Strategy Performance Measure

Reading Notes of Efficiently Inefficient #Ch2

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[1] Lasse Heje Pedersen, 2015. "Efficiently Inefficient: How Smart Money Invests and Market Prices Are Determined," Economics Books, Princeton University Press, edition 1, number 10441.

Alpha and Beta

$$R_t^e = \alpha + \beta R_t^{M,e} + \varepsilon_t \tag{1}$$

where:

- $R_t^e = R_t - R^f$ is the excessive return of the strategy.
- $R_t^{M,e} = R_t^M - R^f$ is the excessive return of the market.
- β (beta) measures the strategy's tendency to follow the market.
- ε_t is the idiosyncratic risk of the strategy (independent of market moves).

Important Facts

- **Diversifications**: If you mix a hedge fund with other nvestments, the beta risk is not diversified away, while idiosyncratic risk largely is.
- **Fees**: Market exposure ("beta risk") is easy to obtain at very low fees, for example, by buying index funds, exchange traded funds (ETFs), or futures contracts.
- **Market Neutral**: $\beta = 0$
  - The hedge fund's performance does not depend on whether the stock market is moving up or down.
  - market-neutral excess return $= R_t^e - \beta R_t^{M,e} = \alpha + \varepsilon_t$.
    $\mathbb{E}$(market-neutral excess return) $= \alpha$.
- α **(alpha)**: measures the strategy's value added above and beyond the market exposure due to the hedge fund's trading skill (or luck, given that alpha is estimated based on realized returns).

Luck or Skill?

t-statistic: alpha divided by the standard error of its estimate

- $t > 2$: the alpha is statistically significant from zero, i.e., evidence of skill that defies the CAPM.
- $t < 2$: corresponds to an alpha estimate which is so noisy that it might have been achieved through luck.

Fama-French Three Factor Model

Include several other risk exposures: eg. Fama and Frence (1993)

$$R_t^e = \alpha + \beta^M R_t^{M,e} + \beta^{\text{HML}} R_t^{\text{HML}} + \beta^{\text{SMB}} R_t^{\text{SMB}} + \varepsilon_t \tag{2}$$

- R^{HML}: return on a value strategy. β^{HML} measures the strategy's tendency to be tilted toward stocks with a high B/M.
- R^{SMB}: return on a size strategy. β^{SMB} measures the tilt toward small stocks.
- α measures the excess return adjusted for market risk, value risk, and size risk. / measures the hedge fund's trading skills beyond simply taking stock market risk and tilting toward small-value stocks (which tend to outperform other stocks, on average).

Question: Is a high positive alpha always better than a low positive alpha?
Answer: Not necessarily.

1. while alpha tells you the size of the market-neutral returns that a strategy delivers, it does not say at what risk.
2. alpha depends on how a strategy is scaled.
   eg. a twice-leveraged strategy has twice the alpha of an unleveraged version of the same strategy.

Follow-up: How to solve?
Answer: Risk-reward Ratio.

Sharpe Ratio (SR)

$$\text{SR} = \frac{\mathbb{E}(R - R^f)}{\sigma(R - R^f)} \tag{3}$$

It measures the investment "reward" per unit of risk.

Information Ratio (IR)

IR is the risk-adjusted abnormal return (alpha)
Definition 1:

$$\text{IR} = \frac{\alpha}{\sigma(\varepsilon)} \tag{4}$$

where α and ε are from

$$R_t^e = \alpha + \beta R_t^{b,e} + \varepsilon_t \tag{5}$$

where $R_t^{b,e}$ is the excess return of some benchmark.

Information Ratio (IR) (continued)

If the hedge fund has a mandate to beat a specific benchmark
Definition 2:

$$\text{IR} = \frac{\mathbb{E}[R - R^b]}{\sigma(R - R^b)} \tag{6}$$

IR measures the extent to which the strategy beats the benchmark per unit of *tracking error* risk.

- *Tracking error* is the difference between the strategy's and the benchmark's returns.
- *Tracking error risk* is the standard deviation of this difference.

Alpha-to-margin (AM) Ratio

$$\text{AM} = \frac{\alpha}{\text{margin}} \tag{7}$$

It measures the return on a *maximally leveraged* version of a market-neutral strategy.

eg. If the margin requirement is 10%, a hedge fund can get 10-to-1 leverage. If alpha is 3% per year, then the AM ratio is 30%.
The hedge fund may prefer this strategy to one trading illiquid securities that cannot be leveraged at all (margin is 100%) with an alpha of 7%. This alternative strategy has a lower AM ratio of 7%, despite its higher alpha.

Link between AM Ratio and IR

$$\text{AM} = \text{IR} \times \frac{\sigma(\varepsilon)}{\text{margin}} \tag{8}$$

The AM ratio is the reward per unit of risk (IR) multiplied by the extent to which the strategy can be leveraged, namely the risk per unit of margin equity.

Ajustment Under Crash Risk

If a hedge fund strategy has a significant crash risk, volatility may not be the best risk measure $\Longrightarrow$ Replace $\sigma(\varepsilon)$ with other indicators such as: **(I)**

The risk-adjusted return on capital (RAROC):

$$\text{RAROC} = \frac{\mathbb{E}(R - R^f)}{\text{economic capital}} \tag{9}$$

Economic capital is the amount of capital that need to set aside to sustain worst-case losses on the strategy with a certain confidence. It can be estimated using value-at-risk (VaR) or stress tests.

Ajustment Under Crash Risk (continued)

(II) Downside Risk:

$$\text{(Sortino ratio) } S = \frac{\mathbb{E}(R - R^f)}{\sigma^{\text{downside}}} \tag{10}$$

$$\sigma^{\text{downside}} = \sigma(R\mathbf{1}_{\{R<\text{MAR}\}}) \tag{11}$$

MAR is the minimum acceptable return. It is often set to be the risk-free rate or zero.

- **Expected Returns**
  - geometric average $= [(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_T)]^T - 1.$
  - arithmetic average $= [R_1 + R_2 + \cdots + R_T]/T.$
- **Variance**

$$\sigma^2 = [(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \cdots + (R_T - \bar{R})^2]/(T - 1)$$

Annualizing

- **Expected Returns**: Simply multiply them by the number n of periods per year:

$$\text{ER}^{\text{annual}} = \text{ER} \times n \qquad \text{(arithmetic average)} \tag{12}$$

$$\text{ER}^{\text{annual}} = (1 + \text{ER})^n - 1 \qquad \text{(geometric average)} \tag{13}$$

- **Variance**: Since returns are (close to) independent over time, their variance is proportional to the time period.

$$\text{var}^{\text{annual}} = \text{var} \times n \tag{14}$$

$$\sigma^{\text{annual}} = \sigma \times \sqrt{n} \tag{15}$$

Annualizing (continued)

- **Sharpe Ratio**:

$$\mathrm{SR}^{\mathrm{annual}} = \mathrm{ER}^{\mathrm{annual}} / \sigma^{\mathrm{annual}} = \mathrm{SR} \times \sqrt{n} \tag{16}$$

- *Interesting Fact*: when you observe the P&L more frequently, the risk is more painful. (SR is lower)
  Assume for simplicity that returns are normally distributed, then the probability of a loss is

$$\mathbb{P}(R^e < 0) = \mathbb{P}(\mathbb{E}(R^e) + \sigma Z < 0) = \mathbb{P}(Z < -\mathrm{SR})$$

  eg. The minute loss probability could be nearly 50%.

TABLE 2.1. PERFORMANCE MEASURES AND TIME HORIZONS

Measurement horizon	Sharpe ratio	Loss probability
Four years	2	2.3%
Year	1	16.0%
Quarter	0.5	31.0%
Month	0.3	39.0%
Trading day	0.06	47.5%
Minute	0.003	49.9%

High Water Mark

A hedge fund's high water mark (HWM) is the highest price P_t (or highest cumulative return) it has achieved in the past:

$$\text{HWM}_t = \max_{s \leq t} P_s \tag{17}$$

Often hedge funds only charge performance fees when their returns are above their HWM.
Hence, if they have experienced losses, they must first make these back and only charge performance fees on the profits above their HWM.

Drawdown

- **Drawdown (DD)**: the amount that has been lost since the peak (i.e., the HWM).

$$\mathrm{DD}_t = (\mathrm{HWM}_t - P_t)/\,\mathrm{HWM}_t \tag{18}$$

- **Max Drawdown (MDD)**:

$$\mathrm{MDD}_T = \max_{t \le T} DD_t \tag{19}$$

Adjustment under Illiquidity

Suppose a company invest 100% in stock market. In January the stock market return is 3%, but the company will report a 3% return in February, which leads to

$$\text{cov}(R_t^e, R_t^{M,e}) = \text{cov}(R_{t-1}^e, R_t^{M,e}) \cong 0$$

Hence, $\beta \cong 0$, and the estimated values of α will be the average stock market return, which is positive over the long term. But actually it has not created values to investors.

This phenomenon is common in illiquid markets. We can adjust for this by

$$R_t^e = \alpha^{\text{adjusted}} + \beta^0 R_t^{M,r} + \beta^1 R_{t-1}^{M,e} + \cdots + \beta^L R_{t-L}^{M,e} + \varepsilon_t \tag{20}$$

$$\beta^{\text{all-in}} = \beta^0 + \beta^1 + \cdots + \beta^L \tag{21}$$

$$\text{IR}^{\text{adjusted}} = \frac{\alpha^{\text{adjusted}}}{\sigma(\varepsilon)} \tag{22}$$

Other Concepts

- **Performance Attribution**: Hedge funds look back over the previous time horizon and review which trades were the main positive return contributors and which ones detracted. It provides insight into the *investment process*, *the drivers of returns*, and *the risk factors to which they are exposed*.
- **Track record**: Hedge funds' realized performance after all fees and costs over its life.
- **Backtests**: Historical simulations of strategy performance under assumptions about how they would have behaved in the past.

Backtest a strategy first $\Longrightarrow$ Adjust with transaction cost