

## Econ Lab: Network Paper Summary

The effect of network density on asymptotic information and welfare in regular random networks

Agent  $i$  observes her own and her neighbors' past successes ( $i \rightarrow j$ )

Before  $\tau$ : both social learn and private experiment?

After  $\tau$ : only social learn

Get optimal cutoff time  $\tau$  by solving  $\phi_\tau = 0$

If  $\tau$  increase by  $\epsilon$ : +payoff from a private success  $p_t(x+y)\epsilon$

=belief  $P(\text{high quality})(\text{payoff})\epsilon$

-expected benefit of future social learning  $p_t(b/r+b)y^*\epsilon$ ,

Where  $b$ =neighbor success arrival rate,  $r$ =welfare discount

-marginal effort cost  $c^*\epsilon$

Asymptotic information  $B$  = total information created by society

Asymptotic learning = ' $B = \inf$ ' = probability that all agents perfectly learn is equal to  $1 =$  'agents learn the state' = an agent learn the successes of all agents?

Second-best benchmark? to provide an upper bound on the equilibrium utility

### Top Figure

Theorem 1(a) =  $B$  decrease, network density increases ( $\lambda \geq 1/\sigma^*$ )

- $\sigma^*$  = the time when an agent perfectly learns and is indifferent about their private experiment at  $t=0$  ( $\phi_0=0$ ) (pessimistic priors  $p_0 < \bar{p}$ ) (optimistic priors  $p_0 \geq \bar{p}$ ,  $\sigma^*=0$ , experimentation incentives are higher and the asymptotic learning obtains as long as  $p=0$ ?)
- $1/\lambda = \lim(\log l / n^l) = \lim(\log \# \text{ of agents} / \# \text{ of links})$  = network's time-diameter = the time for social information to travel between two random agents in the network (continuous)
- Why  $\tau$  comparable to  $B$  (on y-axis)?
- Why use  $v$ ,  $\lambda$ ,  $p$  sequentially to describe increasing network density?

### Middle Figure

Theorem 1(b) =  $V$  rises in  $v$ , attains  $V^*$  iff  $x = \inf$  and  $p=0$ , falls in  $p$

- $v = \#$  of links
- $p = \#$  of links /  $\#$  of agents = proportion of connection among all
- Welfare  $V = V^*$ : min(perfect learn immediately =  $p_0 y$ , some other agent generate the social info =  $V(0,0) = p_0(x+y) - c$ )
- When  $0 < \lambda < 1/\sigma^*$  (time diameter  $1/\lambda \geq \sigma^*$  = perfectly learn time), network sparse enough to accommodate asymptotic learning and dense enough to fully crowd out pre-cutoff social learning (no experiment? Or no neighbor learn before cutoff?) so attain  $V^*$
- Intuitively  $v = \inf$  and  $p=0$ ?

### Bottom Figure

$B^*$ ? Relationship to  $\inf$ ?

**Spare networks:**  $v$  is finite

- Contagion phase accelerates over time(convex)
- Info  $B=\text{inf}$ : each agent experiments time  $>0$  ensure asymptotic learning?

**Intermediate networks:** information spreads in finite time  $=\text{finite} > \lambda > 1/\sigma^*$

- As  $v \rightarrow \text{inf}$ , individual experiment vanish  $\tau \rightarrow 0$ , convex cumulative learning curve  $B$  converges to a step function, constant equal to 0 below  $\sigma^*$ , and inf above?
- Denser, time diameter  $1/\lambda \leq \sigma^* = \text{perfectly learn time}$ , social learning is faster than perfect learning, agents are exposed before perfect learning and eq information  $B < \text{inf}$  (asymptotic learning fails), but  $V^*$  still attains
- As  $\lambda$  grows, the corner of the stepfunction slides along the dashes line?

**Dense networks:** proportion of links is finite  $=0 < p < 1$

- Analogous to the clique: given total info  $B$ , agents learn  $pB$  before stopping and  $(1-p)B$  immediately after stopping
- As  $p \rightarrow 1$ , we approach the clique and  $B \rightarrow \tau_{\text{bar}}$  = maximized cutoff time = agent receive all their social info before stopping, speed of diffusion crowd off discovery

Lemma 1: higher social learning  $B$  increases value  $V$  and decreases the cutoff  $\tau$  (stop earlier)

Lemma 2: the agent's value  $= V(\text{optimal cutoff } \tau, \text{pre-cutoff learning } B_{\text{tau}})$ , which decreases (post-cutoff learning decreases) when  $B_{\text{tau}}$  increase if fix optimal stopping time  $\tau$

So  $V(\tau=0, B_{\text{tau}}=0) = \text{no private experiment and no pre-cutoff learning} > V(\tau > 0, B_{\text{tau}} > 0)$

Lemma 4: assume  $v=\text{inf}$ . Agent  $i$  gets exposed at time  $\sigma$  or never.

(All agents observe the first success at the same deterministic time?)

$p_0$  = all agents have a common prior  $P$  (high quality) at  $t=0$

$c$  = cost of an agent's private efforts to generate successes with arrival rate

$x$  = the payoff received by agent's own successes (discounted at rate  $r$  in welfare calculation)

$y$  = future benefit of success  $= (x-c)/r$

A agent receive  $x+y$  when she succeeds, receive  $y$  when a neighbor succeeds

$p_{\text{bar}} = c/x$  = myopic threshold belief = when agent stop if it ignore the future benefit of success  $y$

An agent's prior

is optimistic if  $p_0 > p_{\text{bar}}$  (always experiment, no matter social learning curve)

is pessimistic if  $p_0 \leq p_{\text{bar}}$

$\phi_t$  = an agent's experimentation incentives at time  $t$

Get optimal cutoff time  $\tau$  by solving  $\phi_{\text{tau}} = 0$  if  $\phi_0$  (initial experiment incentive)  $> 0$

$\tau = 0$  if  $\phi_0$  (initial experiment incentive)  $\leq 0$

$\tau_{\text{bar}}$  = maximized cutoff time in the absence of social learning ( $B=0$ ) = single-agent solution

### 1. Fsolve vs vpsolve

`fsolve` is part of the Optimization Toolbox, and it uses a trust-region algorithm to find the roots of a nonlinear system of equations. It is more robust than `vpsolve` and is generally the preferred method when the problem is well-behaved, i.e., has unique solutions and is not too ill-conditioned. `fsolve` requires an initial guess and can be slower than `vpsolve` for simple problems, but it is more reliable for complex problems.

`vpsolve`, on the other hand, is part of the Symbolic Math Toolbox, and it uses a variant of the Newton-Raphson method to find the roots of a symbolic equation. It is faster than `fsolve` for simple problems, but it is less robust and can fail to find solutions for more complex problems. `vpsolve` does not require an initial guess, but it can only solve for a single variable at a time.

In general, you should use `fsolve` when you have a well-behaved problem and are confident that the initial guess is close to the solution. You should use `vpsolve` when you have a simple problem and do not have an initial guess, or when you need to solve for a single symbolic variable.