1 Calculating coflow completion time

We use linear programming to solve multi-commodity flow problem and minimize the coflow completion time t, which is equivalent to maximize $\alpha = 1/t$.

$$\text{minimize } t, \quad \text{maximize } \alpha = \frac{1}{t} \tag{1}$$

Suppose a network graph $G = (V, E)$. The edge in the network graph (u, w) has bandwidth $c(u, w)$. The ith flow in the coflow C has origin node b_i, destination node e_i and a volume of v_i. Let $f_i(u, w)$ denote the rate of flow i transfered on edge (u, w). We can the following constrains for a linear programming problem according to the flow conservation and capacity constrain of the overall net graph.

$$f_i(u, w) \geq 0, \quad \forall u, w \in V, \forall i \in C \tag{1a}$$

$$\sum_{u \in V} f_i(b_i, u) = \alpha v_i, \quad \forall i \in C. \qquad f_i(u, b_i) = 0, \quad \forall u \in V, \forall i \in C \tag{1b}$$

$$\sum_{u \in V} f_i(u, e_i) = \alpha v_i, \quad \forall i \in C. \qquad f_i(e_i, u) = 0, \quad \forall u \in V, \forall i \in C \tag{1c}$$

$$\sum_{w \in V} f_i(u, w) + f_i(w, u) = 0, \quad \forall u \neq b_i, e_i, \forall i \in C \tag{1d}$$

$$\sum_{i \in C, f_i(u,w) > 0} f_i(u, w) \leq c(u, w), \quad \forall u, w \in V \tag{1e}$$

Here we can set $c(u, w) = c(w, u)$ so that the bandwidth on both direction is the same. Or we can have a more general case where $c(u, w) \neq c(w, u)$, which means the bandwidths are different on each direction.

Equations (1a), (1b), (1c) and (1d) are formulated based on conservation of flows on each edge and each node. Origin node and destination node are different from other nodes since they send out and take in flow data respectively. Equations (1e) are formulated based on the bandwidth constrains of each edge in the network graph.

Now we consider another situation, where the bandwidth of an edge is shared between both direction, namely the sum of the bandwidth from both direction should be under the constrain of $c(u, w)$. Then the constrain (1e) listed above for the bidirectional linked network would be changed. Instead, we will have a new constrain (1e′), but the linear programming still works.

$$\sum_{i \in C, u \neq w} f_i(u, w) + f_i(w, u) \leq c(u, w), \quad \forall u, w \in V \tag{1e'}$$