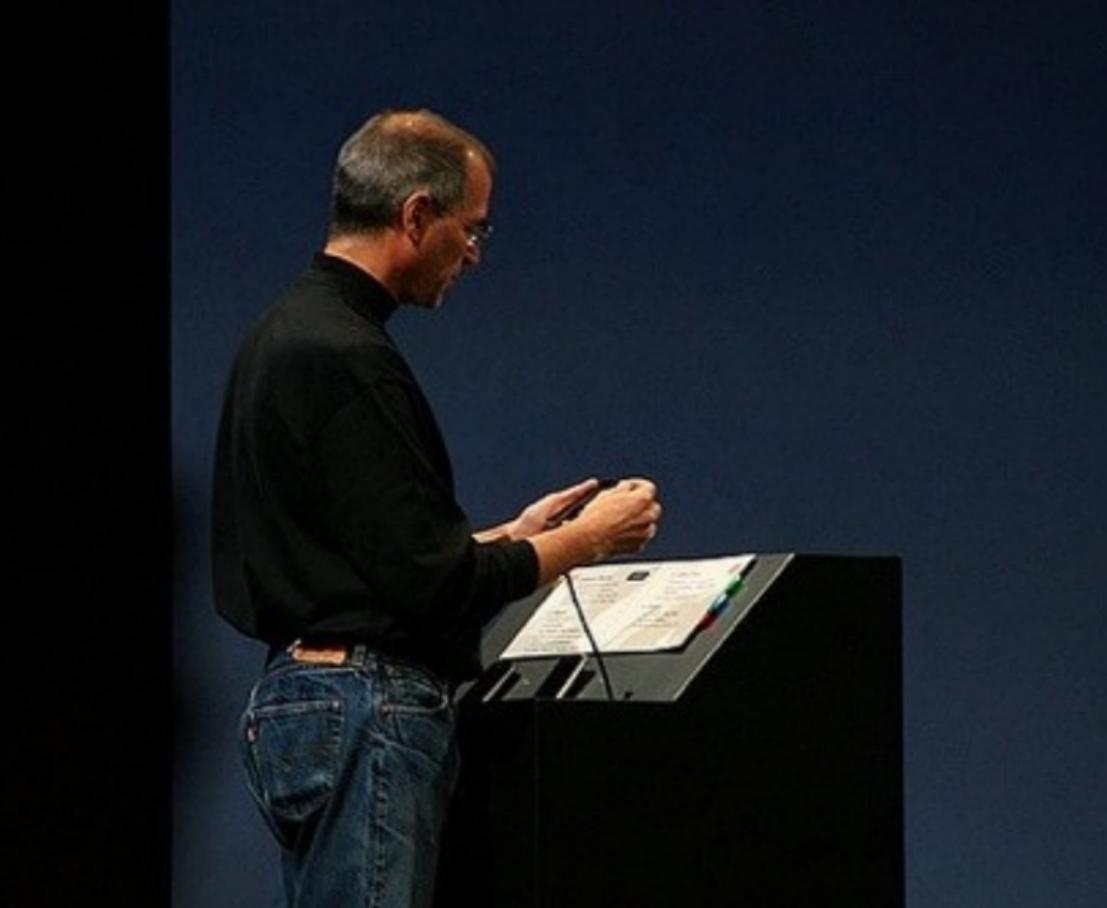


Channel Manipulation as a Coding Technique

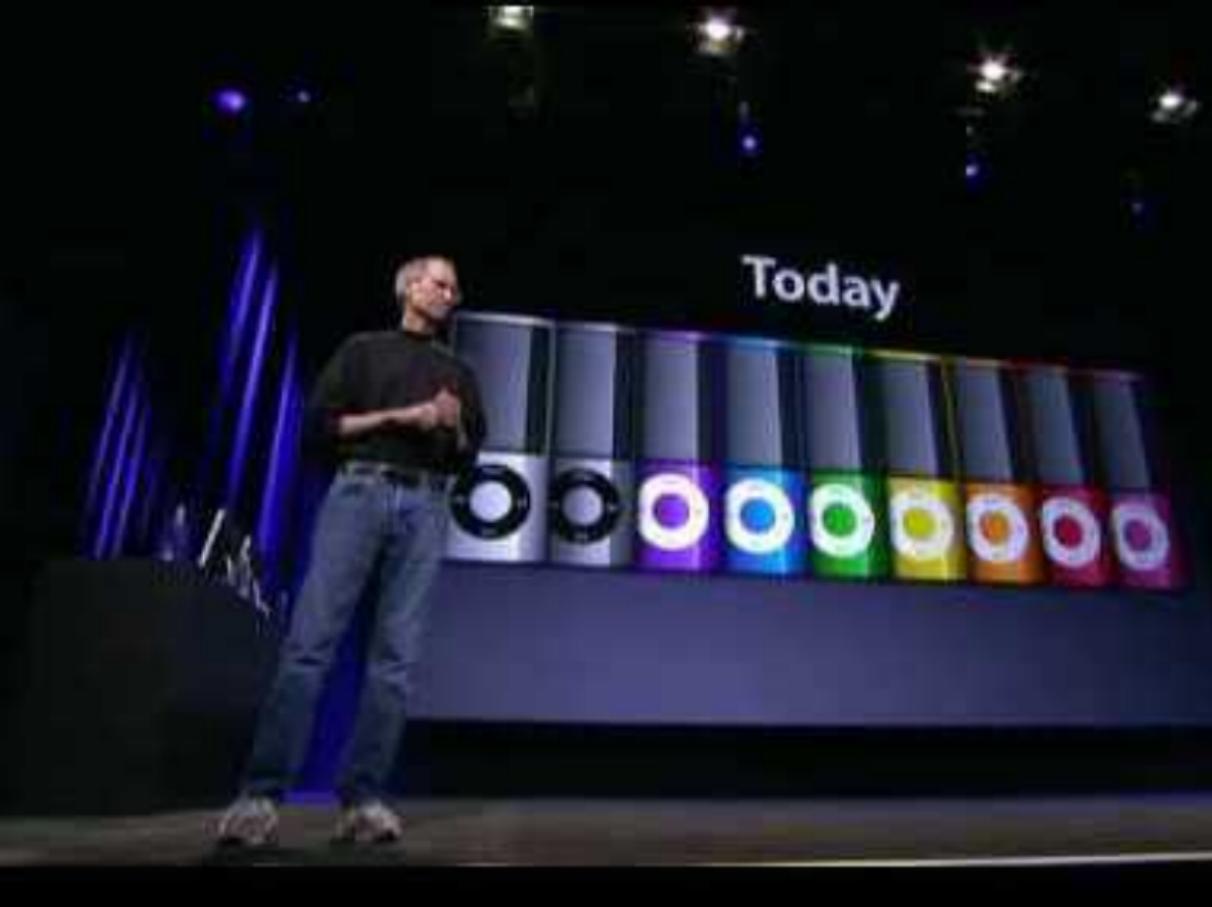
Hsin-Po Wang (EECS, UC Berkeley)

What do the
following pictures
have in common?

(Images from Wired, Youtube, iPhone in Canada)



(Images from Wired, Youtube, iPhone in Canada)



(Images from Wired, Youtube, iPhone in Canada)



(Images from Wired, Youtube, iPhone in Canada)



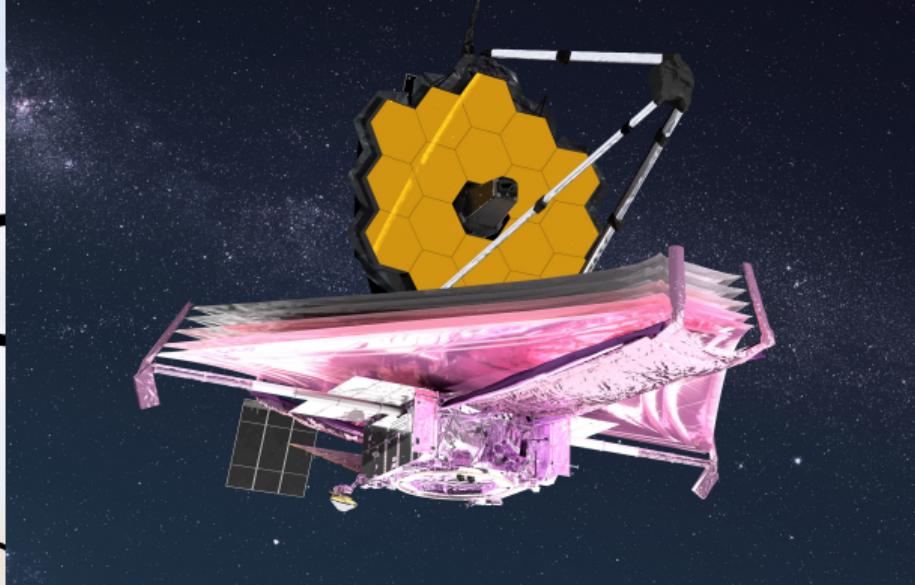


<https://jobblk.symbol.codes>

Coding

= adding redundancies
in a smart way

Coding
= adding ~~redundancies~~
in a smart way



(Images from Wikipedia, NASA, IBM, Qualcomm)



Redundancies in a smart way

(Images from Wikipedia, NASA, IBM, Qualcomm)



roadmap
smart way



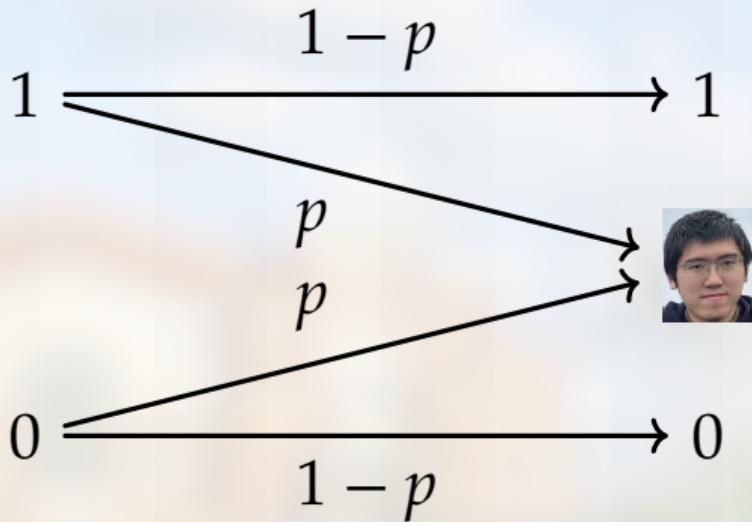
(Images from Wikipedia, NASA, IBM, Qualcomm)







Math Framework of Coding



Binary erasure channel with erasure probability p : BEC(p)

$$N = 78$$

$$T = 84$$

$$U = 85$$

$$E = 69$$

The “NTUEE” polynomial:

$$f(x) = 78 + 84x + 85x^2 + 69x^3 + 69x^4$$

N = 78

T = 84

U = 85

E = 69

The “NTUEE” polynomial:

$$f(x) =$$



$$f(-3) = 274 \quad f(-2) = 964 \quad f(-1) = 86 \quad f(0) = 78$$

$$f(1) = 418 \quad f(2) = 624 \quad f(3) = 254 \quad f(4) = 906$$

N = 78

T = 84

U = 85

E = 69

The “NTUEE” polynomial:

$$f(x) =$$



$$f(-3) = 274 \quad f(-2) = 964 \quad f(-1) = 86 \quad f(0) = ?$$



$$f(1) = 418 \quad f(2) = 624 \quad f(4) = 254 \quad f(4) = 906$$

$N = 78$ $T = 84$ $U = 85$ $E = 69$

The “NTUEE” polynomial:

$$f(x) =$$



$$f(-3) = 274 \quad f(-2) = 964 \quad f(-1) = 86 \quad f(0) = ?$$



$$f(1) = ? \quad f(2) = 624 \quad f(4) = 254 \quad f(4) = 906$$

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U = 85

E = 69

The “NTUEE” polynomial:

$$f(x) =$$



$$f(-3) = 274 \quad f(-2) = 964 \quad f(-1) = 86 \quad f(0) = ?$$



$$f(1) = ? \quad f(2) = 624 \quad f(4) = ? \quad f(4) = 906$$



$N = 78$ $T = 84$ $U = 85$ $E = 69$

The “NTUEE” polynomial:

$$f(x) =$$



$$f(-3) = 274$$

$$f(-2) = 9$$



$$f(-1) = 86$$

$$f(0) = ?$$



$$f(1) = 4$$

$$f(2) = 624$$



$$f(4) = ?$$

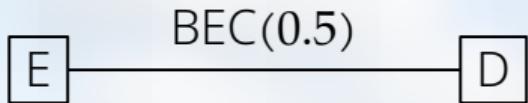
$$f(4) = 906$$



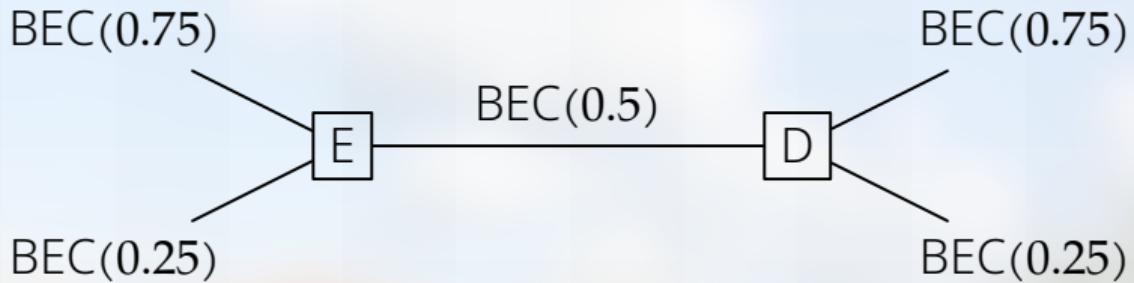
New Idea



Polar Codes



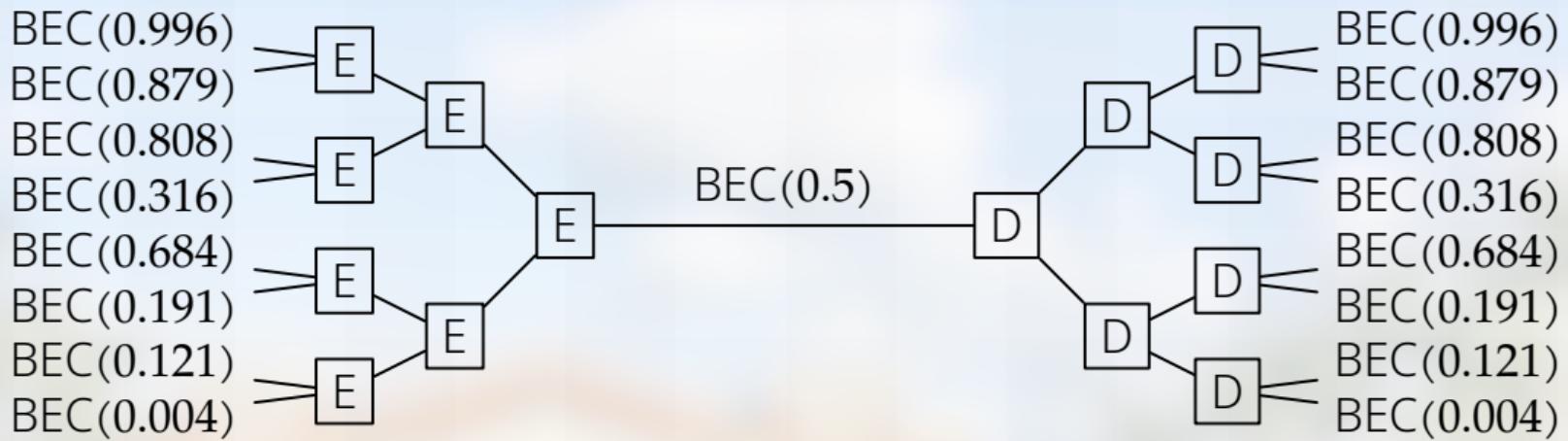
Suppose there are magic devices \boxed{E} and \boxed{D} that turns $\text{BEC}(x)$ into $\text{BEC}(x^2)$ and $\text{BEC}(2x - x^2)$.



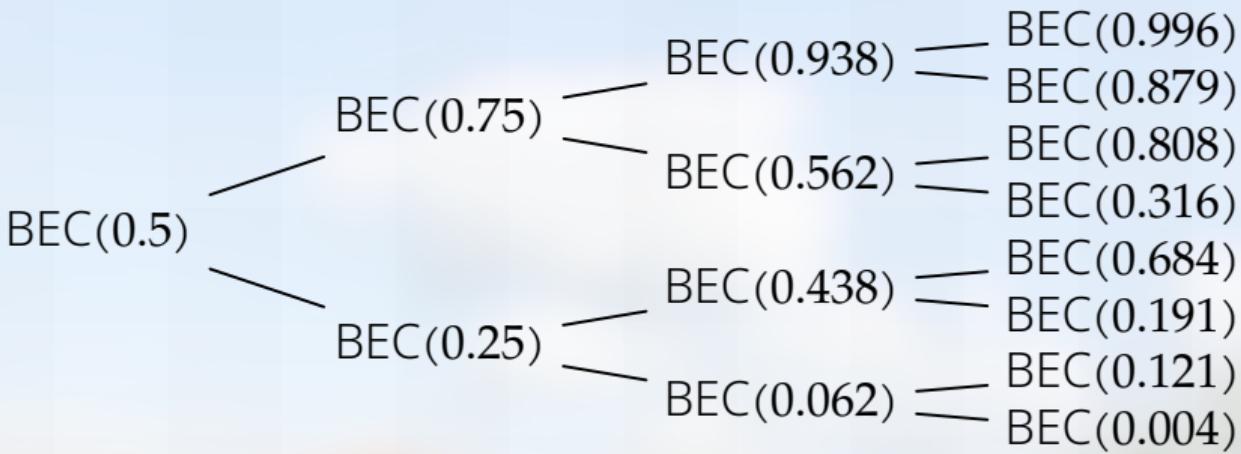
Suppose there are magic devices \boxed{E} and \boxed{D} that turns $\text{BEC}(x)$ into $\text{BEC}(x^2)$ and $\text{BEC}(2x - x^2)$.



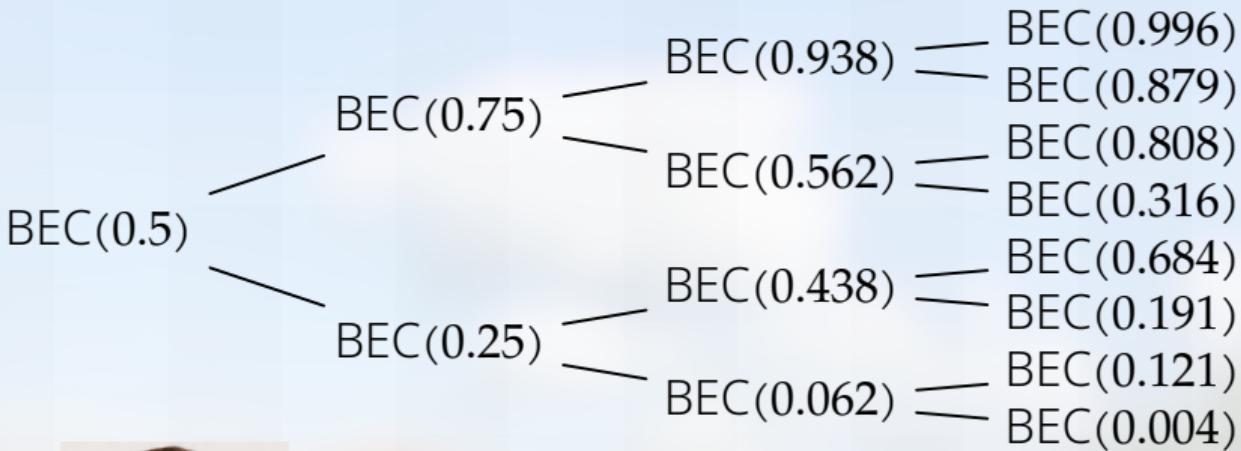
What if we apply more magic devices?



What if we apply apply more magic devices?
And more and more and more???

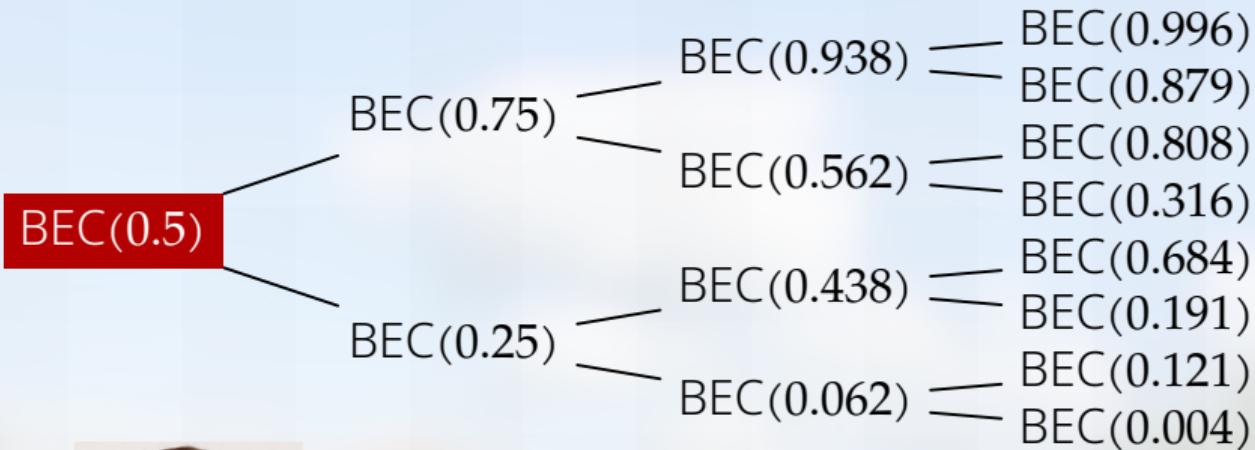


This is a tree



: This is a martingale

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$



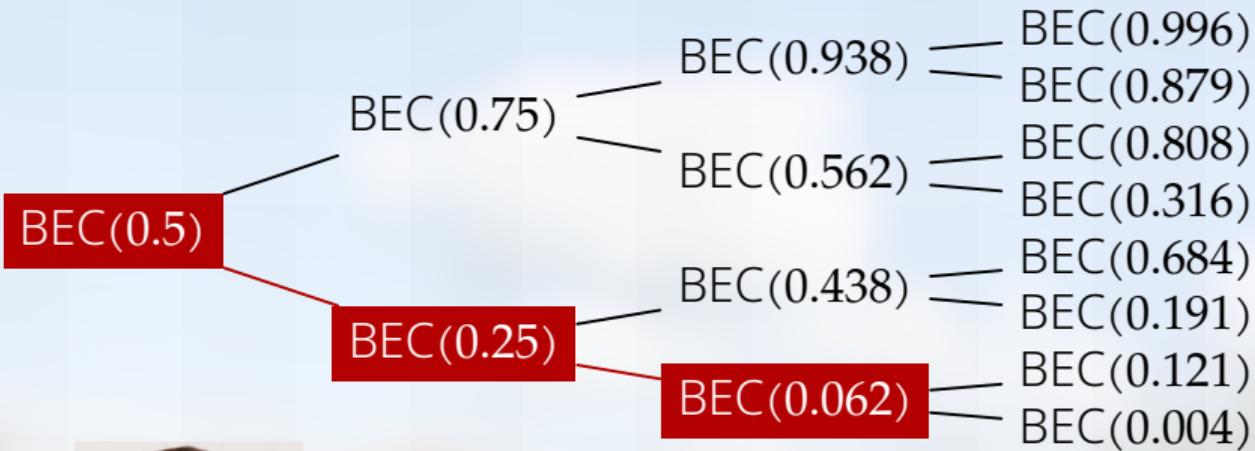
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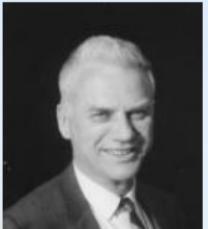
: This is a martingale

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$



: This is a martingale

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$



: Bounded martingale converges.

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$

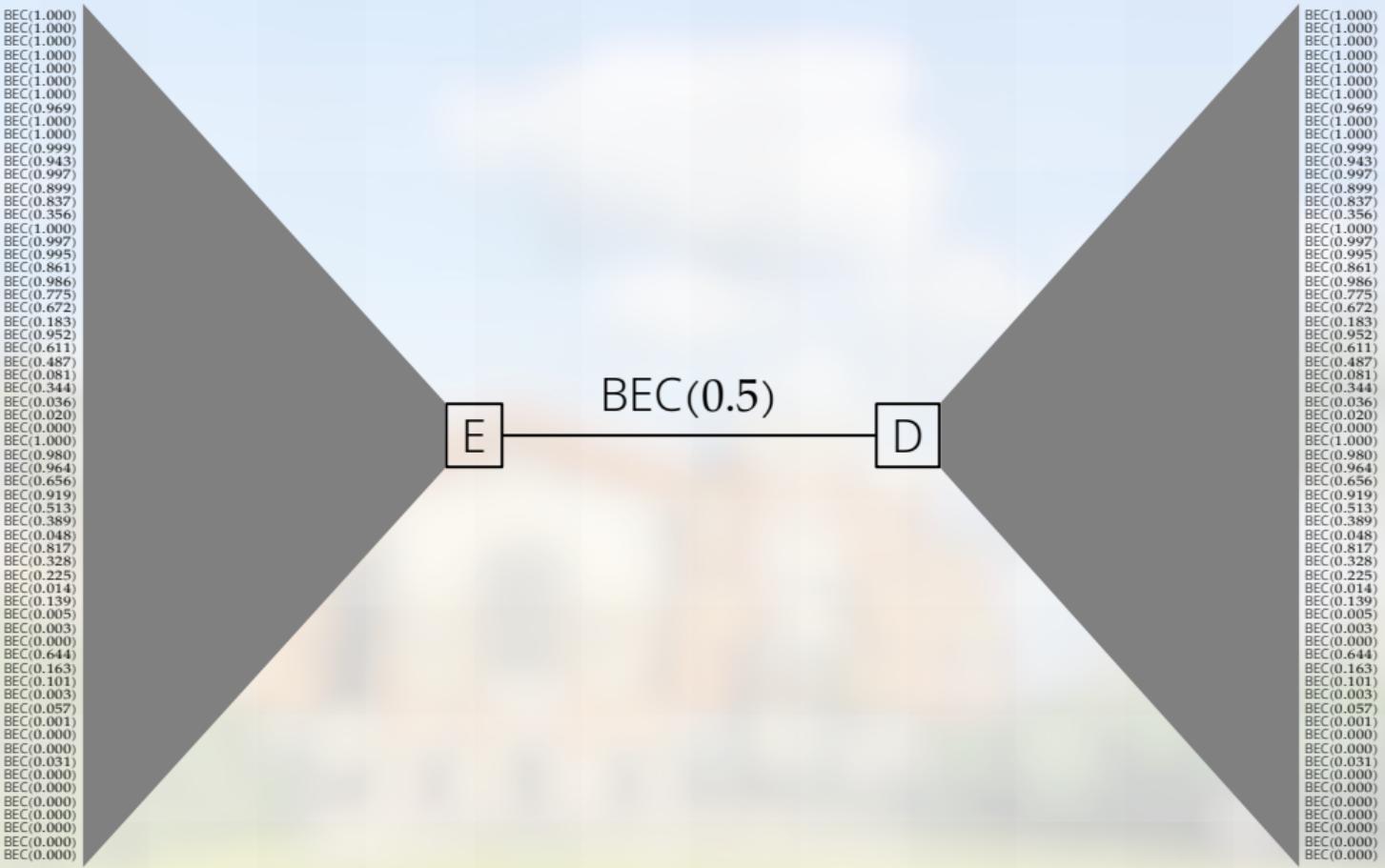


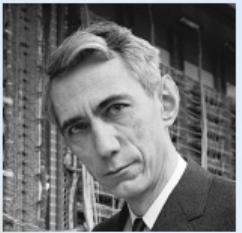
: Bounded martingale converges.



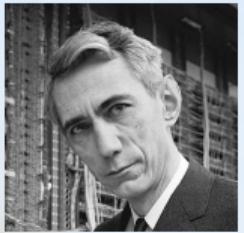
: BECs converges to 0 or 1.

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$





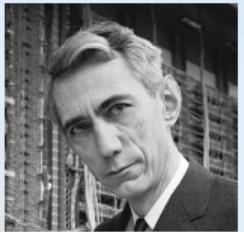
: 5000 bits / 10000 channel uses



: 5000 bits / 10000 channel uses



: Polar 4900 bits / 10000 uses

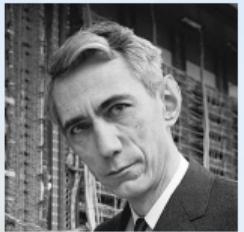


: 5000 bits / 10000 channel uses



: Polar 4900 bits / 10000 uses

In general, $(1/2 - \varepsilon)N$ bits / N uses



: 5000 bits / 10000 channel uses



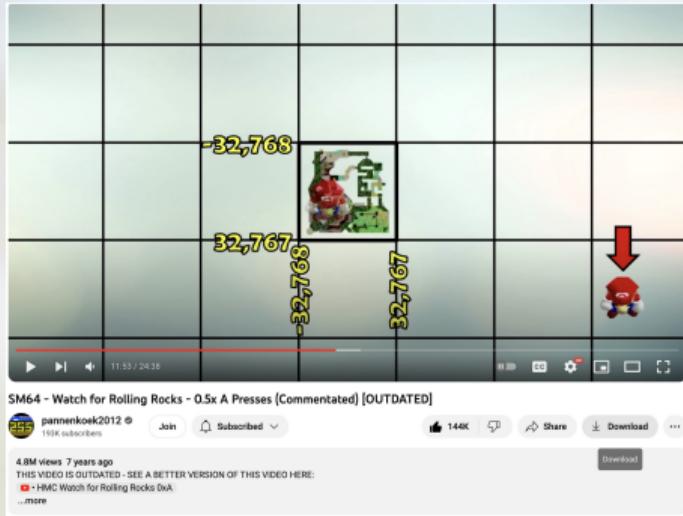
: Polar 4900 bits / 10000 uses

In general, $(1/2 - \varepsilon)N$ bits / N uses

THE END?

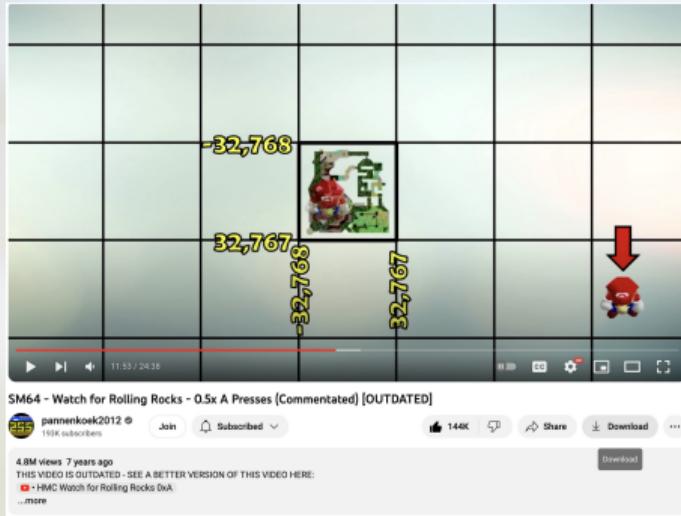
5G is for

5G is for



video streaming

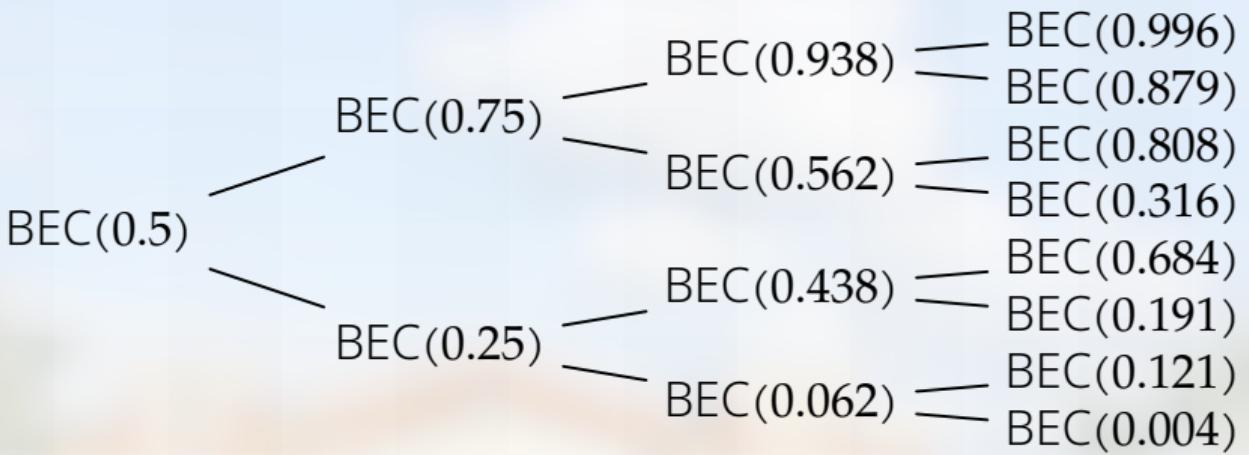
5G is for

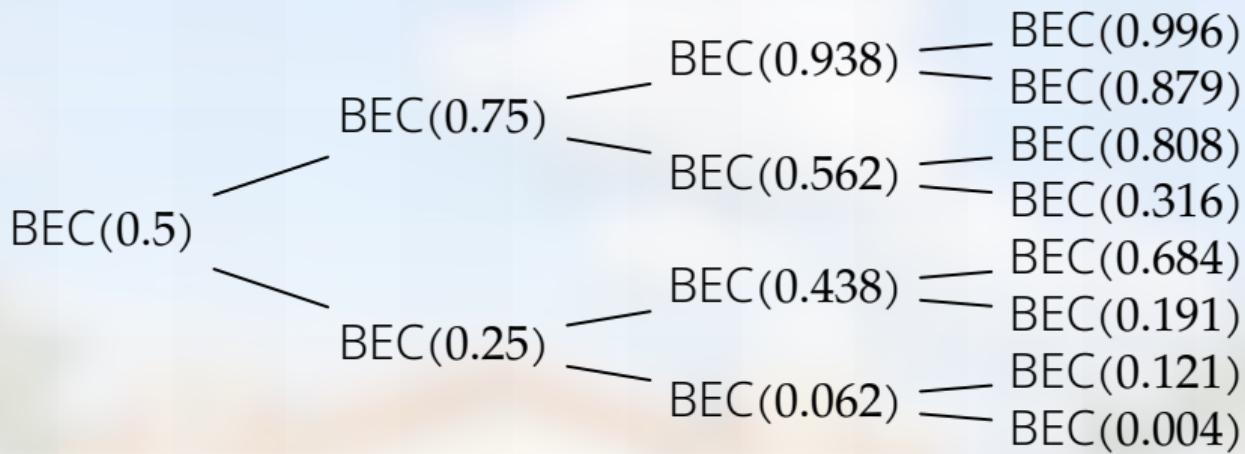


video streaming

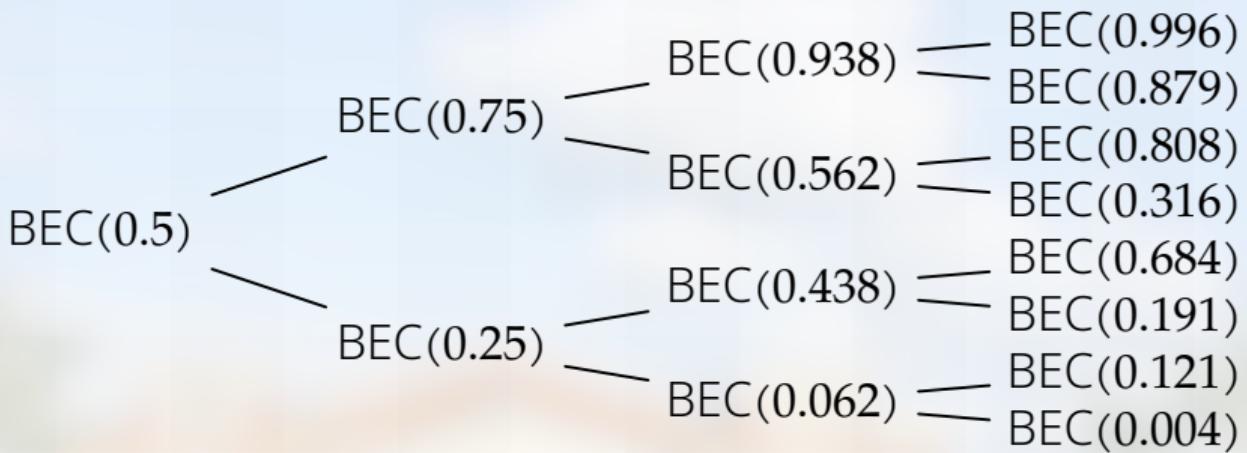


flying drone





Recall tree: early channels not useful.



Recall tree: early channels not useful.
How to accelerate polarization?

Martingale-Code Rate Thm [folklore]

Martingale-Code Rate Thm [folklore]

$$\text{Prob}\{4^{-n} < M_n < 1 - 4^{-n}\} < 2^{-\rho n}$$

implies

$(1/2 - N^{-\rho})N$ bits / N channel uses.

$(1/2 - N^{-\rho})N$ bits / N channel uses

History of ρ over BMSC:0

1/2

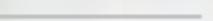
$$(1/2 - N^{-\rho})N \text{ bits} / N \text{ channel uses}$$

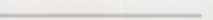
History of ρ over BMSC:0

1/2

2015 Guruswami-Xia 

2012 Goli-Hassani-Urbanke 

2014 Hassani-Alishahi-Urbanke 

2014 Goldin-Burshtein 

2016 Mondelli-Hassani-Urbanke 

$(1/2 - N^{-\rho})N$ bits / N channel uses

History of ρ over BMSC: 0

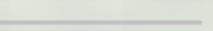
1/2

2015 Guruswami-Xia 

2012 Goli-Hassani-Urbanke 

2014 Hassani-Alishahi-Urbanke 

2014 Goldin-Burshtein 

2016 Mondelli-Hassani-Urbanke 

2022  -Lin-Vardy-Gabrys 

1/2

Improve ρ over BEC: 0

Improve ρ over BEC: 0

1/2

- 2010 Hassani–Alishahi–Urbanke 2×2 
- 2010 Korada–Montanari–Telatar–Urbanke 2×2 
- 2014 Fazeli–Vardy 8×8 
- 2021 Trofimiuk–Trifonov 16×16 
- 2021 Trofimiuk 24×24 
- 2021 Yao–Fazeli–Vardy 32×32 
- 2021 Yao–Fazeli–Vardy 64×64 

Improve ρ over BEC: 0

1/2

- 2010 Hassani–Alishahi–Urbanke 2×2 x
- 2010 Korada–Montanari–Telatar–Urbanke 2×2 .
- 2014 Fazeli–Vardy 8×8 .
- 2021 Trofimiuk–Trifonov 16×16 .
- 2022 Duursma–Gabrys–Guruswami–Lin– 2×2 /GF4 .
- 2021 Trofimiuk 24×24 .
- 2021 Yao–Fazeli–Vardy 32×32 .
- 2021 Yao–Fazeli–Vardy 64×64 .

The optimal ρ : 0

1/2

1/2

The optimal ρ : 0

2019 Pfister–Urbanke
 q -ary erasure channel, $q \rightarrow \infty$

2021 Fazeli–Hassani–Mondelli–Vardy
binary erasure channel

2022 Guruswami–Riazanov–Ye
binary symmetric memoryless channel

The optimal ρ : 0

1/2

2019 Pfister–Urbanke
 q -ary erasure channel, $q \rightarrow \infty$

2021 Fazeli–Hassani–Mondelli–Vardy
binary erasure channel

2022 Guruswami–Riazanov–Ye
binary symmetric memoryless channel

2021 –Duursma
discrete memoryless channel

2011 Alamdar-Yazdi-Kschischang:
Prune the tree to reduce complexity.

2017 El-Khamy-Mahdavifar-Feygin-Lee-Kang:
Pruning reduces complexity by a scalar; still $O(N \log N)$.

2021 Mondelli-Hashemi-Cioffi-Goldsmith,
2021 Hashemi-Mondelli-Fazeli-Vardy-Cioffi-Goldsmith:
Study parallelism vs latency.

2011 Alamdar-Yazdi-Kschischang:
Prune the tree to reduce complexity.

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Pruning reduces complexity by a scalar; still $O(N \log N)$.

2021  -Duursma: $O(N \log \log N)$
if relax the performance requirement.

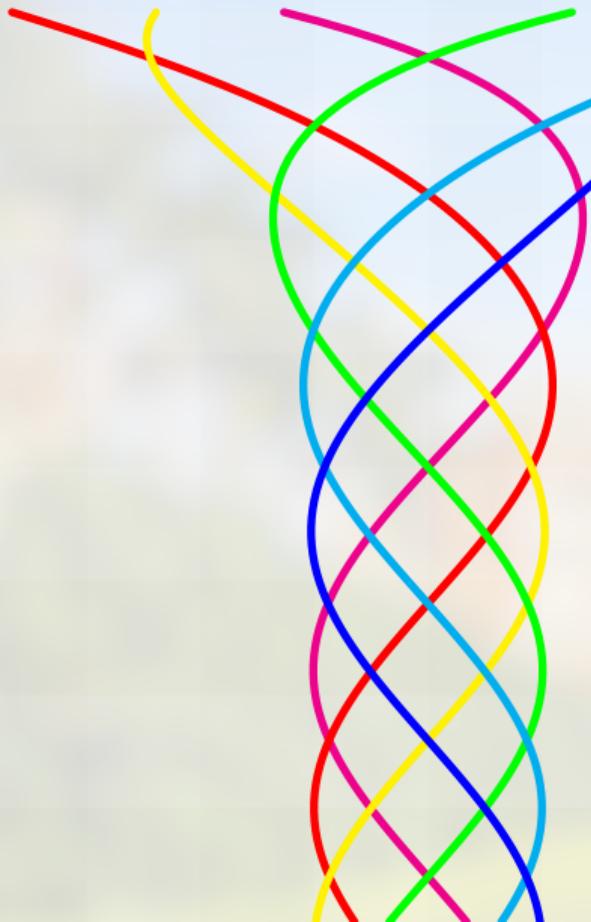
Trade-off: complexity $\approx O(N \log(-\log(\text{decode error})))$.

2021 Mondelli-Hashemi-Cioffi-Goldsmith,
2021 Hashemi-Mondelli-Fazeli-Vardy-Cioffi-Goldsmith:
Study parallelism vs latency.

Polar code is a mathy code

Polar code is a mathy code

Polar achieves the capacity of



Asymmetric channel

Multiple access channel

Lossless compression

Lossy compression

Slepian-Wolf

Lossless compression w/ helper

Wiretap channel (degradation)

Deletion channel ... (good error prob)

Broadcast channel ... (good error prob)

Channel with memory ... (good error prob)

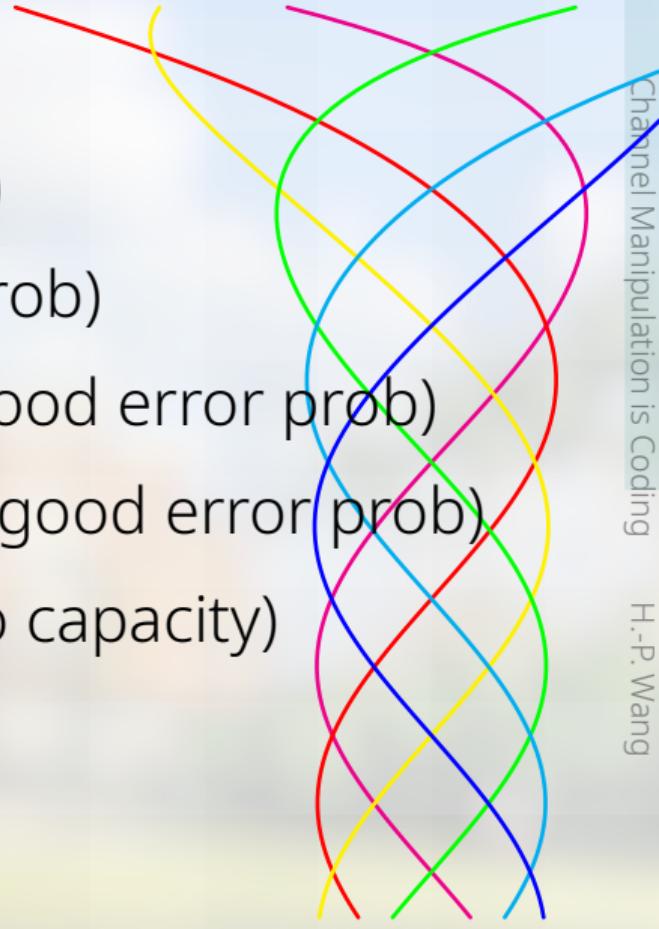
Wiretap channel (no degradation) ... (good error prob)

Hidden Markov chain channel state ... (good error prob)

Non-stationary channel ... (good gap to capacity)

Classical-Quantum channel ... (yes)

Quantum-Quantum channel ... (?)

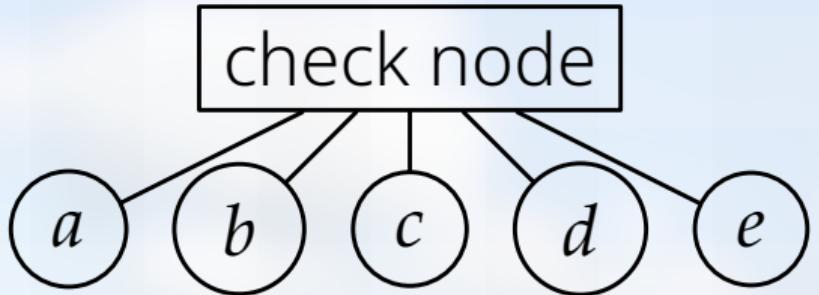


5	3		7			
6			1	9	5	
	9	8				6
8			6			3
4		8	3			1
7		2				6
	6			2	8	
		4	1	9		5
		8			7	9

Wikipedia

Low-Density Parity-Check (LDPC) Codes

Rule: Every
Sum to an even number



Rule: Every
Sum to an even number

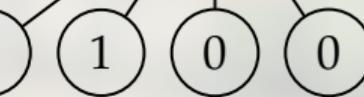
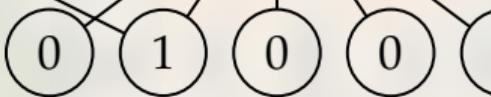
check node

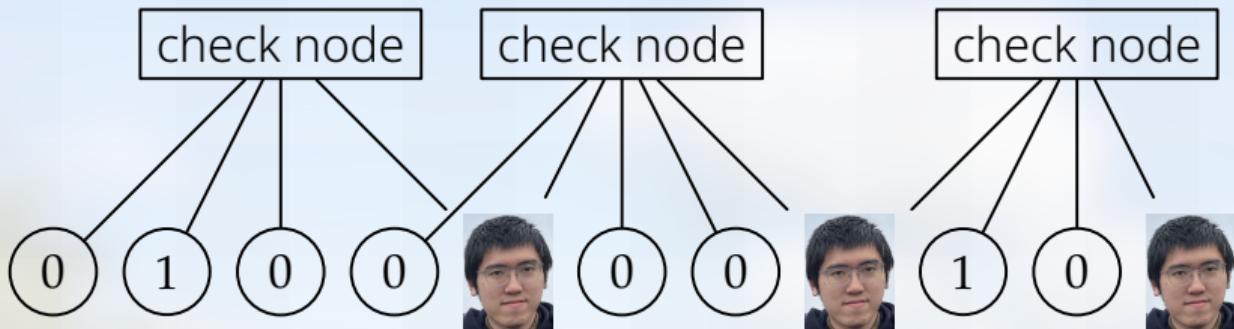


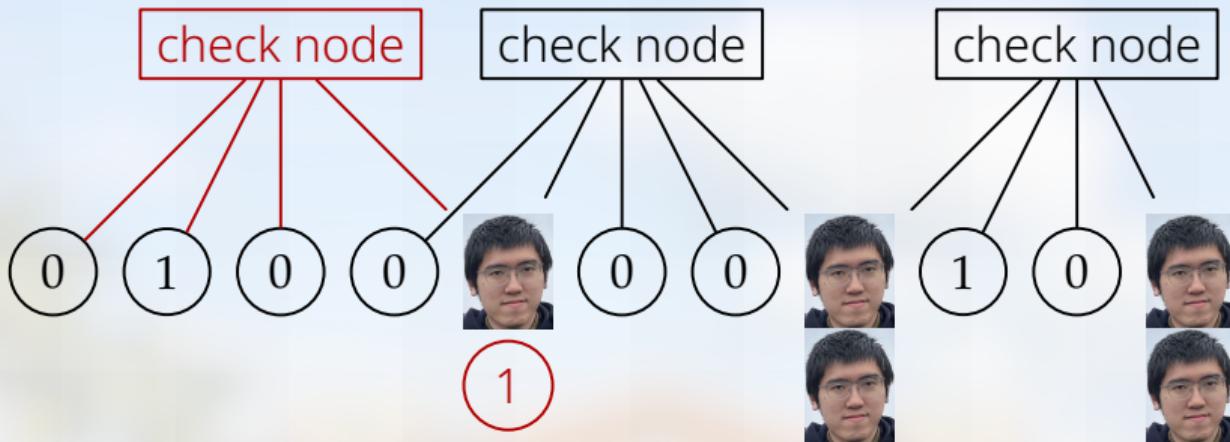
check node

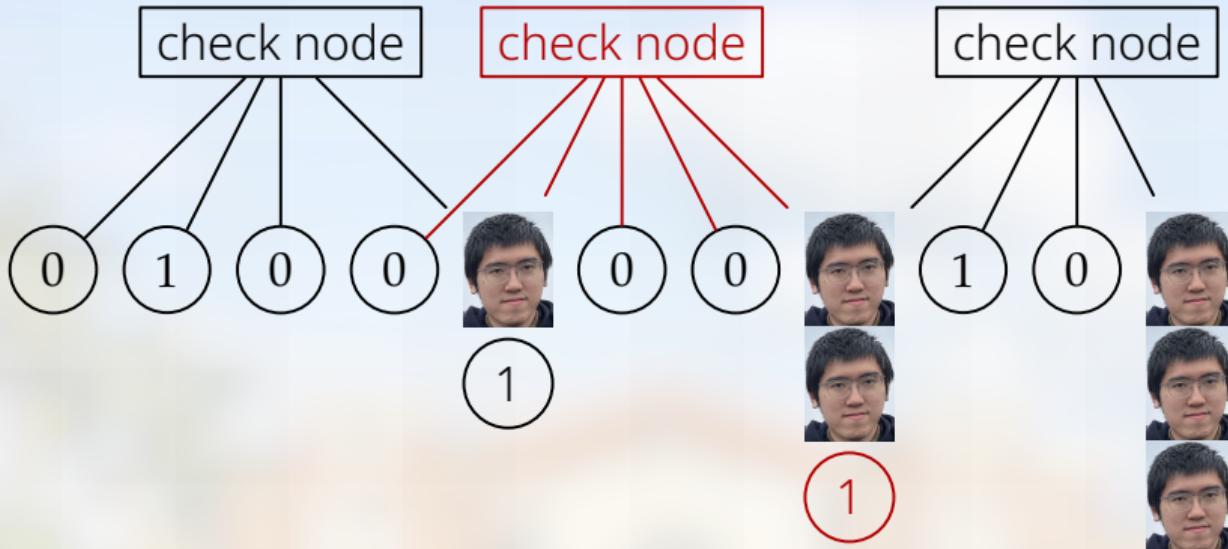
check node

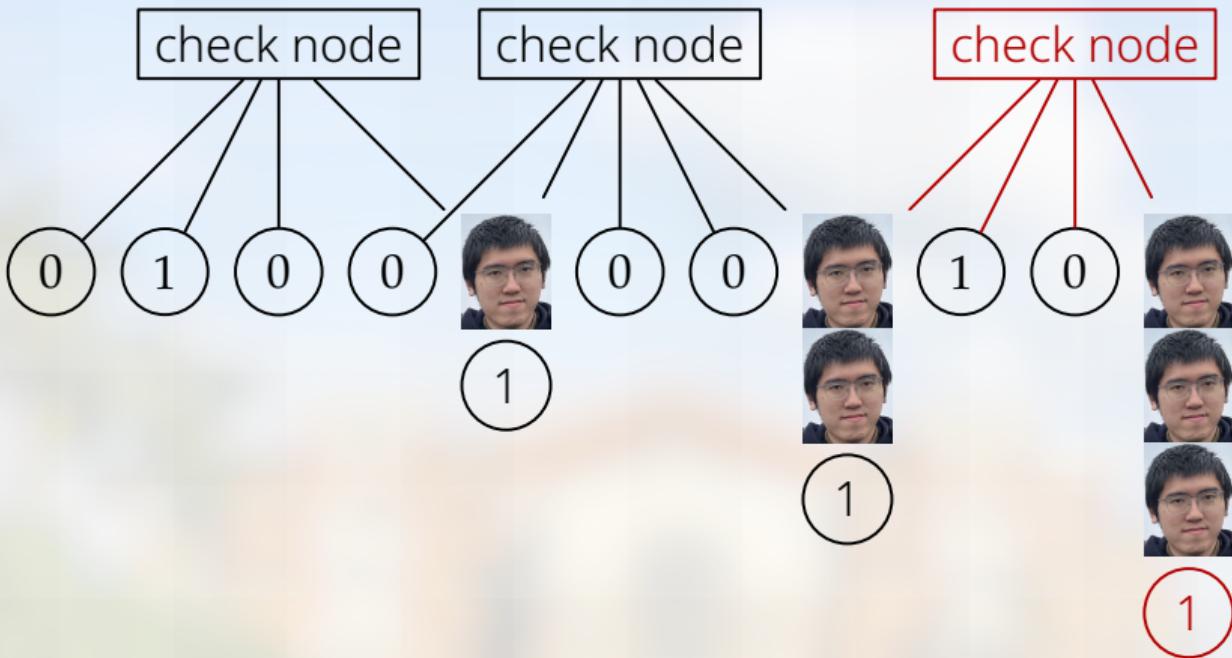
check node

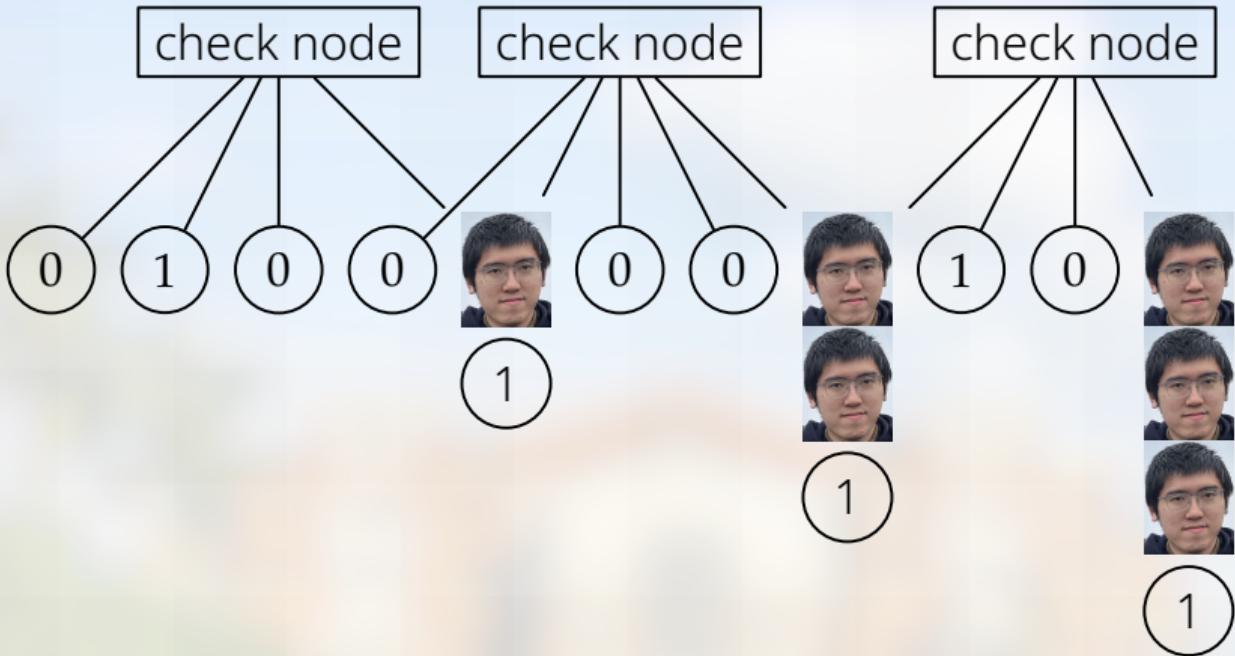




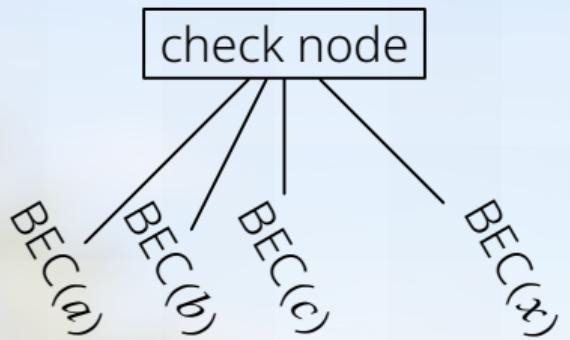


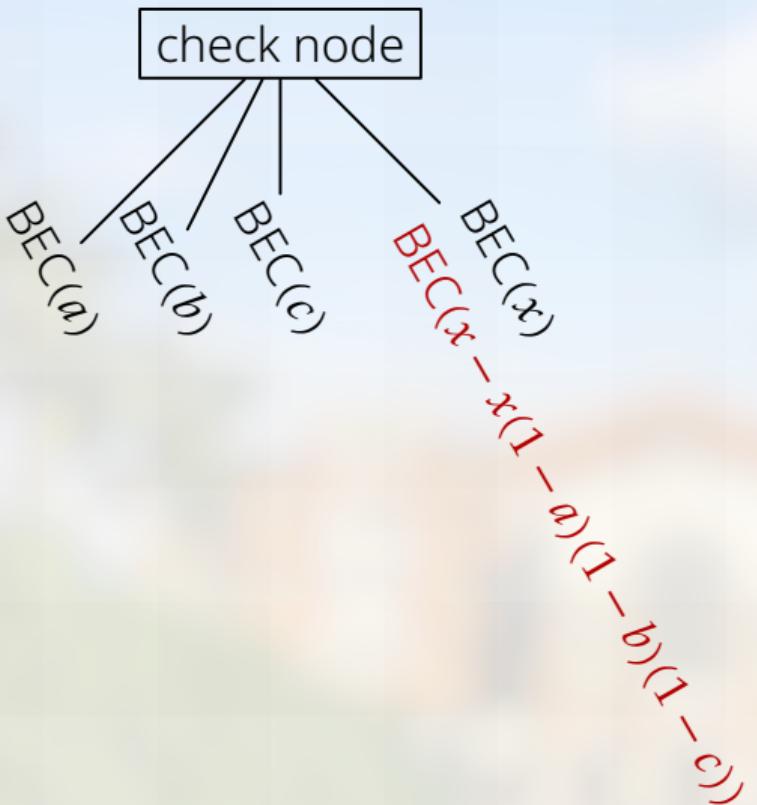


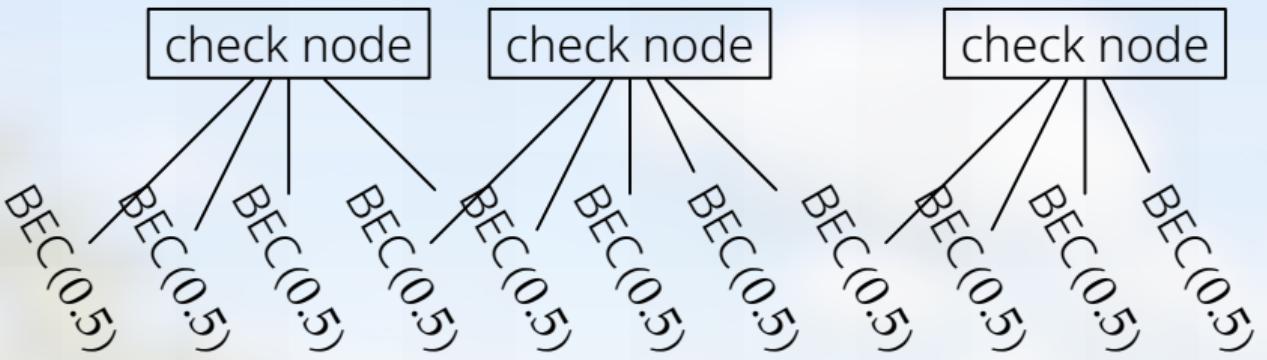


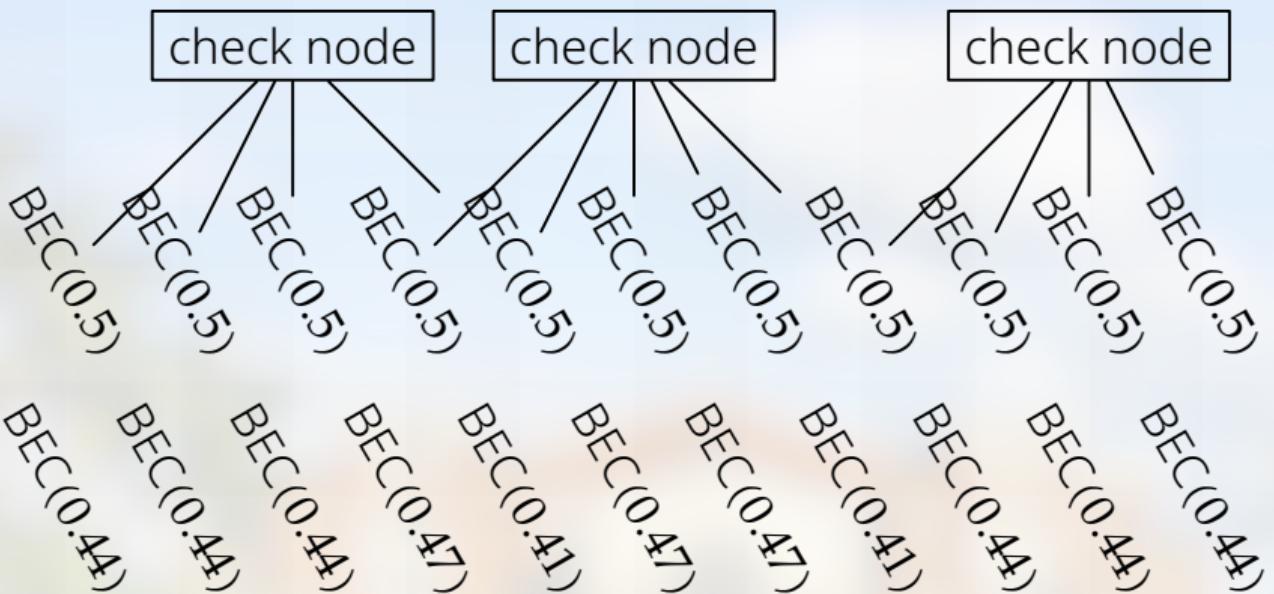


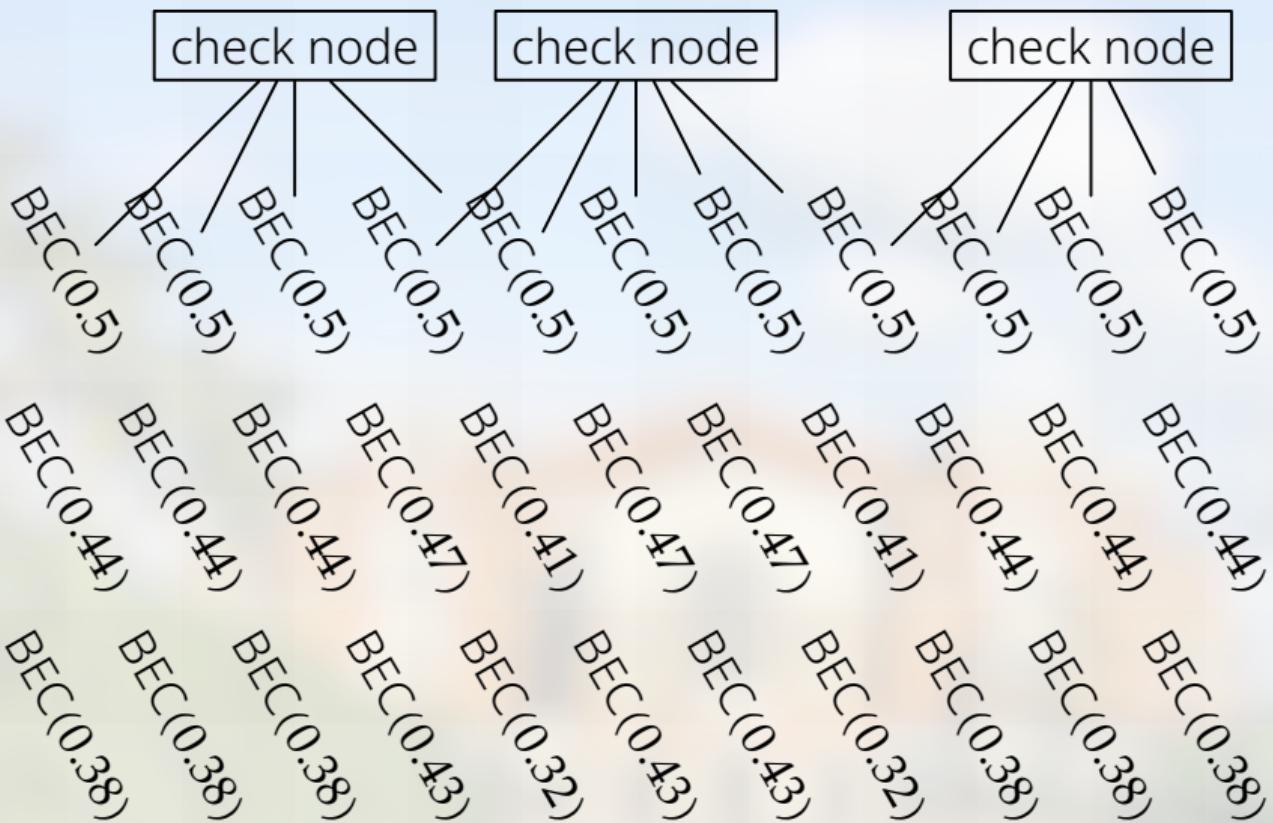
How to analyze, mathematically?













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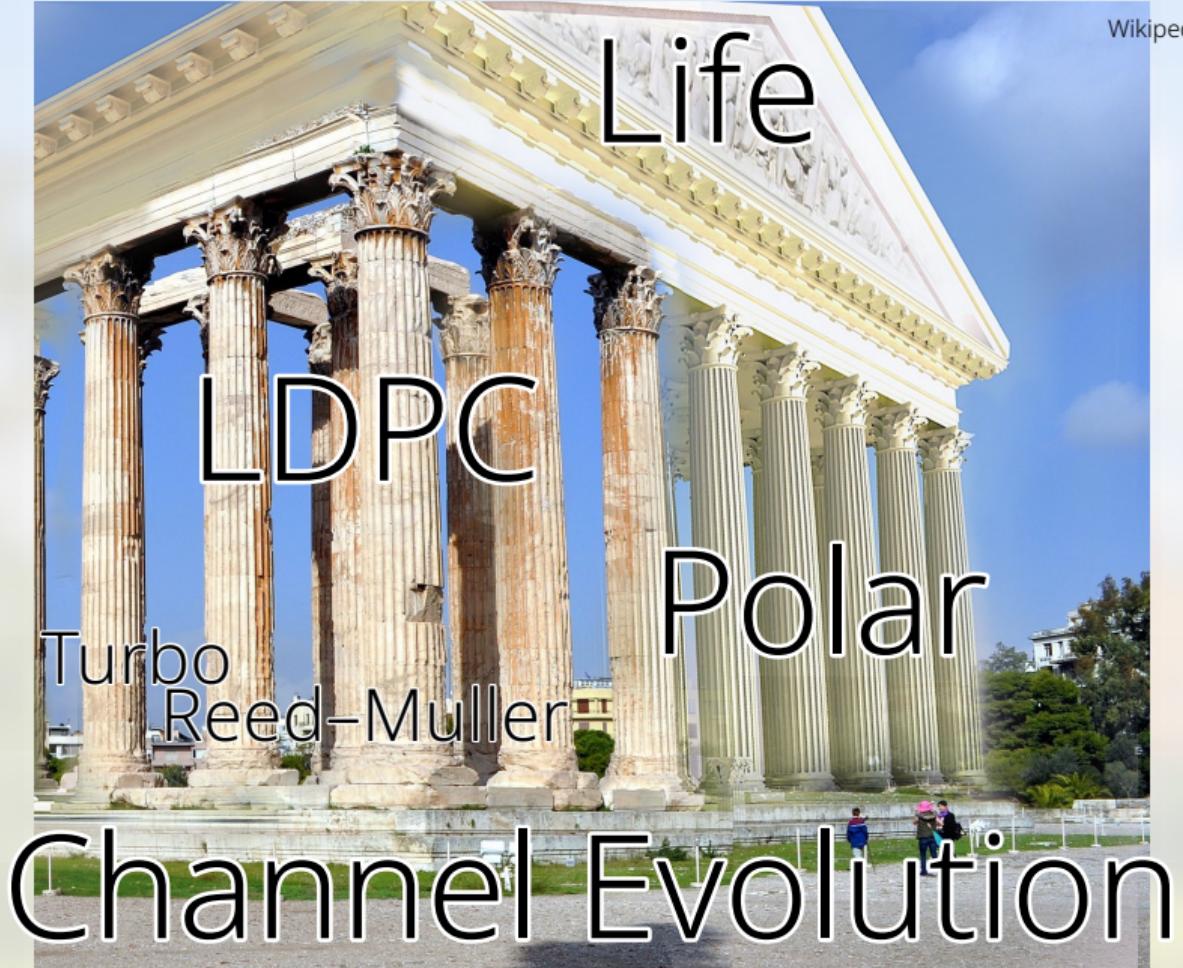
Problem: Theory–Reality Duality

Theory:

Keep track of
channel capacity

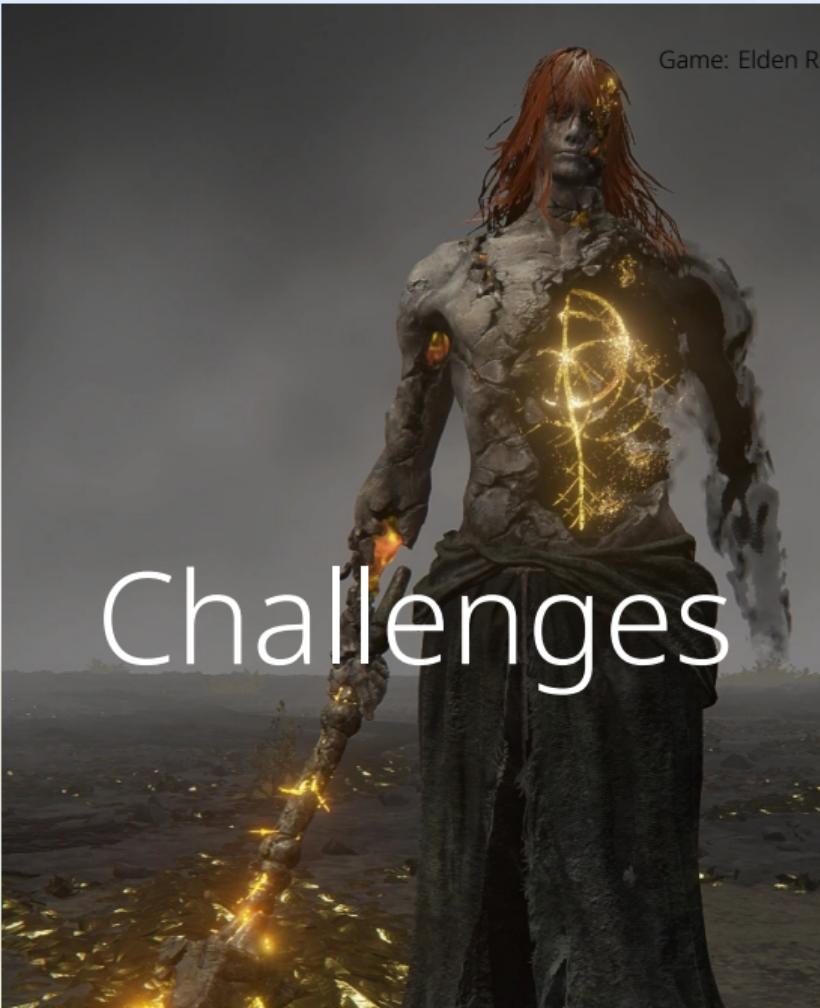
Reality:

“Shape” of
channels matters
(BEC/BSC/AWGN...)



Game: Elden Ring

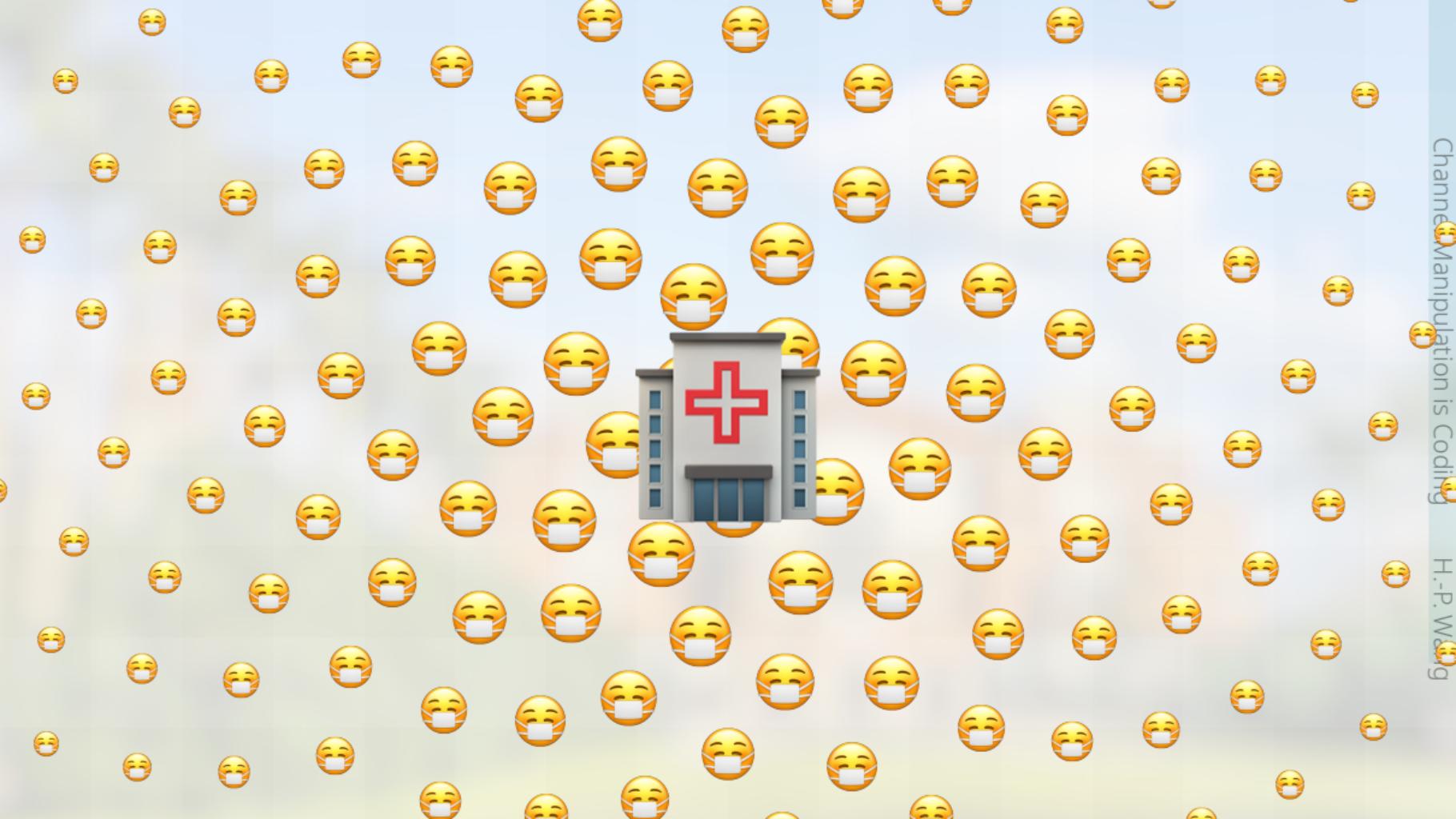
Challenges



Channel Manipulation is Coding

H.-P. Wang





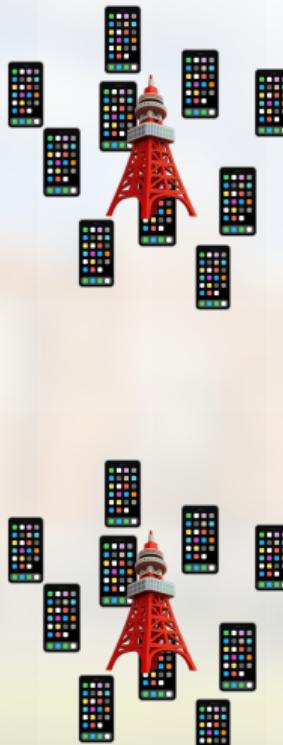
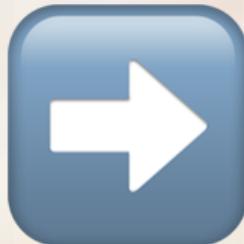
Channel Manipulation is Coding

H.-P. Wang

Problem	There are n	k of them	To solve, use m
Radio protocol	cellphones	want to talk	frequencies
Disease control	people	are sick	virus tests
Search engine	webpages	contain keywords	bits per keyword

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Radio protocol	cellphones	want to talk	frequencies
Disease control	people	are sick	virus tests
Search engine	webpages	contain keywords	bits per keyword
Radio protocol	messages	will be sent	frequencies
Genotyping	genes	cause cancer	gene tests
Computer forensics	files	will be modified	bits of storage
Property-preserving hash	properties	appears in a file	bits per file
Image compression	wavelet coefficients	are nonzero	bits per digit
Traitor tracing	users	resell keys	keys
Heavy hitter / DoS	users	are spamming	virtual servers

Proposal: Prove the following.



Proposal: Prove the following.

Lemma [Your Name Here]

If we can solve (n, k) -RAC
(RAC = random access channel),

then we can solve $(3n, 2k)$ -RAC.

DNA Coding

Substitution

Insertion

Deletion

Transposition

Amplification

Permutation

ATTCCG



ATTACGG

DNA Coding

Substitution

Insertion

Deletion

Transposition

Amplification

Permutation

DNA Coding

ATT CCG

G

Substitution

Insertion

Deletion

Transposition

Amplification

Permutation

DNA Coding

ATT~~X~~CG

Substitution

Insertion

Deletion

Transposition

Amplification

Permutation

DNA Coding

ATTCCG



AT~~C~~T_TCG

Substitution

Insertion

Deletion

Transposition

Amplification

Permutation

DNA Coding



Substitution

Insertion

Deletion

Transposition

Amplification

Permutation

DNA Coding

Substitution

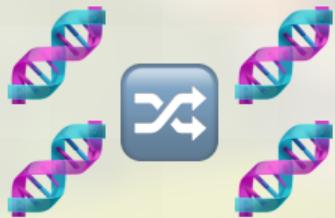
Insertion

Deletion

Transposition

Amplification

Permutation





<https://jobtalk.symbol.codes>

List of Publications

- [1] polar codes' ρ : symbolic calculus, interval arithmetics
- [2] polar codes' improved ρ : symbolic experiments, compactness argument
- [3] polar's optimal ρ : large deviation toolbox, analytic continuation
- [4] polar codes' complexity: optional stopping time theorem
- [5] ordering bit-channels: Picard–Lindelöf, IVP version of Green's
- [6] ordering bit-channels: real-root counting, Galois theory
- [7] ordering bit-channels: fundamental theorem of algebra
- [8] group testing: generating function, Möbius transformation
- [9] group testing: tropical arithmetic, count-min sketch
- [10] cloud storage: chain complex, tensor and alternating algebras
- [11] cloud storage: symmetric algebra, polynomial identity test
- [12] cloud matrix multiplication (MM): fast MM + distributed MM

-  Hsin-Po Wang, Ting-Chun Lin, Alexander Vardy, and Ryan Gabrys.
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