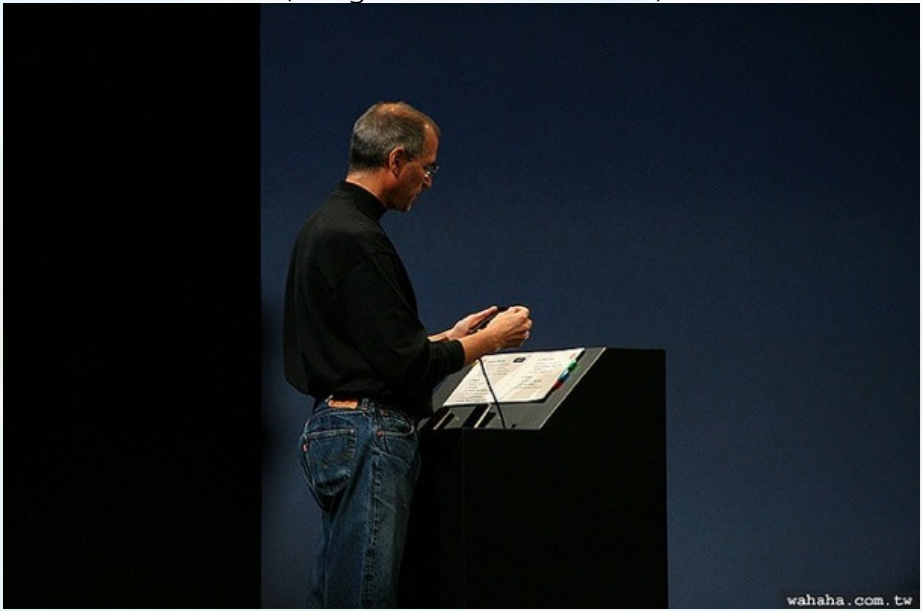
Channel Manipulation as a Coding Technique

Hsin-Po Wang (EECS, UC Berkeley)

What do the following pictures have in common?

(Images from the internet)



(Images from the internet)

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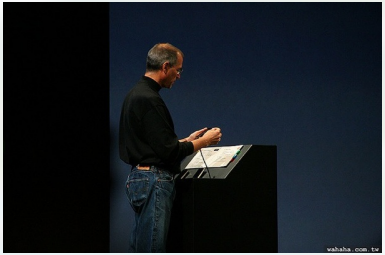


https://jobtalk.symbol.codes

https://jobta  mbol.codes

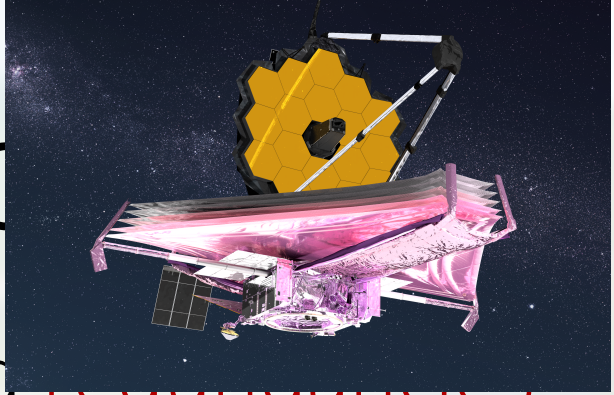
Channel Manipulation is Coding

H.-P Wang

Coding

= adding redundancies
in a smart way

Co



= adding redundancies
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(Images from Wikipedia, NASA, IBM, Qualcomm)

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Redundancies
smart way

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Channel Manipulation is Coding

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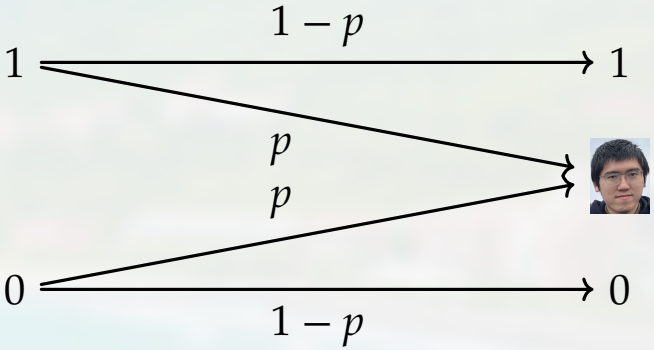
Math Framework of Coding



Binary erasure channel with erasure probability p: BEC(p)

N = 78 S = 83 Y = 89 U = 85

The “NSYSU” polynomial:

$$f(x) = 78 + 83x + 89x^2 + 83x^3 + 85x^4$$

N = 78 S = 83 Y = 89 U = 85

The “NSYSU” polynomial:

$$f(x) = 78 + 83x + 89x^2 + 83x^3 + 85x^4$$

$$f(-3) = 274 \quad f(-2) = 964 \quad f(-1) = 86 \quad f(0) = 78$$

$$f(1) = 418 \quad f(2) = 624 \quad f(4) = 254 \quad f(4) = 906$$

N = 78 S = 83 Y = 89 U = 85

The “NSYSU” polynomial:

$$f(x) = 7$$  $$.4$$

$f(-3) = 274$ $f(-2) = 964$ $f(-1) = 86$ $f(0) =$ 

$f(1) = 418$ $f(2) = 624$ $f(4) = 254$ $f(4) = 906$

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$$f(x) = 7$$  ^{4}

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$f(1) =$  $f(2) = 624$ $f(4) = 254$ $f(4) = 906$

N = 78 S = 83 Y = 89 U = 85

The “NSYSU” polynomial:

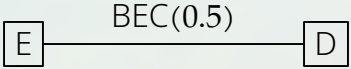
$f(x) = 7$  ^{4}

$f(-3) = 274$ $f(-2) = 964$ $f(-1) = 8$  $f(0) =$ 

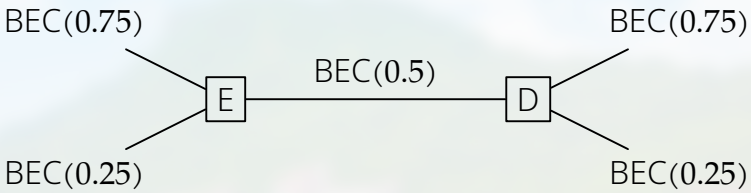
$f(1) = 4$  $f(2) = 624$ $f(4) = 254$ $f(4) = 906$

New Idea

Polar Codes



Suppose there are magic devices E and D that turns BEC(x) into BEC(x^2) and BEC($2x - x^2$).



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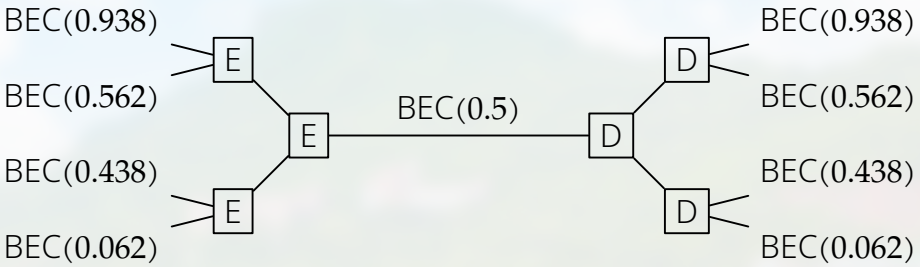
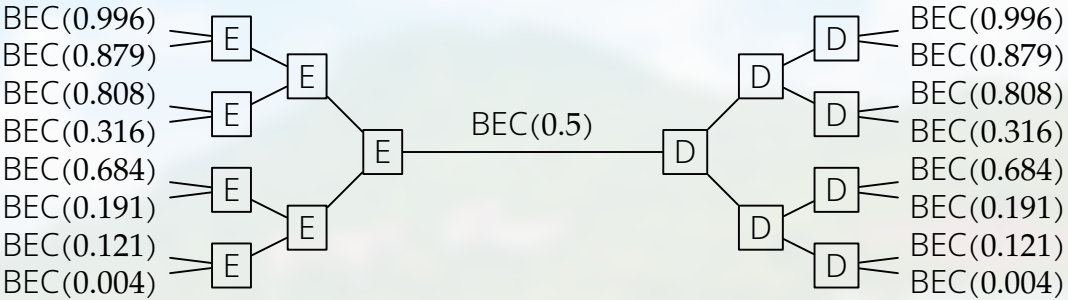
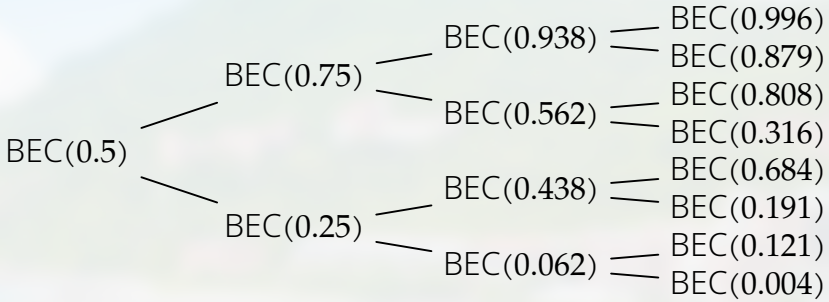


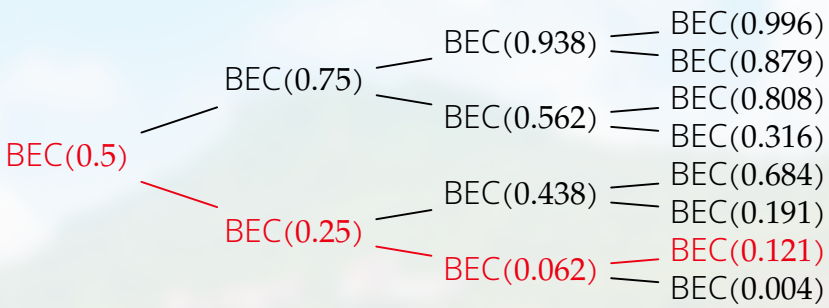
What if we apply apply more magic devices?



What if we apply apply more magic devices?
And more and more and more???

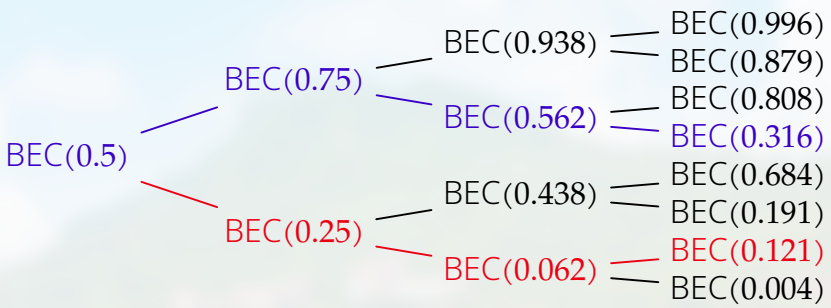


This is a tree



: This is a martingale

$$M_{n+1} := \begin{cases} 2M_n - M_n^2 & \text{with prob. } 1/2, \\ M_n^2 & \text{with prob. } 1/2. \end{cases}$$



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: Bounded martingale converges.

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: Bounded martingale converges.

: BECs converges to 0 or 1.

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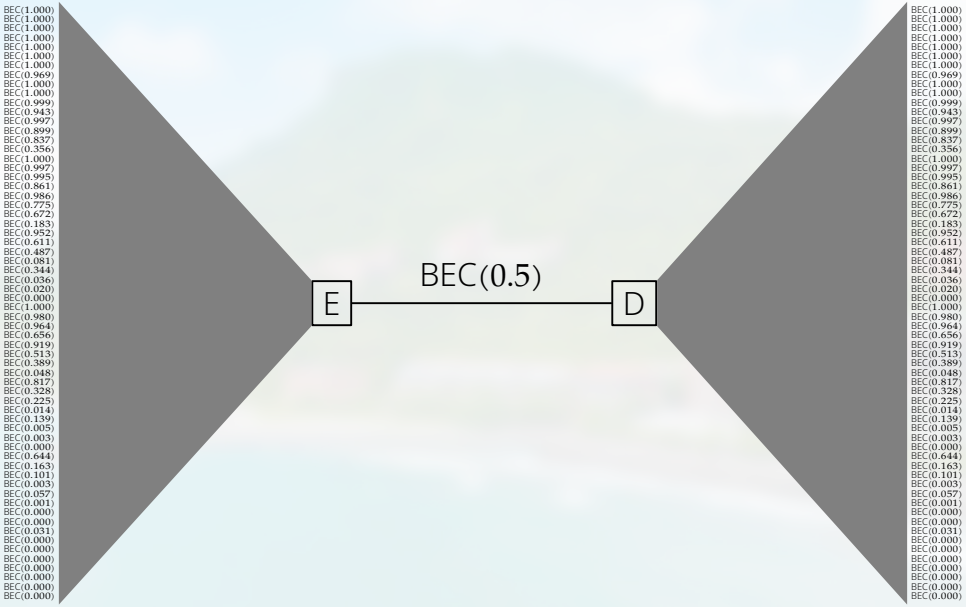
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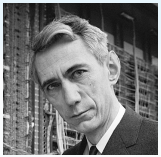
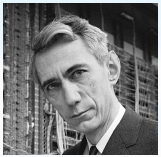
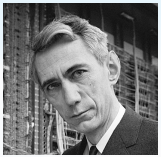


: 5000 bits / 10000 channel uses



: 5000 bits / 10000 channel uses

: Polar code 490bits/10000uses



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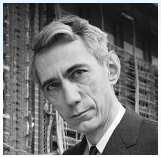
In general, $(1/2 - \varepsilon)N$ bits / N uses



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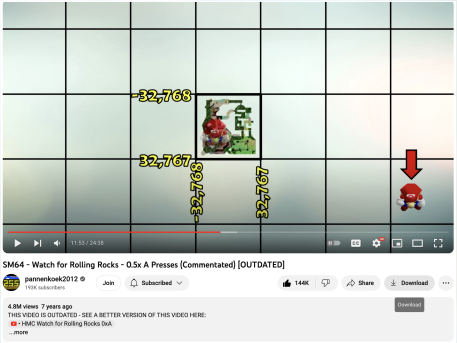
In general, $(1/2 - \varepsilon)N$ bits / N uses

THE END?

5G is for

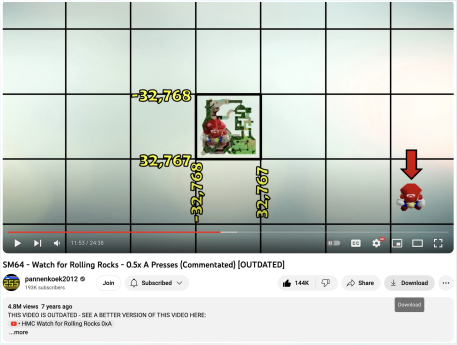
H.-P. Wang

5G is for



video streaming

5G is for



video streaming

flying drone

Channel Manipulation is Coding

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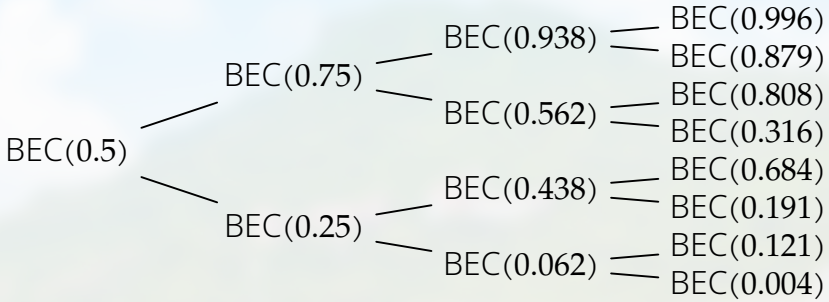
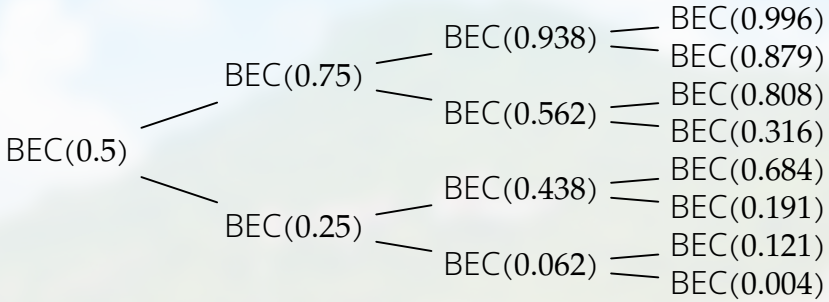
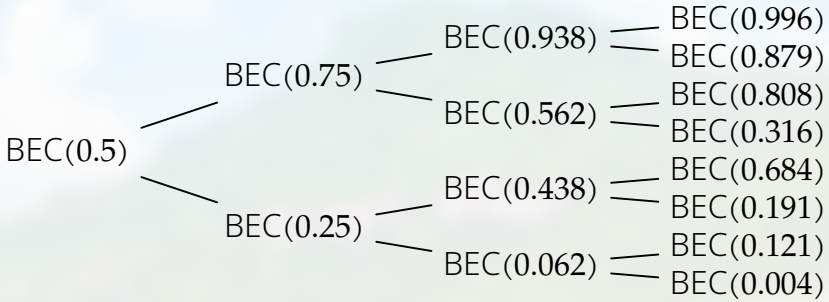

BEC(0.5)
BEC(0.75)
BEC(0.25)
BEC(0.938)
BEC(0.562)
BEC(0.438)
BEC(0.062)
BEC(0.996)
BEC(0.879)
BEC(0.808)
BEC(0.316)
BEC(0.684)
BEC(0.191)
BEC(0.121)
BEC(0.004)

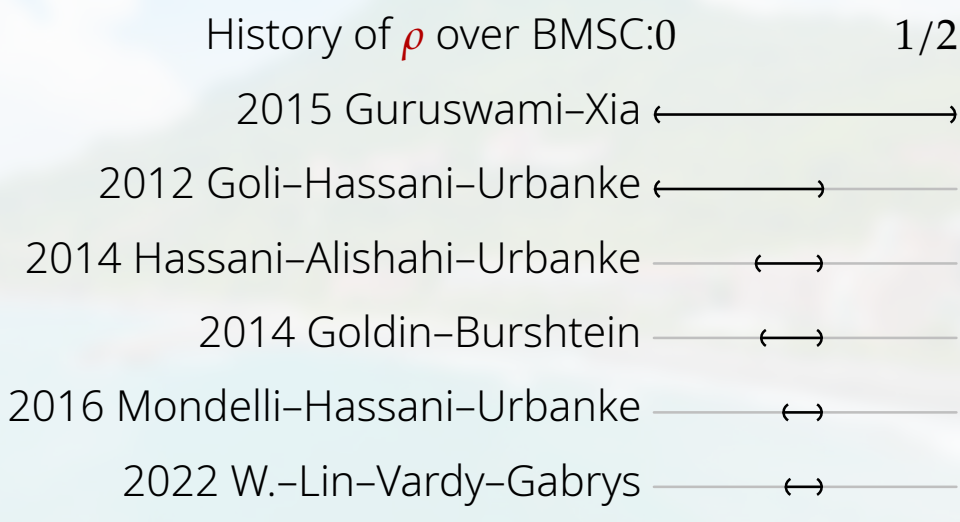


Early channels are not useful.



Early channels are not useful. How to accelerate polarization?

$(1/2 - N^{-\rho})N$ bits / N channel uses




Improve ρ over BEC: 0
1/2
2010 Hassani–Alishahi–Urbanke 2×2
2010 Korada–Montanari–Telatar–Urbanke 2×2
2014 Fazeli–Vardy 8×8
2021 Trofimiuk–Trifonov 16×16
2022 Duursma–Gabrys–Guruswami–Lin–W. 2×2/GF4
2021 Trofimiuk 24×24
2021 Yao–Fazeli–Vardy 32×32
2021 Yao–Fazeli–Vardy 64×64

The optimal ρ: 0 $1/2$

2019 Pfister–Urbanke
q-ary erasure channel, $q \to \infty$

2021 Fazeli–Hassani–Mondelli–Vardy
binary erasure channel

2022 Guruswami–Riazanov–Ye
binary symmetric memoryless channel

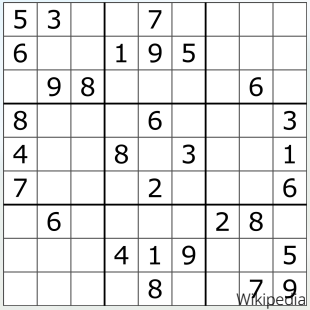
2021 W.–Duursma
discrete memoryless channel

2011 Alamdar-Yazdi–Kschischang:
Orune the tree to reduce complexity.

2017 El-Khamy–Mahdavifar–Feygin–Lee–Kang:
Oruning reduces complexity by a scalar;
still $O(N \log N)$.

2021 W.–Duursma: $O(N \log \log N)$
if relax the performance requirement.

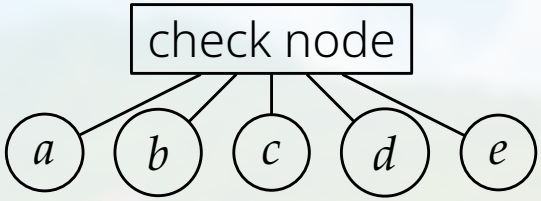
Trade-off: complexity $\approx O(N \log(-\log(\text{decode error})))$.

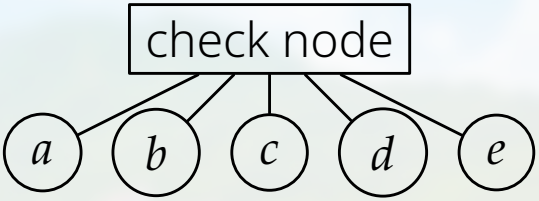


Wikipedia

Low-Density Parity-Check (LDPC) Codes

Rule: Every Sum to an even number





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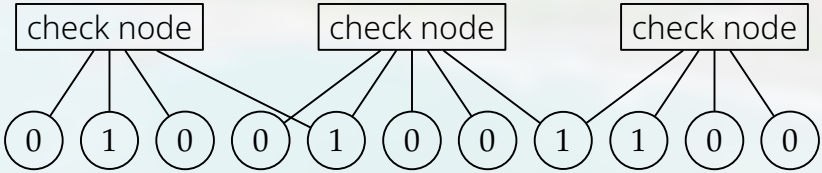

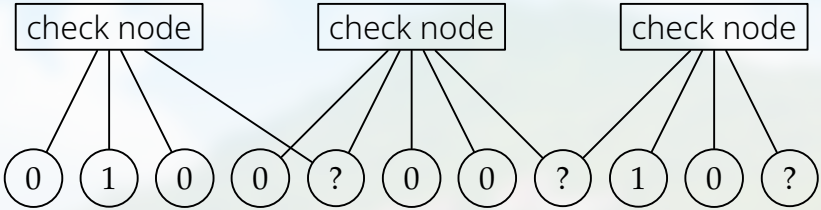
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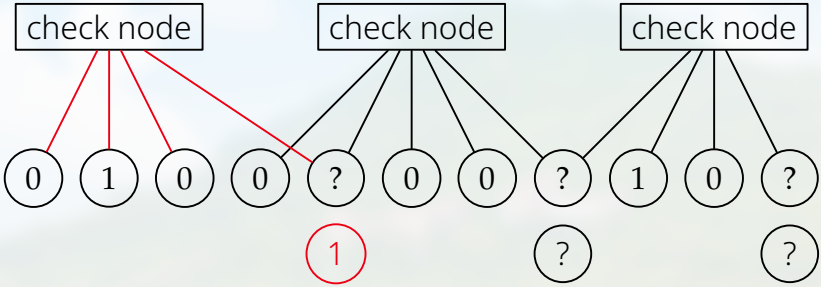
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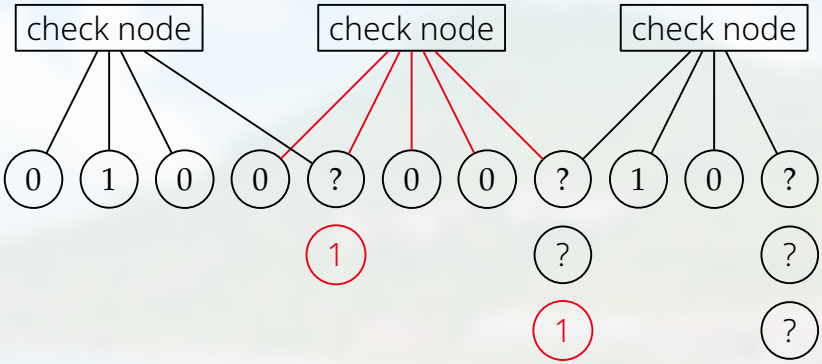
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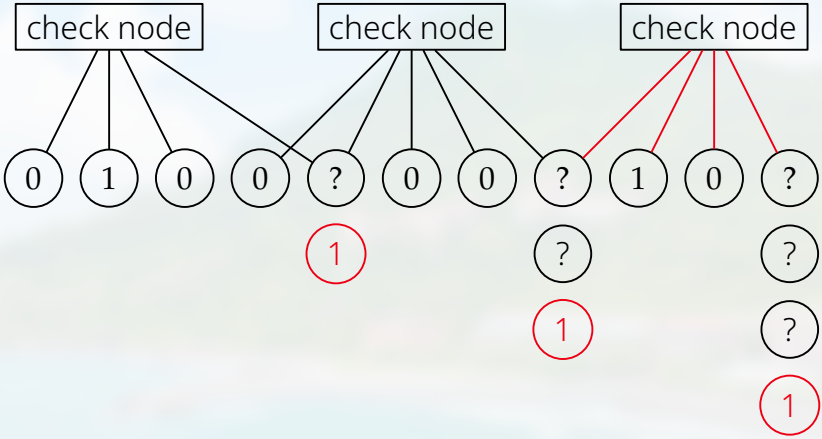
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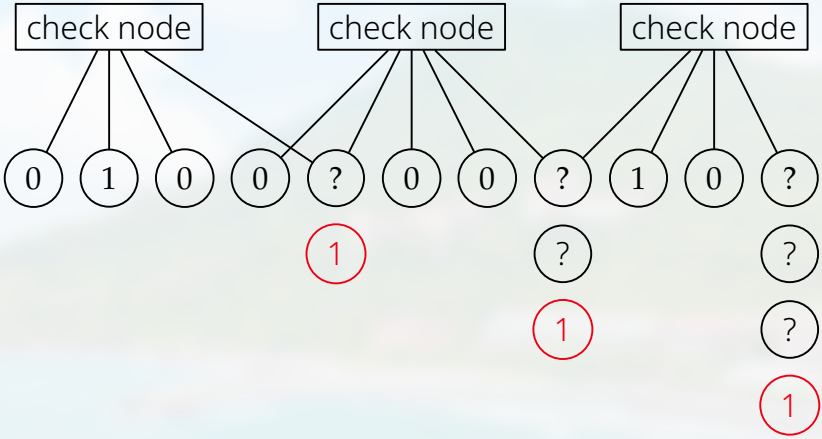
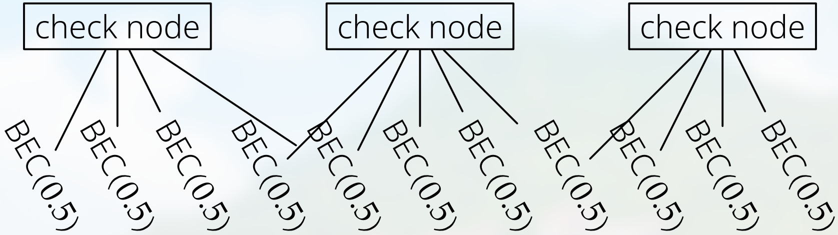


How to analyze, mathematically?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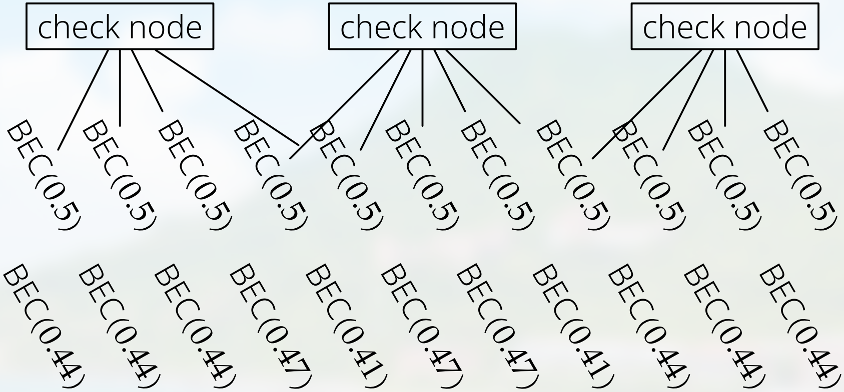
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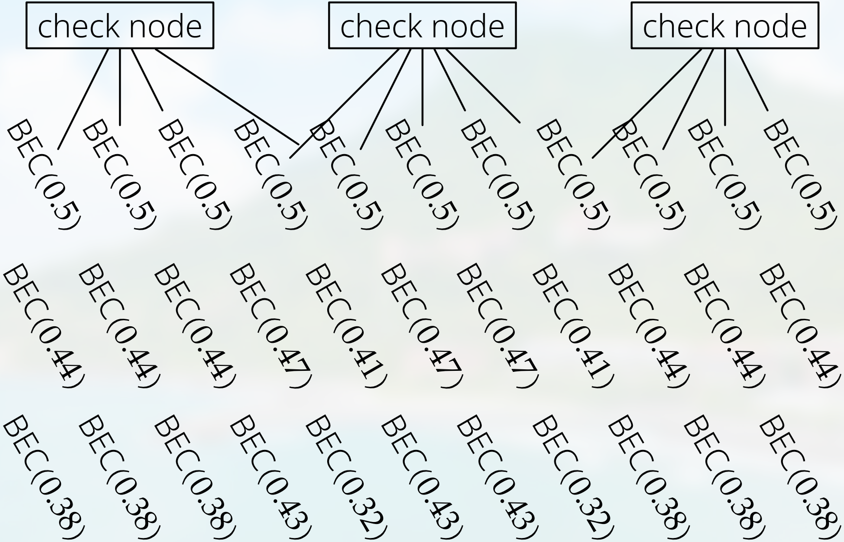
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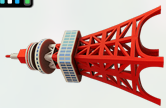
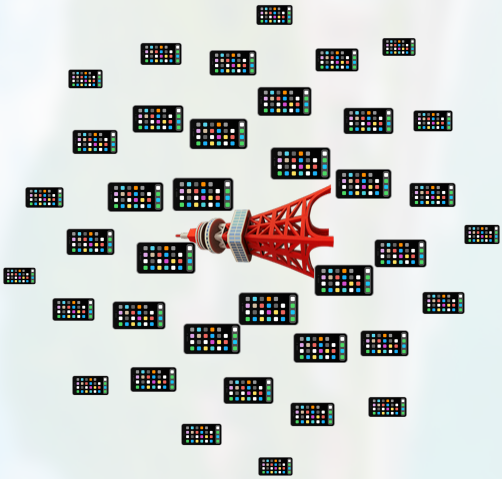
Challenges





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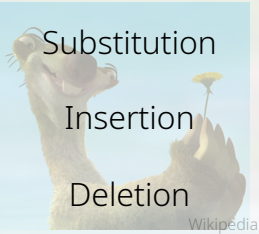

Permutation