Elements of Programming Interviews
Task 16.9
Proof

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Task

Suppose that we have a board for sudoku game of size $n \times n$ and it is empty. How many possible moves can we make?

Solution

We will try to solve this problem by defining a recurrence relation. At first step we can choose among n different cells and each one will lead us to the board with $n-1$ not used cells. This can be written as equation.

$$g(n) = n + ng(n-1)$$

Than we can find some of the first numbers of this recurrence with restriction that $g(0) = 0$.

$$\begin{aligned}
&g(0) = 0 \\
&g(1) = 1 + 1g(0) = 1 \\
&g(2) = 2 + 2g(1) = 4 \\
&g(3) = 3 + 3g(2) = 15 \\
&g(4) = 4 + 4g(3) = 64
\end{aligned}$$

The pattern in recurrence relation is still not clear so we will unfold the relation a few times.

$$\begin{aligned}
g(n) &= n + ng(n-1) \\
&= n + n((n-1) + (n-1)g(n-2)) \\
&= n + n(n-1) + n(n-1)g(n-2) \\
&= n + n(n-1) + n(n-1)((n-2) + (n-2)g(n-3)) \\
&= n + n(n-1) + n(n-1)(n-2) + n(n-1)(n-2)g(n-3)
\end{aligned}$$

We can see the pattern.

$$g(n) = \sum_{i}^{n-1} \prod_{k=0}^{i} (n-k) = en\Gamma(n,1)$$