Theory

- Indifference condition for core agents k

$$\psi_{k,\tau_k} = P^{\emptyset}(I\tau_k)\left(x + y\left(1 - \left(1 - e^{-(r+L)(\tau_\ell - \tau_k)}\right)\frac{L}{r+L}\right)\right) - c \leq 0 \tag{1}$$

  with equality if $\tau_k > 0$. Since (1) falls in τ_k and τ_ℓ, its solution is described by a decreasing function $\tau_k = \phi(\tau_\ell)$.[1]

- Indifference condition for peripheral agents ℓ

$$\psi_{\ell,\tau_\ell} := P^{\emptyset}\left(B_{\tau_\ell} + \tau_\ell\right)\left(x + ry\int_{\tau_\ell}^{\infty} e^{-r(t-\tau_\ell)-(B_t - B_{\tau_\ell})}dt\right) - c = 0. \tag{2}$$

  where the learning curve of peripherals is given by $B_t = Kt$ for $t < \tau_k$, and

$$B_t = K\tau_k - \log\left(1 - \int_0^{\min\{t,\tau_\ell\}}(L-1)e^{-(L-1)s}(1 - e^{-K(t-\max\{s,\tau_k\})})ds\right) \tag{3}$$

  for $t > \tau_k$.[2] The solution of (2) (after substituting (3)) is described by a decreasing function $\tau_\ell = \psi(\tau_k)$.[3]

Simulations

- Parameter restrictions
  - $x > c > 0$, $r > 0$, and $y = (x - c)/r$
  - K, L positive integers
- Graph the functions $\phi(\tau_\ell), \psi(\tau_k)$
  - Conjecture: $\phi'(\tau_\ell)\psi'(\tau_k) < 1$, so they cross exactly once
- Graph the function B_t (given the solution (τ_k, τ_ℓ) of (1) and (2)) for $I = 1000$ (say), and varying levels of $K = 1, 10, 100$ (and $L = I - K$)

[1] This solution is only meaningful when $\phi(\tau_\ell) \leq \tau_\ell$, which implies a lower bound for τ_ℓ.

[2] The chance peripheral ℓ has not socially learned by t (e^{-B_t}) is the product of the chance that no core agent succeeded during their experimentation ($e^{-K\tau_\ell}$) times one minus the probability that some other of the $L-1$ peripherals succeeded at $s \leq \min\{t, \tau_\ell\}$ ($(L-1)e^{-(L-1)s}$), and then one of the K core agents succeeded, between $\max\{s, \tau_k\}$ and t.

[3] Again, this is only meaningful when $\psi(\tau_k) > \tau_k$, which implies an upper bound on τ_k.

- Graph welfare

$$\mathcal{V}(\tau_\ell, B_{\tau_\ell}) = \frac{p_0 x - c}{r} + e^{-r\tau_\ell}\left(p_0 e^{-B_{\tau_\ell}-\tau_\ell}(x-c) - (1-p_0)c\frac{r-1}{r}\right)$$

as a function of K (fixing $I = 1000$ and setting $L = I - K$)