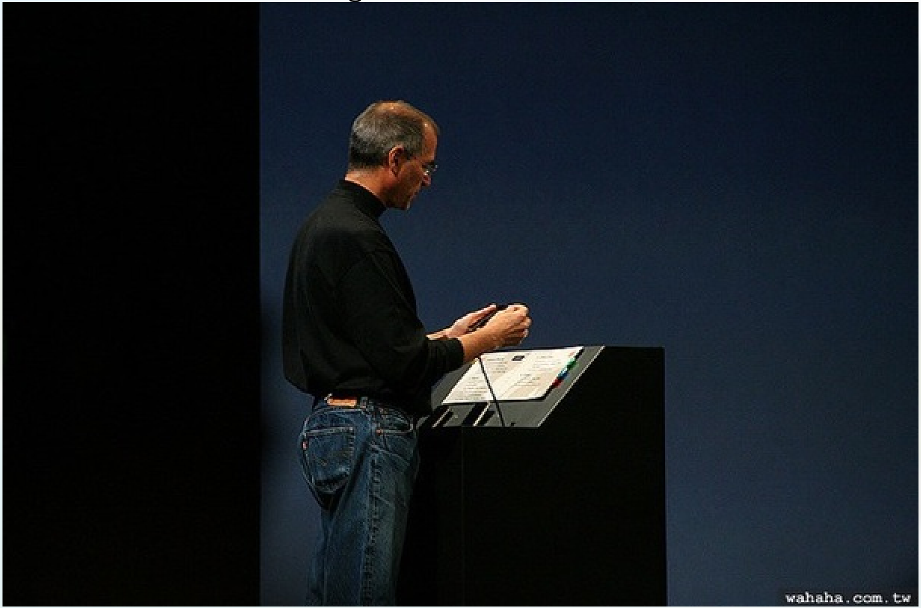
Channel Manipulation as a Coding Technique

Hsin-Po Wang (EECS, UC Berkeley)

What do the following pictures have in common?

(Images from the internet)



(Images from the internet)

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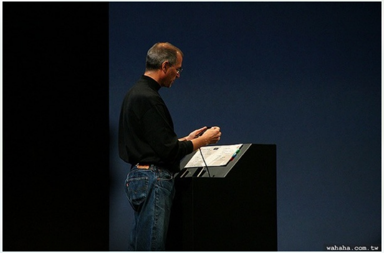
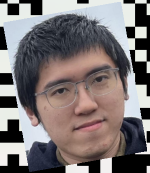


https://jobtalk.symbol.codes



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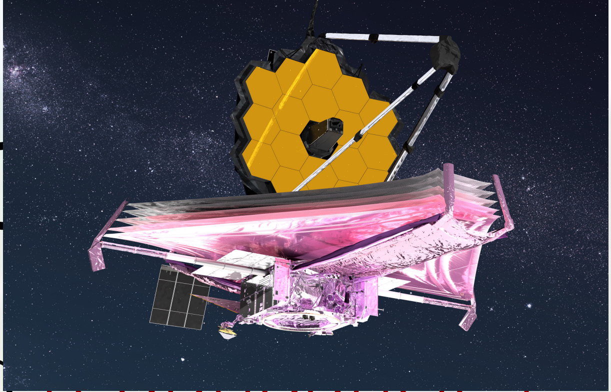
Coding

= adding redundancies
in a smart way

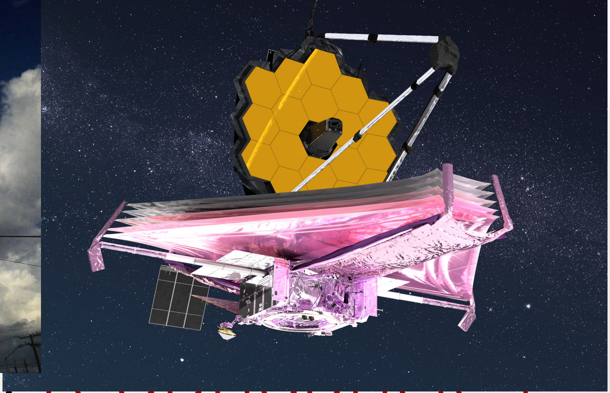
Co



= adding

in a smart way

(Images from Wikipedia, NASA, IBM, Qualcomm)



– adding redundancies
in a smart way

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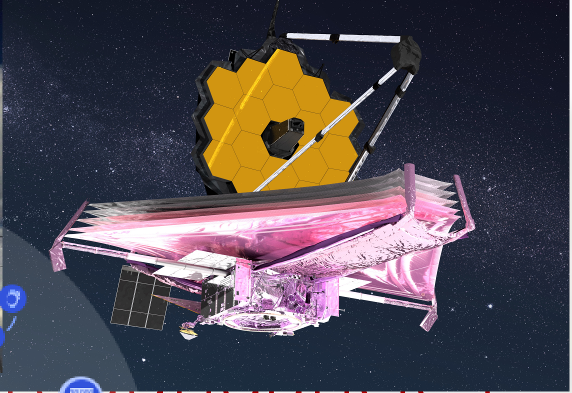

smart way

ipedia, NASA, IBM, Qualcomm)

Channel Manipulation is Coding

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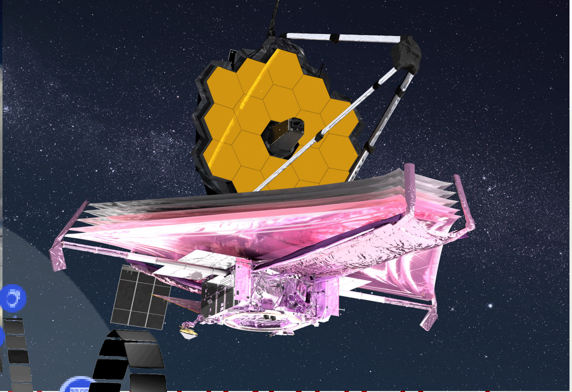

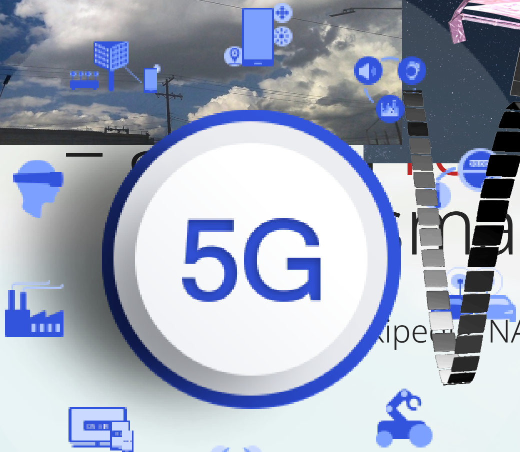

smart

M, Qualcomm)

Channel Manipulation is Coding

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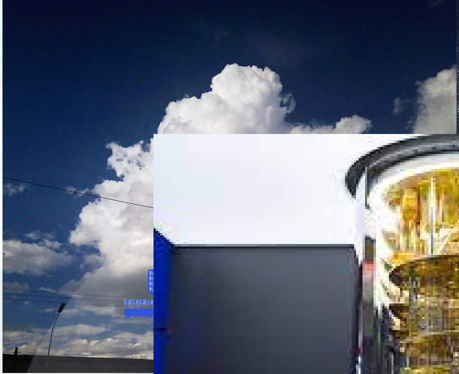

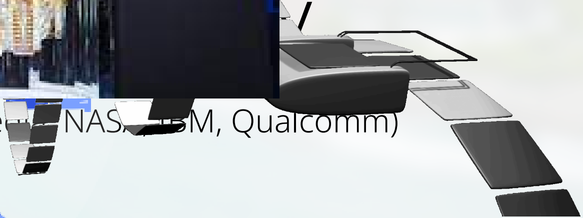

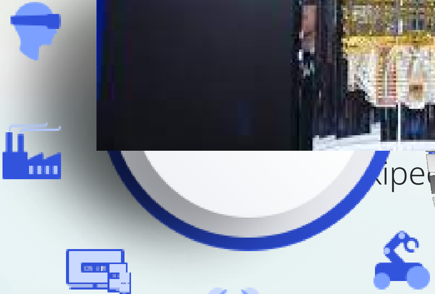

ipe NAS BM, Qualcomm)

Channel Manipulation is Coding

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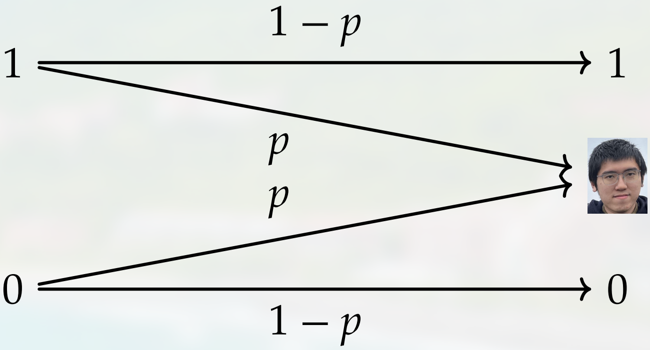
Math Framework of Coding



Binary erasure channel with erasure probability p: BEC(p)

N = 78 S = 83 Y = 89 U = 85

The “NSYSU” polynomial:

$$f(x) = 78 + 83x + 89x^2 + 83x^3 + 85x^4$$

N = 78 S = 83 Y = 89 U = 85

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$$f(x) = 78 + 83x + 89x^2 + 83x^3 + 85x^4$$

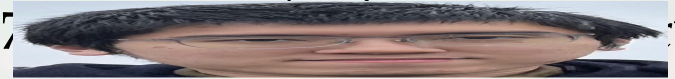
$f(-3) = 274$ $f(-2) = 964$ $f(-1) = 86$ $f(0) = 78$

$f(1) = 418$ $f(2) = 624$ $f(4) = 254$ $f(4) = 906$

H.-P. Wang

N = 78 S = 83 Y = 89 U = 85

The “NSYSU” polynomial:

$f(x) = 7$  4

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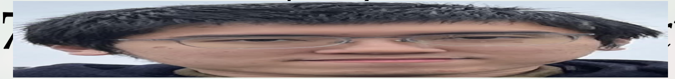
$f(x) = 7$  4

$f(-3) = 274$ $f(-2) = 964$ $f(-1) = 86$ $f(0) = 7$ 

$f(1) = 4$  $f(2) = 624$ $f(4) = 254$ $f(4) = 906$

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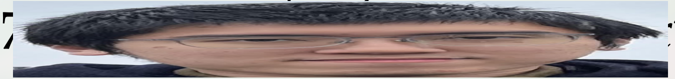
The "NSYSU" polynomial:

$f(x) = 7$  4

$f(-3) = 274$ $f(-2) = 964$ $f(-1) = 8$  $f(0) = 7$ 

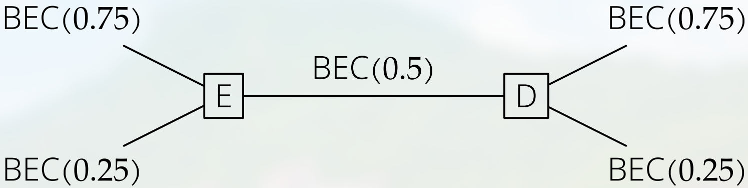
$f(1) = 4$  $f(2) = 624$ $f(4) = 254$ $f(4) = 906$

New Idea

Polar Codes

$$\boxed{\mathrm{E}} \overset{\mathrm{BEC}(0.5)}{\longrightarrow} \boxed{\mathrm{D}}$$

Suppose there are magic devices $\boxed{\mathrm{E}}$ and $\boxed{\mathrm{D}}$ that turns $\mathrm{BEC}(x)$ into $\mathrm{BEC}(x^2)$ and $\mathrm{BEC}(2x - x^2)$.



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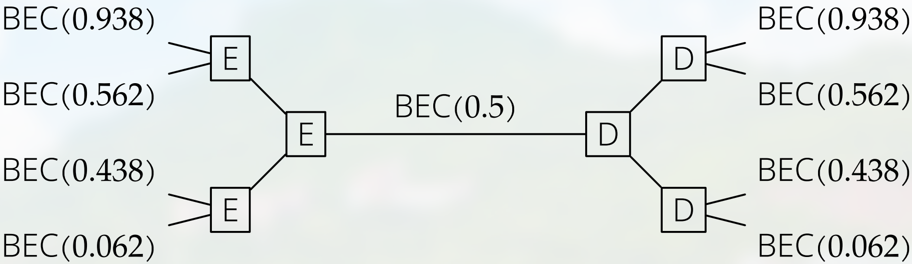
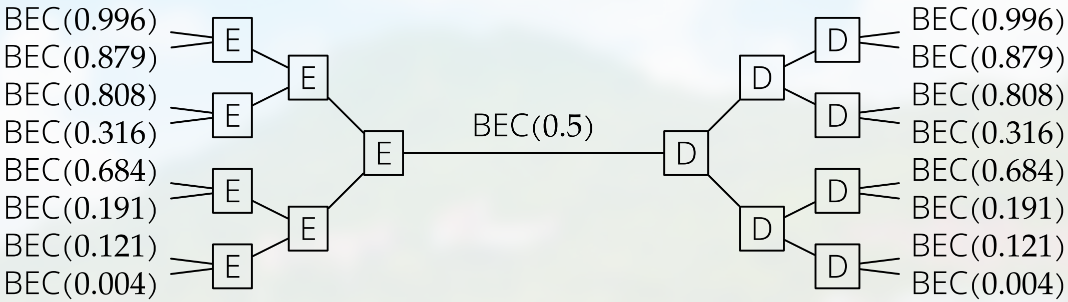
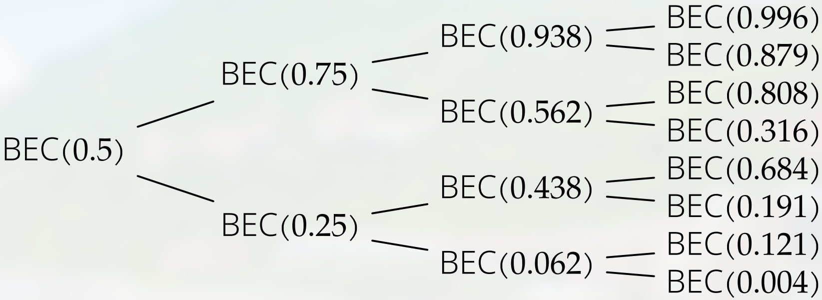


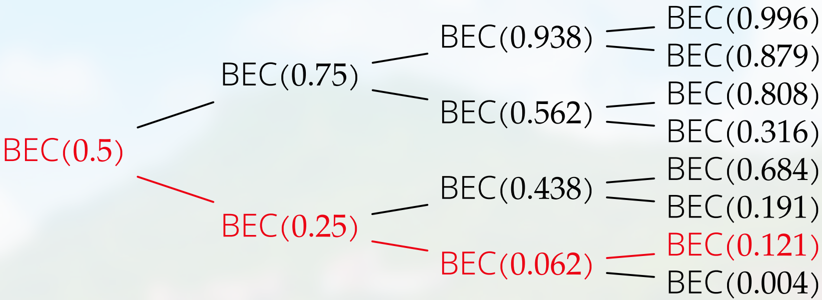
What if we apply apply more magic devices?



What if we apply apply more magic devices?
And more and more and more???



This is a tree




: This is a martingale

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$

BEC(0.5)

- BEC(0.75)
  - BEC(0.938)
    - BEC(0.996)
    - BEC(0.879)
  - BEC(0.562)
    - BEC(0.808)
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: Bounded martingale converges.

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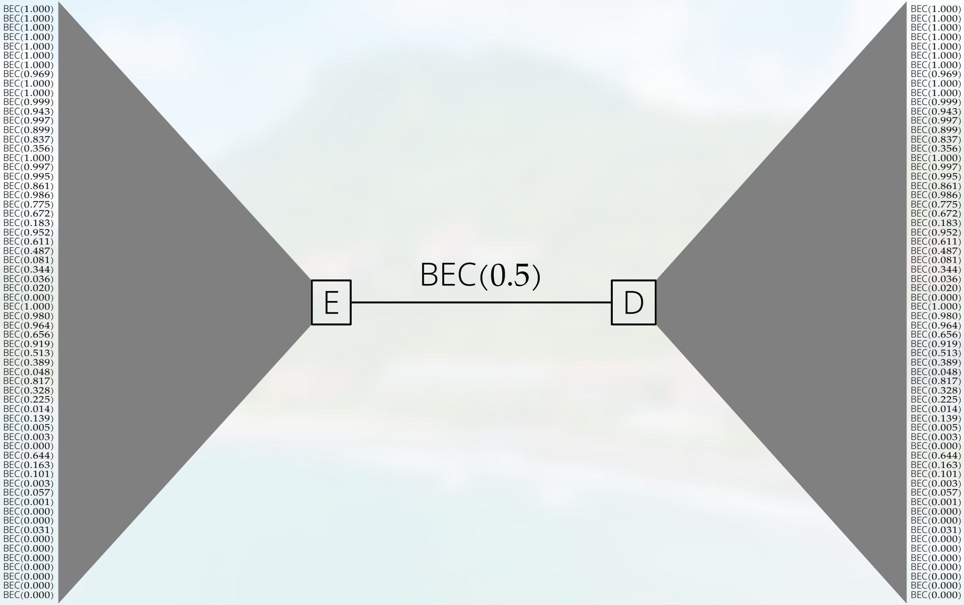
: Bounded martingale converges.

: BECs converges to 0 or 1.

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$


BEC(1.000)
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BEC(0.969)
BEC(1.000)
BEC(1.000)
BEC(0.999)
BEC(0.943)
BEC(0.997)
BEC(0.899)
BEC(0.837)
BEC(0.356)
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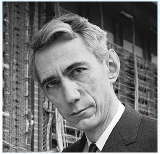
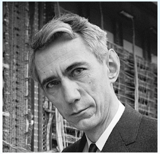
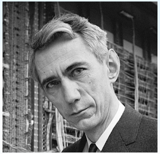
 : 5000 bits / 10000 channel uses



: 5000 bits / 10000 channel uses

: Polar code 490bits/10000uses



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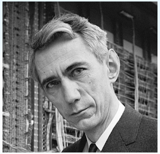
In general, $(1/2-\varepsilon)N$ bits / N uses



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In general, $(1/2 - \varepsilon)N$ bits / N uses

THE END?

5G is for

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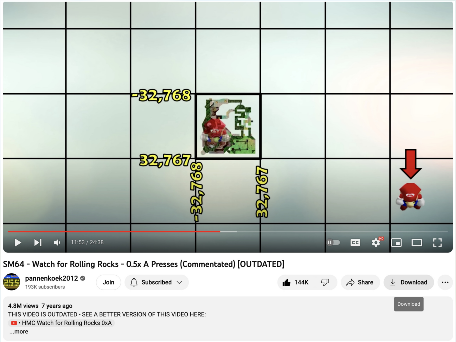

video streaming

Channel Manipulation is Coding

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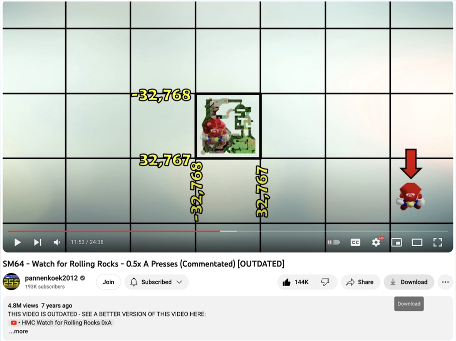
5G is for



video streaming

BBC

flying drone

Channel Manipulation is Coding

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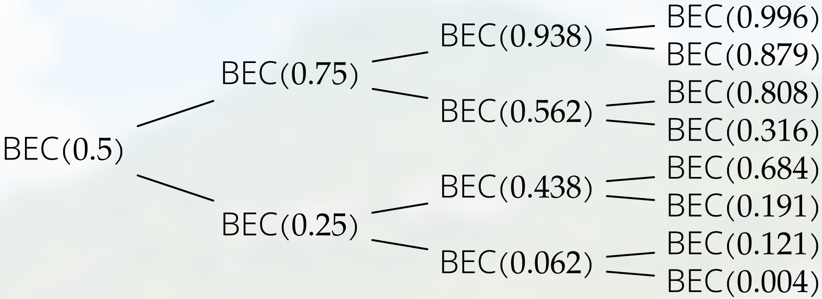
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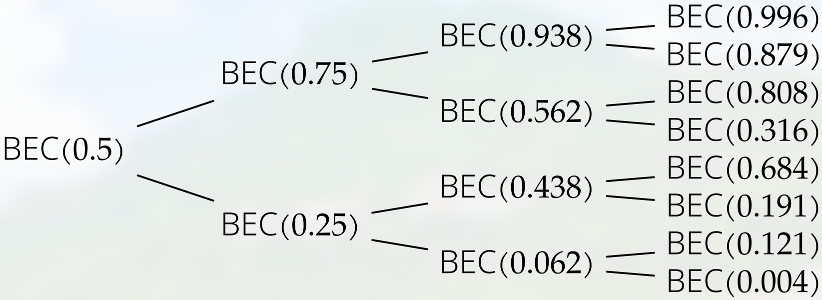
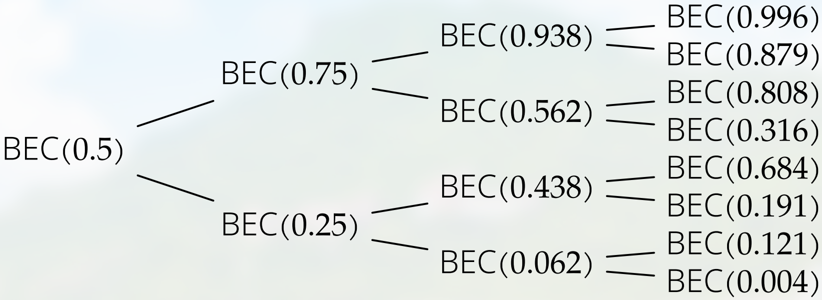


Early channels are not useful.



Early channels are not useful.
How to accelerate polarization?

$(1/2 - N^{-\rho})N$ bits / N channel uses

History of ρ over BMSC: 0 $1/2$

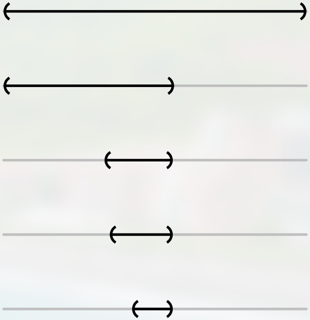
2015 Guruswami–Xia

2012 Goli–Hassani–Urbanke

2014 Hassani–Alishahi–Urbanke

2014 Goldin–Burshtein

2016 Mondelli–Hassani–Urbanke



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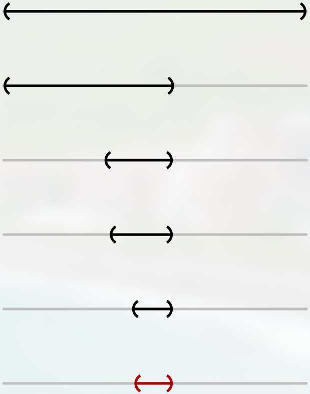
2022 W.–Lin–Vardy–Gabrys



Improve ρ over BEC: 0 $1/2$

2010 Hassani–Alishahi–Urbanke 2×2

2010 Korada–Montanari–Telatar–Urbanke 2×2

2014 Fazeli–Vardy 8×8

2021 Trofimiuk–Trifonov 16×16

2021 Trofimiuk 24×24

2021 Yao–Fazeli–Vardy 32×32

2021 Yao–Fazeli–Vardy 64×64

Improve ρ over BEC: 0 $1/2$

2010 Hassani–Alishahi–Urbanke 2×2

2010 Korada–Montanari–Telatar–Urbanke 2×2

2014 Fazeli–Vardy 8×8

2021 Trofimiuk–Trifonov 16×16

2022 Duursma–Gabrys–Guruswami–Lin–W. $2 \times 2/\mathrm{GF}4$

2021 Trofimiuk 24×24

2021 Yao–Fazeli–Vardy 32×32

2021 Yao–Fazeli–Vardy 64×64

The optimal ρ: 0 $1/2$

2019 Pfister–Urbanke
q-ary erasure channel, $q \to \infty$

2021 Fazeli–Hassani–Mondelli–Vardy
binary erasure channel

2022 Guruswami–Riazanov–Ye
binary symmetric memoryless channel

The optimal ρ: 0 $1/2$

2019 Pfister–Urbanke
q-ary erasure channel, $q \to \infty$

2021 Fazeli–Hassani–Mondelli–Vardy
binary erasure channel

2022 Guruswami–Riazanov–Ye
binary symmetric memoryless channel

2021 W.–Duursma
discrete memoryless channel

2011 Alamdar-Yazdi–Kschischang:
Prune the tree to reduce complexity.

2017 El-Khamy–Mahdavifar–Feygin–Lee–Kang:
Pruning reduces complexity by a scalar; still $O(N \log N)$.

2021 Mondelli–Hashemi–Cioffi–Goldsmith,
2021 Hashemi–Mondelli–Fazeli–Vardy–Cioffi–Goldsmith:
Study parallelism vs latency.

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Pruning reduces complexity by a scalar; still $O(N \log N)$.

2021 W.–Duursma: $O(N \log \log N)$
if relax the performance requirement.

Trade-off: complexity $\approx O(N \log(-\log(\text{decode error})))$.

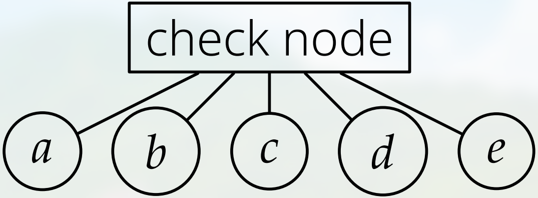
2021 Mondelli–Hashemi–Cioffi–Goldsmith,
2021 Hashemi–Mondelli–Fazeli–Vardy–Cioffi–Goldsmith:
Study parallelism vs latency.

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

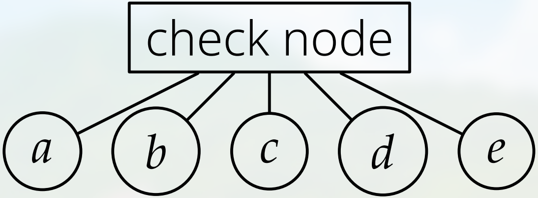
Wikipedia

Low-Density Parity-Check (LDPC) Codes

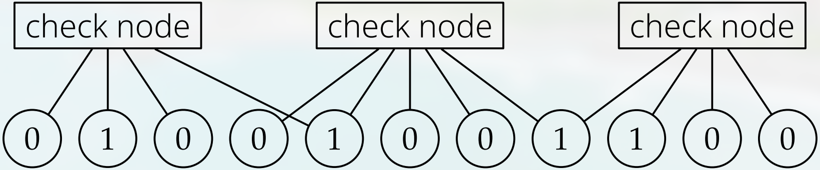
Rule: Every

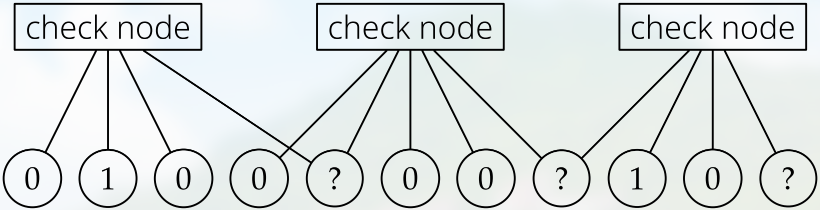


Sum to an even number

Rule: Every 

Sum to an even number




check node
check node
check node
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0
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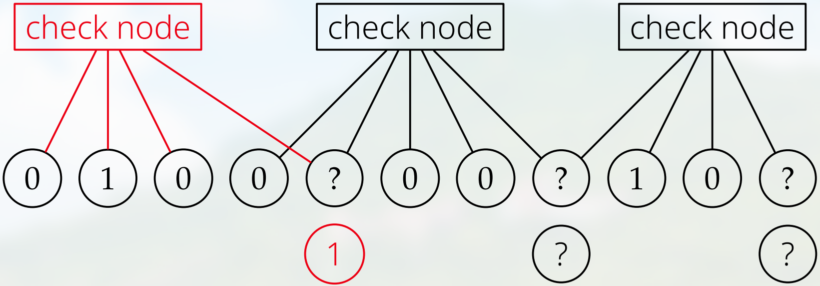
?
1
0
?


check node
check node
check node
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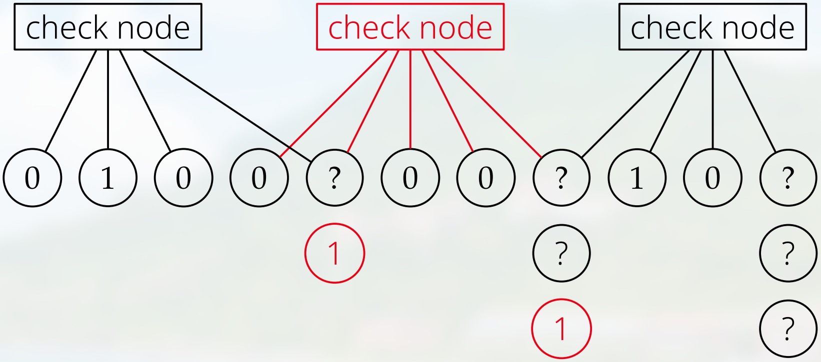
check node
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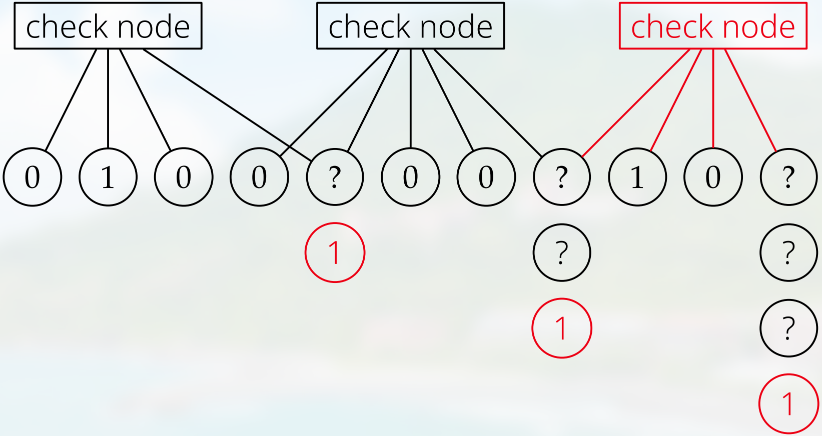
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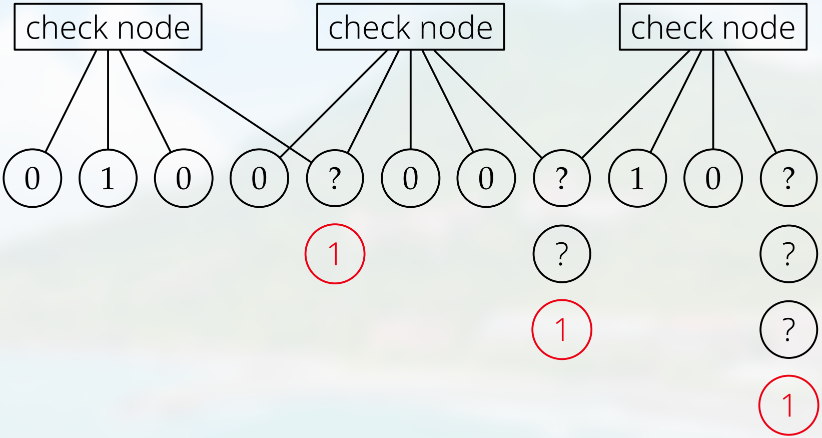
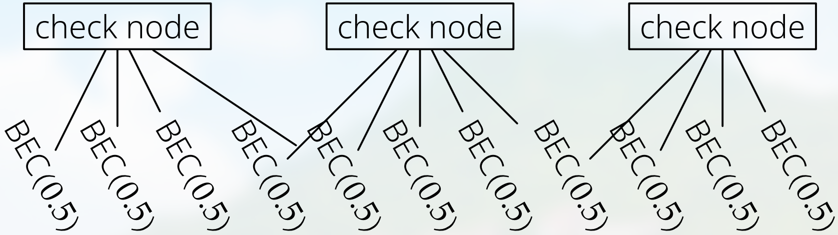


How to analyze, mathematically?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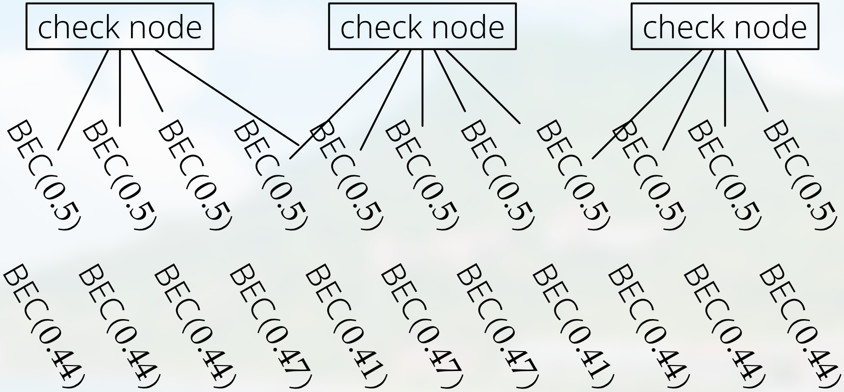
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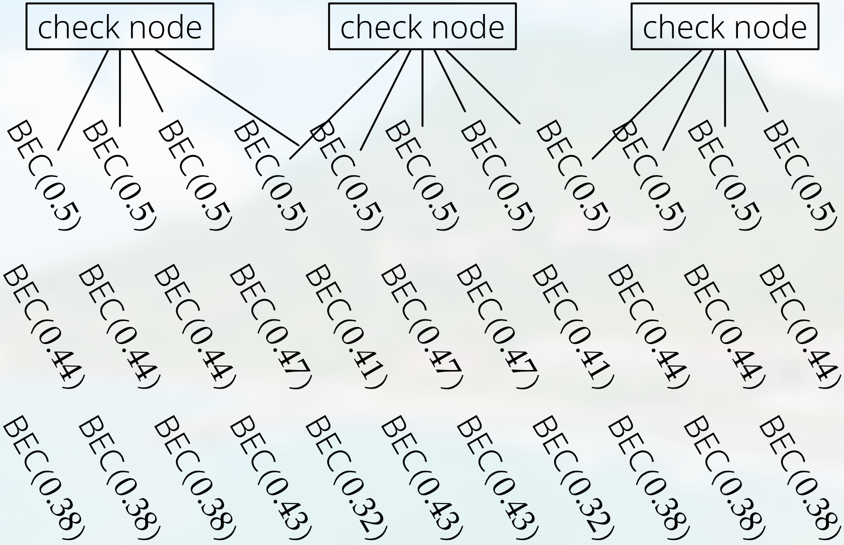
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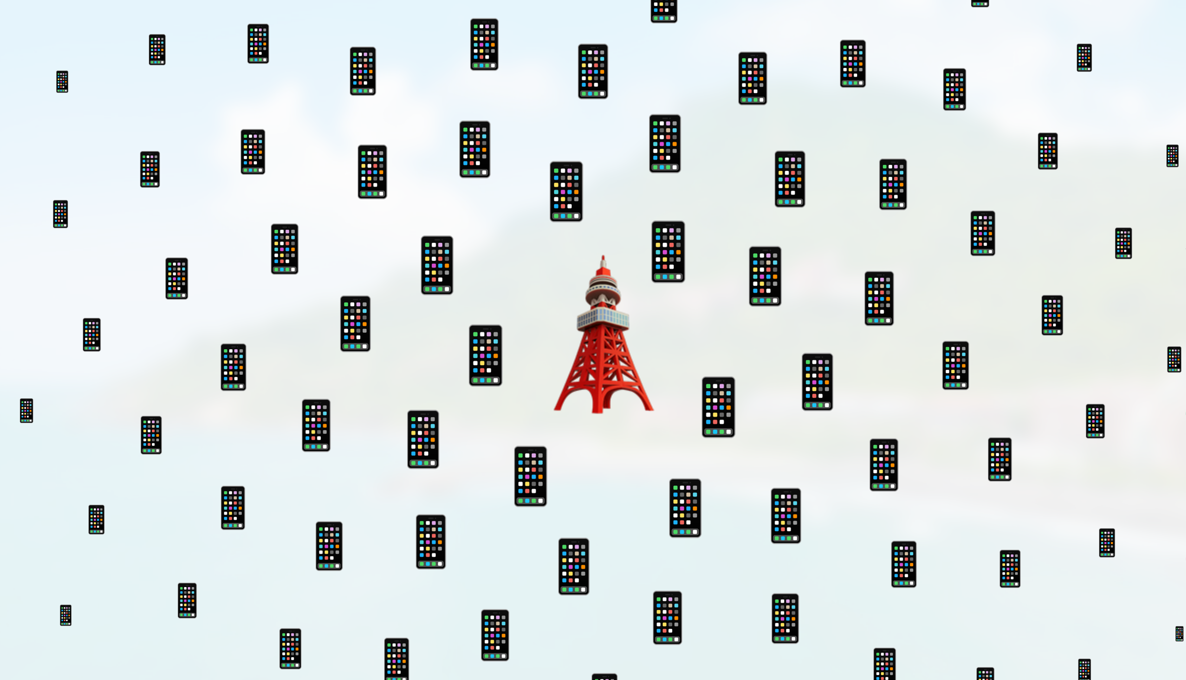
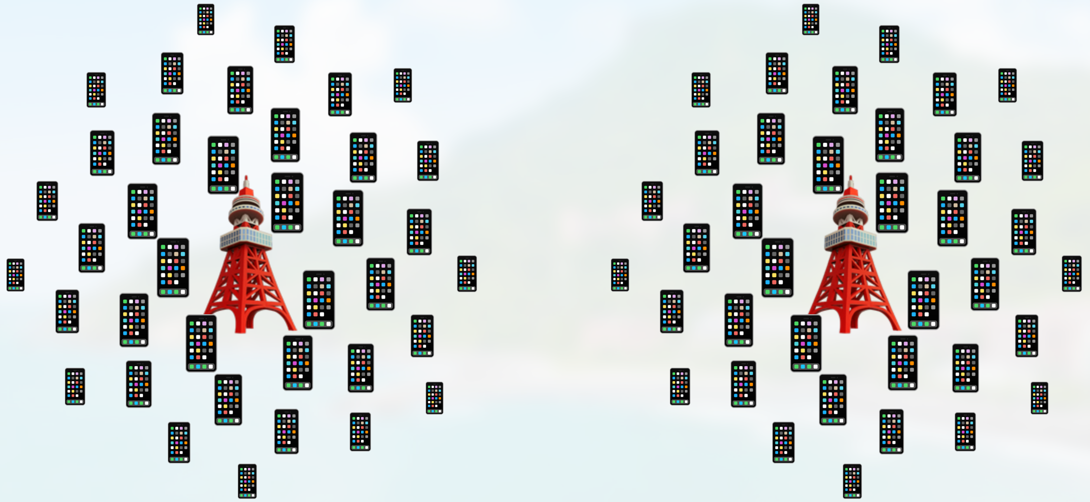
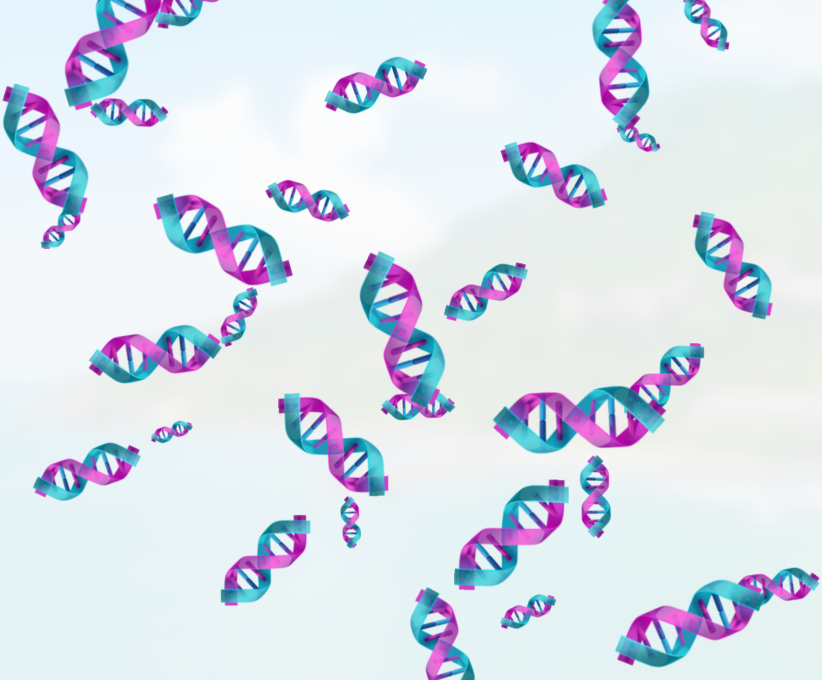
Challenges



Channel Manipulation is Coding

H.P. Wang





DNA coding

Permutation

Amplification

Substitution

Insertion

Deletion