Learning From Data
Homework 1

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Task 3

We have 2 opaque bags, each containing 2 balls. One bag has 2 black balls and the other has a black ball and a white ball. You pick a bag at random and then pick one of the balls in that bag at random. When you look at the ball, it is black. You now pick the second ball from that same bag. What is the probability that this ball is also black?

Solution to task 3

We will use Bayes' theorem to solve this task.

$$\mathsf{P}(H_k|\,A) = \frac{\mathsf{P}(H_k)\mathsf{P}(A|\,H_k)}{\sum\limits_{i=1}^{\infty}\mathsf{P}(H_i)\mathsf{P}(A|\,H_i)}.$$

Let B_1 and B_2 be first and second bags. Then $P(B_1) = P(B_2) = 0.5$ Let A be black marble. Then $P(A|B_1) = 1$ and $P(A|B_2) = 0.5$. Then the answer is:

$$P(B_1|A) = \frac{P(B_1) * P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2)} = \frac{0.5 * 1}{0.5 * 1 + 0.5 * 0.5} = \frac{2}{3}$$

Task 4

Consider a sample of 10 marbles drawn from a bin containing red and green marbles. The probability that any marble we draw is red is $\mu = 0.55$ (independently, with replacement). We address the probability of getting no red marbles ($\nu = 0$) in the following cases:

4. We draw only one such sample. Compute the probability that $\nu = 0$. The closest answer is ('closest answer' means: $|\text{your answer} - \text{given option}|$ is closest to 0):

Solution to task 4

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print(0.45**10)
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0.00034050628916015635

Task 5

5. We draw 1,000 independent samples. Compute the probability that (at least) one of the samples has $\nu = 0$. The closest answer is:

Solution to task 5

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1-(1-(0.45**10))**1000
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Out[15]: 0.28863119784980995

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