

Elements of Programming Interviews

Task 16.1

Variant 6

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Task

You have $2n$ disks of n different sizes, two of each size. You cannot place a larger disk on a smaller disk, but can place a disk of equal size on top of the other. Compute the minimum number of moves to transfer the $2n$ disks from $P1$ to $P2$.

Solution

To move $2n$ disks from $P1$ to $P2$

1. Move $2n - 2$ disks from $P1$ to $P3$
2. Move 2 disks from $P1$ to $P2$
3. Move $2n - 2$ disks from $P3$ to $P2$

We get a recurrence relation

$$a_{2n} = 2a_{2n-2} + 2$$

Which can be solved using plug-and-chug method

$$a_{2n} = 2 * (2a_{2n-4} + 2) + 2$$

$$a_{2n} = 4a_{2n-4} + 4 + 2$$

$$a_{2n} = 4 * (2a_{2n-6} + 2) + 4 + 2$$

$$a_{2n} = 8a_{2n-6} + 8 + 4 + 2$$

Which gives us the equation

$$a_{2n} = 2^{k-1} * (2a_{2n-2k} + 2) + 2^{k-1} + 2^{k-2} + \dots + 2^1$$

As $a_0 = 0$ we get

$$\begin{aligned}
a_{2n} &= 2^k + 2^{k-1} + 2^{k-2} + \cdots + 2^1 \\
&= \sum_{k=1}^n 2^k \\
&= \left(\frac{1 - 2^{n+1}}{1 - 2} \right) - 1 = 2^{n+1} - 2
\end{aligned}$$