedging a put option, hedge production risk in commodity markets.
3. **Institutional Frictions**: such as bond downgrading, futures contract change near the expiry.
4. **Behavioral Biases**: such as active interaction with customers of certain companies.

Running a Backtest: Trading Rules and Beyond

A backtest can teach you about the risk of a strategy, and it can give you ideas about how to improve it. Components

- **Universe**: The universe of securities to be traded.
- **Signals**: The data used as input, the source of the data, and how the data are analyzed.
- **Trading rule**: How you trade on your signals, including how frequently you review them and rebalance your positions, and the sizes of your positions.
- **Time lags**: To make the strategy implementable, the data used as input must have been available at the time it is used.
  For instance, if you use the gross domestic product (GDP) for any year, you must account for the fact that this number was not available on January 1 the following year; it is released with a delay. Also, if you use the closing price as a signal, it is not realistic to assume that you can trade at that same closing price (although academics often do). It is more prudent to assume that the trade is put on with a time lag, for example, using the closing price one or two days later.

Trading Rule Types

1. **Portfolio rebalance rule**: This trading rule looks at the entire portfolio of securities and defines how it is rebalanced. This trading rule is backtested as follows. For each time period,
   - Determine the optimal portfolio of securities.
   - Make a (paper) trade to rebalance to this portfolio.
2. **Enter–exit trading rule**: Another approach is to think in terms of discrete trades:
   - For each asset, determine when to enter a new trade and how to size the initial position.
   - Determine how the position is resized over time, depending on the circumstances.
   - Determine when to exit the trade.

Data Mining and Biases

Backtests typically look a lot better than the real-world trading performance that is realized after the trade is put on. Why?

1. The world is changing and trading strategies that worked in the past may no longer work as well.
   This could be because more people are pursuing these strategies and the competitive pressure adjusts prices and reduces profitability.
2. All backtests suffer from data mining biases (unavoidable).
   These unavoidable biases mean that we should discount backtest returns and place more weight on realized returns. Furthermore, we should discount backtests more if they have more inputs and have been tweaked or optimized more.

Example of Bias (I) - Biased Universe of Securities

For instance, if you only consider the current stocks in the Standard & Poor's (S&P) 500 index, then you have a biased sample. You did not know 15 years ago which stocks would be included today. Stocks often get included because they performed well, and you could not have known 15 years ago which stocks would perform well enough to be included. If you want to use the S&P 500 stocks, you should use stocks that were in the index at the time of your backtest, just as you would have done if you did the trade back then.

Example of Bias (II) - Biased Trading Signals and Rules

You need appropriate time lags. For instance, many announcements happen after the event they are describing (first-quarter earnings are reported some time in the second quarter, and macroeconomic numbers such as the GDP and inflation arrive with a delay), and revisions of these numbers are known even later.

Example of Bias (III) - Optimized Parameters

When parameters have been optimized or estimated, this naturally creates a bias.

For instance, if expected returns have been estimated by running a regression from 1990 to 2010, and you backtest a strategy based on these parameters over the same time period, then the performance is biased to look unrealistically good. The parameters were estimated to be optimal, but you could not have known this in advance, nor do you currently know the parameters that will be optimal in the future.

Example of Bias (III) - Optimized Parameters (continued)

This cheating method is called an **in-sample test**, in contrast to an **out-of-sample test**, where the parameters are estimated using data from before the simulated trading time. Out-of-sample tests are carried out in many ways:

1. Split the sample in two, pick parameters using the first sample, and simulate trading using the second.
2. Use a "rolling" window: Each time period (say, each month), you pick parameters based on older data; you then simulate the trading over the next month, pick new parameters based on the now longer window of older data, simulate the next month of trading, and so on.
3. Of course, the easiest way to avoid in-sample biases is to have a strategy that is simple enough that it does not rely on specific parameter values.

Takeaways

You should always keep in mind that the goal is to find a strategy that works in the future and not to have the best possible backtest. You should strive for a robust process that works even if you adjust it a little.

Adjusting Backtests for Trading Costs

1. Compute the return on the portfolio,
2. Compute the new security positions and the implied trades,
3. Compute the expected trading costs for every security and add them up, and
4. Subtract the total expected trading cost from the portfolio return.

Metatheorem

Any predictive regression can be expressed as a portfolio sort, and any portfolio sort can be expressed as a predictive regression. Specifically:

1. A time series regression corresponds to a market timing strategy.
2. A cross-sectional regression corresponds to a security selection strategy.
3. A univariate regression corresponds to sorting securities by one signal; a bivariate regression corresponds to double-sorting securities by two signals, allowing you to determine whether one signal adds value beyond the other; and a multivariate regression corresponds to sorting by multiple signals.

Time Series Regression

A time series regression corresponds to a market timing strategy.

Consider first a time series regression of the excess return R^e of one security, say, the overall stock market, on a forecasting variable F, say, the dividend-to-price ratio:

$$R^e_{t+1} = a + bF_t + \varepsilon_{t+1} \tag{5}$$

OLS estimates:

$$\hat{b} = \frac{\sum_t (F_t - \bar{F}) R_{t+1}}{\sum_t (F_t - \bar{F})^2} = \sum_{t=1}^{T} x_t R_{t+1} \tag{6}$$

which can be seen as the cumulative return on a **long–short timing strategy**, where the trading position x is given by

$$x_t = k(F_t - \bar{F}) \qquad k = \frac{1}{\sum_t (F_t - \bar{F})^2}$$

Time Series Regression (continued)

The scaling factor k does not affect the Sharpe ratio of the timing trade.

$$x_t = k(F_t - \bar{F}) \qquad k = \frac{1}{\sum_t (F_t - \bar{F})^2}$$

- The timing trade is long in the security when the signal F_t is above its average value, $\bar{F}$, and short in the security when the signal is below its average.
- The timing strategy is profitable when the regression coefficient is positive and unprofitable otherwise.
  This result shows the close link between a regression and a timing strategy—in fact, the regression coefficient is the average profit of a timing strategy!
- (Furthermore, the risk-adjusted return of the strategy is closely related to the t-statistics of the regression coefficient.)

Cross-sectional Regression

A cross-sectional regression corresponds to a security selection strategy.

We have a forecasting variable F_t^i for every security i:

$$R_{t+1}^i = a + bF_t^i + \varepsilon_{t+1}^i \tag{7}$$

We can run this regression across securities at any time t.

$$\hat{b}_t = \frac{\sum_i (F_t^i - \bar{F}_t) R_{t+1}^i}{\sum_i (F_t - \bar{F})^2} = \sum_i x_t^i R_{t+1}^i$$

The only difference from before is that now we are summing over securities i, not time t. This regression coefficient is the profit of a long–short security selection strategy, which is realized between time t and $t+1$. The position in security i is

$$x_t^i = k_t(F_t^i - \bar{F}) \qquad k = \frac{1}{\sum_i (F_t^i - \bar{F}_t)^2}$$

Cross-sectional Regression (continued)

This strategy selects a long position for securities with signals that are better than the average across securities at that time, and a short position for securities with low signals. The overall estimate of the regression

coefficient b_t using the Fama–MacBeth (1973) method is simply the average of all the estimates for each time period:

$$\hat{b} = \frac{1}{T} \sum_{t=1}^{T} \hat{b}_t$$

This is the average profit of the long–short trading strategy over time. The risk of the strategy is the volatility of the profits, that is, the volatility of the regression coefficients:

$$\hat{\sigma} = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (\hat{b}_t - \hat{b})^2}$$

Cross-sectional Regression (continued)

The Sharpe ratio of the security selection strategy is

$$\mathrm{SR} = \frac{\hat{b}}{\hat{\sigma}}$$

which corresponds closely to the t-statistic of the regression estimate:

$$\text{t-statistic} = \sqrt{T}\frac{\hat{b}}{\hat{\sigma}}$$

Recall that a regression coefficient is considered statistically significant if its t-statistic is above two in absolute value. We see that statistical significance corresponds to realizing a high Sharpe ratio over a long time period T. This is intuitive: **A strategy is more likely to work for reasons beyond luck if it has worked well for a long time.**

Regression and Security Sorting

A univariate regression corresponds to sorting securities by one signal; a bivariate regression corresponds to double-sorting securities by two signals, allowing you to determine whether one signal adds value beyond the other; and a multivariate regression corresponds to sorting by multiple signals.

Regression:

$$R^i_{t+1} = a + b^F F^i_t + b^G G^i_t + \varepsilon^i_{t+1} \tag{8}$$

b^F corresponds to the profit from trading on F, given that you are already trading on G.

One advantage of regressions is that it is easy to add many variables on the right-hand side, while it becomes impracticable to quadruple-sort securities.

Problem of Timing Strategy

Timing strategies are more susceptible to biases than security selection strategies.

Indeed, the time series regression corresponds to a "cheating" in-sample backtest, since the position size depends on the average forecasting variable over time $\bar{F}$, but this average was not known at the beginning of the time period.

Similarly, considering whether the signal was in the top, middle, or bottom third is also cheating, because this was also not known in advance.

A more correct backtest is to ask at any time whether the signal was high, medium, or low relative to the signals that had been seen up until that time (or other out-of-sample forecast methods).

Security selection strategies do not suffer from this problem.