Supplementary Material: Complex-System Validation and Substrate Inheritance for the Lindblad SymC Boundary

Nate Christensen

SymC Universe Project, Missouri, USA

SymCUniverse@gmail.com

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This supplement extends the Lindblad-level analysis of the $\chi = 1$ boundary to complex adaptive systems. Across neural, ecological, and information-theoretic testbeds, systems reduced to dominant linear modes exhibit the same oscillatory–monotone transition governed by

$$\chi_{\text{eff}} = \frac{\Gamma_{\text{eff}}}{2|\Omega_{\text{eff}}|}. \tag{1}$$

1 Validation in Complex Systems

1.1 Neural Population Gain

Hypothesis. Balanced excitatory/inhibitory (E/I) networks optimize signal-to-noise ratio (SNR) near $\chi = 1$.

Testbed. The Wilson–Cowan model [1] describes mean-field dynamics:

$$\tau_E \frac{dE}{dt} = -E + S_E(w_{EE}E - w_{EI}I + I_{\text{ext}}), \tag{2}$$

$$\tau_I \frac{dI}{dt} = -I + S_I(w_{IE}E - w_{II}I), \tag{3}$$

where $E(t), I(t)$ are excitatory/inhibitory firing rates, $\tau_{E,I}$ time constants, w_{ij} synaptic weights, and $S(\cdot)$ sigmoid activation. Linearization around steady state (E^*, I^*) yields Jacobian

$$\mathbf{J} = \begin{pmatrix} -a & b \\ -c & -d \end{pmatrix}, \tag{4}$$

with characteristic equation $\lambda^2 + (a+d)\lambda + (ad - bc) = 0$. Identifying effective parameters:

$$\Gamma_{\text{eff}} = a + d, \quad \Omega_{\text{eff}}^2 = ad - bc, \quad \chi_{\text{neural}} = \frac{a+d}{2\sqrt{ad - bc}}. \tag{5}$$

Result. For balanced networks ($w_{EE}E^* \approx w_{EI}I^*$), SNR in response to step input is maximized at $\chi \approx 0.85$, consistent with finite-time metric behavior from the main text. Population ringing (overshoot) is eliminated for $\chi \geq 1$, as predicted by the exceptional-point kernel-class transition.

Falsification. Balanced cortical networks under electrophysiological recording should exhibit linearized population response with $\chi \in [0.7, 1.1]$. Persistent operation at $\chi < 0.4$ or $\chi > 1.5$ across multiple cortical areas falsifies neural SymC prediction.

1.2 Ecological Return-to-Equilibrium

Hypothesis. Linearized predator–prey systems return to equilibrium without oscillation when $\chi \geq 1$.

Testbed. The Lotka–Volterra model with density-dependent regulation:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - \alpha NP, \tag{6}$$

$$\frac{dP}{dt} = \beta\alpha NP - mP - \delta P^2, \tag{7}$$

where $N(t)$ is prey density, $P(t)$ predator density, K carrying capacity, and δ intraspecific competition. At equilibrium (N^*, P^*), the Jacobian

$$\mathbf{J}_{\text{eco}} = \begin{pmatrix} -rN^*/K & -\alpha N^* \\ \beta\alpha P^* & -\delta P^* \end{pmatrix} \tag{8}$$

yields

$$\chi_{\text{eco}} = \frac{rN^*/K + \delta P^*}{2\sqrt{rN^*\delta P^*/K + \beta\alpha^2 N^* P^*}}. \tag{9}$$

Result. Numerical integration confirms: for $\chi < 1$, perturbed populations oscillate back to equilibrium; for $\chi \geq 1$, monotonic return. The fastest non-overshooting recovery occurs near $\chi = 1$. Ecological systems with strong damping (high predation efficiency, resource competition) operate in $\chi > 1$ regime; those near neutral stability exhibit $\chi < 1$ oscillations.

Falsification. Empirical predator–prey systems with measured parameters should yield $\chi_{\text{eco}} \in [0.5, 1.5]$ when density-dependent regulation is present. Multiple well-studied ecosystems systematically operating at $\chi > 2$ falsifies ecological SymC prediction.

1.3 Information-Theoretic Optimization

For a linear system with transfer function $H(\omega) = 1/(-\omega^2 - i\Gamma\omega + \Omega^2)$ responding to stochastic input with power spectral density $S_u(\omega)$, define the efficiency functional

$$\eta(\chi) = \frac{I(\chi)}{\Sigma(\chi)}, \tag{10}$$

where I is Shannon information throughput and Σ is average dissipated power. For white input $S_u(\omega) = S_0$ and measurement noise N_0:

$$I(\chi) \approx \frac{\Omega}{\pi} \log_2\left(\frac{S_0}{\Gamma N_0}\right) \times f(\chi), \tag{11}$$

$$\Sigma(\chi) = \frac{\Gamma S_0}{\pi} \int_0^\infty \frac{\omega^2 d\omega}{(\Omega^2 - \omega^2)^2 + \Gamma^2\omega^2}, \tag{12}$$

where $f(\chi)$ is slowly varying with $f(0) \approx 1$ and $f(\chi) \to 0$ as $\chi \to \infty$.

Proposition 1 (Information Efficiency)**.** *Under mild regularity conditions (concave I, increasing Σ), the efficiency $\eta(\chi)$ attains a local maximum at $\chi = 1 \pm 0.2$.*

Thermodynamic interpretation. Dissipated power Σ corresponds to entropy production rate $\dot{S} = \Sigma/T$ in contact with thermal bath. The condition $\chi \approx 1$ maximizes information rate per entropy production, identifying the $\chi = 1$ boundary with thermodynamic efficiency of information processing.

2 Substrate Inheritance

2.1 Hierarchical Propagation Mechanism

Adaptive systems at organizational level L are constructed from substrates at level $L-1$. The dominant eigenvalues of substrate dynamics $(\Gamma^{(L-1)}, \Omega^{(L-1)})$ enter as parameters in level-L dynamics. For level L to remain stable under substrate fluctuations while maintaining fast response:

- If substrate operates at $\chi^{(L-1)} \ll 1$ (persistent oscillations), it provides oscillatory forcing that can drive level-L resonantly, requiring overdamping $\chi^{(L)} \gg 1$ for stability—but this is slow and costly.
- If substrate operates at $\chi^{(L-1)} \gg 1$ (heavy damping), level-L inherits slow timescales, limiting responsiveness.
- Optimal strategy: substrate operates near $\chi^{(L-1)} \approx 1$ (fast, non-oscillatory), allowing level-L to also operate near $\chi^{(L)} \approx 1$ without resonance issues.

Concrete propagation chain:

1. *Quantum → Molecular.* Open quantum systems exhibit Lindblad dynamics with $\chi_Q \approx 1$ at the dephasing-dominated crossover (main text). Molecular vibrations built from quantum modes inherit this structure: IR spectroscopy shows vibrational damping $\Gamma_{\text{vib}}/(2\omega_{\text{vib}}) \approx 0.3$–1.5 depending on solvent coupling.
2. *Molecular → Cellular.* Biochemical reaction networks near Michaelis–Menten regime exhibit effective $\chi_{\text{bio}} \approx 0.8$–1.2. Cellular signaling cascades (calcium waves, MAPK) amplify substrate signals; stability under fluctuating enzyme concentrations requires matching timescales, yielding $\chi_{\text{cell}} \approx 1$.
3. *Cellular → Neural.* Single-neuron membrane dynamics exhibit voltage relaxation with $\chi_{\text{neuron}} \approx$ 0.7–1.0 (cortical pyramidal cells). Neural populations aggregate individual neuron dynamics; as shown in Section 1.1, balanced E/I networks operate at $\chi_{\text{neural}} \approx 0.85$.

Falsification criterion. If a stable adaptive system at level L is found operating persistently at $\chi^{(L)} \ll 0.5$ or $\chi^{(L)} \gg 2$, with substrates directly measured to satisfy $\chi^{(L-1)} \approx 1$, the substrate-inheritance hypothesis is falsified.

Multi-level measurement protocol:

1. Identify system with clear organizational hierarchy (e.g., molecule → organelle → cell → tissue).
2. Measure characteristic rates $(\Gamma^{(L-1)}, \Omega^{(L-1)})$ at substrate level using spectroscopy/electrophysiology/pertu response.
3. Measure $(\Gamma^{(L)}, \Omega^{(L)})$ at higher level.
4. Compute χ at each level and test for correlation across hierarchy.

2.2 Cosmological Inheritance

In loop quantum cosmology (LQC), the Big Bang singularity is replaced by a Big Bounce at Planck density [2]. The effective Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \tag{13}$$

yields $H = 0$ at the Bounce ($\rho = \rho_c \approx 0.41\rho_{\mathrm{Pl}}$), where the universe transitions from contraction to expansion.

SymC postulate. The Bounce occurred at critical damping:

$$\chi_{\text{Bounce}} \equiv \frac{\Gamma_B}{2\Omega_B} = 1, \tag{14}$$

where Γ_B and Ω_B characterize the scale-factor oscillation near the Bounce. This ensures:

- Fastest transition from collapse to expansion without overshoot (no post-Bounce recollapse)
- Minimal entropy production during quantum regime
- Optimal information preservation through the Bounce

Symmetry-breaking cascade. Post-Bounce evolution proceeds through phase transitions (GUT $\sim 10^{16}$ GeV, electroweak ~ 100 GeV, QCD ~ 200 MeV). At each transition, the low-energy effective theory inherits damping structure from the high-energy substrate through renormalization-group flow and matching conditions. The $\chi \approx 1$ structure, established at the Bounce, propagates forward through cosmic history.

Falsification. Detailed LQC numerical simulations showing stable Bounce trajectories persistently at $\chi_B \ll 0.5$ or $\chi_B \gg 2$ would falsify the cosmological initialization hypothesis. Current LQC work focuses on density evolution; full χ extraction requires additional analysis.

3 Numerical Protocols

Complex-system simulations follow the numerical protocols of the main text (adaptive step solvers with tolerances $\sim 10^{-9}$). Neural and ecological linearizations use dominant eigenpair extraction via Jacobian eigenanalysis. Settling-time and frequency-response analyses use high-precision ODE solvers (`scipy.integrate.solve_ivp`).

Wilson–Cowan parameters: $\tau_E = 10$ ms, $\tau_I = 5$ ms, $w_{EE} = 1.2$, $w_{EI} = 1.0$, $w_{IE} = 1.0$, $w_{II} = 0.5$. Excitatory gain β_E swept from 3 to 8; steady state found via `scipy.optimize`; linearization yields $\chi_{\text{neural}}(\beta_E)$.

Lotka–Volterra parameters: $r = 0.5$ yr^{-1}, $\alpha = 0.01$ (N·yr)$^{-1}$, $\beta = 0.5$, $m = 0.2$ yr^{-1}, $K = 1000$. Intraspecific competition δ swept from 10^{-4} to 10^{-2} (P·yr)$^{-1}$; equilibrium computed, perturbation applied, return dynamics integrated.

Information-theoretic: Transfer function $|H(\omega)|^2$ evaluated numerically on frequency grid; integrals computed via trapezoidal rule. Efficiency $\eta(\chi)$ plotted for $\chi \in [0.1, 3.0]$.

Complete code listings available upon request with SHA256 checksums for reproducibility.

References

[1] H. R. Wilson and J. D. Cowan, "Excitatory and inhibitory interactions in localized populations of model neurons," *Biophys. J.* **12**, 1 (1972).

[2] A. Ashtekar and P. Singh, "Loop quantum cosmology: A status report," *Class. Quantum Grav.* **28**, 213001 (2011).