Learning From Data
Exercise 1.13
Proof

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Task

Exercise 1.13

Consider the bin model for a hypothesis h that makes an error with prob ability μ in approximating a deterministic target function f (both h and f are binary functions). If we use the same h to approximate a noisy version of f given by

$$P(y \mid \mathbf{x}) = \begin{cases} \lambda & y = f(\mathbf{x}), \\ 1 - \lambda & y \neq f(\mathbf{x}). \end{cases}$$

(a) What is the probability of error that h makes in approximating y?

(b) At what value of λ will the performance of h be independent of μ? *[Hint: The noisy target will look completely random.]*

Solution

0.1 (a)

In deterministic case the target function was giving the results like this $f(x) = y$ with probability 100%. Now instead of $y = f(x)$ we take an output y to be a random variable that is affected by x. We should consider two types of errors:

- $h(x) = f(x)$ and $f(x) \neq y \rightarrow (1 - \mu) * (1 - \lambda)$
- $h(x) \neq f(x)$ and $f(x) = y \rightarrow \mu * \lambda$

This can be presented in table form.

$h(x)$ \ $f(x)$	+	-
+	correct	$(1-\mu)*(1-\lambda)$
-	$\mu\lambda$	correct

Now the error will be the sum of errors which is:

$$(1-\mu)*(1-\lambda)+\mu*\lambda = 1-\lambda-\mu+2\mu\lambda$$

0.2 (b)

In order to make h independent of μ we should get rid of it. We can achieve this if $\lambda = \frac{1}{2}$

$$1-\lambda-\mu+2\mu\lambda = 1-\frac{1}{2}-\mu+2\mu\frac{1}{2} = \frac{1}{2}$$