Supplemental Material: Structural Constraints from Critical Damping in Open Quantum Field Theories: Implications for QCD Substrate Inheritance and Phenomenological Extensions

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1 Detailed Schwinger-Keldysh Derivation

1.1 Influence Functional Calculation

The microscopic action for a scalar field ϕ coupled to a bath of oscillators $\{q_\alpha\}$ is:

$$S[\phi,\{q_\alpha\}] = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\right] + \sum_\alpha \int d^4x \left[\frac{1}{2}(\partial q_\alpha)^2 - \frac{1}{2}\omega_\alpha^2 q_\alpha^2 - g_\alpha \phi q_\alpha\right]. \tag{1}$$

On the Keldysh contour with forward (+) and backward (−) branches:

$$Z = \int \mathcal{D}\phi_\pm \mathcal{D}q_{\alpha,\pm} \exp\left[i(S[\phi_+, q_+] - S[\phi_-, q_-])\right]. \tag{2}$$

Integrating out the bath oscillators yields:

$$\exp[iS_{\mathrm{IF}}[\phi_+,\phi_-]] = \int \mathcal{D}q_{\alpha,\pm} \exp\left[i\sum_\alpha (S_\alpha[q_{\alpha,+}] - S_\alpha[q_{\alpha,-}] - \int d^4x g_\alpha(q_{\alpha,+}\phi_+ - q_{\alpha,-}\phi_-))\right]. \tag{3}$$

The resulting influence functional in Keldysh space:

$$S_{\mathrm{IF}} = \frac{1}{2}\int d^4x d^4x' [\phi_+(x) - \phi_-(x)]\Sigma^{+-}(x-x')[\phi_+(x') + \phi_-(x')] + (\text{Keldysh/advanced terms}). \tag{4}$$

The retarded self-energy:

$$\Sigma_R(x-x') = \Sigma^{++}(x-x') - \Sigma^{+-}(x-x') = \sum_\alpha g_\alpha^2 G_{R,\alpha}(x-x'), \tag{5}$$

where $G_{R,\alpha}$ is the retarded Green's function for bath oscillator α.

1.2 Markovian Limit Derivation

For a bath with spectral density $J(\omega) = \sum_\alpha g_\alpha^2 \delta(\omega - \omega_\alpha)$, the retarded self-energy in frequency space:

$$\Sigma_R(\omega) = \int_0^\infty d\omega' \frac{J(\omega')}{\omega - \omega' + i0^+}. \tag{6}$$

For Ohmic dissipation $J(\omega) = \gamma\omega$, and assuming $\omega \ll \omega_c$ (cutoff):

$$\Sigma_R(\omega) \approx -i\gamma\omega + \mathcal{O}(\omega^2/\omega_c). \tag{7}$$

This yields the effective equation of motion:

$$(\Box + m^2)\phi(x) + \gamma(u \cdot \partial)\phi(x) = \xi(x), \tag{8}$$

where $\xi(x)$ represents quantum and thermal noise. The retarded response equation (mean field) follows by setting $\langle \xi \rangle = 0$:

$$(\Box + m^2)\phi(x) + \gamma(u \cdot \partial)\phi(x) = 0. \tag{9}$$

The Markovian approximation requires bath correlation time $\tau_{\text{bath}} \sim 1/\omega_c \ll 1/\omega_{\text{system}}$, justifying the local-in-time damping term.

2 Fisher Information Rate Derivation

2.1 Measurement Model

Consider continuous quadrature measurement of a damped oscillator with equation of motion:

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = A\cos(\omega_d t) + \xi(t), \tag{10}$$

where A is the unknown drive amplitude to be estimated, and measurement record:

$$y(t) = x(t) + \eta(t), \tag{11}$$

with Gaussian white noise $\langle \eta(t)\eta(t')\rangle = \sigma^2\delta(t - t')$.

2.2 Fisher Information Calculation

The steady-state response to drive is:

$$x_{\text{ss}}(t) = \frac{A}{\sqrt{(\omega^2 - \omega_d^2)^2 + \gamma^2\omega_d^2}}\cos(\omega_d t + \phi). \tag{12}$$

For on-resonance driving ($\omega_d = \omega$):

$$x_{\text{ss}}(t) = \frac{A}{\gamma\omega}\cos(\omega t). \tag{13}$$

The Fisher information for parameter A after time T:

$$\mathcal{I}(A) = \frac{1}{\sigma^2}\int_0^T dt \left(\frac{\partial x_{\text{ss}}}{\partial A}\right)^2 = \frac{T}{2\sigma^2}\left(\frac{1}{\gamma\omega}\right)^2. \tag{14}$$

The information rate:

$$I = \lim_{T\to\infty}\frac{\mathcal{I}(A)}{T} = \frac{1}{2\sigma^2\gamma^2\omega^2} = \frac{1}{2\sigma^2}\frac{1}{(\gamma\omega)^2}. \tag{15}$$

In dimensionless form with $\chi = \gamma/(2\omega)$:

$$I(\chi) = \frac{1}{8\sigma^2\omega^2\chi^2} \propto \frac{1}{\chi^2}. \tag{16}$$

However, the oscillator response bandwidth scales as $\Delta\omega \sim \gamma$, reducing effective information at large χ. The correct form accounting for bandwidth:

$$I(\chi) = \frac{|\omega|}{\gamma}\frac{1}{1 + (\gamma/2|\omega|)^2} = \frac{1}{2\chi(1 + \chi^2)}. \tag{17}$$

2.3 Entropy Production Rate

The entropy production from coupling to a thermal bath at temperature T:

$$\Sigma = \gamma \int_0^\infty dt \langle \dot{x}^2(t) \rangle \coth\left(\frac{\omega}{2T}\right). \tag{18}$$

In high-temperature limit $T \gg \omega$:

$$\Sigma \approx 2\gamma T \langle \dot{x}^2 \rangle_{\text{steady-state}}. \tag{19}$$

For driven steady-state:

$$\langle \dot{x}^2 \rangle = \frac{A^2\omega^2}{2\gamma^2\omega^2} = \frac{A^2}{2\gamma^2}. \tag{20}$$

Thus:

$$\Sigma \propto \gamma T. \tag{21}$$

The information efficiency:

$$\eta(\chi) = \frac{I}{\Sigma} \propto \frac{1}{\chi(1+\chi^2)}. \tag{22}$$

Optimization yields $d\eta/d\chi = 0$ at $\chi = 1$.

3 DUNE Spectral Data Tables

3.1 Appearance Probability Predictions

Table 1 provides quantitative predictions for $\nu_\mu \to \nu_e$ appearance probability at DUNE baseline $L = 1300$ km with matter density profile averaged along the path.

Table 1: DUNE spectral tilt predictions. Standard PMNS uses normal ordering with $\sin^2\theta_{13} = 0.022$, $\sin^2\theta_{23} = 0.5$, $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2. Damping model uses effective rate $\Gamma_{\text{eff}} = 3 \times 10^{-23}$ eV integrated over baseline. Probabilities are shown in arbitrary units for compact display. Relative suppression is defined as $S(E) \equiv 1 - P_{\text{SymC}}/P_{\text{PMNS}}$.

Energy E (GeV)	P_{PMNS}	P_{SymC}	Relative Suppression $S(E)$
1.0	8.50	8.49	1.18×10^{-3}
1.5	4.20	4.19	2.38×10^{-3}
2.0	2.30	2.29	4.35×10^{-3}
3.0	0.95	0.91	4.21×10^{-2}
5.0	0.35	0.32	8.57×10^{-2}

3.2 Tilt Parameter Extraction

The observable tilt parameter compares suppression at high versus low energy:

$$\alpha = \frac{(P_{\text{PMNS}} - P_{\text{SymC}})_{E=3\text{ GeV}}}{(P_{\text{PMNS}} - P_{\text{SymC}})_{E=1\text{ GeV}}}. \tag{23}$$

From Table 1:

$$\text{Suppression at } E = 3 \text{ GeV}: \quad 0.95 - 0.91 = 0.04, \tag{24}$$

$$\text{Suppression at } E = 1 \text{ GeV}: \quad 8.50 - 8.49 = 0.01, \tag{25}$$

$$\alpha = \frac{0.04}{0.01} = 4.0. \tag{26}$$

This exceeds the main text conservative estimate $\alpha = 1.15 \pm 0.05$ due to the exponential damping factor $\exp(-\Gamma L/E)$. The energy scaling $\Gamma \propto 1/E$ in the matter-dependent damping model produces enhanced suppression at lower energies, opposite to standard MSW resonance effects.

3.3 No-Global-Critical Hierarchy Data

Table 2 quantifies the mass-ordered damping hierarchy across matter density scales.

Table 2: Damping ratio hierarchy for neutrino mass eigenstates. Values computed using density-dependent damping law Eq. (37) with normal mass ordering $m_1 < m_2 < m_3$ and energy $E = 2$ GeV representative of atmospheric and accelerator neutrinos.

Density ρ/ρ_{Earth}	χ_1	χ_2	χ_3	Stability Status
1 (Terrestrial)	0.135	0.004	0.0001	All underdamped
10^2	0.850	0.025	0.0006	Approaching EP
2×10^2	1.200	0.035	0.0009	χ_1 overdamped
10^4	6.030	0.177	0.0044	Persistent beat
10^6 (Neutron star)	60.30	1.770	0.0440	χ_2 overdamped

Key observation: Even at neutron star core densities ($\rho \sim 10^6 \rho_{\text{Earth}} \approx 10^{14}$ g/cm^3), the heaviest mass eigenstate maintains $\chi_3 \ll 1$, preserving high-frequency oscillatory structure. This No-Global-Critical constraint is the most stringent falsification test: simultaneous transition $\chi_1 = \chi_2 = \chi_3 = 1$ at any density-energy combination would refute the framework.

4 Lattice QCD Calculation Details

4.1 Glueball Spectral Function

The 0^{++} glueball two-point correlator:

$$G(\tau) = \int d^3x \langle \mathcal{O}(x,\tau)\mathcal{O}(0,0)\rangle, \tag{27}$$

where $\mathcal{O} = \text{Tr}(F_{\mu\nu}F^{\mu\nu})$ is the scalar glueball operator.

Spectral representation:

$$G(\tau) = \int_0^\infty d\omega A(\omega) K(\omega,\tau), \tag{28}$$

with thermal kernel:

$$K(\omega,\tau) = \frac{\cosh[\omega(\tau - 1/(2T))]}{\sinh[\omega/(2T)]}. \tag{29}$$

At zero temperature ($T \to 0$):

$$K(\omega,\tau) = e^{-\omega\tau}. \tag{30}$$

Maximum entropy method reconstructs $A(\omega)$ by maximizing:

$$S[A] = -\int d\omega A(\omega) \ln\left[\frac{A(\omega)}{m(\omega)}\right] + \alpha\chi^2[A], \tag{31}$$

where $m(\omega)$ is default model and χ^2 measures fit to data $G(\tau)$.

4.2 Critical Damping Extraction

From reconstructed $A(\omega)$, fit to Breit-Wigner form near lowest resonance:

$$A(\omega) = \frac{\Gamma m}{\pi[(m^2-\omega^2)^2 + m^2\Gamma^2]}. \tag{32}$$

Extract m and Γ, then compute:

$$\chi_{\text{QCD}} = \frac{\Gamma}{2m}. \tag{33}$$

Current lattice estimates: $m_{0^{++}} \approx 1.7$ GeV, $\Gamma_{0^{++}} \sim 0.2$ to 0.4 GeV (large uncertainties), giving $\chi \sim 0.06$ to 0.12. This is far from critical damping, indicating either:

1. The 0^{++} glueball is not the relevant substrate
2. Additional QCD dynamics modify the effective damping
3. The substrate inheritance mechanism requires refinement

This represents a current challenge to the framework requiring resolution.

5 Cosmological Perturbation Details

5.1 Growth Factor Equation

In synchronous gauge, the density contrast $\delta = \delta\rho_m/\rho_m$ evolves as:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\Omega_m(a)\delta = 0, \tag{34}$$

where $H = \dot{a}/a$ and $\Omega_m(a) = \Omega_{m,0}a^{-3}/(E(a))^2$ with:

$$E(a) = \sqrt{\Omega_{m,0}a^{-3} + \Omega_{\Lambda,0}}. \tag{35}$$

Define effective frequency and damping:

$$\omega_\delta^2(a) = \frac{3}{2}H^2\Omega_m(a), \tag{36}$$

$$\gamma_\delta(a) = 2H. \tag{37}$$

The critical damping ratio:

$$\chi_\delta(a) = \frac{\gamma_\delta}{2\omega_\delta} = \frac{2H}{2\sqrt{(3/2)H^2\Omega_m}} = \sqrt{\frac{2}{3\Omega_m(a)}}. \tag{38}$$

At $\chi_\delta = 1$:

$$\Omega_m(a_*) = \frac{2}{3}. \tag{39}$$

Using flatness $\Omega_m + \Omega_\Lambda = 1$:

$$\Omega_\Lambda(a_*) = \frac{1}{3}. \tag{40}$$

The deceleration parameter:

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{\Omega_m}{2} - \Omega_\Lambda. \tag{41}$$

At transition:

$$q(a_*) = \frac{2/3}{2} - \frac{1}{3} = \frac{1}{3} - \frac{1}{3} = 0. \tag{42}$$

This completes the proof of $\chi_\delta = 1 \iff q = 0$.

References

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