The χ=1 Critical–Damping Boundary: A Cross–Domain Principle of Stability, Information Efficiency, and Gravitational Structure

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v.2

Abstract

Revision Note (v2). This version clarifies the distinction between the mathematical boundary ($\chi = 1$) and the empirical adaptive optimum ($\chi \approx 0.8$–0.9), expands the justification for $\omega = \sqrt{4\pi G\rho_m}$, and repositions the glossary for accessibility.

This paper extends the quantum-field–level results of the Symmetrical Convergence (SymC) framework into a cosmological context. The previous study established the critical-damping boundary, $\chi = 1$, as a universal condition governing field stability and information efficiency. The present work explores its macroscopic counterpart, demonstrating that the same boundary appears in cosmological dynamics, where $\chi_\delta = 1$ coincides with the onset of acceleration ($q = 0$) in ΛCDM. The title evolved to reflect this broader scope—from a quantum-only analysis to a unified description linking field stability and cosmic evolution. The goal is not to redefine dark energy but to show that the critical-damping principle identified in QFT persists across scales and yields falsifiable cosmological predictions.

Note on Optimal vs. Critical

The boundary $\chi = 1$ marks the critical-damping condition—the fastest non-oscillatory return to equilibrium. In practice, empirical systems across domains—from laboratory oscillators to cosmic structure—optimize near $\chi \approx 0.8$–0.9, balancing stability with feedback capacity. Throughout this paper, "$\chi = 1$" denotes the mathematical transition point, while "$\chi \approx 1$" refers to the adaptive range $0.7 < \chi < 1.2$.

1 Unified Statement and Methodological Status

Central thesis. The dimensionless ratio $\chi \equiv \gamma/(2|\omega|)$ provides a universal separatrix between oscillatory (underdamped) and monotone (overdamped) moment dynamics; $\chi = 1$ is the critical boundary. The framework is homologous across domains, not merely analogous, because the same second-order operator structure governs each system [1]. In classical control theory, this ratio is directly equivalent to the damping ratio $\zeta = 1$, providing an explicit proof of homology across scales. SymC acts as a falsifiable boundary postulate: it constrains admissible microphysics without committing to specific environmental models. It further serves as a *phenomenological organizing law*—agnostic to microphysical origin but capable of organizing observations and defining structural stability across physical regimes.

Glossary of Key Quantities

$\chi \equiv \gamma/(2|\omega|)$: SymC ratio; χ=1 boundary. $\chi_\delta \equiv H/\sqrt{4\pi G\rho_m}$: Cosmological SymC ratio. $\eta \equiv I/\Sigma$: Information–efficiency. Γ: Memory bandwidth. λ: Quartic coupling (one-loop RG). u^μ: Environment four-velocity. G^R: Retarded propagator. $A_k(\Omega)$: Spectral function. q: Deceleration parameter.

2 Covariant Boundary and Propagator

A dissipative scalar field satisfies $(\Box + m^2)\phi + \gamma(u^\mu\partial_\mu)\phi = 0$, giving $G^R(k;u) = [-(k^2 - m^2) - i\gamma(k\cdot u)]^{-1}$. Pole coalescence yields the invariant boundary $\gamma = 2|k\cdot u|$ with impulse $g_k(t) = \Theta(t)\, t\, e^{-|\omega_k|t}$ and spectral merger at $\Omega = 0$ [2, 3].

3 Continuous Crossover and Decoherence

SymC governs moment dynamics, complementary to environment-induced decoherence [4]. Decoherence time τ_D depends on state size, whereas the SymC transition depends only on (γ, ω). Thus a system may be decohered yet dynamically underdamped ($\chi < 1$).

4 Information–Efficiency Principle and FDT Symmetry

Consistency Theorem: Efficiency Extremum at $\chi = 1$. Define $\eta(\chi) = I(\chi)/\Sigma(\chi)$. Under mild regularity conditions ($I''(1) < 0$, $\Sigma''(1) > 0$), η has a strict local maximum at $\chi = 1$. Unified fluctuation–dissipation scaling $\mathrm{Var}[O] \propto 1/\gamma$ supports the cost-minimization assumption [5–7].

5 Renormalization and Memory Robustness

In $\lambda\phi^4$ with dissipation, $d\chi/d\ell = -(5\lambda/64\pi^2)\chi$ to one loop, so $\chi = 1$ is an approximate fixed line at weak coupling. With exponential memory bandwidth Γ, $\gamma_{\text{eff}} = \gamma_0/\sqrt{1 + (\omega/\Gamma)^2}$ and the EP broadens to a narrow band; the merger persists. See [8, 9].

6 Cosmological Mapping and Tests

Linear growth obeys $\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0$, mapping to $\chi_\delta = H/\sqrt{4\pi G\rho_m} = \sqrt{2/(3\Omega_m)}$. In flat ΛCDM, the identity

$$\chi_\delta = 1 \iff q = 0 \tag{1}$$

follows by: (i) $H^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3}$, (ii) $\Omega_m = \frac{8\pi G\rho_m}{3H^2}$, and (iii) $q = \frac{1}{2}(\Omega_m - 2\Omega_\Lambda)$ with $\Omega_\Lambda = 1 - \Omega_m$. From $\chi_\delta = 1 \Rightarrow \Omega_m = 2/3$ and $q = 0 \Rightarrow \Omega_m = 2/3$, the equivalence holds. Departures from ΛCDM or GR yield falsifiable signatures: the Δa split between $a_{q=0}$ and $a_{\chi=1}$ for $w(a) \neq -1$, and a SymC tilt $\chi_\delta(k, z) = \chi_{\rm GR}(z)[1 + \mu(k, z)]^{-1/2}$ in modified gravity [10–12].
Why ρ_m only? The linear growth equation $\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\,\delta = 0$ maps to a damped oscillator with $\gamma = 2H$ and $\omega^2 = 4\pi G\rho_m$ because matter provides the restorative self-gravity that drives structure formation, whereas dark energy enters only through H (the friction term). Since ρ_Λ does not cluster, it supplies no local restoring force. Thus the assignment $\omega^2 = 4\pi G\rho_m$ is dictated by the growth equation itself rather than chosen by convention.

7 Experimental Verification

This section outlines a *complete, multi-domain program for empirically testing the SymC framework.*

Direct lab falsifiers. Spectral merger at $\Omega = 0$ occurs if and only if $\chi = 1$; the time-domain impulse switches from $e^{-\gamma t/2}\cos(\omega_d t)$ to $te^{-|\omega|t}$.
Derived observational signatures. In cosmology, $\Delta a \equiv a_{q=0} - a_{\chi=1}$ tests dark-energy dynamics, and a scale-dependent SymC tilt in $f\sigma_8(k, z)$ tests modified gravity. Platforms are summarized in Table 1, domain mappings in Table 2, and cross-domain falsifiers in Table 3.

Platform	Control	Observable	Indicator
Circuit QED [13]	Dephasing rate γ	Transmission $S_{21}(\Omega)$	$\gamma = 2\lvert\omega_r\rvert$
Exciton–Polariton	Cavity loss	Photoluminescence	$\gamma_c = 2\lvert\omega_c\rvert$
Magnon–Photon	Bias damping	Level merger	$\gamma_m = 2\lvert\omega_m\rvert$
Optomechanical	Feedback	Displacement PSD	$\gamma_{\rm opt} = 2\lvert\omega_{\rm mec}\rvert$

Table 1: Platforms for SymC falsification (mode-resolved EP).

8 Multi-Mode Exceptional-Point Map

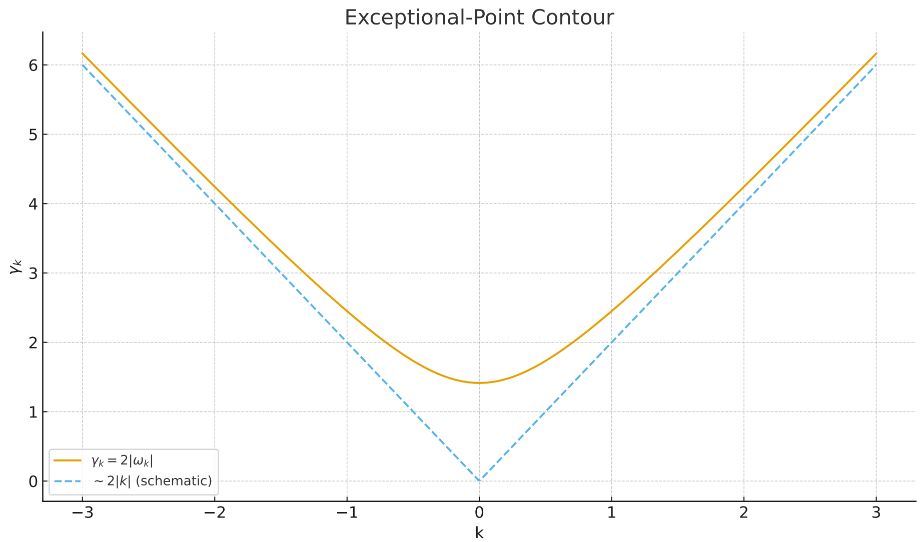

Figure 1: Exceptional-point contour $\gamma_k = 2|\omega_k|$ across modes. Annotations indicate exemplar experimental domains.

9 Unified Domain Tables

Domain	Damping Γ	Drive Ω	χ=1 Interpretation	Notes
QFT / Open	γ_k	ω_k	EP / spectral merger	Non-Hermitian EP2
Cosmology	$2H$	$\sqrt{4\pi G\rho_m}$	$q = 0$ (onset accel.)	Flat ΛCDM identity
AIF / Info	γ, γ_m	ω, ω_0	$\max \eta = I/\Sigma$	Efficiency extremum
Gravity (spec.)	horizon γ	local ω	near-horizon criticality	speculative

Table 2: SymC mapping across domains.

Domain	Test	Observable	Signature at $\chi = 1$
QFT (lab)	Spectral merger	$A_k(\Omega)$	two peaks $\to$ one at $\Omega = 0$
QFT (lab)	Impulse kernel	$g_k(t)$	$e^{-\gamma t/2}\cos \to te^{-\lvert\omega\rvert t}$
Cosmology	Δa split	$a_{q=0}$ vs $a_{\chi=1}$	nonzero if $w(a) \neq -1$
Cosmology	SymC tilt	$f\sigma_8(k, z)$	scale-dependent residual
AIF (qubit)	Info peak	$I(\zeta_Q)$	local max near 1
AIF (qubit)	Fidelity plateau	$F(\zeta_Q)$	plateau near 1

Table 3: Unified falsification signatures for SymC across domains.

10 Supplementary Figures (Figure Atlas)

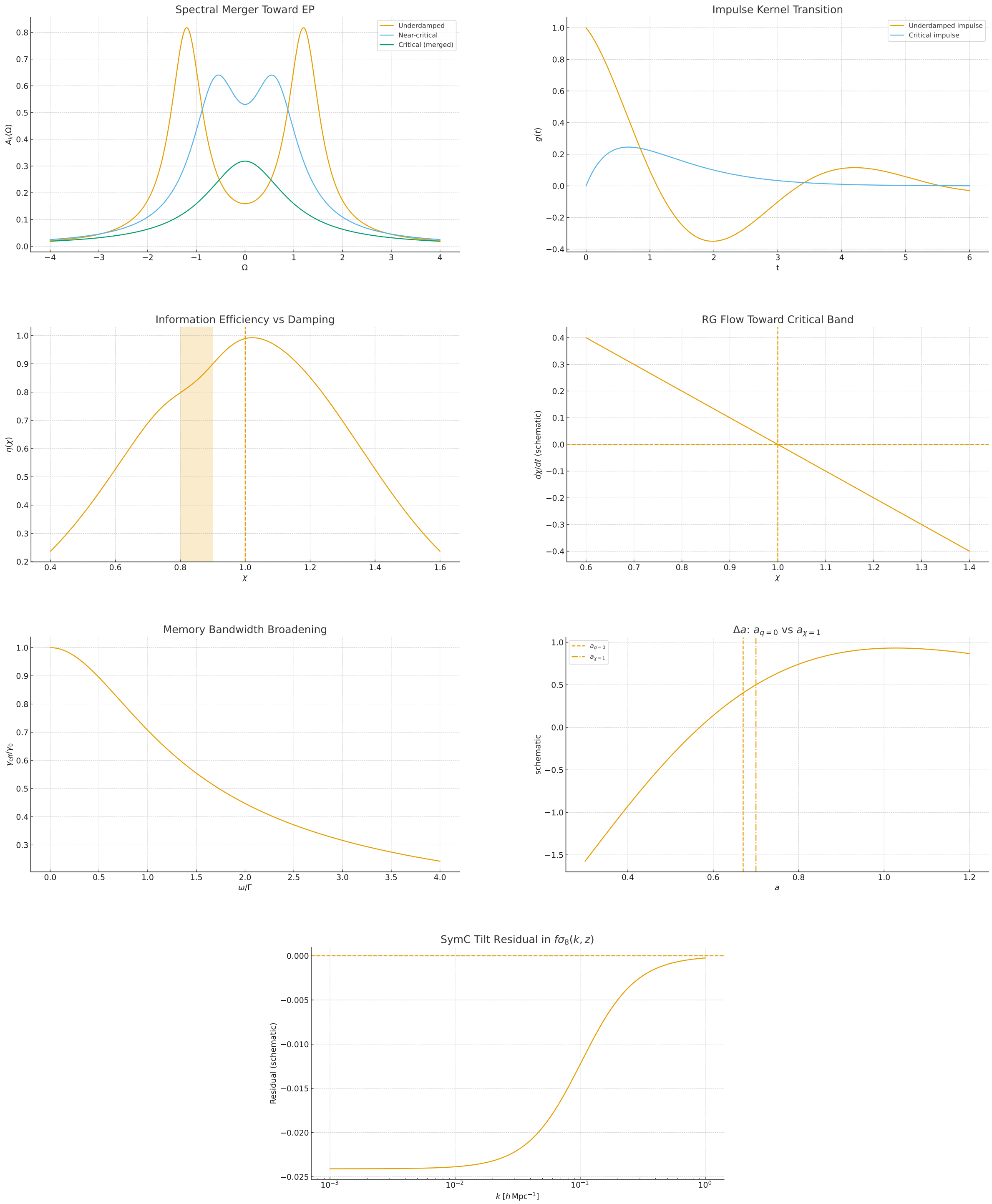

Figure 2: Expanded figure atlas: spectral merger, impulse transition, efficiency peak, RG and memory robustness, and cosmology tests (Δa and SymC tilt). Each file is auto-detected as PDF/PNG; a labeled placeholder appears if missing.

Declaration of Open Intent

11 Data and Code Availability

Numerical simulations use QuTiP [14] and standard Python libraries. A complete archive (manuscript, figures, and notebooks) will be deposited on Zenodo under DOI: 10.5281/zenodo.17507544. All materials will be released under CC–BY 4.0.

12 Conclusion

The SymC boundary $\chi = 1$ provides a covariant, RG-stable, and falsifiable organizing law for open-system dynamics. Its apparent alignment with cosmological acceleration and information-efficiency extremization suggests a unifying role for critical damping across physics. One could hypothesize, based on the pattern across scales, that the correlation between the critically damped state ($\chi_\delta = 1$) and cosmic acceleration warrants empirical investigation. This study derives two falsifiable tests: the Δa split and SymC tilt, measurable with precision $\sigma(\Delta a) \sim 0.007$ and $\sigma(\mu_0) \sim 0.013$.

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