Learning From Data
Exercise 1.3
Proof

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Task

Exercise 1.3

The weight update rule in (1.3) has the nice interpretation that it moves in the direction of classifying $\mathbf{x}(t)$ correctly.

(a) Show that $y(t)\mathbf{w}^{\mathsf{T}}(t)\mathbf{x}(t) < 0$. *[Hint: $\mathbf{x}(t)$ is misclassified by $\mathbf{w}(t)$.]*

(b) Show that $y(t)\mathbf{w}^{\mathsf{T}}(t+1)\mathbf{x}(t) > y(t)\mathbf{w}^{\mathsf{T}}(t)\mathbf{x}(t)$. *[Hint: Use (1.3).]*

(c) As far as classifying $\mathbf{x}(t)$ is concerned, argue that the move from $\mathbf{w}(t)$ to $\mathbf{w}(t+1)$ is a move 'in the right direction'.

Solution

Equation for perceptron:

$$h(x) = sign\left(\sum_{i=0}^{d} w_i x_i\right) = w^{\intercal}x$$

Equation for updating weights:

$$w(t+1) = w(t) + y(t)x(t)$$

0.1 (a)

In task **(a)** there are 2 possibilities:

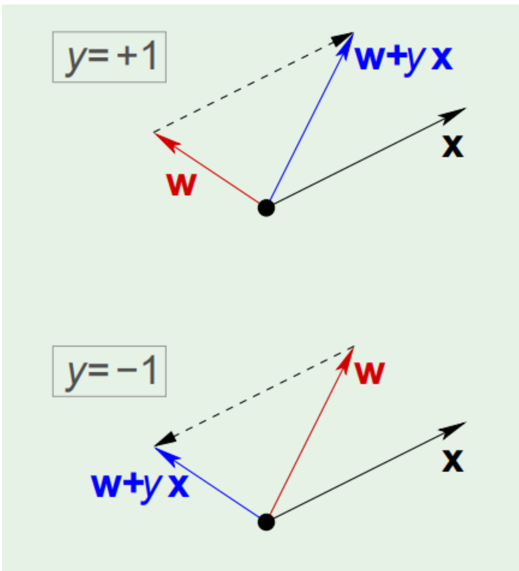
- $w(t)^\intercal x(t) = 1$, then $y(t) = -1$. So $1 * (-1) = -1 < 0$
- $w(t)^\intercal x(t) = -1$, then $y(t) = 1$. So $-1 * (1) = -1 < 0$

0.2 (b)

Since $w^\intercal(t)$ became $w^\intercal(t+1)$ we can say that $w(t)^\intercal x(t)$ classification was not correct. There are two possibilities:

- $w(t)^\intercal x(t) = 1$, then $y(t) = -1$. So $w(t+1) = w(t) - x(t)$. As result $w(t+1) < w(t)$ and as $y(t)w(t)^\intercal x(t) < 0$ we can say that $y(t)w(t+1)^\intercal x(t) > y(t)w(t)^\intercal x(t)$
- $w(t)^\intercal x(t) = -1$, then $y(t) = 1$. So $w(t+1) = w(t) + x(t)$. As result $w(t+1) > w(t)$ and as $y(t)w(t)^\intercal x(t) > 0$ we can say that $y(t)w(t+1)^\intercal x(t) > y(t)w(t)^\intercal x(t)$

0.3 (c)



At the first image dot product of $w \cdot x < 0$ and $y = 1$ so we increase w
At the second image dot product of $w \cdot x > 0$ and $y = -1$ so we decrease w