Elements of Programming Interviews
Task 16.1
Variant 7

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Task

You have $2n$ disks which are colored black or white. You cannot place a white disk directly on top of a black disk. Compute the minimum number of moves to transfer the $2n$ disks from $P1$ to $P2$.

Solution

As we are not told anything about disks' sizes, we will assume that every disk has a unique size. It is said that white disk cannot be placed on top of a black disk, so we assume that in initial position on $P1$ disks are divided into two groups: black (at the top) and white (at the bottom). At this point we know that all disks have unique sizes, so we can forget about colors as all white disks are bigger than black ones. So, the algorithm is as follows

1. Move $2n - 1$ disks from $P1$ to $P3$
2. Move 1 disk from $P1$ to $P2$
3. Move $2n - 1$ disks from $P3$ to $P2$

Which gives us a recurrence relation

$$a_{2n} = a_{2n-1} + 1 + a_{2n-1} = 2a_{2n-1} + 1$$

The solution to the recurrence relation is

$$
\begin{aligned}
a_{2n} &= 2a_{2n-1} + 1 \\
a_{2n} &= 2(2a_{2n-2} + 1) + 1 = 4a_{2n-2} + 2 + 1 \\
a_{2n} &= 4(2a_{2n-3} + 1) + 2 + 1 = 8a_{2n-3} + 4 + 2 + 1 \\
a_{2n} &= 2^k a_{2n-k} + 2^{k-1} + \cdots + 2^0
\end{aligned}
$$

As $a_0 = 0$ we get

$$a_{2n} = \sum_{k=0}^{2n-1} 2^k = \frac{1 - 2^{2n}}{1 - 2} = 2^{2n} - 1$$